① NumOpt Ex 04 09.11.17

Nr 1

$$\lim_{\alpha \to 0} \arg\min_x \frac{1}{2}\|\eta - Jx\|_2^2 + \underbrace{\frac{\alpha}{2}(x-\bar{x})^T Q (x-\bar{x})}_{\text{pure quadratic due to Q symmetric and pos. def}}$$

~~Eq. 6.23:~~

$$\cancel{x^* = (J^TJ + \alpha I)^{-1} J^T \eta}$$

~~Lemma 6.1:~~

$$\cancel{\lim_{\alpha \to 0} (J^TJ + \alpha I)^{-1} J^T \eta}$$

$$\cancel{= J^+ \eta}$$

$$\cancel{= VS^+U^T\eta}$$

$$\sum_{i=1}^{n} b_i (x_i - \bar{x}_i)^2 = \|\sqrt{B}\,(x-\bar{x})\|_2^2$$

$(x-\bar{x})^T Q (x-\bar{x}) =$ [redacted]

where $B = \begin{pmatrix} b_1 & & 0 \\ & \ddots & \\ 0 & & b_n \end{pmatrix}$ [redacted] $B \succ 0$ (all signs of qu.-function pos.)

~~Eq. 6.23:~~ $\cancel{x^* = (J^TJ + \alpha I)^{-1} J^T \eta}$

~~Lemma 6.1:~~ $\cancel{\lim_{\alpha \to 0} (J^TJ + \alpha I)^{-1} J^T \eta = J^+\eta = VS^+U^T\eta}$

~~Where S^+~~

$$\nabla f(x) = J^TJx - J^T\eta + \alpha \sqrt{B}^T \sqrt{B}\,\bar{x} + \alpha \sqrt{B}^T \sqrt{B}\, x$$

$$\sqrt{B}^T \sqrt{B} = R$$

$$\nabla f(x) = J^TJx - J^T\eta + \alpha R\bar{x} + \alpha R x$$

$$= (J^TJ + \alpha R)x - J^T\eta + \alpha R \bar{x}$$

$$x^* = \text{[redacted]} (J^TJ + \alpha R)^{-1}(J^T\eta - \alpha R\bar{x})$$

$$\xrightarrow{\alpha \to 0} (J^TJ)^{-1} J^T \eta = J^+ = VS^+U$$

where $S^+ = \begin{pmatrix} \sigma_1^{-1} & & & 0 \\ & \ddots & & \vdots \\ & & \sigma_r^{-1} & 0 \\ 0 & & \cdots\ 0 & 0 \end{pmatrix}$

②

Nr 3

a.) $\min_{a,b} \sum_{i=1}^{N} |(ax_i + b - y_i)|$

$\Rightarrow \min_{a,b,s} \sum_{i=1}^{N} s_i$

s.t. $-s_i \leq ax_i + b - y_i \leq s_i$

$-s_i \leq 0$

$\Rightarrow s_i = |ax_i + b - y_i|$

$z = \begin{pmatrix} a \\ b \\ s_1 \\ \vdots \\ s_N \end{pmatrix}$

~~$ax_i + b - y_i \leq s_i$~~

~~$ax_i + b - s_i - y_i \leq 0$~~

~~$\begin{pmatrix} J & \begin{matrix} -1 & & 0 \\ & \ddots & \\ 0 & & -1 \end{matrix} \end{pmatrix} \begin{pmatrix} a \\ b \\ s_1 \\ \vdots \\ s_N \end{pmatrix} - y \leq 0$~~

~~$A = \begin{pmatrix} J & \begin{matrix} -1 & & 0 \\ & \ddots & \\ 0 & & -1 \end{matrix} \end{pmatrix}$ $b = y$~~

~~$f = \begin{pmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$~~

~~$|ax_i + b - y_i| \leq s_i$~~

~~$\underbrace{|ax_i + b - y_i|}_{} - s_i \leq 0$~~

~~is <0 then overall <0, if not it still has to be less then s_i~~

~~$ax_i + b - y_i - s_i \leq 0$~~

~~$\begin{pmatrix} J & \begin{matrix} -1 & & 0 \\ 0 & \ddots & -1 \end{matrix} \end{pmatrix} \begin{pmatrix} a \\ b \\ s_1 \\ \vdots \\ s_N \end{pmatrix} - y \leq 0$ $A = \begin{pmatrix} J & \begin{matrix} -1 & & 0 \\ 0 & \ddots & -1 \end{matrix} \end{pmatrix}$ $b = y$ $f = \begin{pmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$~~

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$-s_i \leq ax_i + b - y_i$

$\Leftrightarrow -s_i - ax_i - b + y_i \leq 0$

$\begin{pmatrix} -J & \begin{matrix} -1 & & 0 \\ & \ddots & \\ 0 & & -1 \end{matrix} \end{pmatrix} z + y \leq 0$

$ax_i + b - y_i \leq s_i$

$\Leftrightarrow ax_i + b - y_i - s_i \leq 0$

$\begin{pmatrix} J & \begin{matrix} -1 & & 0 \\ & \ddots & \\ 0 & & -1 \end{matrix} \end{pmatrix} - y \leq 0$

$$\underbrace{\begin{pmatrix} -J & \begin{matrix} -1 & & 0 \\ & \ddots & \\ 0 & & -1 \end{matrix} \\ J & \begin{matrix} -1 & & 0 \\ & \ddots & \\ 0 & & -1 \end{matrix} \end{pmatrix}}_{A} z - \underbrace{\begin{pmatrix} -y \\ y \end{pmatrix}}_{b} \leq 0$$