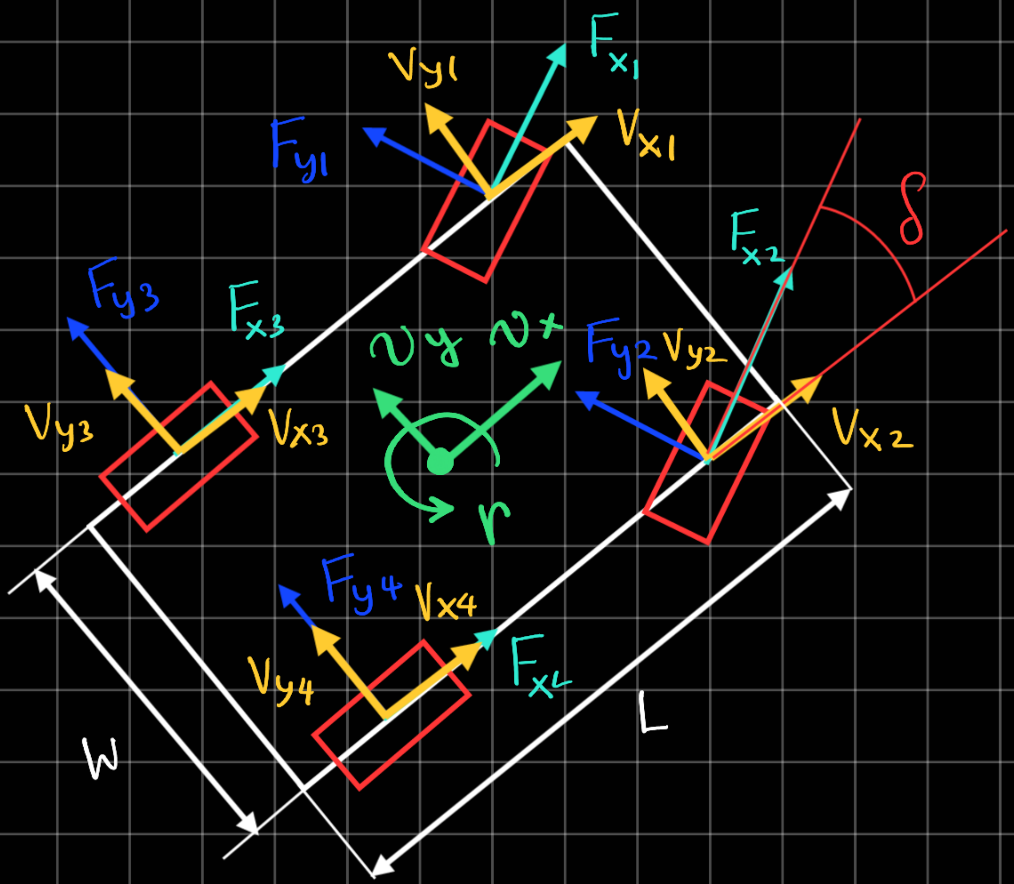
Vehicle Dynamical Model



I. Tire Dynamics and Model

1. Longitudinal Tire Slip (Slip Ratio)

The difference between the actual longitudinal velocity at the axle of the wheel V_x and the equivalent rotational velocity ωR of the tire is called the longitudinal slip. The slip ratio is defined as follows:

$$S = \frac{\omega R - V_x}{\max(V_x, \omega R)}$$

2. Lateral Tire Slip (Slip Angle)

Slip Angle is defined as the inverse tan of the ratio between the lateral and longitudinal velocities at the axle of the wheel. The equation for slip angle is shown below:

$$\tan\alpha = \frac{-V_y}{V_x}$$

Vehicle Modeling

MPC Model

Dynamic equations

$$\dot{X} = v_x \cos\psi - v_y \sin\psi$$
$$\dot{Y} = v_x \sin\psi + v_y \cos\psi$$

$$\dot{\psi} = \frac{v_x}{L} \tan\delta$$

$$m\dot{v}_x = F_x + m v_y \dot{\psi} - 2 F_{cf} \sin\delta$$
$$m\dot{v}_y = - m v_x \dot{\psi} + 2 F_{cf} \cos\delta + 2 F_{cr}$$
$$I\ddot{\psi} = 2 l_f F_{cf} \cos\delta - 2 l_r F_{cr}$$

$$F_{cf} = C_c\left(\delta - \arctan\left(\frac{v_y + L_f\dot{\psi}}{v_x}\right)\right)$$

$$F_{cr} = C_c\left(- \arctan\left(\frac{v_y - L_r\dot{\psi}}{v_x}\right)\right)$$

Assume longitudinal forces are zero for the MPC model

Main simulation model

Ground point tire velocities :

$F = 1 ; L = 1$
$R = 2 ; R = 2$

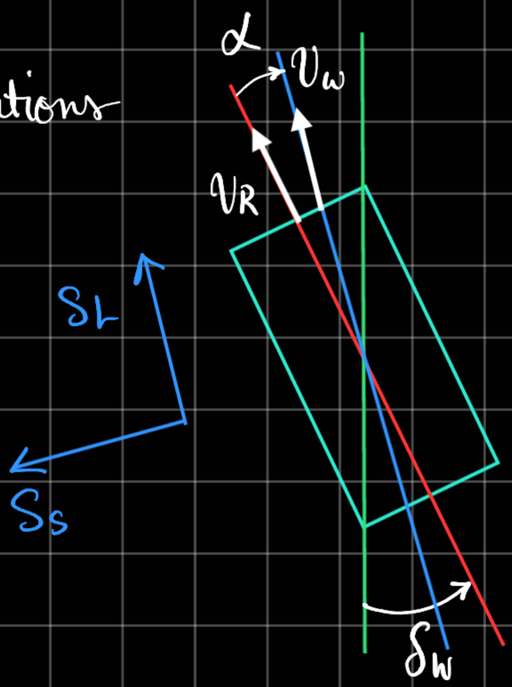
$$v_{\omega FL} = v_{cog} - r\left(\frac{b_f}{2}\cos\beta - l_f \sin\beta\right)$$

$$v_{w_{FR}} = v_{cog} + r\left(\frac{b_f}{2}\cos\beta + l_f \sin\beta\right)$$

$$v_{w_{RL}} = v_{cog} - r\left(\frac{b_f}{2}\cos\beta + l_f \sin\beta\right)$$

$$v_{w_{RR}} = v_{cog} + r\left(\frac{b_f}{2}\cos\beta - l_f \sin\beta\right)$$

Wheel slip calculations



Longitudinal Slip :

$$S_L = \frac{v_R \cos\alpha - v_W}{\max\{v_W, v_R\cos\alpha\}}$$

$$S_S = \tan\alpha \, \mathbb{1}(S_L) + \frac{v_R}{v_W}\sin\alpha \, \mathbb{1}(-S_L)$$

$$S_{Res}^2 = S_L^2 + S_S^2$$

Single-Track vehicle model for slip calculation :

Slip angle calculations:

$$\tan(\delta - \alpha_F) = \frac{l_f r + v_{cog} \sin\beta}{v_{cog} \sin\beta}$$

$$\tan(\alpha_R) = \frac{l_r r - v_{cog} \sin\beta}{v_{cog} \sin\beta}$$

Simplified ($\beta \to 0$)

$$\alpha_F = -\beta + \delta - \frac{l_f r}{v_{cog}} \; ; \; \alpha_R = -\beta + \frac{l_r r}{v_{cog}}$$

$$\beta = \arctan\left(\frac{l_r \tan\delta}{l_r + l_f}\right) \approx \frac{l_r \tan\delta}{l_r + l_f}$$

Adhesion coefficient μ:

$$\mu(S_{Res}) = C_1\left(1 - e^{-C_2 S_{Res}}\right) - C_3 S_{Res}$$

$$\mu_L = \mu(S_{Res}) \cdot \frac{S_L}{S_{Res}}$$

$$\mu_S = \mu(S_{Res}) \cdot \frac{S_S}{S_{Res}}$$

$$F_{WL} = \mu(S_{Res}) \frac{S_L}{S_{Res}} F_Z$$

$$F_{WS} = \mu(S_{Res}) \frac{S_S}{S_{Res}} F_Z$$

$$F_L = F_{WL}\cos\alpha + F_{WS}\sin\alpha$$
$$F_S = F_{WS}\cos\alpha - F_{WL}\sin\alpha$$

$$F_L = \frac{\mu(S_{Res})}{S_{Res}} F_z (S_L\cos\alpha + S_S\sin\alpha)$$

$$F_S = \frac{\mu(S_{Res}) F_z}{S_{Res}} (S_S\cos\alpha - S_L\sin\alpha)$$

Front and rear tire forces

- $S_{LFL} = \dfrac{v_{RFL}\cos\alpha_F - v_{WFL}}{\max\{v_{WFL}, v_{RFL}\cos\alpha_F\}}$ $\quad v_{RFL} = \omega_{FL} R$

- $S_{SFL} = \tan\alpha_F \mathbb{1}(S_{LFL}) + \dfrac{v_R}{v_W}\sin\alpha_F \mathbb{1}(-S_{LFL})$

- $S_{ResFL} = \sqrt{S_{LFL}^2 + S_{SFL}^2}$

- $F_{LFL} = \dfrac{\mu(S_{ResFL})}{S_{ResFL}} F_z (S_{LFL}\cos\alpha_F + S_{SFL}\sin\alpha_F)$

- $F_{SFL} = \dfrac{\mu(S_{ResFL})}{S_{ResFL}} F_z (S_{SFL}\cos\alpha_F - S_{LFL}\sin\alpha_F)$

- $F_{XFL} = F_{LFL}\cos\delta - F_{SFL}\sin\delta$

- $F_{YFL} = F_{SFL}\cos\delta + F_{LFL}\sin\delta$

$$\cdot\ S_{LFR} = \frac{v_{RFR}\cos\alpha_F - v_{WFR}}{\max\{v_{WFR}, v_{RFR}\cos\alpha_F\}} \qquad v_{RFL} = \omega_{FL} R$$

$$\cdot\ S_{SFR} = \tan\alpha_F \mathbb{1}(S_{LFR}) + \frac{v_R}{v_W}\sin\alpha_F \mathbb{1}(-S_{LFR})$$

$$\cdot\ S_{ResFR} = \sqrt{S_{LFR}^2 + S_{SFR}^2}$$

$$\cdot\ F_{LFR} = \frac{\mu(S_{ResFR})}{S_{ResFR}} F_z (S_{LFR}\cos\alpha_F + S_{SFR}\sin\alpha_F)$$

$$\cdot\ F_{SFR} = \frac{\mu(S_{ResFR})}{S_{ResFR}} F_z (S_{SFR}\cos\alpha_F - S_{LFR}\sin\alpha_F)$$

$$\cdot\ F_{XFR} = F_{LFR}\cos\delta - F_{SFR}\sin\delta$$

$$\cdot\ F_{YFR} = F_{SFR}\cos\delta + F_{LFR}\sin\delta$$

$$\cdot\ S_{LRL} = \frac{v_{RRL}\cos\alpha_R - v_{WRL}}{\max\{v_{WRL}, v_{RRL}\cos\alpha_F\}} \qquad v_{RRL} = \omega_{RL} R$$

$$\cdot\ S_{SRL} = \tan\alpha_R \mathbb{1}(S_{LRL}) + \frac{v_R}{v_W}\sin\alpha_R \mathbb{1}(-S_{LRL})$$

$$\cdot\ S_{ResRL} = \sqrt{S_{LRL}^2 + S_{SRL}^2}$$

$$\cdot\ F_{LRL} = \frac{\mu(S_{ResRL})}{S_{ResRL}} F_z (S_{LRL}\cos\alpha_R + S_{SRL}\sin\alpha_R)$$

$$\cdot\ F_{SRL} = \frac{\mu(S_{ResRL})}{S_{ResRL}} F_z (S_{SRL}\cos\alpha_R - S_{LRL}\sin\alpha_R)$$

$$F_{XRL} = F_{LRL} \qquad F_{XRR} = F_{LRR}$$

$$F_{YRL} = F_{SRL} \qquad F_{YRR} = F_{SRR}$$

Wheel dynamics:

$$J_\omega \dot{\omega}_{FL} = -R F_{LFL} + \tau_\omega$$

$$J_\omega \dot{\omega}_{FR} = -R F_{LFR} + \tau_\omega$$

$$J_\omega \dot{\omega}_{RL} = -R F_{LFR} + \tau_\omega$$

$$J_\omega \dot{\omega}_{RR} = -R F_{LRR} + \tau_\omega$$

Vehicle dynamics equations:

$$\dot{X} = v_x \cos\psi - v_y \sin\psi$$

$$\dot{Y} = v_x \sin\psi + v_y \cos\psi$$

$$\dot{\psi} = \frac{v_x}{L} \tan\delta$$

$$m\dot{v}_x = m v_y r + F_{XFL} + F_{XFR} + F_{XRL} + F_{XRR} - F_D$$

$$m\dot{v}_y = -m v_x r + F_{YFL} + F_{YFR} + F_{YRL} + F_{YRR}$$

$$I_z \dot{r} = l_F (F_{YFL} + F_{YFR}) - l_R (F_{YRL} + F_{YRR}) + \frac{b_R}{2} (F_{XRR} - F_{XRL}) + \frac{b_F}{2} (F_{XFR} - F_{XFL})$$

$$J_\omega \dot{\omega}_{FL} = -R F_{LFL} + \tau_\omega$$

$$J_\omega \dot{\omega}_{FR} = -R F_{LFR} + \tau_\omega$$

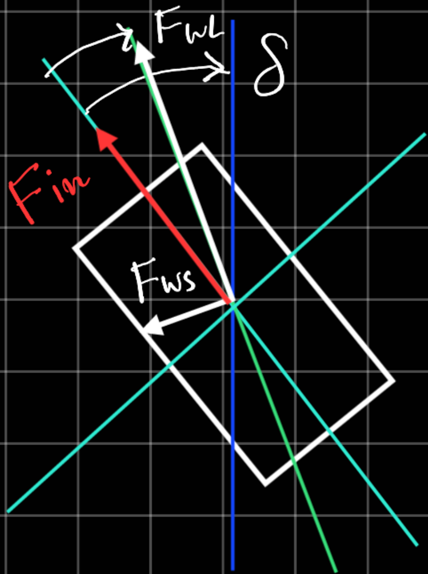
$$J_\omega \dot{\omega}_{RL} = -R F_{LFR} + \tau_\omega$$

$$J_\omega \dot{\omega}_{RR} = -R F_{LRR} + \tau_\omega$$

$$\dot{\delta} = \omega$$

$$X = [X \; Y \; \psi \; v_x \; v_y \; r \; \omega_{FL} \; \omega_{FR} \; \omega_{RL} \; \omega_{RR} \; \delta]^T \in \mathbb{R}^{11}$$

$$u^{(1)} = [F_{in} \;\; \omega]^T_d \quad \text{or} \quad u^{(2)} = [\tau_\omega \;\; \omega]^T$$



$F_{wl} \cos\alpha + F_{ws} \sin\alpha$

$-F_{wl} \sin\alpha + F_{ws} \cos\alpha$

$C_f = 151550 \; N/m$

$C_r = 52020 \; N/m$

17130 Kg

9.8 m/s^2

$C_1 = 1.2801$

$C_2 = 23.99$

$C_3 = 0.52$

33.7405 times

98.2963 times

Model Predictive Control for autonomous driving

Consider the discrete time system dynamics:

$$x_{k+1} = f(x_k, u_k)$$

Consider the state $x_0 \in \mathcal{X}$ and $u_0 \in \mathcal{U}$

$\hat{x}_0(k)$ is the state trajectory obtained by applying the input sequence $u(k) = u_0 \ \forall k > 0$ to the system with $x_0(0) = x_0$

This system can be approximated by the following LTV system:

$$\delta x(k+1) = A_{k,0}\, \delta x(k) + B_{k,0}\, \delta u_k$$

Where $A_{k,0} = \left.\dfrac{\partial f}{\partial x}\right|_{\hat{x}_0(k),\, u_0}$, $B_{k,0} = \left.\dfrac{\partial f}{\partial u}\right|_{\hat{x}_0(k),\, u_0}$

$$\tilde{x}_{k+1} = A_k \tilde{x}_k + B_k \tilde{u}_k$$

Now, given references signals x_{ref} and u_{ref} we formulate the MPC as follows:

$$\min_{u} \ \tilde{x}_N^T S \tilde{x}_N + \sum_{k=0}^{N-1} \tilde{x}_k^T Q \tilde{x}_k + \tilde{u}_k^T R \tilde{u}_k$$

$$\text{s.t.} \quad \tilde{x}_{k+1} = A_k \tilde{x}_k + B \tilde{u}_k$$

$$\tilde{x}_k = x_k - x_k^r$$
$$\tilde{u}_k = u_k - u_k^r$$
$$C\tilde{x}_i + D\tilde{u}_k \leq b$$

LTV Model predictive control

Given the error dynamics:

$$\dot{e} = A(t)e + B(t)\omega$$

where $e = x - x_r$ and $u = u - u_r$

$$A(T) = \left.\frac{\partial f}{\partial x}\right|_{x_r(T),\, u_r(T)}$$

$$B(T) = \left.\frac{\partial f}{\partial u}\right|_{x_r(T),\, u_r(T)}$$

Runtime cost

$$l_k(e_k, \omega_k) = \frac{1}{2} e_k^T Q e_k + \frac{1}{2}\omega_k^T R \omega_k$$

Terminal cost

$$l_N(e_N) = \frac{1}{2} e_N^T S e_N \qquad \frac{a(1+s)}{1+sa}$$

$$1 - \frac{1-a}{1+sa} \qquad \frac{\cancel{1}+sa-\cancel{1}+a}{1+sa}$$