CS 449

Homework 1

Group 10
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Question 1 Part a:

Additive Inverse yields

$$ {}^{F}p_{F}^{Sink\,1} + {}^{Sink\,1}p_{F}^{item\,1} = {}^{F}p_{F}^{item}. $$

Simulation values are,

$$ {}^{F}p_{F}^{Sink\,1} = [-0.5 \quad 2.0 \quad 0.4], {}^{Sink\,1}p_{F}^{item\,1} = [-0.1 \quad -0.1 \quad 0.52] $$

and thus

$$ {}^{F}p_{F}^{item} = [-0.5 \quad 2.0 \quad 0.4] + [-0.1 \quad -0.1 \quad 0.52] = [-0.6 \quad 1.9 \quad 0.92] $$

As for orientation, sink1 has no rotation to world frame, and the same applies for item1. Thus, item1 is oriented the same as sink1.

Question 1 Part c:

The relative position of “item1” does not change according to “tray” and “item2”. This is because while we transition “tray” from “stove1” to “table1”, we carry both item1 and item2 without any relative change to the tray. This can be verified by the simulation as well. We have not changed any details of “item1” or “item2”, but made “trey” a child of “table1”.

Question 1 Part d:

It is assumed that we are to calculate the relative pose of item1 to sink1 (${}^{Sink\,1}p_{F}^{item} = ?$) after the tray translation in part c.

$$ {}^{Sink\,1}p_{F}^{item\,1} = {}^{Sink\,1}p_{F}^{F} + {}^{F}p_{F}^{table\,1} + {}^{table\,1}p_{F}^{tray} + {}^{tray}p_{F}^{item\,1} $$
$$ {}^{Sink\,1}p_{F}^{item\,1} = -{}^{F}p_{F}^{Sink\,1} + {}^{F}p_{F}^{table\,1} + {}^{table\,1}p_{F}^{tray} + {}^{tray}p_{F}^{item1} $$

The following values are taken from the simulation, and hence, may vary a little from answer to answer:

$$ {}^{Sink\,1}p_{F}^{item\,1} = -[-0.5 \quad 2.0 \quad 0.4] + [2.0 \quad 0.0 \quad 0.385] + [0.0 \quad 0.0 \quad 0.42] + [-0.1 \quad -0.1 \quad 0.15] $$

Which gives

$$ {}^{Sink\,1}p_{F}^{item\,1} = [2.4 \quad -2.1 \quad 0.555] $$

Question 2 Part c:

To compute the Jacobian matrix, we can use the formulas derived in part b,

$$\begin{gathered} y=\sin(q_0)+\sin(q_0+q_1) \\ z=\cos(q_0)+\cos(q_0+q_1) \end{gathered}.$$

These yield

$$J(q)=\frac{\partial f(q)}{\partial q}=\begin{bmatrix} \partial y/\partial q_0 & \partial y/\partial q_1 \\ \partial z/\partial q_0 & \partial z/\partial q_1 \end{bmatrix}=\begin{bmatrix} \cos(q_0)+\cos(q_0+q_1) & \cos(q_0+q_1) \\ -\sin(q_0)-\sin(q_0+q_1) & -\sin(q_0+q_1) \end{bmatrix}.$$

We have verified the Jacobian Matrix works using joint-angles= [1.95205226, 3.15753276, 0]

Question 2 Part d:

Taking the determinant of the Jacobian matrix would yield

$$\begin{gathered} -\sin(q_0+q_1)\cos(q_0)-\sin(q_0+q_1)\cos(q_0+q_1)+\sin(q_0)\cos(q_0+q_1)+\sin(q_0+q_1)\cos(q_0+q_1) \\ = \\ \sin(q_0)\cos(q_0+q_1)-\sin(q_0+q_1)\cos(q_0) \end{gathered}$$

We expect singularity to appear in when the determinant of the Jacobian matrix is 0. This would appear when either $q_1=0$or $q_1=\pi$.

When $q_1=0$we have the determinant as

$$\sin(q_0)\cos(q_0)-\sin(q_0)\cos(q_0)=0.$$

When $q_1=\pi$we have the determinant as

$$\sin(q_0)\cos(q_0+\pi)-\sin(q_0+\pi)\cos(q_0)=-\sin(q_0)\cos(q_0)+\sin(q_0)\cos(q_0)=0.$$

This means that whenever our two-link manipulator looks like a straigth line, we lose a degree of freedom, which makes sense, and then singularity appears.

Question 3 Part a:

For the three-link planar manipulator, we need three angles for joints, in which y and z coordinates of the end effector depends on. Therefore, we need to have a 2x3 matrix. We need two rows for x and y axis. We need three columns for q_0 , q_1 , and q_2 .

Question 3 Part b:

The size of the translational Jacobian determines the type of inverse that can be computed, that is, whether an exact inverse can be computed or not. If the Jacobian is a square matrix, the number of joints is the same as the degrees of freedom of the manipulator, and if its determinant is non-zero, no singularity observed, then you can calculate the exact inverse. If it is not a square matrix, the number of joints is not the same as the degrees of freedom of the manipulator, then we need to use different methods, such as pseudoinverses, to approximate the inverse [2].

In planar two-link manipulator, we have a square matrix. Provided that we observe no singularity, then we can calculate the exact inverse.

In planar three-link manipulator, we do not have a square matrix, and we can not calculate the exact inverse.

Question 3 Part c:

When a singularity occurs, we get close to the situation in which a degree of freedom is lost. Therefore, since we are working in two dimensions (x-y dimensions), we expect to have a manipulability ellipsoid in the circular shape. Thus, if we lose a degree of freedom, the manipulability ellipsoid would get narrower and eventually look like a **straight line**. An important remark is that, even though we have a three-link manipulator, it would have a manipulability ellipsoid in the circular shape since we are working in the two-dimension medium. Thus, the shape of the manipulability ellipsoid appears to have a relationship with the dimensions we work with rather than the number of joints in the manipulator.

Question 4 Part a:

$$^{B}p^{C}(0)=\begin{bmatrix}1\\0\\0\end{bmatrix}, {}^{A}p^{C}(0)={}^{A}p^{B}(0)+{}^{B}p^{C}(0)={}^{A}p^{B}(0)+\begin{bmatrix}1\\0\\0\end{bmatrix}.$$

$^{A}p^{B}(0)=\begin{bmatrix}0\\l_1+l_2\\l_0\end{bmatrix}$ as at t=0, the link(arm) is aligned with y-axis of frame A and that of B. Hence,

$$^{A}p^{C}(0)=\begin{bmatrix}0\\l_1+l_2\\l_0\end{bmatrix}+\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}1\\l_1+l_2\\l_0\end{bmatrix}.$$

We rotate in z-axis, hence,

$$^{A}R^{B}(0)=\begin{bmatrix}\cos(\theta(t)) & -\sin(\theta(t)) & 0\\ \sin(\theta(t)) & \cos(\theta(t)) & 0\\ 0 & 0 & 1\end{bmatrix}.$$

Question 4 Part b:

Rotation Matrices are orthogonal matrices with determinant 1, that is, for a rotation matrix R, we have

$$R^T = R^{-1}.$$

Hence, we have

$$\hat{\omega} = \dot{R} R^{-1} = \dot{R} R^T \rightarrow -\hat{\omega}^T = -\left(\dot{R} R^{-1}\right)^T = -R \dot{R}^T.$$

It suffices to show that $\dot{R} R = -R \dot{R}^T$(1) to prove $\hat{\omega} = -\hat{\omega}^T$.
Since rotation matrices have determinant 1, they are a part of Special Orthogonal Group[1].
For Special Orthogonal Group we have

$$R R^T = I \rightarrow \frac{d}{dt}\left(R R^T\right) = \frac{d}{dt}(I) = 0 \rightarrow \dot{R} R^T + R \dot{R}^T = 0 \rightarrow \hat{\omega} = \dot{R} R^T = -R \dot{R}^T = -\hat{\omega}^T.$$

Since we have $R^T = R^{-1}$, $\hat{\omega} = -\hat{\omega}^T$is proven for $\hat{\omega} = \dot{R} R^{-1}$.

Question 4 Part c:

We are trying to find the spatial velocity ${}^A V^B(t)$. To find it we take the derivative of the transformation matrix ${}^A X^B(t)$ as

$$\frac{d}{dt} {}^A X^B(t) = {}^A V^B(t).$$

We have

$${}^A X^B(t) = \begin{bmatrix} \cos(\theta(t)) & -\sin(\theta(t)) & 0 & -l_2 \sin(\theta(t)) \\ \sin(\theta(t)) & \cos(\theta(t)) & 0 & l_1 + l_2 \cos(\theta(t)) \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

3x3 matrix beginning from the top left corner represents the translation matrix. 3X1 matrix starting from the top right corner represents the rotation matrix. The fourth row is added to maintain consistency and make it a 4x4 matrix.

If we take the derivative of ${}^A X^B(t)$, we will get the spatial velocity ${}^A V^B(t)$ as

$$\frac{d}{dt}{}^{A}X^{B}(t)={}^{A}V^{B}(t)=\begin{bmatrix} -\sin(\theta(t)) & -\cos(\theta(t)) & 0 & -l_2\cos(\theta(t)) \\ \cos(\theta(t)) & -\sin(\theta(t)) & 0 & -l_2\sin(\theta(t)) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Question 5:

While optimization based methods can be used in well defined problems, computational methods are preferred when the problem is learning based and data driven. In our case inverse kinematics with optimization does the calculations and decides the movements with respect to joint positions. The computations are handled by the methods invoked on a defined KOMO object. However, in inverse kinematics with computational methods, the working principles of the inverse kinematics are calculated with well defined equations.In optimization based problems, the results are precise and optimally built. On the other hand, generalized data is used in the computational methods.

Bibliography:

[1] “Rotation matrix”. https://en.wikipedia.org/wiki/Rotation_matrix. [Accessed: October 14, 2023].

[2] “Jacobian”. https://byjus.com/maths/jacobian/#:~:text=Jacobian%20matrix%20is%20a%20matrix,in%20the%20transformation%20of%20coordinates. [Accessed: October 14, 2023].