Parametric Families of Discrete Distributions

Name	pdf	Parameter Space	Mean	Variance	Moment Generating Function = $\mathsf{E}[e^{tX}]$
Uniform	$\frac{1}{n+1} I_{\{0,1,\dots,n\}}(x)$	$n = 1, 2, \dots$	$n/2$	$n(n+2)/12$	$\sum_{j=0}^{n} \frac{1}{n+1} e^{jt} = \frac{1-e^{(n+1)t}}{(n+1)(1-e^t)}$
Bernoulli	$p^x(1-p)^{1-x} I_{\{0,1\}}(x)$	$0 \le p \le 1$	p	$p(1-p)$	$(1-p) + pe^t$
Binomial	$\binom{n}{x} p^x (1-p)^{n-x} I_{\{0,1,\dots,n\}}(x)$	$0 \le p \le 1$ $n = 1, 2, \dots$	np	$np(1-p)$	$[(1-p) + pe^t]^n$
Hyper-geometric	$\frac{\binom{K}{x}\binom{M-K}{n-x}}{\binom{M}{n}} I_{\{0,1,\dots,n\}}(x)$	$M = 1, 2, \dots$ $K = 0, \dots, M$ $n = 1, \dots, M$	$n\frac{K}{M}$	$n\frac{K}{M}(1-\frac{K}{M})\frac{M-n}{M-1}$	not useful
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!} I_{\{0,1,\dots\}}(x)$	$\lambda > 0$	λ	λ	$exp[\lambda(e^t - 1)]$
Geometric	$p(1-p)^x I_{\{0,1,\dots\}}(x)$	$0 < p < 1$	$(1-p)/p$	$(1-p)/p^2$	$\frac{p}{1-(1-p)e^t}$ for $t < -ln(1-p)$
Geometric	$p(1-p)^{x-1} I_{\{1,2,\dots\}}(x)$	$0 < p < 1$	$1/p$	$(1-p)/p^2$	$\frac{pe^t}{1-(1-p)e^t}$ for $t < -ln(1-p)$
Negative Binomial	$\binom{r+x-1}{x} p^r (1-p)^x I_{\{0,1,\dots\}}(x)$	$0 < p < 1$ and $r > 0$	$r(1-p)/p$	$r(1-p)/p^2$	$\left[\frac{p}{1-(1-p)e^t}\right]^r$ for $t < -ln(1-p)$
Negative Binomial	$\binom{x-1}{r-1} p^r (1-p)^{x-r} I_{\{r,r+1,\dots\}}(x)$	$0 < p < 1$ and $r > 0$	r/p	$r(1-p)/p^2$	$\left[\frac{pe^t}{1-(1-p)e^t}\right]^r$ for $t < -ln(1-p)$
Beta-binomial	$\binom{n}{x} \frac{\mathcal{B}(x+a,n-x+b)}{\mathcal{B}(a,b)} I_{\{0,\dots,n\}}(x)$	$a > 0, b > 0$ $n = 1, 2, \dots$	$\frac{na}{a+b}$	$\frac{nab(n+a+b)}{(a+b)^2(a+b+1)}$	not useful
Logarithmic	$\frac{(1-p)^x}{(-xlnp)} I_{\{1,2,\dots\}}(x)$	$0 < p < 1$	$\frac{(1-p)}{(-p\ lnp)}$	$\frac{(1-p)(1-p+lnp)}{-(p\ lnp)^2}$	$\frac{ln[1-(1-p)e^t]}{lnp}$ for $t < -ln(1-p)$
Discrete Pareto	$\frac{(1/x^{\gamma+1})}{\sum_{j=1}^{\infty}(1/j^{\gamma+1})} I_{\{1,2,\dots\}}(x)$	$\gamma > 0$	$\frac{\sum_1^\infty (1/j)^\gamma}{\sum_1^\infty (1/j)^{\gamma+1}}$ for $\gamma > 1$		does not exist

Parametric Families of Continuous Distributions

Name	pdf $= f(x)$ cdf $= F(x)$	Parameter Space	Mean	Variance	Moment Generating Function $= \mathsf{E}[e^{tX}]$
Uniform	$f(x) = \frac{1}{\beta} I_{(\alpha,\alpha+\beta)}(x)$	$-\infty < \alpha < \infty$ $\beta > 0$	$\alpha + \frac{\beta}{2}$	$\frac{\beta^2}{12}$	$\frac{e^{(\alpha+\beta)t} - e^{\alpha t}}{\beta t}$
Normal	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$	$-\infty < \mu < \infty$ $\sigma > 0$	μ	σ^2	$\exp[\mu t + \frac{1}{2}\sigma^2 t^2]$
Exponential (rate λ)	$f(x) = \lambda e^{-\lambda x} I_{(0,\infty)}(x)$	$0 < \lambda < \infty$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda - t}$ for $t < \lambda$
Bilateral exponential	$f(x) = \frac{1}{2}\beta e^{-\beta\|x-\alpha\|}$	$-\infty < \alpha < \infty$ $0 < \beta < \infty$	α	$\frac{2}{\beta^2}$	$e^{t\alpha}/(1 - t^2/\beta^2)$ for $\|t\| < \beta$
Gamma	$f(x) = \frac{1}{\Gamma(\alpha)} \beta^\alpha x^{\alpha-1} e^{-(\beta x)} I_{(0,\infty)}(x)$	$\alpha > 0$ $\beta > 0$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$	$\left(\frac{\beta}{\beta - t}\right)^\alpha$ for $t < \beta$
Weibull	$f(x) = \frac{\gamma}{\beta}\left(\frac{x-\alpha}{\beta}\right)^{\gamma-1} e^{-\left(\frac{x-\alpha}{\beta}\right)^\gamma} I_{(\alpha,\infty)}(x)$	$-\infty < \alpha < \infty$ $\beta > 0,\ \gamma > 0$	$\alpha + \beta\,\Gamma\left(1 + \frac{1}{\gamma}\right)$	$\beta^2 \left[\Gamma\left(1 + \frac{2}{\gamma}\right)\right.$ $\left.-\Gamma^2\left(1 + \frac{1}{\gamma}\right)\right]$	not useful $\mathsf{E}[(X-\alpha)^k] =$ $\beta^k \Gamma\left(1 + \frac{k}{\gamma}\right)$
Beta	$f(x) = \frac{1}{\mathcal{B}(a,b)} x^{a-1}(1-x)^{b-1} I_{(0,1)}(x)$	$a > 0$ $b > 0$	$\frac{a}{a+b}$	$\frac{ab}{(a+b)^2(a+b+1)}$	not useful $\mathsf{E}[X^k] =$ $\frac{\mathcal{B}(a+k,b)}{\mathcal{B}(a,b)}$
Pareto	$f(x) = \frac{\gamma}{(1+x)^{\gamma+1}} I_{(0,\infty)}(x)$	$\gamma > 0$	$1/(\gamma - 1)$ for $\gamma > 1$	$\gamma/[(\gamma-2)(\gamma-1)^2]$ for $\gamma > 2$	does not exist
Cauchy	$f(x) = \frac{1}{\beta\pi} \frac{1}{1 + \left(\frac{x-\alpha}{\beta}\right)^2}$	$-\infty < \alpha < \infty$ $\beta > 0$	does not exist	does not exist	does not exist
Logistic	$F(x) = \left[1 + e^{-\left(\frac{x-\alpha}{\beta}\right)}\right]^{-1}$	$-\infty < \alpha < \infty$ $\beta > 0$	α	$\beta^2\pi^2/3$	$e^{\alpha t}\beta\pi t\ \csc(\beta\pi t)$
Gumbel (Extreme value)	$F(x) = \exp\left[-e^{-\left(\frac{x-\alpha}{\beta}\right)}\right]$	$-\infty < \alpha < \infty$ $\beta > 0$	$\alpha + \beta\gamma$ where $\gamma \approx 0.577216$	$\beta^2\pi^2/6$	$e^{\alpha t}\Gamma(1 - \beta t)$ for $t < 1/\beta$
Log normal	$F(x) = \Phi\left(\frac{ln\ x - \mu}{\sigma}\right) I_{(0,\infty)}(x)$	$-\infty < \mu < \infty$ $\sigma > 0$	$\exp[\mu + \frac{1}{2}\sigma^2]$	$\exp[2\mu + 2\sigma^2] -$ $\exp[2\mu + \sigma^2]$	does not exist $\mathsf{E}[X^k] =$ $\exp[k\mu + \frac{1}{2}k^2\sigma^2]$