Multivariate Rational Functions in Julia

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Project Definition

The project is basically about implementing an algorithm that simplifies the multivariate rational functions that are too complex by using techniques such as interpolation.

$$\frac{f(x_1, x_2, ..., x_n)}{g(x_1, x_2, ..., x_n)} \in K(x_1, x_2, ..., x_n)$$

Project Definition

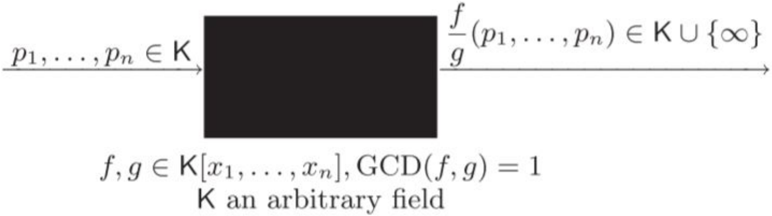

Figure 1: Black box for rational function evaluation

[KY07]

Evaluation of Numerator and Denominator

Input:
- $\frac{f(x_1,x_2,\ldots,x_n)}{g(x_1,x_2,\ldots,x_n)} \in \mathsf{K}(x_1,x_2,\ldots,x_n)$ input as a black box (see above)
- $B_2,\ldots,B_n$: $n-1$ shift elements that are randomly chosen from a sufficiently large finite set $S_1 \subseteq \mathsf{K}$
- $p_1,\ldots,p_n$: n evaluation points that are randomly chosen from a sufficiently large finite set $S_2 \subseteq \mathsf{K}$
- $\bar{d},\bar{e}$: degree bounds $\bar{d} \geq \deg(f)$ and $\bar{e} \geq \deg(g)$
- d,e (optional): the degrees of f and g, respectively (with high probability)
- $\tau_1,\ldots,\tau_n$: a given exponent vector with $1 \leq \tau_i \leq \min(\bar{d},\bar{e})$

Output:
- the value of $f(p_1^{\tau_1},\ldots,p_n^{\tau_n})/c$ and $g(p_1^{\tau_1},\ldots,p_n^{\tau_n})/c$ (with high probability), where c is the leading coefficient of $g(X,B_2X,\ldots,B_nX)$ (with high probability)
- or "failure," in which case the random values input are diagnosed as unusable

[KY07]

Project Design

Rational Function Interpolation

Input:
- $\frac{f(x_1,x_2,\ldots,x_n)}{g(x_1,x_2,\ldots,x_n)} \in \mathsf{K}(x_1,x_2,\ldots,x_n)$ input as a black box
- $(x_1,\ldots,x_n)$: an ordered list of variables in f/g.
- $\bar{d},\bar{e}$: degree bounds $\bar{d} \geq \deg(f)$ and $\bar{e} \geq \deg(g)$

Output:
- $f(x_1,\ldots,x_n)/c$ and $g(x_1,\ldots,x_n)/c$ (with high probability), where $c \in \mathsf{K}$.
- Or "failure", in which case unlucky random elements have been selected (one can rerun the algorithm with new random values) or the black box does not evaluate a rational function of the given degree bounds.

[KY07]

Project Requirements

*Julia lang(MultivariatePolynomials.jl,Interpolations.jl)
*Data sets (Multivariate rational functions)
*Testing methods

Success Criteria

- Minimizing the number of evaluations.
- Rational interpolation.

References I

Erich Kaltofen and Zhengfeng Yang, *On exact and approximate interpolation of sparse rational functions*, ISSAC '07: Proceedings of the 2007 international symposium on Symbolic and algebraic computation (2007), 203–210.