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Fullest COLREGs Evaluation Using Fuzzy Logic for Collaborative Decision-Making Analysis of Autonomous Ships in Complex Situations

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***Abstract*— Maritime Autonomous Surface Ships (MASSs) will reshape the fast-evolving ecosystem for their attractive socio-economic benefits and potential to improve safety. However, their new systems and technology need thorough verifications to identify unintended components of risk. The interaction between MASS cyber-physical systems and the existing regulatory framework is currently unpredictable; AI-powered intelligent situation awareness and autonomous navigation algorithms must safely and efficiently adhere to the regulations which are only designed for human interpretation without MASSs consideration. This paper contributes to algorithmic regulations and particularly algorithmic COLREGs in real-world MASS applications. It focuses on codifying COLREGs into a machine-executable system applicable to MASSs. This fullest COLREGs evaluation is modelled in form of a fuzzy expert system based on ordinary seamanship practice. The full input space spans 21 features derived from maneuverability-dependent risk, AIS traffic data, vessel information, maps and nautical charts, water-depth, visibility, and sea conditions. The model assesses pairwise vessel encounters over the full time-window of a situation from entrance to exit. 42 fuzzy rules are designed in 6 criteria that represent COLREGs Rules 2–19 and model their logical connections, priorities, and relationships. This algorithmic COLREGs form satisfies the crucial needs in simulation, collision-avoidance, complexity monitoring, and compliance quantification in MASS applications. The fullest COLREGs evaluation model is verified on a large database of historical encounters using real data from multiple sources.**

***Index Terms*— AIS, algorithmic regulation, big data, collision risk, COLREGs, fuzzy logic, fuzzy preferences, grounding risk, maritime autonomous surface ships (MASSs), sea-state, traffic scenario, traffic separation schemes, visibility, wind speed, water-depth to draught ratio.**

I. Introduction

THE current developments in maritime intelligent transportation thrust along three main directions: digitalization, automation, and autonomous algorithms for systems and logistics. These enthusiastically growing progresses power the extensive use of data-driven solutions through Artificial Intelligence (AI) and machine learning to provide new services and maximize benefits. Consequently, computational algorithms are increasingly adopted to make decisions in Maritime Autonomous Surface Ships (MASSs). However, this progress leaves safety and compliance questionable, particularly in complex scenarios.

The debate around ethics of cyber-physical systems introduced the need for *algorithmic regulations* [1]; thus, the need for codified regulations, or at least an algorithmic form that applies firmly to cyber-physical systems, and/or the need for regulating various black-box decision-making algorithms in order to enforce compliance and promote safety. For example, the International Regulations for Preventing Collisions at Sea (COLREGs) [2] were not drafted with MASSs in mind, they do not cover MASSs' operations, and may be interpreted differently in computational algorithms. Full verification is then essential to protect against unintended harmful effects of AI in MASSs applications while still encouraging and further honing the developments in these innovative technologies.

Particularly, new actors are joining and reshaping the maritime transportation ecosystem through AI-powered data-driven intelligent situation awareness [3] and autonomous navigation algorithms [4]. Parallel to this momentum, the regulatory bodies initiated new plans for integrating advanced technologies in their regulatory framework to cope with the fast-evolving ecosystem such as the regulations of MASSs [5] to guard safety and security. In this direction, this paper presents an executable expert system that fully evaluates COLREGs in simulations and big-data applications in a seaman-like interpretation of the Rules. It contributes to a needed component that complements advanced computer simulations of real-world scenarios –extracted from big data– to fully test and verify the interaction between emerging MASSs innovations and the regulatory framework.

MASS verification aims at identifying and also explaining possible vulnerabilities or safety-blind hotspots that should be fixed by the ship designers; otherwise, providing evidence of trustworthiness to the investors and 'Right to Explanation' to the public. It is also essential to anticipate any unintended impact and understand possible elements of risk or incompliance concerns. Advanced simulation of COLREGs in realistic scenarios will also provide futureproof skills to ensure safe human–machine interaction. It is also useful to assess current regulations for MASS applications and determine

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which and how some regulations may need to be amended or revised.

This paper presents explainable models for fullest COLREGs evaluation using fuzzy logic with particular contributions of:

- Fullest scenario and environment representation using the full set of features that influence COLREGs decision-making.
- COLREGs evaluation over the full duration of the scenario subject to dynamic time-variations.
- Evaluation of the full Ruleset and modelling the connections, priorities, and relationships between the Rules.
- Seamanship-like context-based interpretation of the variables and the Rules.
- Validation through a large database of real-world scenarios extracted from historical data.

Due to COLREGs central role, their fullest codification is a strong bridge to both MASSs' design and verification. In this work, 'Rules' refer specifically to original COLREGs Rules#2–19 as referred to in COLREGs, whereas 'rules' refer to general and legal rules beyond COLREGs and fuzzy rules. In fact, the Rules will be considered as the *criteria* that should be satisfied by every vessel in every situation at any time. A (sub-)criterion might translate to multiple fuzzy conditional statements to cover all low-level conditions. This paper focuses on the development of the fullest pairwise COLREGs evaluation models. The models are validated through a large database of historical conflicts extracted from Automatic Identification System (AIS) traffic data.

II. Motivation and Background

COLREGs specify the core conducts between vessels to prevent accidents. An *accident* is an unintended event that may result in *negative consequences* including personal injury, equipment damage, or environmental impact such as oil spill. Ship navigation accidents include *collision* with another vessel; *grounding* or contact such as hitting seabed, shore, rocks, base of an offshore wind turbine, or a bridge. COLREGs are primarily designed to prevent collision, but grounding also falls under "other navigational hazards" in the Rules and it is taken into full consideration in this work, especially to assess 'safe passage' and "sufficient sea-room". This work is hence map-based, i.e., high-resolution maps and *digital nautical charts* are utilized to assess all physical and virtual obstructions that restrict the maneuverable space. Maps define the physical obstructions whereas nautical charts determine complementary smaller obstructions like fish facilities, wind farms, rocks, and bridges in addition to restricted navigation areas and *Traffic Separation Schemes* (TSSs) like traffic lanes, separation zones, and inshore zones. Map-based features are novel in this work to evaluate COLREGs Rules 8–10 which were not considered before.

Preventing navigational accidents is transformed into avoiding *navigation risk* in COLREGs Rules. This work hence presents a risk-based fullest COLREGs evaluation model where navigation risk is a fundamental input variable; the term "risk" is explicitly mentioned in most of the COLREGs Rules under part A and implied in other Rules using closely-related terms such as "danger" and "safe". However, without a unified risk evaluation method [6], risk remains a fuzzy variable with an interpretation that depends mostly on the methods and factors that are used in its determination and the purpose of their use. The theoretical definition of risk is based on the probabilities of encounter and failure and the negative consequences. However, the more practical concept is based on a reactive approach [7], i.e., determine if the risk is high and react according to COLREGs. The latter concept measures the nearness –in space and in time– of a *near-miss* situation, i.e., an incident that did not cause a structural impact but it would have turned into an accident given a slight change in position, time, Speed Over Ground (SOG), or Course Over Ground (COG). Common examples are near-collision and near grounding situations with respective *collision risk* (CR) and *grounding risk* (GR). A common technique that is mostly used in the literature is the Closest Point of Approach (CPA) [8].

However, the same values of CPA parameters, distance at- and time to CPA, have radically different risk interpretations in different *contexts* such as various vessels' length (L), SOG, encounter type, and pose at CPA. A more solid concept of near-miss situations is defined in COLREGs as *Close-Quarters Situations* (CQS), e.g., Rule#8 defines collision-avoidance actions and it requires that if sufficient sea-room is available then actions to avoid a CQS should be substantial and made in good time. Additionally, a CQS is defined as a situation of two vessels dangerously approaching one another such that the actions by one vessel alone are insufficient to solve the situation. This concept depends on many influencing factors to determine the risk [7]: maneuverability of each vessel, i.e., the ability to turn and stop depending on vessel size and SOG; availability of maneuverable space around each vessel (sea-room); and vessels' primary responsibility to start collision-avoidance actions. This approach is commonly implemented using *vessel safety domain* [9]. However, CQSs should be predicted before they actually develop, they should be avoided according to COLREGs, explicitly in Rules#8 and 19.

The fullest approach of navigation risk assessment was reported in [10] to efficiently predict a CQS and assess its level of risk. The CR estimation method is based on the famous concept of ship safety domain. The CR is quantified based on the area of intersection between the two vessels' domains. The size of each vessel domain depends on the ship dimensions; The domain is not symmetrical since it considers Rules 13, 14, 15. Reference [7] proposed a ship domain that is adaptive on ship speed and course to account for varying maneuverability parameters. Moreover, [10] presented a computationally efficient method to quantify risk and the ship domain concept was improved to forecast risk before it takes place. This work also considers the available maneuverable space, such as grounding risk and TSSs.

A *conflict* is defined as an encounter where a CQS is predicted to take place shortly; the approach in [10] predicts the time to CQS t_{cqs} (i.e., the available time to avoid a CQS) and the level of collision risk CR during the CQS in a collision conflict; similarly, it estimates GR and determines the available time t_g to avoid GR in a grounding conflict. This risk assessment method is compliant to COLREGs since it makes full appraisal of the situation and of CR at all times as required by Rule#5. All levels of risk are evaluated

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precisely and CQSs are predicted early to allow more time to assess the situation and make actions. Moreover, this approach considers traffic separation schemes as required by COLREGs Rules# 1 and 10. In determining risk, it takes into account vessel maneuverability according to vessel size and speed as mentioned in COLREGs Rule#6 about safe speed. However, the risk assessment in [10] did not take into account the prevailing navigation conditions such as the states of visibility, wind, sea, and currents. These conditions are considered in this work to ensure full compliance to the fullest Ruleset.

This paper presents a fuzzy expert system that ensures the fullest COLREGs evaluation; the fullness is in terms of context features, set of Rules, compliance, and seamanship practice. The state-of-the-art COLREGs models are incomplete by considering only a subset of input variables, a subset of the Rules, and evaluation limited to particular contexts, e.g., open water and good weather conditions. Fuzzy-logic-based models are explainable and incorporate experts' knowledge, unlike data-driven machine learning algorithms which result in black-box models that may replicate human erroneous actions that are present in the training data. Using fuzzy logic is also motivated by its advantage to quantify the vagueness of COLREGs which are open to various linguistic interpretations of many input variables and extra-linguistic interpretations of the Rules and their connections as investigated in [11]. Fuzzy reasoning is advantageous to avoid any assumed specifications to hone COLREGs such as baselessly defining 'head-on' by a 6° sector.

Fuzzy logic has been widely used for approximate risk estimation and path planning in maritime applications. Reference [12] extended the potential field algorithm for collision avoidance using four fuzzy variables to evaluate COLREGs Rule#8. Reference [13] used fuzzy logic to determine collision avoidance actions based on COLREGs Rules#13–15. Reference [14] used fuzzy logic to assess risk levels. More generally, [15] simulated COLREGs Rules#8, 13–15 using fuzzy logic based on CPA parameters while the rest of the Rules are not covered and the full bathymetric and environment features are not considered.

In this work, the input variables span the *context features* and the *conflict features* to fully describe the narrative of any scenario. The context is characterized by background information about the environment and the weather conditions. This includes variables of: visibility distance; sea-state; wind speed; water depth; and sea-surface currents. The conflict features are related to traffic information and risk assessment including: relative bearing; relative COG; collision risk; available time to CR; grounding risk; available time to GR; and dynamic change of risk. The fullest set of input variables is collected, processed, and matched from multiple data sources. Linguistic variables are designed based on their influence on ship dynamics and their interpretation in COLREGs.

The fullness of the *input feature space* is crucially important. The context variables determine the vessels' abilities such as maneuverability, stability, and controllability and the conflict features determine the severity of the situation. More importantly, all the variables jointly determine which and how COLREGs Rules should be evaluated. Moreover, COLREGs Rules should be evaluated together since the Rules exhibit many logical connections and priorities. The full list of input variables and interpretation are considered in section III in the fuzzification process design whereas the fuzzy reasoning behind the Rules will be designed in section IV.

III. Linguistic COLREGs Interpretation

Fuzzy logic is used in this paper for its advantages of flexible quantitative modelling of all types of *fuzziness* including (*i*) *inaccuracy; (ii) uncertainty; (iii) ambiguity*, and *(iv)confusion* which are attributed to data, Rules, models, and decisions. This work does not repeat the theory of *fuzzy inference process* which is well reported in the literature and implemented in most artificial intelligence libraries and software. This paper focuses only on the novel design of the expert system; the standard Mamdani fuzzy inference [17] is used to execute the designs. Similarly, COLREGs Rules are not repeated here and interested readers are referred to [2].

Inaccuracy is the difference between the actual and available values of physically measured variables such as water-depth for instance. Inaccuracies are attributed to different factors including: *value resolution* such as the AIS position in decimal degrees with resolution $10^{-4\circ} \cong 11$m in lateral direction; *spatial resolution* such as visibility and tide measurements collected from few stations (positions), spatial interpolation and extrapolation introduce inaccuracies to estimate their values at particular locations; and similarly *temporal resolution*. Uncertainty is similar to inaccuracy where the background truth is not precisely known. Aleatory and epistemic uncertainties are attributed to: (*1*) *noise and errors* present in real-world data; (*2*) *estimated quantities* such as collision and grounding risk are estimated without a universal concept; and (*3*) *open interpretation* that is a unique characteristic of COLREGs such as the uncertain interpretation of the value "Restricted" in the variable "visibility" and "*channel*"= "*narrow*" in COLREGs. Inaccuracies and uncertainties are inevitable in big data collected from multiple sources; also, MASSs use computer vision and sensor fusion where uncertainty is a solid factor.

Ambiguity is a characteristic of a policy problem and it is typical in COLREGs, there is no standard structure for the connection between the (sub-)Rules. Ambiguities are in interpreting the Rule purpose; the functionality of the Rule; the relationships such as the connection logic and the priorities between the Rules. E.g., "Notwithstanding …" and "When …in any doubt" in Rule#13(a,c) introduce strong connections with the rest of Rules, other connections are weak and unclear. The evaluation of vague Rules leads to fuzzy outputs, called fuzzy *preferences* over *alternative states* {Give-way, Stand-on} in this work. Confusion results from unclear preferences due to: (1) uncertainty in the preferences; (2) *multi-Rule situations*, ex. both impeding safe passage Rules and collision-avoidance Rules apply simultaneously and to limited degrees; and (3) the *confliction* between pairwise COLREGs-defined preferences in *multi-vessel situations* (>2 vessels in risk).

A. Linguistic Variables

The Algorithmic COLREGs evaluation using fuzzy systems involves more than 20 linguistic variables of multiple terms.

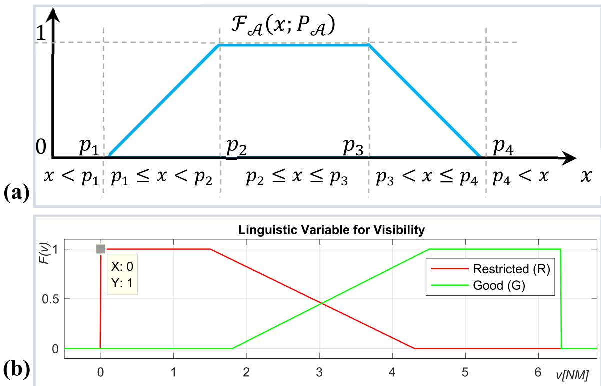

Fig. 1. Generic trapezoidal FMF and a linguistic variable. (a) $\mathcal{F}_{\mathcal{A}}(x)$ is the degree to which x belongs to $\mathcal{A}$. (b) Linguistic variable representation for visibility using 2 FMFs for 2 fuzzy sub-sets with $P_G = [1.8, 4.5, 6.3, 6.3]$ for Good and $P_R = [0, 0, 1.5, 4.3]$ for restricted conditions.

Hence, this paper first presents a generic parametric model that is then tailored to each variable using new different parameters.

Unlike a crisp set X where an element x either belongs to the set $x \in X$ or not $x \notin X$, a *fuzzy set* $\mathcal{A}$ is defined as a nonempty collection of elements $x \in X$ with a degree of *membership* $\mathcal{F}(x)$ described by any number in the unit interval [17],

$$I = [0, 1], \mathcal{F} : X \to I, x \to \mathcal{F}_{\mathcal{A}}(x), \quad (1)$$

where $\mathcal{F}(x)$ is called the *Fuzzy Membership Function* (FMF) and the fuzzy set is expressed as

$$\mathcal{A} = \{x, \mathcal{F}(x) \,|\, x \in X\}. \quad (2)$$

A particular type of fuzzy sets is a *fuzzy number* defined over a *range* of real numbers $x \in R$ with a membership function $\mathcal{F} : R \to I$; where $\mathcal{F}$ is upper semi-continuous, $\mathcal{F}(x) = 0$ outside the range R, and $\mathcal{F}(x) = 1$ within a subrange inside R. A common membership function is the trapezoidal FMF that is used in this work; It is defined for fuzzy set $\mathcal{A}$ over a numerical variable x through four parameters $P_{\mathcal{A}} = [p_1 p_2 p_3 p_4]$, with $p_1 \leq p_2 \leq p_3 \leq p_4$, over the range $R = [p_1, p_4]$ as

$$\mathcal{F}_{\mathcal{A}}(x; P_{\mathcal{A}}) = \max\left(\min\left(\frac{x - p_1}{p_2 - p_1}, 1, \frac{p_4 - x}{p_4 - p_3}\right), 0\right), \quad (3)$$

as shown in Fig. 1.

This work uses the trapezoidal FMF for its computational, explainability, and practical advantages. First, this function is controlled through 4 parameters which allow designing many forms including certain and uncertain ranges as desired; e.g., a triangular FMF has 3 parameters and it can be easily obtained as a particular case of the trapezoidal FMF if the latter satisfies $p_2 = p_3$. Moreover, these FMFs have many advantages over other alternatives including exponential FMFs such as sigmoidal or Gaussian FMFs. First, the trapezoidal FMF-based fuzzy systems are computationally very efficient in the inference and defuzzification processes, they are advantageous for large-scale fuzzy inference problems and big data applications. More importantly, the parameters of the exponential FMFs are abstract and do not have a direct practical interpretation. Fortunately, the four parameters of the trapezoidal FMF are fully explainable since they have direct min/max interpretations; they can be easily tuned by the experts or learned from historical data during the design phase; they also make the overall fuzzy system fully interpretable.

A *linguistic variable* y is defined through the quintuple

$$y := (y, Y, T(y), M, \mathcal{G}), \quad (4)$$

- y is the name of the linguistic variable, e.g., "visibility" or v in nautical miles (NM) *unit* as shown in Fig. 1(b);
- Y is the *universe of discourse*, e.g., interval $V = [0, 6.3]\, NM$;
- y takes *linguistic values* in $T(y)$ which is the set of *labels* of y, e.g., $T(v) = \{'Good', 'Restricted'\}$;
- $M : T(y) \to \mathcal{F}(Y; P)$ is the *semantic rule* which assigns a fuzzy set (or subset) of Y to each label in $T(y)$, e.g., the FMFs of *Restricted* (R) and *Good* (G) visibility terms are $M(R) = \mathcal{F}(v; [0, 0, 1.5, 4.3])$ and $M(G) = \mathcal{F}(v; [1.8, 4.5, 6.3, 6.3])$.
- $\mathcal{G}$ is the syntactic grammar which produces extra labels (linguistic values) for y through composition of its fuzzy sets with fuzzy modifiers called *hedges*, e.g. '*Very*' in 'Very Good' visibility (VG) such that $M(G) = \mathcal{F}(v; P)$ and $M(VG) = \mathcal{F}^2(v; P)$ obtained as the squared FMF $\mathcal{F}$.

The universe of discourse can be clipped to a range of interest, e.g., $v > 6.3$ NM is considered totally 'Good' and rounded to 6.3, or to the scope of COLREGs, e.g., $\frac{dCR(t)}{dt} > 0$.

In the context of COLREGs, v is a linguistic variable where 3 NM is practically known inside an *uncertainty range* that can neither be considered as absolutely Good nor as absolutely Restricted; values in this range are partially members of both conditions in a comparable degree.

This idea of multiple and partial memberships of the linguistic variables offers a mathematical tool to model the fuzziness. Assigning fuzzy sets to linguistic terms allows computing with words to make full and subjective COLREGs evaluation with high accuracy and reduced complexity.

For most variables, p_1 and/or p_4 with p_2 and/or p_3 are straightforward to specify since they depend on the overall universe of discourse. E.g., since $0 \leq v \leq 6.3$ NM, visibility parameters are set as $p_1 = p_2 = 0$ for P_R and $p_3 = p_4 = 6.3$ for P_G as shown in Fig. 1(b). The remaining parameters represent the range of uncertainty and they need contextual knowledge or historical data to tune, the next section will focus only on such parameters. Using the same example, experienced seafarers can answer direct simple questions such as: What is the largest value of visibility that you fully consider 'Restricted'? and what is the smallest value of visibility that you fully consider 'Good'? Or what is the range of visibility values that you consider between Restricted and Good? The various answers allow the direct tuning of p_3 and p_4 for P_R and p_1 and p_2 for P_G. Different sources use different visibility tables, e.g., the range of 'moderate' that is between 'poor' and 'good' visibility was considered 2–5 NM in [18] compared to 2–5 miles, i.e., 1.74–4.34 NM in [19]. Hence, the uncertain range parameters are tuned manually between these values in Fig. 1(b).

TABLE I

DEFINITION OF COLREGS' LINGUISTIC VARIABLES FOR A CONFLICT $CC_{i,j}$

	Fuzzy variable	Description	COLREGs Rules	Labels	FMF parameters P $[p_1, p_2, p_3, p_4]$ $(unit)$
Traffic features	RB_i	RB wrt i at "t_f"	7, 13	Overtaken (O)	[105, 117.5, 243.5, 256.5](°)
	ΔCOG_i	Relative COG wrt i	"9–10", 14, "15"	CrossS (CS) Head-on (HO) CrossP (CP)	[35, 75, 155, 170] (°) [155, 170, 190, 205] [190, 205, 285, 325]
	L_i	Length i	9, 10	Less20 (L)	[0, 0, 19, 21] (m)
Risk features	CR	Collision risk	2, 5– 8, 14, 15, 17–19	Low (L) High (H)	[0, 0, .04, .08] (r_0) [.04, .08, .17, .17]
	t_{cqs}	Available time to CQS	7, 10, 16, 19	Short (R) Sufficient (S)	[0, 0, 2, 5] (minutes) [2, 5, 15, 15]
	Δ_{CR}	Change of CR	"17"	Insufficient (U) Improving (P)	[0,0,.008,.012] $\left(\frac{r_0}{min}\right)$ [.008, .012, .2, .2]
	GR_i	Grounding risk for i	"8–10"	Low (L) High (H)	[0, 0, .07, .1] (–) [.07, .1, 1, 1]
	t_{gi}	Time to GR for i	"8–10"	Before (B) After (A)	[0.1, 0.2, 1, 1.15] [1.1, 1.2, 2, 2] (–)
Weather & Sea conditions	v	Visibility distance	4, 6, 19	Restricted (R) Good (G)	[0, 0, 1.5, 4.3] (NM) [1.8, 4.5, 6.3, 6.3]
	SS	Sea-State (SWH)	6	Calm (C) Rough (R)	[0, 0, 0.5, 1.25] [0.5, 2.5, 4, 4] (m)
	w	Wind-Speed	6	Calm (C) Gale (G)	[0, 0, 1.5, 10.7] (m/s) [5.5, 13.8, 20, 20]
	SC	Sea-Surface-Current	6	Green (G) Red (R)	[0, 0, .4, .75] (m/s) [.55, .85, 1, 1]
	H/T	Depth-to Draught	6, "9", 18	Shallow (S) Deep (D)	[0, 0, 2, 3] [2, 3, 5, 5] (–)
Navigational States	NC	Navigation condition	5 to 7	Normal (N) Rough (R)	[0, 0, .25, .75] [.25, .75, 1, 1] (–)
	"CAA"	Collision avoiding action	6, 8, 13–17, 19	Normal (N) Early (E) Urgent (U)	[0, 0, .15, .4] [.2, .4, .6, .8] [.6, .85, 1, 1] (–)
	d	Impede safe passage	8–10	A–B (A–B) Normal (N) B–A (B–A)	[0, 0,.2, .35] [.25, .4, .6, .75] [.65, .8, 1, 1] (–)
	P_i	Normal pref. for i	13–19	SO GW	[0, 0, .3, .7] [.3, .7, 1, 1] (–)
		Urgent Preference	13–19	SO High (SOH) GW High (GWH)	[0, 0, .2, .6] [.2, .6, 1, 1] (–)
		Relaxed Preference	13–19	SO Low (SOL) GW Low (GWL)	[0, 0, .4, .8] [.4, .8, 1, 1] (–)

Quoted expressions "x" are implicitly mentioned in COLREGs. 'wrt' stands for "with respect to".

For simplicity, the table lists fuzzy variables wrt vessel i only, but variables wrt both vessels are used jointly. '(–)' means unitless.

B. COLREGs Variables

In this paper, every Collision Conflict (CC), denoted $CC_{i,j}$, between vessels i and j is fully characterized by a set of 23 features listed in Table I. These features are defined in details in the following and their designs are explained. It is important to bear in mind that this work aims at verifying millions of nested conflicts in all contexts and not a single pair (i, j); the evaluation is updated over time but the time index t is dropped for simplicity.

All features are represented by linguistic variables using trapezoidal FMFs of their fuzzy subsets $\mathcal{A}$, $\mathcal{F}_{\mathcal{A}}(x; P_{\mathcal{A}})$ as in (3), the parameters of each term's FMF are summarized in Table I. Fuzzy logic based COLREGs evaluation entails an expert system where various sorts of domain and experts' knowledge can be incorporated in the design. The design of each linguistic variable is based on: (i) the analysis and observations from historical data and statistical analyses as in [8], [10]; (ii) technical experimental reports about the main influence of a specific feature on vessel dynamics and on collision-avoidance practices as in [20], [21]; and (iii) contextual knowledge as in [18], [19]. The design can also be tailored and improved through surveys from expert mariners. Most linguistic variables are designed with two FMFs only to ensure simplicity and satisfy the bivalent logic in legal rules. Table I entails 156 parameters; they are tuned in this work based on (i, ii, iii). The values of these parameters are not an international convention; they should be adjusted by experts and/or learned from historical data. $RB_i\left(t_f\right)$ is the *Relative Bearing* (RB) between vessels i and j with respect to vessel i, it is determined at t_f which is the *first time* when the two vessels became in sight of one another,

$$D_{i,j}\left(t_f\right) = \left(x_i\left(t_f\right) - x_j\left(t_f\right), y_i\left(t_f\right) - y_j\left(t_f\right)\right),$$
$$AB_i\left(t_f\right) = mod\left(angle\left(D_{i,j}\left(t_f\right)\right), 360\right), \tag{5}$$
$$s.t. \begin{cases} AB_i \in [0\ 360) \\ 0^\circ \equiv North \\ 90^\circ \equiv East, \end{cases} \tag{6}$$
$$RB_i\left(t_f\right) = mod\left(AB_i\left(t_f\right) - COG_i\left(t_f\right), 360\right), \tag{7}$$

as shown in Fig. 2(a), where (x_i, y_i) is the position of vessel i, $D_{i,j}$ is the distance vector from the position of vessel i to the position of vessel j; AB_i is the absolute bearing in clockwise direction from the north. Similarly, $RB_j\left(t_f\right)$ can be obtained using (5–7) by interchanging the indices i and j; and both RB_i and RB_j are used in this paper.

RB_i specifies whether vessel i is being '*overtaken*' (O) or '*not overtaken*'. The overtaking sector is defined in Rule#13(b) by 22.5° above the beam of the vessel being overtaken $112.5 < RB_i < 247.5$, see Fig. 2(b). However, there are three motivations for the fuzzy design of RB_i. First, RB_i is in fact calculated using the *heading* which is difficult to measure and inaccurate in all databases, it is approximated by COG in (7); RB_i is imprecisely assessed visually in practice. Second, Rule#13(c) addresses an uncertainty just-outside the above crisp sector "when …in any doubt". Third, Rule#13(d) insists that the overtaking situation should not become a crossing situation despite any subsequent alterations in $RB_i(t)$; this work therefore uses only the initial value $RB_i\left(t_f\right)$ without updates despite changes over the entire duration of the maneuver, whereas all the other features are updated at t. t_f is not strictly defined and two vessels may spot one another at different timestamps, where $RB_i(t)$ may change substantially since it depends on position, COG, and SOG of both vessels. It may cause confusion whether the situation is crossing or overtaking as reported multiple times as in this example [22]. $RB_i\left(t_f\right)$ is fundamental to not only Rule#13. Given its importance and the above three sources of uncertainty, the crisp 'Overtaken' range 112.5°–247.5° is extended to the fuzzy set with FMF $\mathcal{F}_{\mathcal{O}}(RB; P_{\mathcal{O}})$ with $P_{\mathcal{O}} = [105, 117.5, 243.5, 256.5]$ as shown in Fig. 2(c). The uncertain range extends 7.5° on starboard side and 9° on the port side

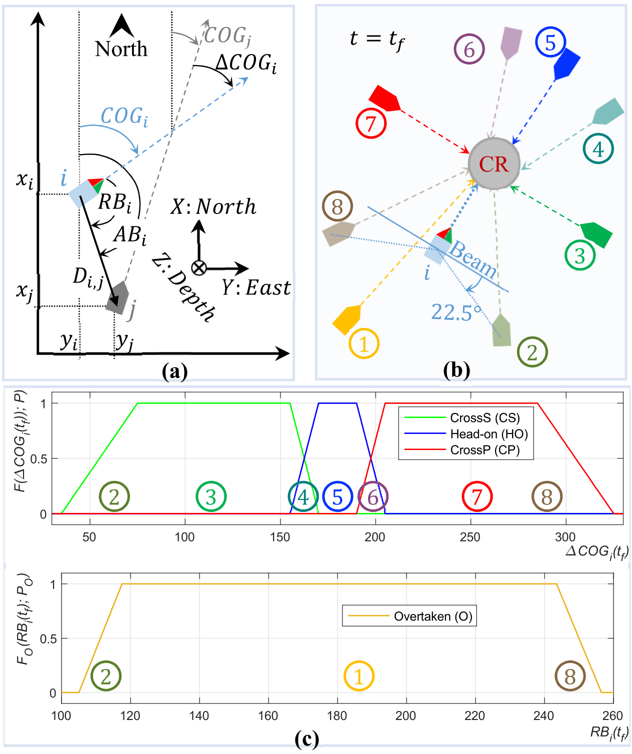

Fig. 2. Illustration of RB and ΔCOG variables and their interpretation in COLREGs. (a) describes the coordinates system, vessels positions, absolute and relative bearing angles, and relative COG. (b) shows pairwise encounters involving a Collision Risk with vessel i at the first time the pair became in sight of one another. (c) shows the fuzzy variables and terms for numerical ΔCOG_i and RB_i variables with a projection of the cases in (b).

because overtaking or crossing on the port side entail almost the same responsibilities.

Similarly, relative COG is defined with respect to vessel i as

$$\Delta COG_i(t) = COG_i(t) - COG_j(t). \quad (8)$$

This feature is more accurate and stable than RB, however, its linguistic value '*reciprocal or nearly reciprocal courses*' in Rule#14(a) is vague, it poorly defines the *Head-On* sector (HO), as shown in Fig. 2(b, c).

The head-on sector angle also tolerates Rule#14(c) where '*any doubt*' between head-on and crossing sectors is mapped in favor of head-on. Recent research papers use different crisp values for the limits of HO, common values vary considerably from ±3° up to ±22.5° as in [23]–[25]. This implies that no standard value can be considered as a convention. Hence, this work models the uncertain limits of HO between ±10° to ±15°, which is the range of reasonable and common values since most of the literature use ±10°, ±13°, and ±15° as in [8, 26, 27]. This results in HO's FMF parameters tuned to $P_{HO} = [155, 170, 190, 205]$, centered around 180° that implies 'reciprocal', as shown in Fig. 2(b, c).

The crossing sector is defined as the residual sector from overtaking and head-on sectors as interpreted from the order of Rules#13–15 and the "…in any doubt …" expressions in both Rule#13(c) and Rule#14(c). Crossing on Port side CrossP (CP) and Crossing on Starboard side CrossS (CS) are both defined herein since they necessitate opposite roles to the vessels. Hence, the FMFs of the residual crossing sectors are designed as shown in Fig. 2(b, c). ΔCOG_i is also implied in Rule#9–10 for 'crossing' narrow channels, fairways, and traffic lanes. The actual vessel length L is crisp, $L < 20$m is used in Rules#9–10, it is fuzzified with an uncertainty of 1m $P_L = [0, 0, 19, 21]$ to tolerate value inaccuracies.

CR is estimated using the fullest approach in [10] with respect to a relative risk factor r_0; t_{cqs} is the predicted available time to avoid the CQS and it is estimated in minutes. Grounding risk GR_i and GR_j are also considered herein to assess the availability of a maneuverable sea-room and to assess the OCLREGs terms related to 'passage' and 'safe passage' in narrow channels, fairways, and traffic lanes in Rule#8–10. Similarly, t_{gi} and t_{gj} are the estimated available time to avoid GR normalized to t_{cqs};

$$t_{gi} = \frac{t_{GRi}}{t_{cqs}}, \quad (9)$$

they are designed to assess the ability of the vessel to conduct a CQS avoidance maneuver 'before' GR avoidance.

The parameters of the FMFs of CR and GR are designed based on statistics of the situations in [10]. It was observed moderate collision risk levels are in the range 0.04 – 0.08 whereas 0.17 is the highest risk. Similarly, moderate GR levels are in the band of 0.07 to 0.1, whereas $GR = 1$ is the max value.

For t_{cqs}, the design of FMFs' parameters are based on statistical analysis of historical collision-avoidance maneuvers in [8], it was observed that the distribution mode of the evacuation time is around 4 to 6 minutes in open waters. The uncertain range is therefore adjusted to 2 to 5 minutes to account for all cases in open and restricted waters. The design of the FMF parameters for t_g is simple since the latter only specifies whether GR should be avoided before or after CR.

The "collision risk drop" feature $\Delta_{CR}(t)$,

$$\Delta_{CR} = \frac{-dCR}{dt}, \quad (10)$$

is designed to monitor the improvement of the conflict, it is implied in Rule#17 to determine whether the Give-Way vessel has been taking sufficient positive actions to avoid CR. The FMFs of this linguistic variable are designed to verify if the change is '*improving*' enough or '*insufficient*'. Since $CR = 0.04$ is the lowest value that is partially considered 'High', and $t_{cqs} = 5$ minutes is the largest value that is partially 'Short', CR should be changing at a rate $\Delta_{CR} > 0.008$ to be sufficient in the best case. Hence, P for Δ_{CR} 'Improving' is designed as $[0.008, 0.012, 0.2, 0.2]$.

Visibility υ is mentioned in Rules#4, 6, and 19, υ determines when/where the two vessels likely spotted one another; it influences risk; and it indistinctly dominates Rules under Sections I, II, and III of part B which respectively apply for '*any*', '*good*', and '*restricted*' visibility conditions. Visibility here represents the maximum distance at which a target vessel or her lights can be seen in daylight or at night, respectively. In maritime transport, visibility can be restricted due to many causes such as fog, mist, falling snow, and heavy rain, and it takes various scales. In a collision conflict, the uncertain interpretation of visibility may leave the navigators of both vessels in doubt whether to execute Rule#19 or Rules#11–18. Visibility FMFs were designed in previous sub-section.

The linguistic variable '*Sea-State'* (SS) is designed using the Douglas sea-state scale based on Significant Wave Height (SWH) [28] of combined wind-waves and swell. Waves are known for their impact on ship structural loads [29], but SS is designed herein to address the impact of SWH on ship maneuverability [30]. Douglas sea-state scale classifies SS condition as 'Slight' or degree 3 for $0.5 < SWH < 1.25m$; Hence $P_C = [0, 0, 0.5, 1.25]$ is used in Table I for 'Calm'. Degree 4 that signifies 'Moderate' is $1.25 \leq SWH < 2.5m$ and 'Rough' is $2.5 \leq SWH \leq 4m$; Hence, $P_R = [0.5, 2.5, 4, 4]$ is used here for the FMF 'Rough'.

Similarly, the *wind* linguistic variable w is designed using wind-speed based on the Beaufort wind scale and the reported analyses of wind impact on vessel speed and resistance. The Beaufort scale classifies wind condition as 'Calm' and 'Light air' in degrees 0 and 1 for $w \leq 1.5m/s$, until 'Fresh Breeze' in degree 5 for $7.9 < w \leq 10.7m/s$. It is degree 6, $10.7 < w$, that is associated with red warning flag. Hence, $P_C = [0, 0, 1.5, 10.7]$ is used for the designed 'Calm' SS FMF. The equivalent certain range of this FMF is Beaufort degrees 0 and 1, the equivalent uncertainty range is Beaufort degrees 2 to 6. The 'Rough' SS FMF covers degree 4 to 12. The Beaufort scale classifies the band $5.5 \leq w \leq 7.9m/s$ as 'Moderate breeze' in degree 4, and 'High wind' as degree 7 for $13.8 < w \leq 17.1m/s$. The FMF of 'Rough' SS is designed with parameters $P_R = [5.5, 13.8, 20, 20]$ using Beaufort classes from degree 4 until degree 7 as uncertain range, while the remaining degrees 8–12 are used as certainly 'Rough'.

The linguistic variable for *Sea-surface-Current* (SC) is designed based on observations of the effects of currents on ship navigation as reported in [31]. In average, moderate SC effects were observed for approximately $0.5 < SC < 0.8$ m/s. Therefore, $P_G = [0, 0, .4, .75]$ and $P_R = [.55, .85, 1, 1]$ are selected for the FMFs of 'Green' and 'Red' SC conditions.

The available *water-depth to draught ratio* H/T is a crucial and complex attribute to analyze since this data varies in time and in 2D space and it has complex effects on the vessel and on the evaluation of the Rules. Shallower water depth increases the flux around the ship which decreases the pressure applied to the hull and the ship actual draught increases compared to the average draught given in AIS data; this is called squat effect and it depends on vessel speed too. Shallow water also impacts ship resistance and stopping ability, course controllability and stability, and robustness to waves and wind. H/T is explicitly mentioned under COLREGs Rules# 6 and 18 to determine risk and vessel responsibility. It is also implicitly mentioned in Rule#9 where '*narrow channel*' and '*fairway'* depend on the channel width and also the available water-depth. On this regard, [20] conducted extensive experimental investigations of H/T effects on maneuverability. Moderate effects on the vessels' ability to stop and turn were observed in the range $2 < H/T < 3$. These observations were confirmed in [21] using advanced course-stability simulations; Smaller values have higher effects whereas large depth ratios have negligible effects; $H/T > 3$ was considered as 'deep water'. Therefore, the parameters $P_S = [0, 0, 2, 3]$ and $P_D = [2, 3, 5, 5]$ are designed for the FMFs that model 'Shallow' and 'Deep' terms.

Each collision conflict between vessels i and $j CC_{i,j}$ is fully characterized by a set of linguistic variables valued to a vector

$$\begin{aligned} CC_{i,j}(t) = \big[&CR, t_{cqs}, RB_i(t_f), RB_j(t_f), \\ &\times\ \Delta COG_i, \Delta_{CR}, GR_i, GR_j, t_{gi}, t_{gj}, \\ &\upsilon, SS, w, SC, H/T, L_i, L_j\big]; \qquad (11) \end{aligned}$$

They evaluate to a vector input of the fuzzy inference systems.

IV. Fuzzy Systemic COLREGs Interpretation

COLREGs Rules and their connections/relationships are also subject to fuzzy interpretation. They are translated into fuzzy inference systems as illustrated in Fig. 3. In addition to the fuzzified input context and conflict variables, this work designs *output variables* and *artificial state variables* in form of linguistic variables over a unit interval $I = [0, 1]$ as their universe of discourse. The state variables are used internally as fuzzy connection variables between the multiple criteria.

The linguistic variable '*Navigation Conditions*' (NC) defines the "*prevailing circumstances and conditions*" as mentioned in Rule#5 to 7; it summarizes sea and weather states which are measured in various units and scales. The '*presumed Collision Avoidance Action*' (CAA) is defined in Rule#8 and it relates most of COLREGs criteria; any action to avoid collision should be: (*i*) positive, (*ii*) made in good time, (*iii*) apparent to the other vessel, and (*iv*) proper {early, compliant, and large enough} to ensure passing at a safe distance. Hence, the design of this state is based on the navigation conditions, available time, and level of risk. This linguistic variable is designed through three equally-separated fuzzy subsets with terms '*Normal*' (N), '*Early*' (E), '*Urgent*' (U) as levels of urgency of the vessels to initiate their collision avoidance maneuvers given all the above circumstances (*i*–*iv*).

The map-based evaluation of Rules#8–10 in narrow channels and traffic lanes is challenging, the outcome is modelled by the linguistic state variable d which stands for '*impeding (safe) passage*', it determines if Rules#8–10 apply to the situation and the roles of the vessels in it. Suppose i and j $(i < j)$ are anonymized vessel identification numbers, d has three fuzzy terms: '$A - B'$ for '*i is impeding (safe) passage of j*$'$, '$B - A'$ for '*j is impeding (safe) passage of i*$'$, and 'N' is the normal case. The FMFs of '$A - B'$ and '$B - A'$ do not overlap since it is not possible to have both vessels impeding the (safe) passage of one another; yet, any of the two cases may hold partially.

Evaluating all COLREGs criteria on the set of input features assigns "partially" every vessel in each conflict a privilege to '*stand-on'* (SO) or a burden to '*give-way*' (GW) at every timestamp; SO and GW are the roles of the vessels and they are called herein the *collision-avoidance states*. At a particular timestamp during the conflict, vessel i should be under one of the two states $S_i = \{SO, GW\}$ which are *bivalent*, i.e., only one of the two states, no third state, not both and not between the two states. This bivalent logic follows from the requirements on SO vessel actions in Rule#17(a)(i) versus the actions to avoid collision in Rule#8, actions to allow

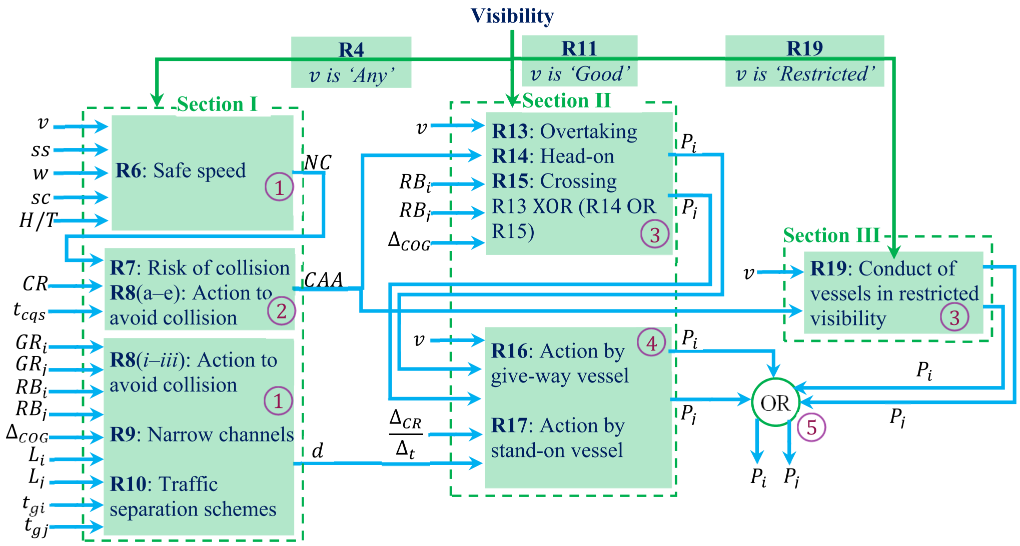

Fig. 3. Fuzzy process for COLREGs evaluation of a collision conflict between vessels i and j.

safe passing in Rule#8–9, and actions by a GW vessel in Rule#16. While SO vessel shall keep constant speed and course; GW vessel actions should be substantial and apparent to the other vessels. It is true the SO vessel '*may*' or '*shall*' switch to a GW state in Rule#17(a)(ii) and 17(b) as considered in the following, but actions under SO and GW are clearly distinguished and small alterations of speed and/or course should be avoided by Rule#8(b).

Yet, the decision (if, when, how) for a vessel to opt for one state over the other may not be definite if the value of an input variable is in the range of uncertainty, or if multiple Rules apply simultaneously but partially. Also, the actions of the GW vessel are not precisely defined, e.g., when to start the COG/SOG alteration. Hence, the degree to which a vessel is obliged by COLREGs to follow a particular state at t (or the degree to which the criteria are satisfied if so) are fuzzy output variables.

This uncertainty over the bivalent states is modeled by a *fuzzy preference* P_i. Fuzzy preferences [17] are fuzzy binary relations defined between pairs of nondominated alternatives in decision-making problems. Fuzzy preferences over finite m states $S = \{S_1, \ldots, S_m\}$ are represented by a *Relation matrix* $\mathcal{R} = \left(p_{k,l}\right) \in \mathbb{R}^{m \times m}$ defined with a membership function

$$f_{\mathcal{R}} : S \times S \to I = [0,1],\ f_{\mathcal{R}}(S_k, S_l) = p_{k,l}, \tag{12}$$

where $p_{k,l}$ is the degree of preference of state S_k over state S_l and it satisfies,

$$p_{k,l} + p_{l,k} = 1, \text{ and } p_{k,k} = 0.5, \tag{13}$$

for all sates $k, l = 1, 2, \ldots, m$. A fuzzy preference measures the degree of preference and not the amount of preference that determines the likelihood of a decision-maker to follow a particular state. Typical values of $p_{k,l} = 0$, $p_{k,l} < 0.5$, $p_{k,l} = 0.5$, $p_{k,l} > 0.5$, $p_{k,l} = 1$ are respectively interpreted as S_l is definitely preferred over S_k, S_l is likely preferred over S_k, indifferent preference, S_k is likely preferred over S_l, and S_k is definitely preferred over S_l.

Here, vessels i and j are the two participants in a single collision conflict $CC_{i,j}(t)$ represented by a set of features and each vessel has a role under two nondominated alternatives $S_i = \{SO, GW\}$, $S_j = \{SO, GW\}$ over which the preferences are respectively defined for i, j, and for the conflict $CC_{i,j}(t)$ as in (14) where $p^{i,j}_{GW} = p\left(GW_{i,j}, SO_{i,j}\right)$ represents the degree of preference of GW state (i shall GW to j) over SO state (i shall SO to j) while $p^{i,j}_{SO}$ is the preference of SO over GW. In this work, $p^{i,j}_{GW} = \mathcal{F}_{\mathrm{GW}}(P_i; P_{\mathrm{GW}})$ is the degree of membership of the preference P_i to the GW term with FMF parameters defined in Table I. Similarly, $p^{i,j}_{SO} = \mathcal{F}_{\mathrm{SO}}(P_i; P_{\mathrm{SO}})$ with P_{SO} parameters in Table I. In the same way, $p^{j,i}_{GW} = \mathcal{F}_{\mathrm{GW}}\left(P_j; P_{\mathrm{GW}}\right)$ and $p^{j,i}_{SO} = \mathcal{F}_{\mathrm{SO}}\left(P_j; P_{\mathrm{SO}}\right)$. Finally, the new preferences are represented in relation matrix form as

$$P_i = \begin{bmatrix} 0.5 & p^{i,j}_{SO} \\ p^{i,j}_{GW} & 0.5 \end{bmatrix} \in I^{2\times 2}$$
$$P_j = \begin{bmatrix} 0.5 & p^{j,i}_{SO} \\ p^{j,i}_{GW} & 0.5 \end{bmatrix} \in I^{2\times 2}$$
$$P_{i,j} = \begin{bmatrix} P_i & \cdot \\ \cdot & P_j \end{bmatrix} \in I^{4\times 4}, \tag{14}$$

The preferences of j towards i are $p^{j,i}_{GW}$ and $p^{j,i}_{SO}$ and they are defined similarly by reversing the vessels indices. The preferences are undefined for states of one vessel over states of another vessel since they are independent. Both $p^{i,j}_{GW} + p^{i,j}_{SO} = 1$ and $p^{j,i}_{GW} + p^{j,i}_{SO} = 1$ must be satisfied for any $CC_{i,j}(t)$ for bivalent logic principle; this is assured by SO and GW FMFs in Table I.

The fuzzy preferences variable, in Table I, includes two hedges for relaxed and urgent preferences; GWH and SOH FMFs give Higher preference to the GW state, whereas GWL and SOL FMFs give Lower preference to the GW state.

The fuzzy criteria are designed in the groups shown in Fig. 3 based on extra linguistic COLREGs interpretation taking into account: (i) the context in which the Rules are mentioned and in which they apply; (ii) the overall purpose of the regulation and its subjective and objective elements through

TABLE II

$C_{1,r}$: FUZZY RULES FOR COLREGs CRITERIA R6 (NC)

r	h	if:	v		SS		w		SC		H/T		NC
1:	1	is	R	|	R	|	G	|	R	|	S	then:	R
2:	1	is	G	&	C	&	C	&	G	&	D		N

TABLE III

$C_{2,r}$: FUZZY RULES FOR COLREGs CRITERIA R7_8 (CAA)

r	h	if:	NC		CR		t_{cqs}		CAA
1:	1	is	N	&	L	&	S	then:	N
2:	1	is	–		H	&	S		E
3:	1	is	–		L	&	R		E
4:	.5	is	R	&	L	&	S		E
5:	.7	is	N	&	H	&	R		U
6:	1	is	R	&	H	&	R		U

the question "what does this set of Rules regulate?"; (*iii*) the function of the Rules that regulate particular aspects (outputs) based on particular information (inputs); (*iv*) the intentional place of the Rule, especially the order within the same section and part, e.g., Rules#13 then 14 then 15 have a strong reason to be in that order; and (*v*) the *logic* within and between the Rules, e.g., "*notwithstanding anything contained in the Rules of part B, …*' in Rule#13(a) is a very strong condition that gives execution priority and logical statements in Rules#8–10 and Rules#13–15 are combined with an exclusive OR (XOR) logic. For simplicity, each criterion is implemented in a separate fuzzy inference system as explained in the following tables.

As shown in Fig. 3, the evaluation process consists of 5 stages ① … ⑤ which represent the time order of evaluation. The six criteria should be evaluated in sequence since some criteria depend on the outcomes of the evaluation of other criteria. First, $Stage\#1 \equiv [R6, \{R8, R9, R10\}]$ assesses the navigation conditions NC from weather/sea data and verifies the case for 'narrow channel or fairway', and assess the 'impeding passage' conditions d using map data, independent of any other rules. $Stage\#2 \equiv \{R7, R8\}$ determines the presumed collision-avoidance actions CAA depending on 'prevailing …conditions' in Rule#7(a) which results from NC in $Stage\#1$; $Stage\#3 \equiv [\{R13, R14, R15\}, R19]$ can then be evaluated to determine the roles of participating vessels at a time t depending on whether the situation requires 'Urgent' or 'Relaxed' actions as specified by CAA from $Stage\#2$. $Stage\#4 \equiv \{R16, R17\}$ combines (i) the roles determined by $\{R13, R14, R15\}$ in $Stage\#3$ with (ii) the roles resulting from narrow channels and TSSs that are summarized in d in $Stage\#2$. $Stage\#4$ also updates the Stand-On vessel role if the actions of a vessel supposed "to keep out of way" are found insufficient to avoid risk. Finally, the preferences resulting from R19 in $Stage\#3$ and $Stage\#4$ are combined in $Stage\#5$. These stages do not represent priorities, but the algorithm execution flow. The building blocks of the fuzzy COLREGs evaluation process are also grouped in sections I, II, III as in COLREGs. Rule#18 is crisp and simple to implement, but it is not explicitly evaluated in this work due to the lack of reliable navigational status information for the vessels.

The COLREGs are translated into 42 fuzzy rules, listed in Tables II to VI. The r^{th} fuzzy rule in the k^{th} criterion is represented by $C_{k,r}$; h is the rule weight, $0 < h \leq 1$. Generally, $h = 1$ has no effect on the fuzzy implication, h can be decreased to reduce the effect of a weaker rule. The symbols '|', '&', and '$\overline{x}$' are respectively the fuzzy 'OR', 'AND', and 'NOT' logical operators implied on the FMFs using max, min, and negation that is expressed as

$$\mathcal{F}_{\bar{x}}(y) = 1 - \mathcal{F}_x(y) \quad \forall y \in Y. \tag{15}$$

The outcomes of multiple rules $C_{k,r}$ in the k^{th} criterion are aggregated into C_k using OR logic expressed as the union

$$C_k\left(CC_{i,j}\right) = \bigcup\nolimits_r C_{k,r}\left(CC_{i,j}\right). \tag{16}$$

For example, the second row of Table II is $C_{1,1}$ and it reads:

$$rule1 : (h = 1) : if\ v\ is\ R\ O\ R\ SS\ is\ ROR\ w\ is\ Gor\ SC\ is\ R\ O\ R\ H\ /T\ is\ S\ , Then\ NC\ is\ R\ ; \tag{17}$$

Refer to Table I for full details of each linguistic variable and term; in full words, this rule reads: 'Criterion 1, rule 1, rule weight is 1: if Visibility is Restricted OR Sea-State is Rough OR Wind-Speed is Gale OR Sea-Surface-Current is Red OR Water-depth to Draught ratio is Shallow, then Navigation condition is Rough' while all these variables and their terms are mathematically defined by their FMFs in Table I; while 'is' sometimes refer to 'indicates' or 'implies'.

Similarly, the implication for $C_{1,2}$ is based on AND logic and we have $C_1\left(CC_{i,j}\right) = C_{1,1}\left(CC_{i,j}\right) \cup C_{1,2}\left(CC_{i,j}\right)$. The relations are also modelled through fuzzy operators AND, OR, NOT, XOR, and composition (∘), e.g.,

$$\begin{aligned} CAA &= C_2\left(NC, CR, t_{cqs}\right) \\ &= C_2\left(C_1\left(v, SS, w, SC, H/T\right), CR, t_{cqs}\right) \\ &= C_1 \circ C_2\left(CC_{i,j}\right). \end{aligned} \tag{18}$$

The first criterion C_1 in Table II summarizes the prevailing circumstances of weather condition and sea states in the NC terms which then determine the presumed necessary CAA defined in Rules#7 and 8 in C_2 (Table III) according to risk and available time. Rules#8, 9, and 10 which regulate conducts in narrow channels and traffic separation schemes are then evaluated in C_3 (Table IV) using all map-based features to determine in an impeding d state exists. The main navigation Rules# 13, 14, and 15 are evaluated in C_4 (Table V) which determines the preferences for vessels i and j under a collision conflict $CC_{i,j}$. These preferences are then modified according to Rules#16 and 17 in C_5 if the situation is urgent, if it is not improving, or if one vessel has a limited safe passage. Notice that Rule#19 represents a whole section in a single criterion in Fig.3 but it is merged with the rules of C_5 and it is represented by the 9$^{\text{th}}$ rule therein, Rule#19≡ $C_{5,9}$ in Table VI. The output preferences are labelled $Po_{i/j}$ in Table VI just to distinguish the input preferences from the modified output preferences, but it is the same linguistic variable.

V. Analysis and Discussion

The presented fuzzy models are integrated into one expert system that yields an algorithmic COLREGs evaluation form.

TABLE IV

$C_{3,r}$: FUZZY RULES FOR COLREGs CRITERIA R8–10 ABOUT IMPEDE (SAFE) PASSAGE

r	h	if:	GR_i	GR_j	t_{gi}	t_{gj}	RB_i	RB_j	$\Delta_{COG\,i}$	L_i	L_j		d
1:	1	is	H &	–	B &	–	$\overline{O}$ &	$\overline{O}$ &	CS &	–	–	then:	B–A
2:	1	is	H &	–	B &	–	$\overline{O}$ &	$\overline{O}$ &	CP &	–	–		B–A
3:	1	is	H &	–	B &	–	$\overline{O}$ &	$\overline{O}$ &	–	$\overline{L}$ &	L		B–A
4:	1	is	–	H &	–	B &	$\overline{O}$ &	$\overline{O}$ &	CS &	–	–		A–B
5:	1	is	–	H &	–	B &	$\overline{O}$ &	$\overline{O}$ &	CP &	–	–		A–B
6:	1	is	–	H &	–	B &	$\overline{O}$ &	$\overline{O}$ &	–	L &	$\overline{L}$		A–B
7:	1	is	L &	L	–	–	–	–	–	–	–		N
8:	1	is					O |	O					N

TABLE V

$C_{4,r}$: FUZZY RULES FOR COLREGs CRITERIA R13–15

r	h	if:	v	CAA	RB_i	RB_j	$\Delta_{COG\,i}$		P_i	P_j
1:	1	is	G &	N &	O			then:	SOL	GWL
2:	1	is	G &	E &	O				SO	GW
3:	1	is	G &	U &	O				SOH	GWH
4:	1	is	G &	N &		O			GWL	SOL
5:	1	is	G &	E &		O			GW	SO
6:	1	is	G &	U &		O			GWH	SOH
7:	1	is	G &	N &	$\overline{O}$ &	$\overline{O}$ &	HO		GWL	GWL
8:	1	is	G &	E &	$\overline{O}$ &	$\overline{O}$ &	HO		GW	GW
9:	1	is	G &	U &	$\overline{O}$ &	$\overline{O}$ &	HO		GWH	GWH
10:	1	is	G &	N &	$\overline{O}$ &	$\overline{O}$ &	CS		GWL	SOL
11:	1	is	G &	E &	$\overline{O}$ &	$\overline{O}$ &	CS		GW	SO
12:	1	is	G &	U &	$\overline{O}$ &	$\overline{O}$ &	CS		GWH	SOH
13:	1	is	G &	N &	$\overline{O}$ &	$\overline{O}$ &	CP		SOL	GWL
14:	1	is	G &	E &	$\overline{O}$ &	$\overline{O}$ &	CP		SO	GW
15:	1	is	G &	U &	$\overline{O}$ &	$\overline{O}$ &	CP		SOH	GWH

TABLE VI

$C_{5,r}$: FUZZY RULES FOR COLREGs CRITERIA R16–19

r	h	if:	Pi_i	Pi_j	v	CAA	Δ_{CR}	d		Po_i	Po_j
1:	1	is	GW						then:	GW	
2:	1	is		GW							GW
3:	.5	is	SO &	–	G &	U &	U			GW	
4:	1	is	SO &		G &		P			SO	
5:	1	is	SO &		G &	$\overline{U}$				SO	
6:	.5	is		SO &	G &	U &	U				GW
7:	1	is		SO &	G &		P				SO
8:	1	is		SO &	G &	$\overline{U}$					SO
9:	1	is			R &	$\overline{N}$				GW	GW
10:	1	is				N &		A–B		GW	SO
11:	1	is				N &		B–A		SO	GW

The validation of the presented models is based on a large database of real-world navigation scenarios designed using the approach of [10]. For visualization and validation, 10^5 real conflicts are selected randomly to cover all possible scenarios in various contexts. This sample spans around 8% of situations extracted from 2-years historical data where the codified COLREGs are simulated in few hours using an ordinary computer (Intel ®6 Core™i7 CPU; Memory 16 GB, MATLAB R2018a). The 10^5 pairs of vessels (i, j) are verified at a random timestamp. The results are depicted in Fig. 4 which visualizes the preferences determined by the fullest COLREGs evaluation model. For simplicity, the GW preferences are shown versus two dominant features only, $RB_i\left(t_f\right)$ and ΔCOG_i. Fig. 4(a) and Fig. 4(b) show respectively the GW preferences of vessel i ($p_{GW}^{i,j}$) and vessel j ($p_{GW}^{j,i}$) for the same conflict $CC_{i,j}(t)$. Full results are provided in Fig. 7 in the appendix.

This evaluation results in 9 regions, $\boxed{1}$to, $\boxed{9}$ characterized by unclear preference; they are explained in Fig. 4(c). These regions represent poorly regulated hotspots which may result in safety vulnerability. The safety-blind hotspots are scattered in a 21-dimensional feature space in this work. Fig. 4(a, b) show minor samples exhibiting the opposite preference compared to their surrounding clusters. Such regions will be difficult to recognize along other dimensions for other variables since their samples make a ungrouped minority classes. A particular interpretation of COLRTEGs may contribute to few unregulated regions (loopholes) or many/wider poorly regulated (indecisive) clusters.

Regions $\boxed{3}$ and $\boxed{4}$ raise confusion to vessel i, whereas regions $\boxed{5}$ and $\boxed{6}$ are confusing to vessel j, and the remaining regions are confusing for both vessels. The regions and their symmetry property may change if vessels i and j are controlled by different algorithms, as in MASSs. In scenario- and simulation-based verification of MASSs, and particularly to test black-box autonomous navigation algorithms, finding such safety-vulnerable regions can be casted into a classification/clustering problem to identify possible ways of failures to improve design and regulations.

The actual vessels' behavior –during the analyzed conflicts– was analyzed to compare the actual conducts against the predicted preferences. For each vessel, course and speed alterations are quantified using their standard deviation (std) during the conflict. Fig. 5 compares actual maneuvers against the predicted preferences. There are few situations where the preference is unidentified. The results show that vessels with higher predicted GW preference made more and larger COG and/or SOG alterations compared to vessels with a predicted stand-on preference. It should be taken into account that not all historical situations are fully and precisely COLREGs compliant at all timestamps; besides there exist also noise and normal route-following actions that may interfere with the collision-avoidance actions. But in general, the model outputs match to a great extent the facts shown by true vessels' behaviors.

Fig. 6 depicts the level of actual transient speed alteration for both GW and SO predictions. The frequencies of occurrence are normalized, but it is worth mentioning that the number of GW predictions is much higher than SO predictions. The analysis is limited to time during the analyzed situations, i.e. larger and sudden changes are not observed before the conflict is solved. In this analysis, 74% of the vessels with stronger SO prediction made indeed no speed change $\Delta SOG < 1knot$; whereas 88% of the vessels with stronger GW prediction made actually an apparent speed-change maneuver $\Delta SOG > 1knot$ during the conflict. These matching rates are significant considering COLREGs vagueness and the presence of route-following maneuvers.

Following the central role of COLREGs, their codification into the presented system has many potential applications:

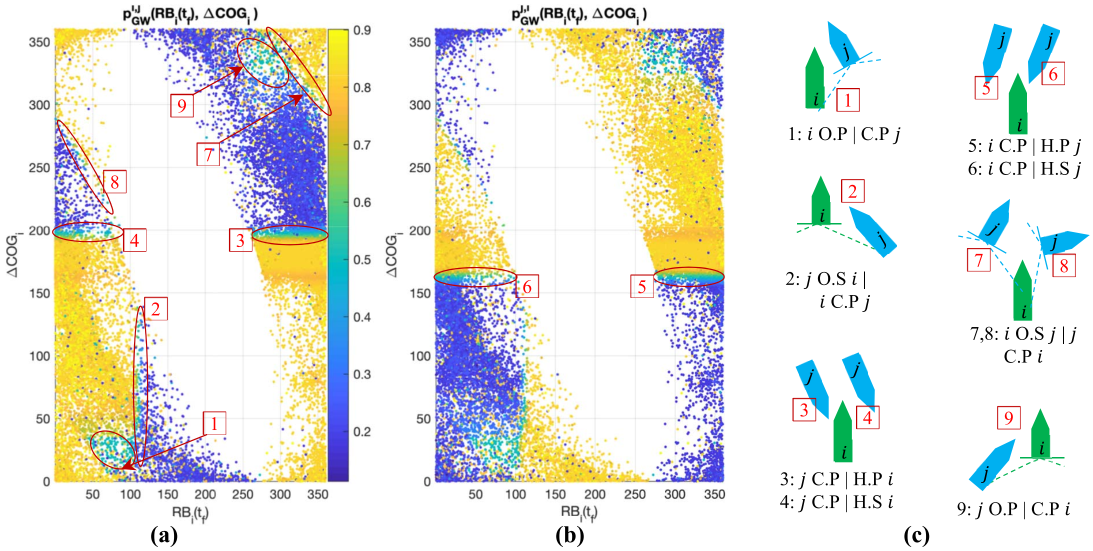

Fig. 4. Results of full COLREGs evaluation of 10^5 conflicts $CC_{i,j}$ randomly sampled from two-years real database. $p_{GW}^{i,j} \approx 1$ (yellow) signifies higher preference that i will GW to j; $p_{GW}^{i,j} \approx 0$ (dark blue) signifies higher preference for i to SO. (a) and (b) show the GW preferences of i and j respectively; (c) visualizes examples from weakly-regulated regions; 'C', 'O', and 'H' stand for 'Crossing', 'Overtaking', and 'Head on' respectively; 'P' and 'S' stand for 'on the Port side of' and 'on the Starboard side of', vessel shape length signifies her relative speed.

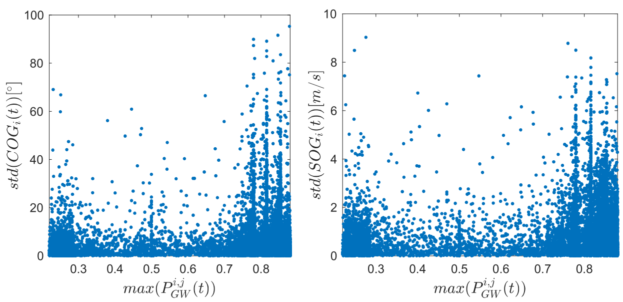

Fig. 5. Companion of actual vessels' conducts against the predicted preferences.

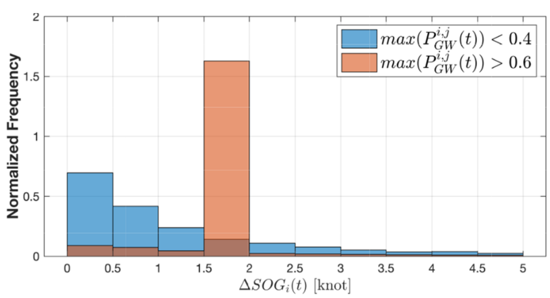

Fig. 6. Comparison of actual speed-alteration maneuvers for SO and GW predictions.

- Simulation-based testing/verification of MASSs interactions.
- Comparing and explaining incompliant or unsafe decisions from black-box autonomous navigation algorithms.
- Embodying and augmenting experts' knowledge into supervising data-driven machine learning algorithms of intelligent situation awareness and autonomous navigation.
- Extending the alternative states to decide actions into an autonomous collision-avoidance algorithm, and
- Post-test analysis of executed actions under the defined states to quantify the degree of compliance to COLREGs.
- Estimating and monitoring traffic complexity for online monitoring, surveillance, and spatial analysis.

VI. Conclusion

COLREGs are the 'Rules of road at sea' and they are the core regulations of conducts between vessels to avoid risk and prevent accidents. However, COLREGs are designed for humans and open for interpretation, they are flexible and their evaluation is context-based, they are in a vague form that is not directly applicable to MASSs. This paper codified COLREGs into an expert system that is applicable to MASSs, especially for simulation, testing, and performance evaluation.

Firstly, the paper defined the fullest input feature space including all the variables that affect ship dynamics and influence the evaluation of COLREGs. The paper focused on the context features that fully represent the narrative of any scenario in addition to collision conflict features and their complete and compliant estimation approaches. The input features included maneuverability-dependent risk estimations, all sea-state and weather condition variables, nautical charts- and maps-based features. Input features are fuzzified into linguistic variables based on uncertainties in their values and in the linguistic interpretation of their terms in COLREGs. Fuzzy state variables and the fuzzy preferences are designed over bivalent states of Give-Way and Stand-On to model any uncertainty in the outputs of the fuzzy decision-making process.

The fullest COLREGs evaluation system covers Rules 2 to 19; these were translated through extra linguistic interpretation into 6 connected criteria using 42 fuzzy conditional statements. The fuzzy-logic models were designed to codify the rules and model their logical and priority connections.

Finally, the connected models were verified on 500,000 situations sampled from a 2-years historical database.

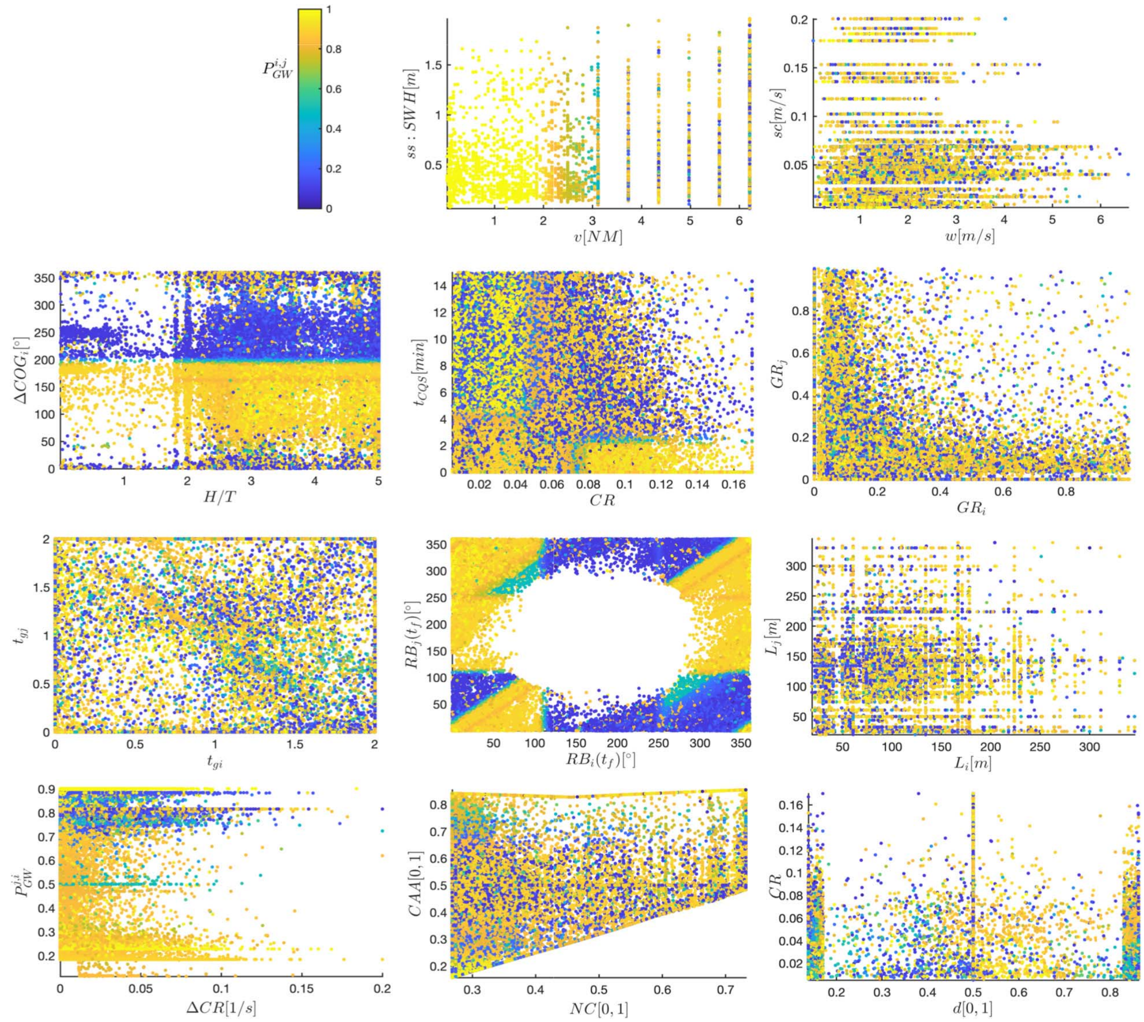

Fig. 7. Visualization of the resulting give-way preferences against the 21 input features.

The evaluation models were proved computationally efficient and their outputs match the observed actual vessel maneuvers.

This method discretized raw COLREGs into 156 parameters where there is room to augment contextual and experts' knowledge into the design. Future works may consider a statistical and/or machine learning approach to estimate the parameters based on historical data and surveys.

VII. Appendix A

Fig. 7 shows the outputs of codified COLREGs in form of $P_{GW}^{i,j}$ versus pairs of input features. Starting from the top left sub-graph, the following observations are drawn.

- $P_{GW}^{i,j}$ is given with a 0 to 1 scale to simulate COLREGS vagueness. The results show the frequency and the strength of a particular preference to verify COLREGs compliance.
- Under restricted visibility (v), the results show absolute preference to GW regardless the other features. GW decisions are more often during rough sea state (ss).
- There are not many situations at rough wind speed and surface current speed (w, sc); the rougher are these conditions, the higher is the frequency of GW decisions. Most situations are decided based on the remaining features.
- The results show weaker GW preferences and less GW decisions for vessels restricted by limited H/T.
- For short t_{cqs}, the results show higher frequency of GW decisions regardless the other features. For both short time and high risk, the results show absolute GW preference regardless the case.
- There are very few situations where both vessels are under grounding conflicts. But otherwise, vessels more often have higher preference to give way to other vessels restricted by a Grounding Risk (GR).
- The available time before grounding risk is insufficient to determine the preferences; the situations are decided based on the remaining features.
- The relative bearings RB are dominant features; the results show that the vessels often stand on for other ships approaching from the aft and the port side and give way to other vessels. In the stand on regions, there are few stronger preferences to GW as per Rules R8, 9, 10, 17, and 19.
- The vessel Length (L) is not a dominant feature, but it is observed that vessels $L < 20\ m$ long tend to have higher preference to GW to bigger vessels $L \gg 20\ m$.
- If the collision risk decay (ΔCR) is not fast enough, the results show a clear preference to always

GW if the target vessels (j) have lower GW preference.

- The collision avoidance actions (CAA) are identified as urgent for rougher navigation conditions (NC). As $CAAs$ become urgent, the results show a higher frequency of GW regardless the situation.
- As one vessel is impeding the safe passage of another vessel and the risk is low, she tends to give way in the model regardless the situation.

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