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## Preface

Placeholder text

Erlend Hestvik, 20.12.2021



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**Abstract**

test acronym for error control: Automatic Identification System (AIS)



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## Acronyms

**AIS** Automatic Identification System

**COLREGs** Convention on the International Regulations for Preventing Collisions at Sea

**dCPA** distance at Closest Point of Approach

**DOF** Degrees Of Freedom

**IPOPT** Interior Point OPTimizer

**NED** North East Down

**NLP** NonLinear Programming

**OS** Own Ship

**tCPA** time to Closest Point of Approach

**TS** Target Ship

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# 1 Introduction

Placeholder text. this is a placeholder citation to remove an error: Eriksen and Breivik 2017.

- something something this thesis is about trajectory planning
- had this idea I wanted to try out
- This chapter explains my motivation for the thesis. discusses previous work of the same subject. Explains the problem as I see it, and my contributions to a solution. lastly an outline of the rest of the thesis for a quick intro of every section.

## 1.1 Motivation

- Worked on the same subject matter for a "fordypningsprosjekt" (finn godt ord).
- Autonomous vehicle control is an important milestone on the journey to a fully autonomous life.
- It's also just fricking cool on a conceptual level.
- Fishing industries and other marine industries are 'behind the curve' and not given as much attention as land based industries.
- A great learning opportunity for practical implementations of theory learned over the past two years.
- All in all a highly relevant project for the career trajectory I want.
- AI is pretty cool too I guess
- wanted to see if there could be a difference if autonomous vessels had more advanced prediction algorithms.
- just make something up.

## 1.2 Previous Work

- boy I sure wish I had read more research.

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### **1.3 Problem Description**

- Many papers on trajectory planning enjoy simple "cpa" prediction.
- Many algorithms end up creating a very 'active' vessel, which is different from how most humans navigate.
- Convention on the International Regulations for Preventing Collisions at Sea (COLREGs) compliance needs more attention.
- 

### **1.4 Contributions**

- Analyse av fordeler med å ha bedre / avansert prediksjon av TS.
- 

### **1.5 Outline**

- Samma stil som på fordypningsoppgaven.

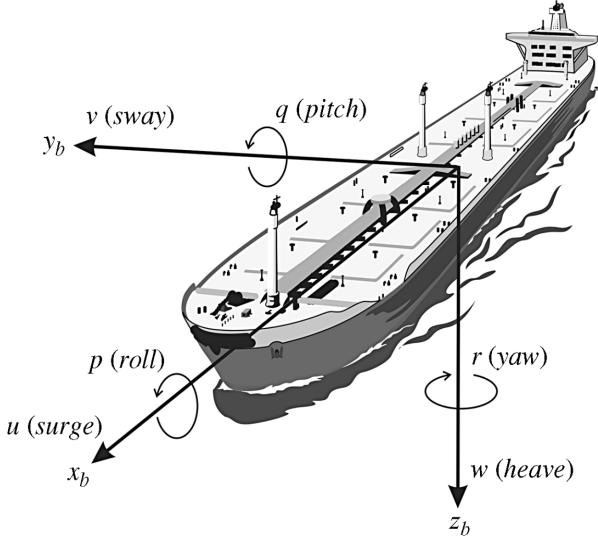


Figure 1: A ships 6 degrees of Freedom, from Fossen 2011

## 2 Background Theory

This chapter will introduce the concepts and theory necessary to understand the design and intent behind the trajectory planning algorithm, as well as the discussion on its functionality. The goal of the chapter is that the reader will have enough intuition of the applied theory that the proposed arguments and solutions make sense. In addition the chapter is structured so that it should be easy to quickly find and read about specifics.

### 2.1 Vessel modelling

A mathematical model is a tool for describing physical systems and expressing how they change over time, respond to external forces, and how stable the system might be. Models are very useful when designing control systems as they translate the physical into equations that computers can understand. When making a model it is often useful to separate the dynamics of the different parts of the system we are interested in, these are the Degrees Of Freedom (DOF) the system has, and is often the directions the system can move, though they can also just be nondescript generalized coordinates. Deciding what DOF to separate out and model the dynamics of is often what separates models from each other, it is pointless to model an aspect of a system that there is no intent to interact with. For example a ship has six DOF, see Figure 1, for modelling a control system for stationkeeping all six are important because stationkeeping involves keeping

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the whole ship as steady as possible. When modelling for path following on the other hand it is not important what the heave, roll, or pitch of our vessel is and so the dynamics of those DOF can safely be ignored. The model used to describe our vessel in this thesis is based on the theory and notation by Fossen 2011, and is a 3DOF nonlinear mass-damper system with thruster dynamics and no external disturbances such as wind or currents. The dynamics of the vessel can be described by the differential equations below:

$$\dot{\boldsymbol{\eta}} = \mathbf{R}(\psi)\boldsymbol{\nu} \quad (2.1)$$

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau} \quad (2.2)$$

Where  $\boldsymbol{\eta}$  is the pose of the vessel, parameterized in the tangential plane North East Down (NED) where the x-axis points towards true north, the y-axis east and the z-axis down towards the center of the planet. The vector  $\boldsymbol{\eta}$  is a column vector with the vessel's North, East and Heading values, which are the three DOF of the system. The  $\boldsymbol{\nu}$  vector is a column vector containing the vessel's velocities parameterized in the BODY frame, namely surge, sway, and yaw rate. In the BODY frame there are no fixed rules for where the axis are pointing, but the common convention for modelling vehicles is that the x-axis points along the longitudinal axis of the vessel, the y-axis points along the lateral axis and the z-axis points along the vertical axis. This is also seen in Figure 1. The anchor point for the BODY frame is arbitrary but always fixed to and moves with the vessel.  $\mathbf{R}$  is a rotational matrix about the heading ( $\psi$ ) of the vessel and it transforms the BODY velocities into NED movement. The Rotation matrix, as well as pose, velocity, and the thruster vector  $\boldsymbol{\tau}$  are:

$$\mathbf{R}(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.3)$$

$$\boldsymbol{\eta} = \begin{bmatrix} x & y & \psi \end{bmatrix}^T \quad (2.4)$$

$$\boldsymbol{\nu} = \begin{bmatrix} u & v & r \end{bmatrix}^T \quad (2.5)$$

$$\boldsymbol{\tau} = \begin{bmatrix} F_X & F_Y & F_N \end{bmatrix}^T \quad (2.6)$$

In 2.2 the  $\mathbf{M}$  matrix is the inertia matrix of the system, which describes how 'heavy'

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the DOF are to nudge, in addition to the vessel's inherent inertia from being massive the vessel must also push water out of the way when it moves, this is what is known as hydrodynamic added mass and is linearly added to the inertia matrix. The coriolis matrix  $\mathbf{C}$  also has to include hydrodynamic added mass, however for the purpose of this thesis it is not important to know the parameters for either of these matrices or for the dampening matrix  $\mathbf{D}$ . That is because a trajectory planning algorithm needs to work regardless of vessel parameters, Andersson et al. 2019 explains more in-depth how the system parameters can be found. Continuing on, the dampening matrix is a linear combination of the linear dampening stemming from water viscosity and non-linear dampening from cross-flow, once again the parameters themselves are not strictly relevant to this thesis, but intuition is important. The result are matrices in the following form:

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \quad (2.7)$$

$$\mathbf{C}(\boldsymbol{\nu}) = \begin{bmatrix} 0 & 0 & c_{13}(\boldsymbol{\nu}) \\ 0 & 0 & c_{23}(\boldsymbol{\nu}) \\ c_{31}(\boldsymbol{\nu}) & c_{32}(\boldsymbol{\nu}) & 0 \end{bmatrix} \quad (2.8)$$

$$\mathbf{D}(\boldsymbol{\nu}) = \begin{bmatrix} d_{11}(\boldsymbol{\nu}) & 0 & 0 \\ 0 & d_{22}(\boldsymbol{\nu}) & d_{23}(\boldsymbol{\nu}) \\ 0 & d_{32}(\boldsymbol{\nu}) & d_{33}(\boldsymbol{\nu}) \end{bmatrix} \quad (2.9)$$

The dampening matrix can be a bit of a computational nightmare and can be simplified to a linear and diagonal matrix without too much of a detrimental impact on our simulations. The justification for this simplification is the underlying assumption that the reference output from the trajectory planner algorithm will be parsed through a final control module that will account for dampening. The risk is that the end result from the trajectory planner turns out to be infeasible, but that's a problem for another thesis.

$$\mathbf{D}(\boldsymbol{\nu}) = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} \quad (2.10)$$

Finally, a word on heading vs course. Throughout this thesis the terms course and

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heading might be used interchangeably, but the words are strictly not synonymous. Heading is equivalent with yaw as both denote a rotation about the vessel's third axis, the difference between the two is that yaw is often a relative term describing a change by some degrees from one arbitrary pose to another. Heading is an absolute term and is often based on compass directions, meaning  $0^\circ$  heading equates to the nose of the vessel pointing towards true north. Neither heading nor yaw is equivalent with course, which is strictly the direction of travel relative to true north. In a simplified world void of disturbances heading and course will align during straight line travel, but external forces such as wind or currents will cause the two angles to deviate. However for the purpose of this thesis disturbances are not accounted for, and so the terms heading and course might be used interchangeably, though it often makes sense to deliberately pick one term over the other.

## 2.2 Trajectory Planning

- Core concept. Path VS Trajectory.
- Many ways of doing it.
- chosen method for this thesis.
- Numerical optimization, OCP, NLP, MPC.

Because the vessel dynamics are described by a model expressed as a set of time-invariant ordinary differential equations, any desired state can be reached by solving for the sequence of inputs that will take the vessel from a given initial condition to said state. In the context of this thesis "state" refers to the pose,  $\eta$ , of the vessel. The simplest application of this would be moving in a straight line from point A to point B. The solution is simply to find the input sequence which turns the vessel to the correct course and then maintaining a forward speed until point B is reached.

We might not have just a desired destination, for example we could have a set of waypoints which we wish to pass through. the set of waypoints form a path, or set of states to pass by to reach the ultimate state, the destination. Due to constraints it might be impossible to follow the path exactly, best we can do is calculate the optimal input sequence, applying the sequence in order results in the final trajectory of the vessel. they can be solved for the input necessary to achieve any desired state.

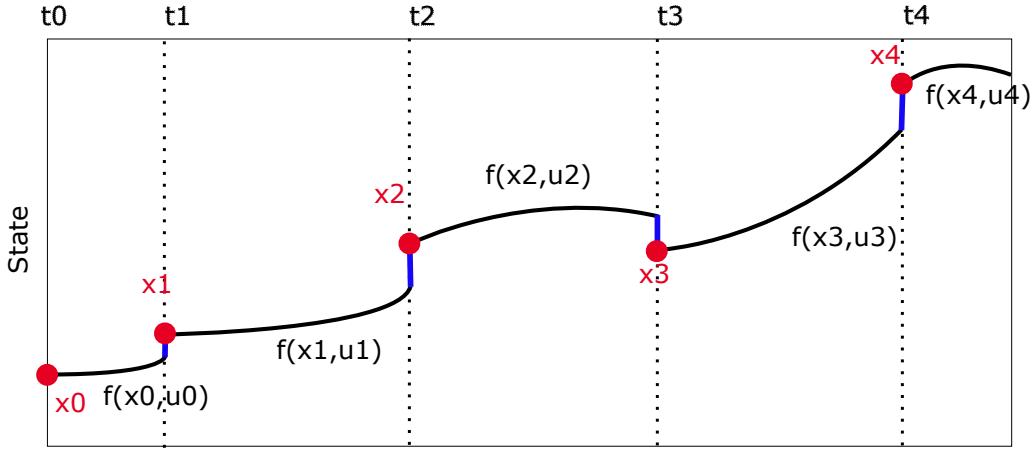


Figure 2: A physically feasible trajectory is formed by "pinching" the shooting gaps so they close. Reproduction of (TODO: CITER GROS), scale arbitrary.

$$\text{Minimize } \mathbf{F}(\boldsymbol{\eta}(t), \mathbf{u}(t)) \quad (2.11a)$$

$$\text{subject to } \dot{\boldsymbol{\eta}}(t) = \mathbf{L}(\boldsymbol{\eta}(t), \mathbf{u}(t)) \quad (2.11b)$$

$$\mathbf{h}(\boldsymbol{\eta}(t), \mathbf{u}(t)) \leq \mathbf{0} \quad (2.11c)$$

$$\boldsymbol{\eta}(t_0) = \boldsymbol{\eta}_0 \quad (2.11d)$$

(TODO: REWRITE THIS, SOME PARTS MAKE LITTLE SENSE)

where  $\mathbf{F}$  is the objective function,  $\boldsymbol{\eta}(t)$  is the position and attitude trajectory of the vehicle,  $\mathbf{u}(t)$  is the control input trajectory and  $\mathbf{L}$  is the kinematic model of the vehicle. Though the OCP can be solved the way it is set up in (2.11) it is more practical to discretize it into a ( NonLinear Programming (NLP)). With a method called direct multiple shooting, both state and control input are defined as decision variables, the NLP with  $N$  control intervals is:

$$\min_{\boldsymbol{\omega}} \mathbf{F}(\boldsymbol{\omega}) \quad (2.12a)$$

$$\text{subject to } \boldsymbol{\omega}_{lb} \leq \boldsymbol{\omega} \leq \boldsymbol{\omega}_{ub} \quad (2.12b)$$

$$\mathbf{g}_{lb} \leq \mathbf{g} \leq \mathbf{g}_{ub} \quad (2.12c)$$

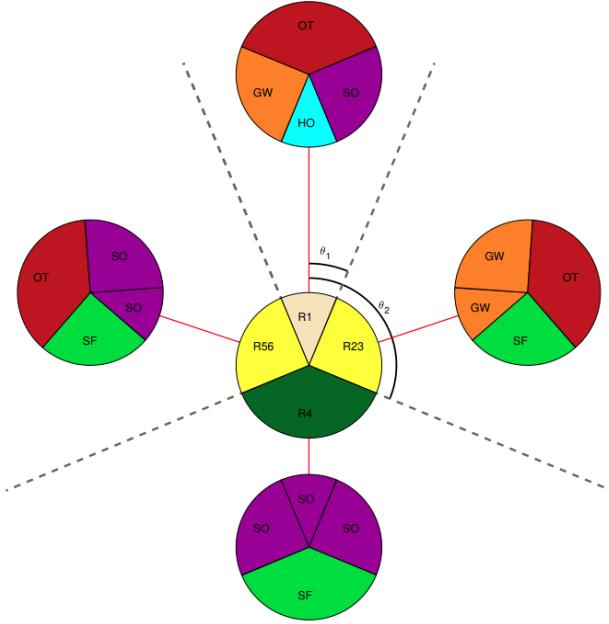


Figure 3: (TODO:ENDRE)Assigning COLREGS flag: with OS in the center we can place the TS in one of four regions. Similarly the relative bearing from TS to OS can be assigned regions with region 1 pointed directly at the OS and the rest following in a clockwise rotation. Courtesy of Emil Thyri.

### 2.3 Collision Avoidance

- COLREGs

Expected behaviour, situation classification, etc etc.

- distance at Closest Point of Approach (dCPA) / time to Closest Point of Approach (tCPA)
- Other risk assessment? Situation complexity? Det er mer som inngår i ”collisions avoidance” som jeg kanskje ikke dekker så veldig bra med min algoritme.

### 2.4 ’The complete system’

- Vet ikke helt om dette kapittelet er nødvendig, men jeg lurer på om det er en god ide å skrive litt om nøyaktig hvor i ett fullt funksjonelt system jeg forventer at min algoritme passer inn. Hva de andre delene jeg ikke kommer til å skrive om har ansvar for, og hva som forventes av systemene rundt mitt eget.
- Hvis systemet mitt var en sort boks, hvilke inputs og outputs ville det ha?

### 2.5 Simulator setup

- liten forklaring av hvordan simulatoren funker selv om jeg ikke har laget den.

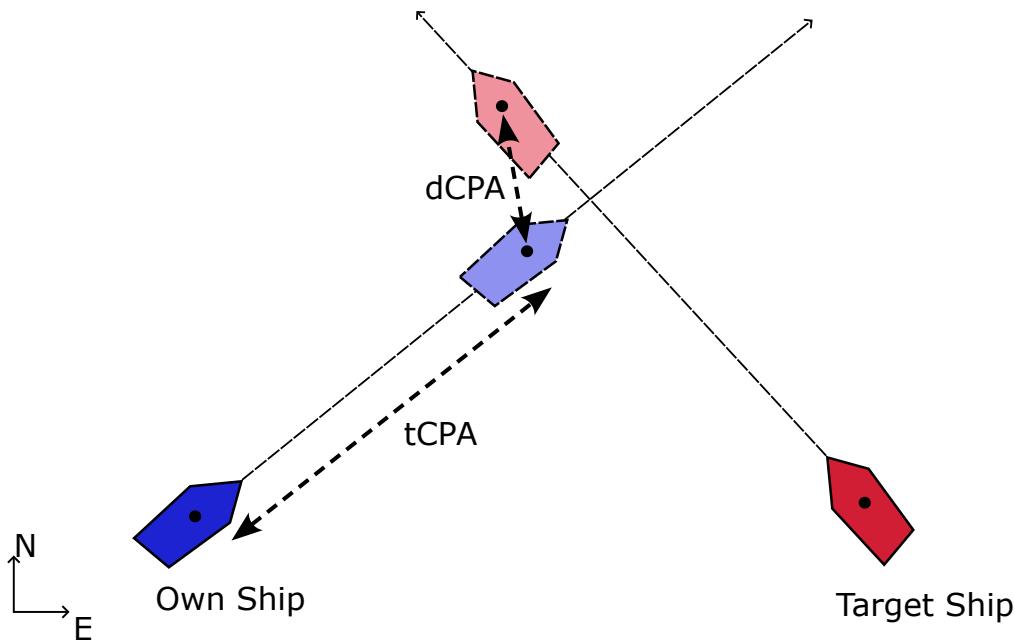


Figure 4: Visualizing dCPA and tCPA.

- f.eks agent structs kan bli viktig.
- Forklar at det er en liten forskjell på hvordan own ship agenten og andre agenter oppdateres, fører til en maksimal 'feil' på 1 meter.

## 2.6 Target Ship prediction

— Genereal thoughts —

- Gjenfortelling fra fordypningsprosjekt, da kalt traffic pattern
- Fant en annen artikkel fra Kina som skrev om nogenlunde det samme, AIS data - prediksjon
- Skiller seg fra fordypningsprosjekt fordi det er egentlig ikke traffic pattern som er den viktige antagelsen, Det er heller viktig at vi antar det finnes en måte å gjette/vite hvor andre båter vil være fremover i tid.
- Andre metoder for target ship prediction kan være f.eks utvidelse av AIS som inkluderer autonav data for de neste 5 minuttene eller noe lignende.
- Disambiguate Simple and Full prediction.

— /General Thoughts —

- Naval navigation is an 'active' task, always looking out for obstacles and making sure the way forward is clear.

- 
- There are no lanes, instead proper conduct is dictated by COLREGs, the rules are laid out so that different situations have different rules
  - Knowing which situation you are in is half the battle, therefore the ability to predict and estimate how encounters will happen is a powerful tool. Human navigators do this by experience.
  - Autonomous vessels could also predict by experience, that would be the machine learning approach. AIS can already provide training data.
  - But it could be easier than that, what if AIS data packets were expanded to send out autonav data for where a vessel intends to traverse. The assumption here is that vessels using autonav will correct their course when spotting a conflict. Vessels not using autonav would still be able to observe the intended path of other vessels and adjust their plans 'manually'.
  - Relying on fully predicting the transit of a target vessel is fragile, relying on a few known waypoints would be much more robust.
  - In the end the best solution would be if all vessels could fully communicate their intended path either peer to peer or through a centralized system. Solving conflicting paths would be done on a higher level than individual vessel.

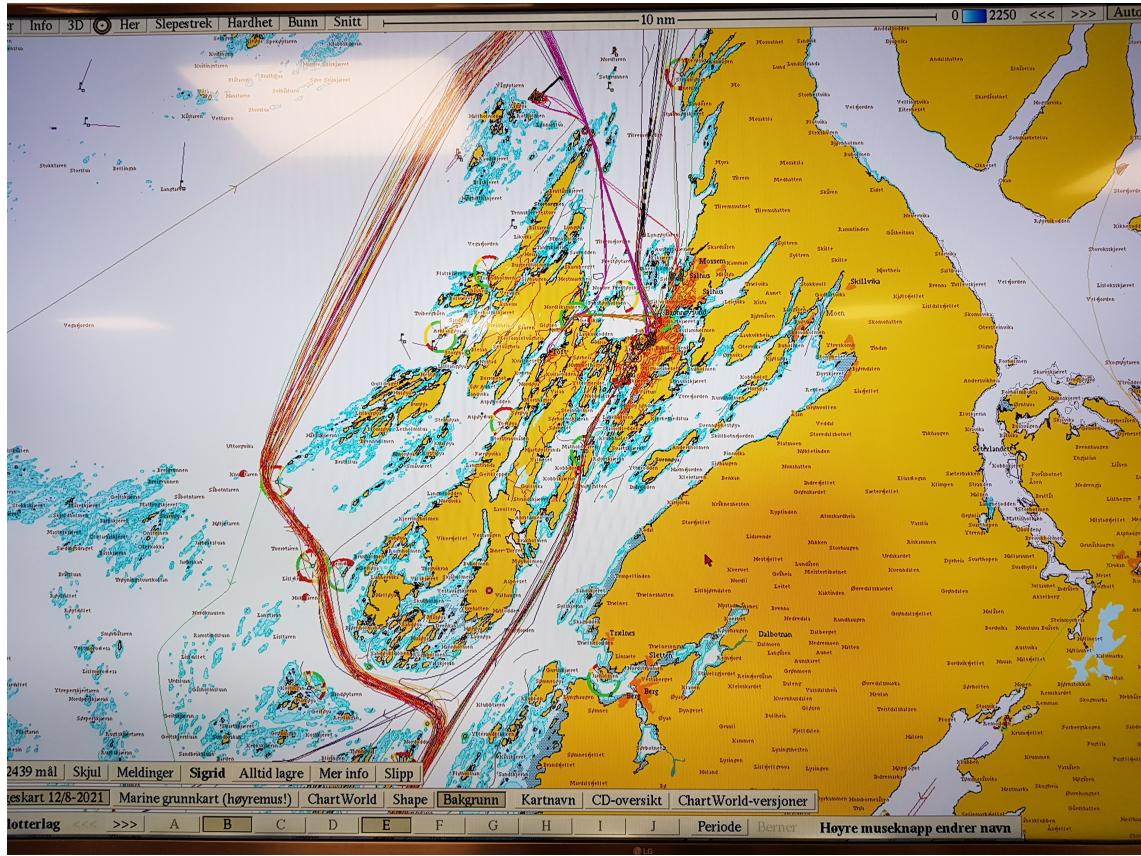


Figure 5: TODO: Skriv. AIS data can show common transit routes. Image courtesy of Olex AS

### 3 Trajectory planner

(TODO: skriv bedre og mer korrekt.) This chapter presents the development of the trajectory planning and collision avoidance algorithm, explaining the design decisions made and analyzing some of the problems that arose during development. First the general dataflow of the algorithm is explained so that an intuition is formed as for how the individual parts are connected. Second a piece by piece construction of the algorithm up until the construction of the NLP. Lastly the NLP is constructed and solved using framework provided by CasADi. The core design of the algorithm is that path following is done through numerical optimization of a cost function, whilst collision avoidance and safety is implemented as hard constraints in the NLP, together an optimal trajectory is formed.

#### 3.1 Dataflow

The dataflow is depicted in Figure 6, to avoid clutter the diagram does not include every subfunction and minor detail, it's a representative diagram, not a blueprint. On the left

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we begin with a higher level system, the algorithm relies on getting information about it's own vessel, as well as information about other ships (called Tracks in the diagram), static obstacles and miscellaneous other settings. These structs need to contain data in some specific fashions which will be discussed more deeply later in this chapter.(TODO: This part). The Tracks structure as it arrives from the higher lever system is assumed to have full prediction, the algorithm can simplify the prediction for testing purposes but there is no internal logic to switch between prediction detail level. Regardless of if the tracks were simplified or not they are parsed through a COLREGs assessment algorithm to determine which Target Ship (TS)s need to be considered active obstacles, as well as finding how long until the tCPA, and which COLREGs situation the encounters would be. Independently of the creation of dynamic obstacles we use the information about our own vessel's position and end goal to determine how long the time horizon for this instance of the algorithm should be. When time horizon and discretization step size is known the CasADi setup can be ran to generate the function 'F' which will be used during the construction of the NLP later. With a dynamic horizon in place a LOS algorithm can be used to generate the final piece of the dynamic obstacles struct; the trajectory of the TSs themselves, which are used to place constraints later in the NLP construction. One crucial detail to take note of is that COLREGs assessment of a TS is conducted using only the waypoints provided by the Tracks struct, while the placement of dynamic constraints are based on the trajectory generated by a LOS guidance law. The feasibility check is not ran in the first instance of the algorithm after starting up the system. The result of the feasibility check is used when generating our Own Ship (OS)'s reference path to determine if the desired velocity should be reduced. the LOS guidance returns as mentioned the positions for the dynamic obstacles constraints as well as the reference positions and velocities in NED.

The NLP can now be initialized. Initial position and velocity forms the first constraint of the NLP (TODO: SKRIV MER)

### 3.2 Setup

- All the stuff before main loop.
- subsubsection for each 'block' as outlined by the dataflow.
- when the trajectory planner is called we need to run through some calculations before constructing the NLP problem

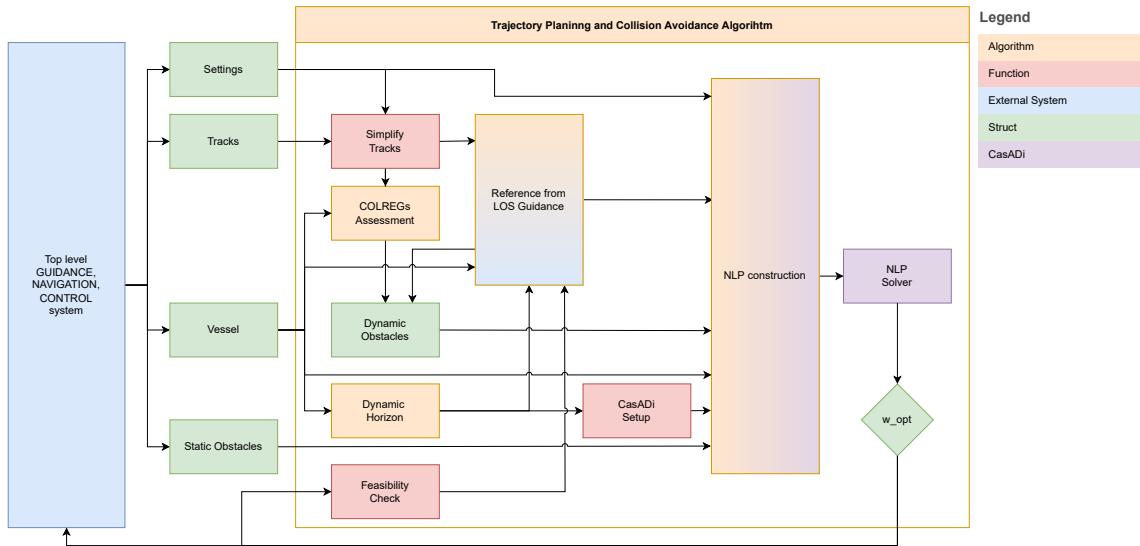


Figure 6: The simple version of the dataflow...

- These calculations are a mix of situation analysis, simulation settings, and CasADI initialization.
- Some of these calculations could be redundant in a complete control and navigation system, where other modules of the system would calculate the same thing.
- It's also important to remember that the value of many parameters are just guesswork, many of the subfunctions would benefit from a more sophisticated design that are tuned based on the situation the vessel finds itself in.

### 3.2.1 Simplify Prediction

This part of the setup is only required in simulations, the aim is to emulate the 'standard' way target ship (TS) prediction is conducted, which is to say constant course and velocity [TODO: Citation needed]. The TS trajectory is changed so that the first waypoint is the current position of the ship, and the next waypoint is one nautical mile in the direction of the ships heading. Ideally course over ground would be used instead of heading, however in the simulator crab angle and sideslip are not accounted for, therefor heading and course are the same angle. Excess waypoints stored in the TS struct are also truncated and the current waypoint index is forcefully set to 1 to prevent index out of range type errors.

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### 3.2.2 COLREGs assessment

The COLREGs assessment function solves two problems; figuring out if when a TS vessel will be in close enough proximity that evasive maneuvers might be considered, and deciding which of the COLREGs rules will apply for the encounter. The design idea is to first find what the distance at closest point (dCPA) of approach with the TS is, and then time until closest point of approach (tCPA) occurs. If both dCPA and tCPA values are under a set threshold we consider the encounter an active event and run the COLREGs situation assessment shown by [TODO: cite paper], COLREGs assessment is also explained in [TODO: fordypningsoppgaven].

Finding the dCPA and tCPA between two vessels with constant velocity and course is easily done with a formula as shown by (eller in?) Kufoalor et al. 2018.

$$t_{AB}^{CPA} = \begin{cases} \frac{\mathbf{P}_{BA} \cdot \mathbf{V}_{A|B}}{\|\mathbf{V}_{A|B}\|^2} & \text{if } \|\mathbf{V}_{A|B}\| > 0 \\ 0 & \text{otherwise} \end{cases} \quad (3.1a)$$

$$d_{AB}^{CPA} = \|(\mathbf{P}_A + t_{AB}^{CPA} \mathbf{V}_A) - (\mathbf{P}_B + t_{AB}^{CPA} \mathbf{V}_B)\| \quad (3.1b)$$

Where  $\mathbf{V}_{A|B} = \mathbf{V}_A - \mathbf{V}_B$  with  $\mathbf{V}_A$ ,  $\mathbf{V}_B$ ,  $\mathbf{P}_A$  and  $\mathbf{P}_B$  being the respective velocities and positions of vessel A and vessel B parameterized in NED. However if we are to utilize the advanced prediction we have on other agents a bit more logic must be applied to achieve full coverage of our intended path. Presume that our path contains a set of waypoints we intend to pass by, and similarly we know of a set of waypoints another agent intends to pass by. To find the true dCPA and tCPA between our own ship and the target vessel we use equations 3.1 as the situation is when each agent passes by one of their waypoints (TODO: TRENGER FIGUR FOR Å VISE BEDRE HVA JEG MENER). Speed is tougher to account for, unless we know better speed should be assumed to remain constant (TODO: dette hører hjemme i background).

TODO: Ikke ferdig

to get a list of all dCPA and tCPAs between two agents, as well as the corresponding positions of both agents as they are when the euqations 3.1 are used. getCPAlist

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**Algorithm 1** getCPAlist. Denne ble jævelig stygg, beholder den for synlighet

---

**Input:**  $Agent1.Agent2$  ▷ Agent is a struct that includes path waypoints

1:  $dCPAlist \leftarrow []$   
2:  $tCPAlist \leftarrow []$   
3:  $pos\_OS\_list \leftarrow []$   
4:  $pos\_TS\_list \leftarrow []$   
5:  $timer \leftarrow 0$  ▷ Initialize timer used to calculate position of Agent2  
6: **for**  $i \leftarrow Agent1.current\_wp : agent\_wplist.length - 1$  **do**  
7:    $[pos_{OS}, vel_{OS}] \leftarrow VesselReadout(Agent1, i)$  ▷ VesselReadout explained in algorithm...  
8:    $DisttonextWP \leftarrow$  Distance to Agent1's next waypoint  
9:    $TimetonextWP \leftarrow DisttonextWP \div$  Agent1's speed over ground  
10:    $[pos_{TS}, vel_{TS}] \leftarrow whereisTS(Agent2, Timer)$  ▷ whereisTS explained in algorithm...  
11:    $[dCPA, tCPA] \leftarrow$  Equation for dCPA & tCPA as shown by...  
12:    $tCPA \leftarrow tCPA + timer$  ▷ Add travel time to reach current wp  
13:    $timer \leftarrow timer + TimetonextWP$   
14:    $pos\_OS\_list \leftarrow [pos\_OS\_list, pos_{OS}]$  ▷ Append all values to respective list.  
15:    $pos\_TS\_list \leftarrow [pos\_TS\_list, pos_{TS}]$   
16:    $dCPAlist \leftarrow [dCPAlist, dCPA]$   
17:    $tCPAlist \leftarrow [tCPAlist, tCPA]$   
18:    $i \leftarrow i + 1$   
19: **end for**  
20: **return**  $pos\_OS\_list, pos\_TS\_list, dCPAlist, tCPAlist$

---

### 3.2.3 Dynamic Horizon

- Dynamic horizon is a balancing act between distance to goal, encompassing all active dynamic obstacles, and not looking too far ahead into the future.
- changing the dynamic horizon is really just changing how many control intervals we want the NLP to have.
- As the distance to goal approaches zero we want the number of control intervals to shrink accordingly, otherwise we end up with too many control intervals stationary at the goal, which can cause problems like the cost function becoming unbalanced.
- If there are active dynamic obstacles we need the dynamic horizon to encompass them.
- we don't want the dynamic horizon to be too short during transit (why not?).

### 3.2.4 CasADi setup

- sym  $x = [N, E, \psi, u, v, r]$
- sym tau as a free variable
- sym xref as reference
- model parameters

- 
- M, C, D matrix
  - xdot [ $\dot{\nu}$ ,  $\dot{\eta}$ ]
  - Error in the correct reference frame.
  - Why is the cost function the way it is.
  - runge-kutta method.
  - the final function F that CasADi needs.
  - Noe av disse greiene blir dekt av Background, forh  pentligvis.

### 3.2.5 Feasibility check

- The feasibility check came from the wish to read out the status report CasADi prints to the command window.
- It's very important to know if the previous iteration of the trajectory planner function yielded a feasible result or not.
- if the result is not feasible the path forward might be completely blocked, in which case reducing our vessel speed is the best option.
- Very simple check, just checks if every point in the previously calculated optimal trajectory is within 5 meters of each other. This is very lenient and should of course change depending on vessel speed.

### 3.2.6 Reference from LOS

- I didn't write this )
- The important part is that the time discretization is consistent with the trajectory planner's time step
- You don't need to use LOS for reference.
- Position reference and speed reference need to be consistent with each other.

## 3.3 NLP construction and solver

- inputs vessel, ref\_trajectory, static\_obs, dynamic\_obs, F, settings, h, N, previous\_w\_opt.

- 
- sub funksjoner
    - Dynamic Obs.
    - Static Obs.
    - integration step.
  - output w\_opt

### 3.3.1 NLP initialization

- Initial conditions and end of interval coditions, we need the end of one control interval and the beginning of the next to match.
- Tror dette delkapittelet er litt unødvendig

### 3.3.2 Integration step

- Getting the correct reference
- make sure all the indexes are correct!
- put the references in w0, speeds up runtime significantly.

### 3.3.3 Dynamic Obstacles constraints

- When to place constraints
- Where to place them
- How to place them

### 3.3.4 Static Obstacles constraints

- Explain static\_obstacles\_check and the theory for convex-free set
- explain why circles, such as the ones used for dynamic obstacles are insufficient.
- this is sort of similar to finding a cross track error, if that helps to explain what is going on.

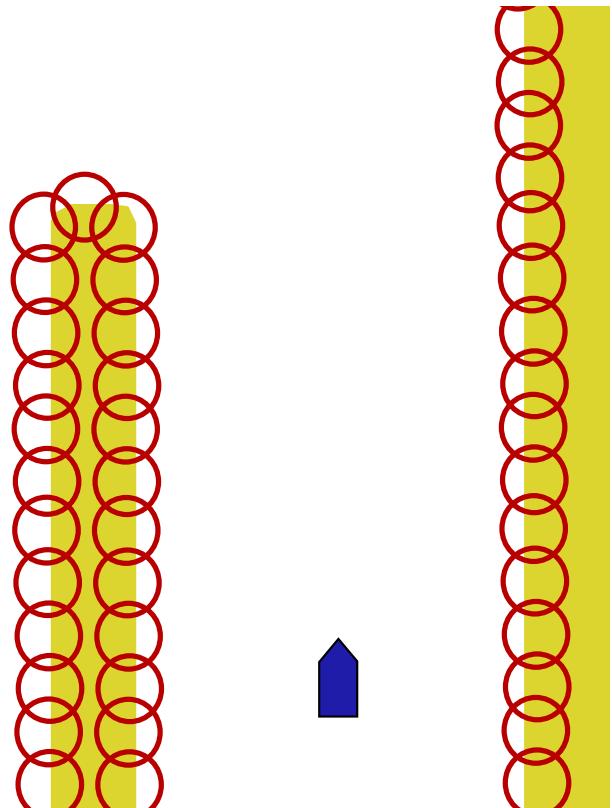
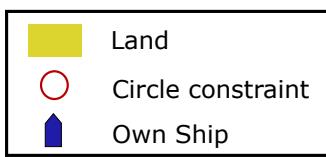


Figure 7: TODO: SKRIV OG REFERER. Naiv approach 1

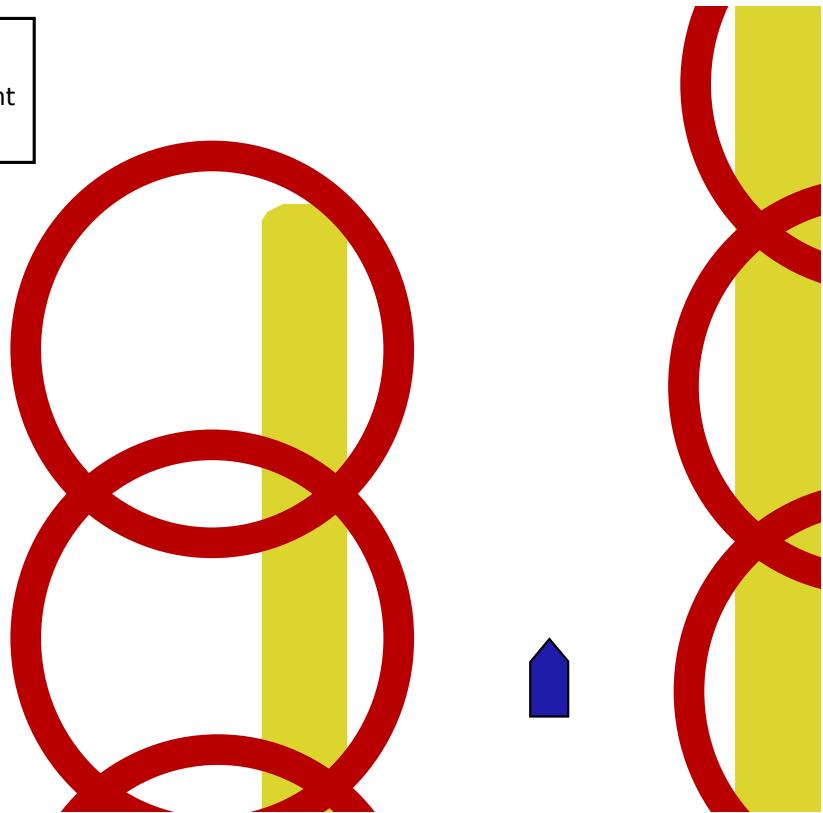
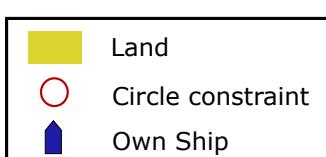


Figure 8: TODO: SKRIV OG REFERER. Naiv approach 2

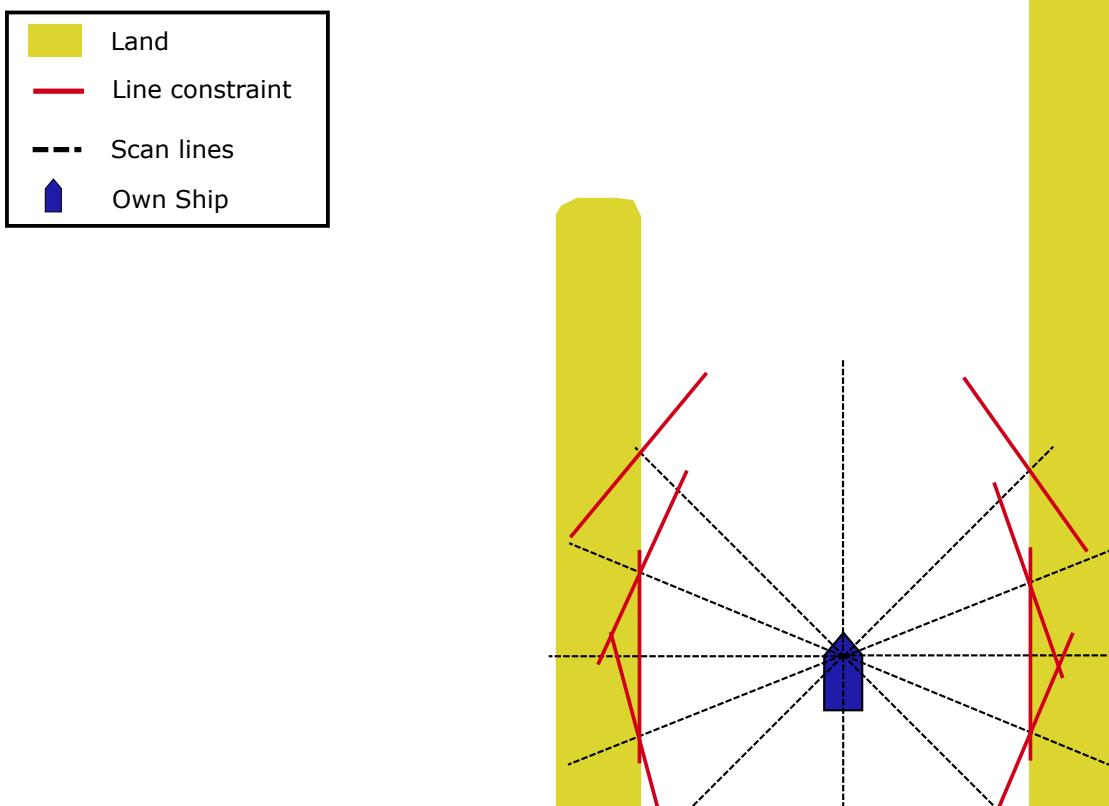


Figure 9: TODO: SKRIV OG REFERER. Convex free set

### 3.3.5 Solver

- Options, there are many options.
- things to try / were tried for optimizing runtime.
- CasADi really does all the hard work.

## 3.4 Alternative ideas and lessons

Burde kanskje heller gå under discussion, og igjen i future work.

- Change w0 based on previous solution runtime.
- Gamle versjoner av Static\_obs.
- eksperimenter med feasibility check.
- Masse styr med COLREGs assessment, tcpa og dcpa.
- ipopt innstillinger.

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## 4 Simulation Results

To test the capabilities of the trajectory planning algorithm it is useful to conduct simulations of various scenarios. With a simulator it is possible to cover a wide assortment of scenarios in a timely fashion, this helps explore the full range of the algorithm's behaviour without having to conduct time consuming full scale tests. NTNU also has a full-scale functional prototype of an autonomous ferry that could be used to conduct real life tests. However during the period of working on this thesis the ferry was out of commission due to a thruster failure. The MATLAB simulator employed for this thesis was developed by Emil Thyri and is used with permission. In this chapter the results are presented with figures to show the development of the scenario over time, in addition to these figures there exists a youtube video compiling all the results in video format, the video can be found as an attachment to the thesis, or by following this link: (TODO: set in link).

All the simulations are conducted under the assumption that the OS has perfect vision for spotting and tracking dynamic obstacles. disturbances are also largely ignored, the simulation features no current or wind induced sideslip, crab angle is also not considered.

### 4.1 scenario overview

The scenarios used for this thesis are constructed to test both trajectory planning and collision avoidance capabilities through a combination of both trivial and complex situations. The scenarios are also designed so that behaviour differences between full and simple TS prediction can be observed. Any time we encounter a TS that maintains a steady course and velocity there will not be any observable difference, therefore most of the scenarios are constructed so that encounters occur when ships are turning. The first set of scenarios are simple situations to establish baseline behaviour in the various COLREGs situations. In these scenarios there are only two agents and there are mostly no meaningful differences observed between simple and full prediction of TSs. The second set of scenarios are more complex by featuring more agents and longer paths to follow. These scenarios often feature multiple COLREGs situations that can even overlap, additionally TSs will not be considerate of the OS and will exhibit reckless behaviour in order to test a sort of worst case scenario. The complex scenarios also incorporate static obstacles to show how the algorithm handles both types of obstacles at the same time.

---

## **Simple COLREGs Situations**

These scenarios feature two agents, the OS and the TS, each entering a fully open space while maintaining a steady course and fixed speed. The agents then cross in manners as described by the COLREGs rules discussed in prior chapters.

## **Turning COLREGs Situations**

Similar to the simple COLREGs situations these scenarios all feature two agents who enter a fully open space. The difference is as the name implies that these scenarios feature a turn by the TS. Shortly after both agents are in motion the TS will alter it's course, changing the COLREGs situation from one apparent situation to another.

## **Canals**

This scenario features a set of canals that form a T-junction as well as a choke point on one of the junction points that restricts the traversable space. There are three agents present and they all meet roughly at the choke point, the scenario is set up so that the dynamic constraints of the TSs completely block the path of the OS if full prediction is used.

## **Fjord**

The fjord is construct as a miniature version of the Trondheimsfjord, this scenario is designed as a stress test of COLREGs situations. With multiple TSs crossing, turning and overtaking the OS simultaneously this scenario will show how the trajectory planning algorithm differs with prediction level.

## **Helloya**

The situation in this scenario is specifically modelled after a spot near Brønnøysund and is not an entirely uncommon situation when in transit along the coast of Norway. Traffic that wishes to avoid the narrow pass leading in to or out of Brønnøysund's will elect to take a wider path on the outside of the local archipelago. The result is a path with a very prominent turn that is invisible at a glance, but very obvious to any experienced navigator. The simulation is conducted with the OS arriving from both the north and

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south direction with both full and simple prediction enabled.

### **Skjærgård With Traffic**

Skjærgård is a Norwegian term for a section of ocean where there are many small islands and skerries, while the term translates to archipelago a skjærgård is generally small in scale. This scenario puts a lot of stress on the trajectory planner which has to deal with both moving dynamic obstacles as well as the static obstacles that are sometimes blocking the reference path.

### **Skjærgård Without Traffic**

A simpler version of the previous scenario, this time with no traffic but with more skerries near the reference path.

### **Miscellaneous**

These scenarios are not meant to simulate any specific situation, rather these are meant to showcase quirks, features, and bugs encountered while developing and testing the algorithm. While some of the problems shown here were taken care of and are no longer present in the current iteration of the algorithm they are nonetheless important to showcase and discuss.

## **4.2 Results**

- 'Dårlig' resultat er fortsatt resultat
- Computational efficiency is also a topic
- First a brief discussion about each scenario result individually, taking a look at both the simple and full prediction level results.
- Then a closer examination of specific behaviours, problems, and observations that are not necessary situation specific.
- Then a qualitative disucssion on the results as a whole, are theese the exected result? why or why not. etc.

- 
- The scale is not uniform across all simulations, sometimes the boats are scaled up to make the figures easier to read.
  - The results are accompanied by MALTAB figures, as well as a youtube video that compiles all the results into a video which sampled the simulations every second.

#### 4.2.1 Simple Head On

- Very straight forward result, behaviour is exactly as one would expect given the placement of the constraint.
- No difference between full and simple prediction because target ship holds steady course and velocity.
- This is the behaviour we can expect every time a target ship is met directly head on and there are no other disturbances.

---

#### 4.2.2 Simple Give Way

- It would be highly unusual to start using the trajectory planner when already this close to a situation
- This is reflected in the strange behaviour where our path is completely blocked when dynamic obstacle constraints are enabled.
- otherwise the behaviour ends up being exactly as expected considering the placements of the constraints.

---

#### 4.2.3 Simple Stand On

- This scenario assumes that the TS plays nice and attempts to follow the COLREGs rules.
- When using simple prediction the optimal path is pushed towards port side behind the TS, and as the TS begins to turn the OS is dragged along by the movement of the constraints.
- This is actually quite unrealistic, there is no reason to assume the TS would continue to yield after observing the OS change course like this. It's also a bad result for simple prediction because the behaviour can cause confusion for other navigator.
- The author realized late that the way prediction is done of the TS is not entirely consistent with the way prediction is handled internally in the algorithm. In the algorithm waypoints are used to predict TSs trajectories, which is not necessary the exact same trajectory as the TS ends up following.
- In order to get the TS to comply with expected COLREGs behaviour a waypoint was placed some meters south of the OS position at the would be tCPA. This waypoint would obviously not exist in a normal transit situation and so this scenario is actually a bit of a cheat in regards to how prediction is argued for in prior chapters.
- The result is still interesting, for this scenario only we can exchange the advanced prediction result with a fully accurate prediction, and the result for simple prediction would hold for the current algorithm in all cases.

#### 4.2.4 Turn Head On

- THIS SCENARIO SHOULD BE MIRRORED HORIZONTALLY TO INCREASE THE CHANCES OF AN OBSERVABLE DIFFERENCE BETWEEN PREDICTIONS.
- otherwise business as usual, turns aren't predicted quite right.

---

#### 4.2.5 Turn Give Way

- Here we finally observe a big difference between full and simple prediction.
- due to how the situation plays out the OS is dragged along by the constraints of the TS.
- being dragged by constraints is not unique to this situation, and is one of the problems that can occur with this method of collision avoidance.

---

#### **4.2.6 Turn Stand On**

- Absolutely nothing happens here. move on.

#### **4.2.7 Canals**

- blocked path
- feasibility check
- forskjell mellom prediction metoder
- When the path is blocked it takes a very very very long time to solve the NLP, users beware.
- Get to see static obstacles in effect.

---

#### **4.2.8 Trondheimsfjord**

- COLREGs stress test
- Full prediction behaves much 'calmer', which is better in the author's opinion.
- Full prediction is also much more computationally efficient, for reasons that will be discussed later.

#### **4.2.9 Helløya**

- Both simple and full prediction actually navigate the 'invisible' turn quite well
- even in the reverse direction it's not really a problem, though simple prediction cuts the turn, which could be considered bad.
- when being overtaken the full prediction exhibits much better behaviour.
- This is one of the scenarios where the scale is very exaggerated.

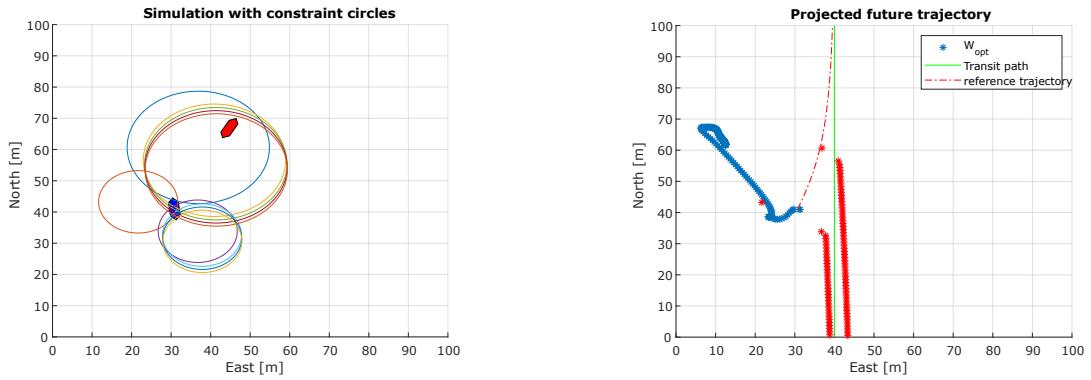
(TODO: Skriv og figurer)

#### **4.2.10 Skjærgård with traffic**

- Both prediction levels successfully navigate this scenario
- with simple prediction the algorithm has an absolute nightmare trying to get past the third TS, this sim took hours to run.
- an interesting problem with 'islands' is that the optimal trajectory could get stuck inside one.
- Tried experimenting with placing bigger and bigger islands on top of the reference path, did not go too well.

#### **4.2.11 skjærgård without traffic**

- Good path tracking.
- this is where I discovered a heading reference problem.
- able to dodge islands that are in the way.
- computationally not too difficult either, algorithm exhibit reasonable execution times.



(a) TODO: SKriv. when prediction is poorly executed the risk of getting entangled in constraints increases.

(b) TODO: Skriv. When entangled in constraints the solver is unable to find a feasible solution.

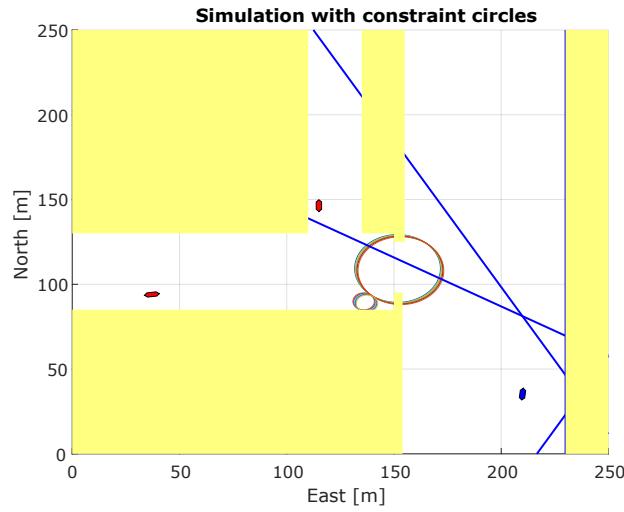
Figure 10: TODO: Skriv om bad prediction

#### 4.2.12 Miscellaneous

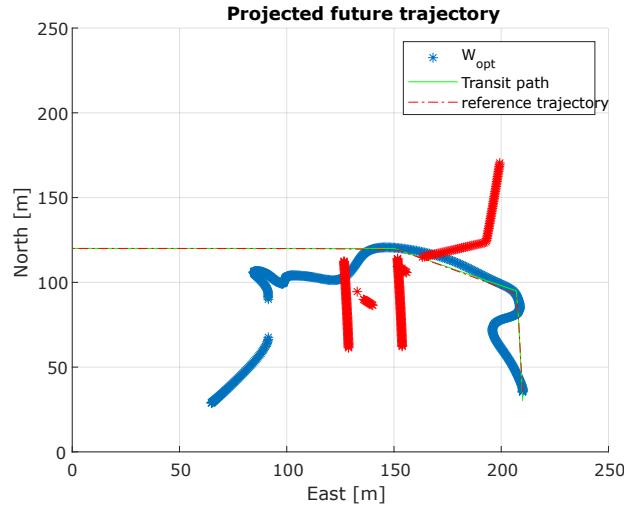
- Bad Prediction, what happens when the target ship does not follow the predicted path
- Blocked Path, a closer look at what happens when the path we intend to take is fully blocked.
- Wrong turn, observe that the optimal trajectory is often to turn the wrong direction slightly when changing course.
- WrapTo2Pi problems, how to explain to an algorithm how course works.
- 'Dragged' along by Target Ships. When does it happen.
- When overtaking or being overtaken the constraints can really mess with the Interior Point OPTimizer (IPOPT) solver.
- the optimal solution could get trapped inside static obstacles with the way the constraints are active 'both ways'. please provide picture.

### 4.3 Discussion

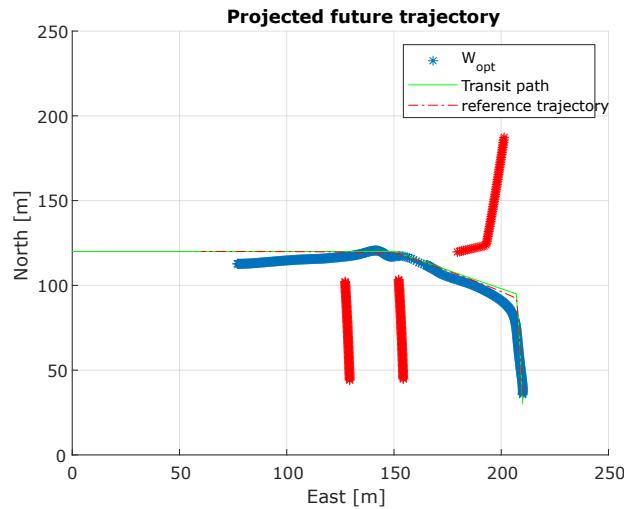
- Hvorfor er viktigere en hva
- ikke overanalyser resultat, ikke dra ville konklusjoner.
- Hvis et resultat er mye værre enn forventet kan det godt være det er bugs.
- i tillegg til det resultatene viser kan jeg også skrive om det jeg kan se med debugging.
- WrapTo2Pi problems (shortest signed angle stuff)



(a) TODO: SKriv. A Target ship is predicted to completely block our path



(b) TODO: Skriv. When the path is blocked, the solution will never be feasible.



(c) TODO: Skriv. The solution for the next iteration, when speed is reduced by a factor of 3

Figure 11: TODO: A blocked path event, and how the solution develops

- 
- Turning the wrong way to get a more even turn, Optimization leads to this problem.
  - if( $\tilde{\text{isempty}}(\text{previous\_w\_opt})$ ) && feasibility ==  
previous\_feasibility && feasibility skaper problemer
  - We really don't want to put a cost on heading reference more than neccessary, heading will often not be correct due to disturbances. heading is also just plain wrong any time we deviate from the reference trajecotry.
  - With 'full' prediction solving the NLP is often computationally more efficient due to a better previous\_w\_opt.

#### 4.4 Improvements over previous version

- Definite improvements in terms of computational efficiency. This greatly increases the likelihood of finding an optimal solution
- Because of the better efficiency the algorithm is also able to handle more control intervals, This means it is better at handling both greater time horizon and shorter control interval steps.
- The new method for handling static obstacles is much less prone to misplaced or inefficnet constraints. (her ta gjerne med figuren som viser problemer med sirkel constraints for statiske hindringer).
- The new way of handling dynamic constraints should in theory make the algorithm better suited for more complex situations with more agents, however the placement of dynamic constraints remains largely unchanged. Dynamic Constraint placement is bigger 'bottleneck' than agent culling for how complex situations are handled.
- More robust when an encounter leads to an infeasible solution.
- Improved COLREGs assessment
- But does it behave *noticeably* better? Yes.

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## 5 Conclusion and Future Work

- conclusion:

oppsummering, forklaring, avsluttende ord.

- future work:

(variabel) Cost funksjon

'grenseverdier', altså verdier som constraint størrelse, distanse fra statiske hindringer, verdier som egentlig burde tunes basert på situasjonen slik den er i øyeblikket.

plassering av dynamiske constraints.

bedre måte å gjøre COLREGs assessment (ikke bare skjekk waypoints slik jeg gjør).

generelt andre metoder jeg ville foreslått å prøve isteden for spaghettijen jeg har kokt sammen.

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