1. Calculating the Advanced Heuristic Value

The advanced heuristic will combine the Manhattan distance with linear conflict. Linear conflict is when two tiles are in their correct row or column but are reversed relative to their goal positions. Heuristics of this kind are more informative and thus more effective than the essential Manhattan distance.

Calculation Steps

1. **Manhattan Distance:** Calculate the sum of the Manhattan distances of the goal piece (2x2) from its current position to its target position. The Manhattan distance is computed as:

$$\text{Manhattan Distance} = |x_{\text{current}} - x_{\text{goal}}| + |y_{\text{current}} - y_{\text{goal}}| \tag{1}$$

   where $(x_{\text{goal}}, y_{\text{goal}})$ is the target position $(1, 3)$.

2. **Advanced:** Identify pairs of tiles in the same row or column that are in their correct row or column but are out of order. Each such pair adds two moves to the heuristic value because the tiles need to be moved out of the way and then back into their goal positions.

3. **Combining Heuristic Values:** The total heuristic value h is calculated as:

$$h = \text{Manhattan Distance} + (\text{number of linear conflicts}) \tag{2}$$

2. Why is the Advanced Heuristic Admissible?

A heuristic function is admissible if the actual cost to reach the goal from the current state is non-negative and never overestimated. The advanced heuristic function is admissible because:

- **Manhattan Distance:** The Manhattan distance itself is admissible since it represents the minimum number of moves required to reach the goal position without any obstacles; and based on the calculation above we can guarantee that the result is always non-negative.

- **Advanced:** The linear conflict considers the number of blocks that are blocking the direct path of the 2x2 piece to the goal state. This function will also guarantee the result is non-negative, and it will never over-estimate since we will always need to move out the blocking piece first, and for n piece, we need at least n moves to move them out of the way.

Now we have shown that components of the heuristic (Manhattan distance and Advanced) are admissible, we want to show the combination is admissible as well by showing the combination never over-estimate. We assume n pieces are blocking the way; we will need at least n moves to move all of them out of the way, and now the path is clear, the move it needs for the 2x2 to get to the goal state would be equal to the Manhattan Distance. Therefore, we showed that the Heuristic is admissible.

3. Why Does the Advanced Heuristic Dominate the Manhattan Distance Heuristic?

We say that a heuristic function h_1 dominates another heuristic function h_2 exactly when, for all state s, the heuristic value $h_1(s)$ is at least as large as the heuristic value $h_2(s)$ for the state, and there exists at least one state s for which $h_1(s)$ is strictly greater than $h_2(s)$. More formally, in this case:

The complete advanced heuristic function is the sum of the Manhattan Distance and the Advanced function. According to the game rule, there must exist cases that contain pieces blocking the direct path. Therefore the Advanced Heuristic function dominated the Manhattan Distance Heuristic.