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Two-Nucleon Problem with Pion Theoretical Potential, III
— *p-p* Scattering at 18.2 Mev —

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p–p scattering experiment at 18 Mev is analysed from the point of view of the pion theory. The pion theoretical potential, which is consistent with all low energy phenomena, gives a precise fit to the experimental data at 18 Mev. Especially, it is found that in the triplet odd state an attractive central potential around the pion range is required from the experimental data. This attractive potential is consistent with the two-pion-exchange potential.

§ 1. Introduction

In the present work, *p-p* scattering experiment at 18.2 Mev is analysed from the point of view of the pion theory.

From this point of view, two-nucleon problems at low energies were discussed by the present authors: The deuteron problem in the paper I[1], the singlet even state in II[2] and the triplet odd state[3] in the paper entitled as "Meson Theoretical Potentials in Triplet Odd State". In the series of these works, the one-pion-exchange potential, which is the first and the most natural consequence of the pion theory, was quantitatively established. The effective coupling constant of the one-pion-exchange potential was determined as $g_e^2/4\pi = 0.080 \pm 0.010$ and all low energy phenomena were explained consistently by the pion theory of nuclear forces.

One might expect that he could gain more and more essential knowledge of nuclear forces by analysing the intermediate and high energy phenomena. However, from the pion theoretical point of view, they are not so suitable for a detailed investigation into nuclear forces as it is usually believed. Many parameters—phase shifts and mixing ratios of the two components of the eigenwaves of scattering — are included in the expression of the scattering cross section. About a half of the parameters are expected to be affected more or less by the quantitatively unreliable potential in the region II $(0.7 \lesssim x \lesssim 1.5)$ and by the little known interactions in the region III $(x \lesssim 0.7)$. Therefore, what we can expect to obtain by analysing the intermediate and high energy phenomena is, at

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the present stage of the pion theory, not quantitative information but only qualitative one. Since we know the quantitative properties of the one-pion-exchange potential in the outer region, some qualitative information on the two-(or more-)pion-exchange potential can be obtained by making full use of the known properties of the one-pion-exchange potential.

As is seen from the above discussion, it seems useful to analyse an accurate experiment of nucleon-nucleon scattering at not so high energy. For this purpose we have chosen the p-p scattering experiment at 18.2 ± 0.2 Mev made at Princeton[4]. The new information we can expect to obtain from it is mainly concerned with the two-pion-exchange potential which is dominant in the region II ($0.7 \lesssim x \lesssim 1.5$), since the impact parameter* of the P-wave, which is mainly responsible to the angular distribution, is nearly equal to 2 at 18 Mev. The same experiment has already been analysed by some of the present authors and their collaborator, i.e. by Otsuki and Fujii[6] and by Otsuki and Tamagaki[7], and by other people, i.e. by Martin and Verlet[8] and by Beretta, Clementel and Villi[9]. However, it is to be emphasized that the present analysis is based on the validity of the one-pion-exchange potential in the outer region in contrast to other phenomenological analyses. Other experiments on nucleon-nucleon scattering at a few tens Mev, especially the n-p scattering, are not so accurate. The photodisintegration of the deuteron at the photon energy of about 20 Mev may serve to elucidate some aspects of nuclear forces, and will be discussed in the forthcoming paper IV.[10]

We describe the method and the results of our analysis in § 2 and § 3 respectively. The discussion and conclusion are given in § 4.

The results support the pion theory of nuclear forces, since the theory which is consistent with all low energy phenomena can also give a precise fit to the data at 18.2 Mev. In particular, the data require an attractive central force in the region II in the triplet odd state and this attractive force is consistent with the prediction of the two-pion-exchange potential and with the result of the low energy p-p scattering.

§ 2. Method

Our procedure for analysing the 18.2 Mev p-p scattering data is as follows:

To assume the $^1S_0$-wave phase shift $^1\delta_0$ as a free parameter.

To determine the $^1D_2$-wave phase shift $^1\delta_2$ by the one-pion-exchange potential.

To calculate the $^3P_{0,1,2}$-wave phase shifts $^3\delta_0^{\,1}$, $^3\delta_1^{\,1}$, $^3\delta_2^{\,1}$ (the suffices 0, 1, 2 denoting the total angular momentum) using the following potential

$$V=(1/3)\,(g_e^2/4\pi)\,\mu c^2(1+S_{12}(1+3/x+3/x^2))\,e^{-x}/x \quad \text{for } x>1,$$

$$\text{(the one-pion-exchange potential)} \qquad (1)$$

$$=V_{OC}+S_{12}V_{OT}\,(V_{OC} \text{ and } V_{OT} \text{ are two constant.}) \quad \text{for } x<1.$$

*) The impact parameter b is defined as $bk=\sqrt{l(l+1)}$.[5]

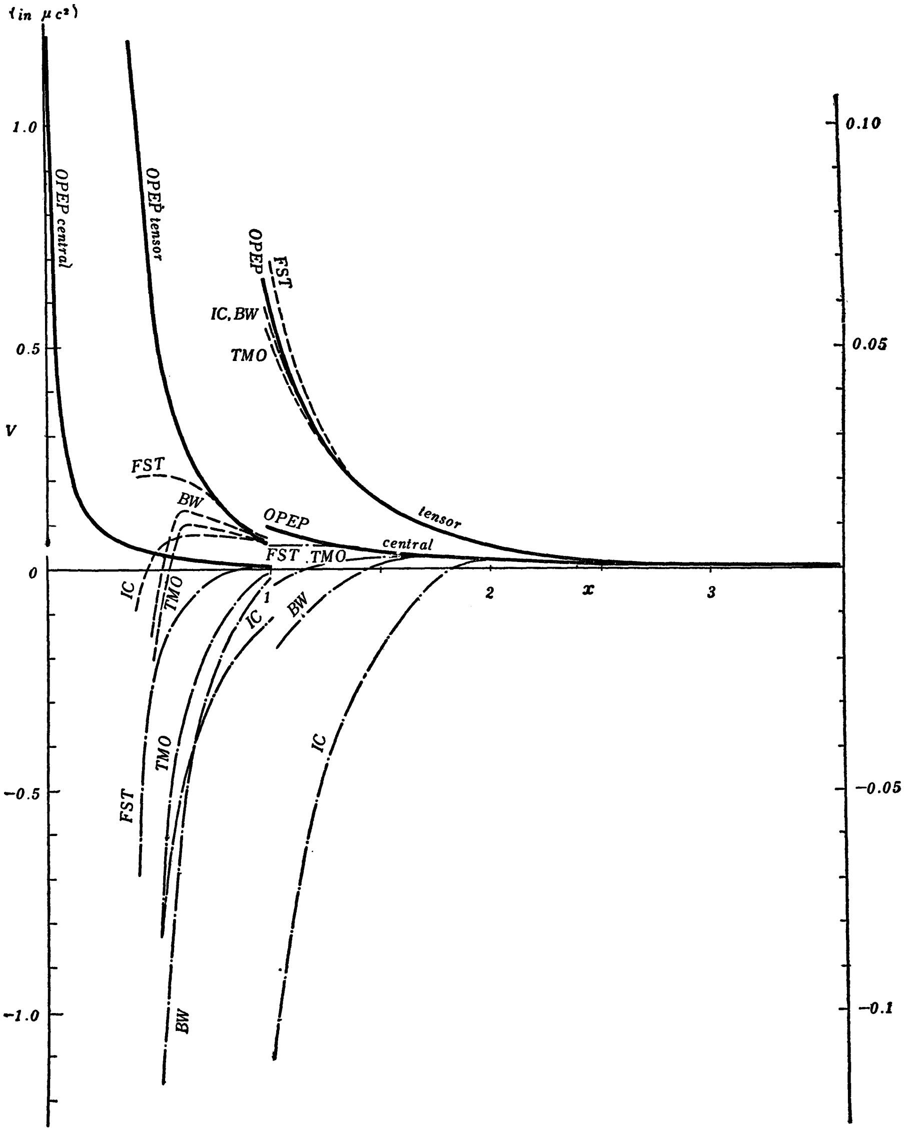

Fig. 1. The pion theoretical potential in the triplet odd state.

OPEP: The one-pion-exchange potential with $g_e^2/4\pi=0.08$.

FST: The potential constructed by Fukuda, Sawada and Taketani's method, with $g_e^2/4\pi=0.08$[11a)11b)]. The probability that the nucleons are to be bare is properly taken into account. The high frequency part of the pion field is cut off by the Gaussian factor with the cut-off momentum $\hbar k_c=6\mu c$.

TMO: The g^2+g^4 potential in the perturbation expansion calculated by Taketani, Machida and Onuma, with $g^2/4\pi=0.08$[11c)] The probability part is also expanded in powers of the coupling constant.

BW: The g^2+g^4 potential calculated by Brueckner and Watson with $g^2/4\pi=0.08$.[11d)] The probability part is approximated by unity.

IC: The one- and two-pion-exchange potential in the intermediate coupling theory with $g_e^2/4\pi=0.08$.[11e)] The cut-off momentum of the Gaussian cut-off factor is $\hbar k_c=4.1\mu c$.

The coupling between the $^3P_2$- and $^3F_2$-waves are neglected. The reason for the above procedure is as follows:

We have examined the singlet even state potential qualitatively in the work II[2] and found that it is consistent with the prediction of the pion theory. However, if we calculate $^1\delta_0$ using this potential the value of this phase shift at 18 Mev is uncertain in a fairly wide range, because the inner potential has a large ambiguity. Therefore we should treat $^1\delta_0$ as a free parameter and adjust it to reproduce the cross section at $\theta=90°$.

On the contrary, $^1\delta_2$ is almost completely determined by the one-pion-exchange potential, since the impact parameter of the D-wave is as large as 3.7 at 18 Mev.

The P-wave phase shifts at 18 Mev are expected to be mainly determined by the one-pion-exchange potential, since the impact parameter is 2.2. However, in contrast with the D-wave phase shift, they are affected also by the two-pion-exchange potential. Since we do not know the exact shape of the two-pion-exchange potential as can be seen from Fig. 1, we take into account the effect of the two-pion-exchange potential by introducing a phenomenological square well potential in the inner region, i.e., $V_{OC}+S_{12}V_{OT}$ of Eq. (1).

From the present view point of the theory of nuclear forces some results of the previous works by two of the authors are out of date:[6),7)] The most serious fault in these works is that the $^1S_0$-wave phase shift was calculated with the g^2+g^4 potential in the perturbation approximation[11c)] everywhere in the region $x>0.6$, and then its effects was subtracted from the experimental data in order to identify the remainder as the triplet scattering. At that time, the g^2+g^4 potential in the singlet even state was regarded much more reliable than that in the triplet odd state. Now the situation has been much changed with respect to the reliability of the pion theoretical potential owing to the new development, as was described in § 1 of our work I.[1)]

§ 3. Results

The value of the singlet D-wave phase shift is calculated to be $^1\delta_2=0.25°$ for $g_e^2/4\pi=0.08$. Since the potential at small distances has very little influence on the phase shift, the value is expected in the first approximation to be proportional to $g_e^2/4\pi$, the strength of the one-pion-exchange potential.

Next, we assume three values for V_{OT}, the depth of the inner tensor potential in the triplet odd state, as

$$\begin{aligned} V_{OT}&=0 && \text{(zero cut-off)},\\ &=(7/3)\,(g_e^2/4\pi)\,\mu c^2 e^{-1} && \text{(straight cut-off of the one-pion-exchange potential at } x=1),\\ &=2\times(7/3)\,(g_e^2/4\pi)\,\mu c^2 e^{-1} && \text{(twice the straight cut-off)}. \end{aligned}$$

Such a choice of V_{OT} is sufficient for the analysis of the 18 Mev data from our point of view since the two-pion-exchange potential is expected to be small (about 10% even at $x=1$) and consequently the one-pion-exchange potential is the main part of the tensor force around and outside the pion range (see Fig. 1).

The situation is complicated for the central force in the triplet odd state, as the potential due to the two-pion-exchange processes is expected to surpass the one-pion-exchange potential outside the pion range (see Fig. 1). Indeed the g^4 potential in the perturbation approximation is a few times stronger than the g^2 potential at $x=1$. Furthermore their signs are different. Therefore, we change the depth of the inner central potential V_{OT} over a wide range, i.e., from 0 to -50 Mev.

The value of the adjustable parameter ${}^1\delta_0$, the singlet S-wave phase shift, is determined so as to reproduce $d\sigma(90°)/d\Omega$ in each case.

The result for the case of $g_e^2/4\pi=0.08$ is tabulated in Table I. We can find from the table that when V_{OT} is large, ${}^1\delta_0$ is comparatively small. The reason is that the strong (positive) tensor potential gives a large isotropic cross section of the triplet scattering, which excludes the large cross section due to the singlet scattering.

On the contrary, the angular distribution around $\theta\sim30°$, where the interference effect between the nuclear and the Coulomb scatterings is appreciable, is sensitive to V_{OC} as is expected. Consequently the upper and lower limits for V_{OC} can be derived from the angular distribution, in particular from the difference $d\sigma(90°)/d\Omega-d\sigma(30°)/d\Omega$.

Table I. Phase shifts and the p–p angular distribution at 18.2 Mev calculated according to the procedure of § 2. The coupling constant of the one-pion-exchange potential is taken as $g_e^2/4\pi=0.08$. Experimental data are

$$d\sigma(90°)/d\Omega-d\sigma(30°)/d\Omega=2.32\pm0.4 \text{ mb},$$
$$\text{and} \quad C_2<0.01$$

where C_2 is the coefficient of $\cos^2\theta$ in the triplet nuclear scattering. $d\sigma(90°)/d\Omega$ can be reproduced since ${}^1\delta_0$ is adjusted. Some examples of the angular distribution given by the phase shifts of this table are shown in Fig. 2.

V_{OT}	zero cut-off		straight cut-off		twice the straight cut-off	
V_{OC} (Mev)	0	-50	0	-50	0	-50
${}^3\delta_0{}^1$ (deg.)	8.4	11.2	9.8	16.7	13.7	27.9
${}^3\delta_1{}^1$ (deg.)	-3.7	-2.9	-3.9	-3.3	-4.1	-3.6
${}^3\delta_2{}^1$ (deg.)	0.1	1.3	0.2	1.4	0.3	1.6
${}^1\delta_0$ (deg.)	51.1	50.6	50.6	48.3	49.6	40.6
${}^1\delta_2$ (deg.)	0.25	0.25	0.25	0.25	0.25	0.25
$d\sigma(90°)/d\Omega$ $-d\sigma(30°)/d\Omega$ (mb.)	0.6_5	2.2	0.8	2.7	1.4	3.2
C_2	0.003	0.007	0.004	0.009	0.005	0.018

By linear interpolation from Table I we can estimate the values of V_{OC} and ${}^1\delta_0$ which fit to the experimental data. For the three assumed values of V_{OT} they are as follows:

V_{OT}	V_{OC} (in Mev)	${}^1\delta_0$ (in degree)	
zero cut-off	$-40\sim-65$	$50.7\sim50.4$,	
straight cut-off	$-28\sim-50$	$50.4\sim48.3$,	(2)
twice the straight cut-off	$-14\sim-37$	$47.0\sim43.3$.	

Therefore in the case of $g_e^2/4\pi=0.08$, it may be reasonable to conclude that

$$V_{OC}=-10\sim-70\text{ Mev and }{}^1\delta_0=51^\circ\sim43^\circ.$$

When V_{OC} is vanishingly small or positive, the effect of the repulsive central part of the one-pion-exchange potential becomes dominating. In such a case the interference minimum around $\theta\sim30^\circ$ becomes less pronounced as the curve III in Fig. 2. On the contrary, when V_{OC} is about -50 Mev, this inner attractive central potential diminishes the effect of the repulsive central part of the one-pion-exchange potential. In such a case, the interference minimum becomes quite appreciacle. This can be understood when one notices that the minimum is caused by the destructive interference between the Coulomb scattering and the P-wave nuclear scattering, where the effects of the tensor force to the latter are likely to cancel out one another.

Calculation was also made for other values of $g_e^2/4\pi$ ($=0.07$ and 0.09). The adjusted values of V_{OC} are scarcely changed, i.e., $V_{OC}=0\sim-70$ Mev. For $g_e^2/4\pi=0.07$ the value of the adjusted ${}^1S_0$-wave phase shift ${}^1\delta_0$ increases by about $1\sim2^\circ$ from the corresponding value for $g_e^2/4\pi=0.08$. For $g_e^2/4\pi=0.09$, it decreases by almost the same amount.

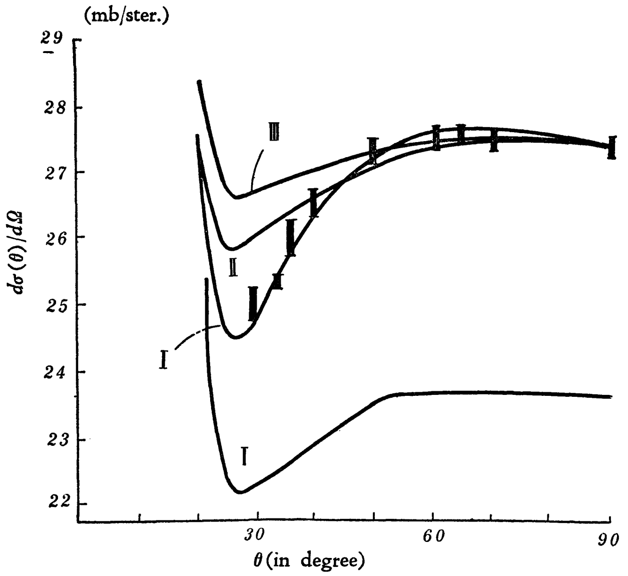

Fig. 2. Theoretical curves of the p–p scattering at 18.2 Mev.

	${}^1\delta_0$	${}^1\delta_2$	${}^3\delta_0{}^1$	${}^3\delta_1{}^1$	${}^3\delta_2{}^1$
I	48.3	0.25	16.7	−3.3	1.4
I′	48.3	0.25	—	—	—
II	54.2	0.25	—	—	—
III	50.7	0.25	9.8	−3.9	0.2

Curves I and III are due to the pion theoretical potential of Eq. (1) with $g_e^2/4\pi=0.08$. V_{OT} is assumed as the straight cut-off of the one-pion-exchange potential at $x=1$. $V_{OC}=-50$ Mev for I and $V_{OC}=0$ for III respectively
Curves I′ and II are the angular distributions when no interaction is present in the triplet odd state.
Experimental data are those of reference 4).
Note: The lowest curve I in the figure should be read as I′.

§ 4. Discussion and conclusion

In the previous section we have obtained the result that the inner central potential in the triplet odd state of Eq. (1) is

$$V_{OC}=0\sim-70\text{ Mev.}\tag{3}$$

Now it is very interesting to point out that the value of this inner central potential is in agreement with that estimated from the negative triplet P-wave phase shift at low energies.[3),7)]

The repulsive one-pion-exchange central potential in the triplet odd state gives a negative P-wave phase shift at low energies, which is, however, somewhat larger than the experimental one in its magnitude, if $g_e^2/4\pi$ of the one-pion-exchange potential is larger than about 0.06. Therefore an attractive interaction must be present in the inner region. If this attractive interaction is represented by the potential of the shape of Eq. (1), the depth V_{OC} estimated from the negative P-wave phase shift at low energies is $V_{OC}=0\sim-50$ Mev. Thus the results of the two independent estimates which were made in two different energy regions agree with each other surprisingly well. It is very likely that this agreement is due to our correct choice of the coupling constant $g_e^2/4\pi\sim 0.08$ for the one-pion-exchange potential.

Of course, the attractive potential V_{OC} is consistent with the prediction of the two-pion-exchange potential, not only in its tendency but also in its magnitude as can be seen from Fig. 1.

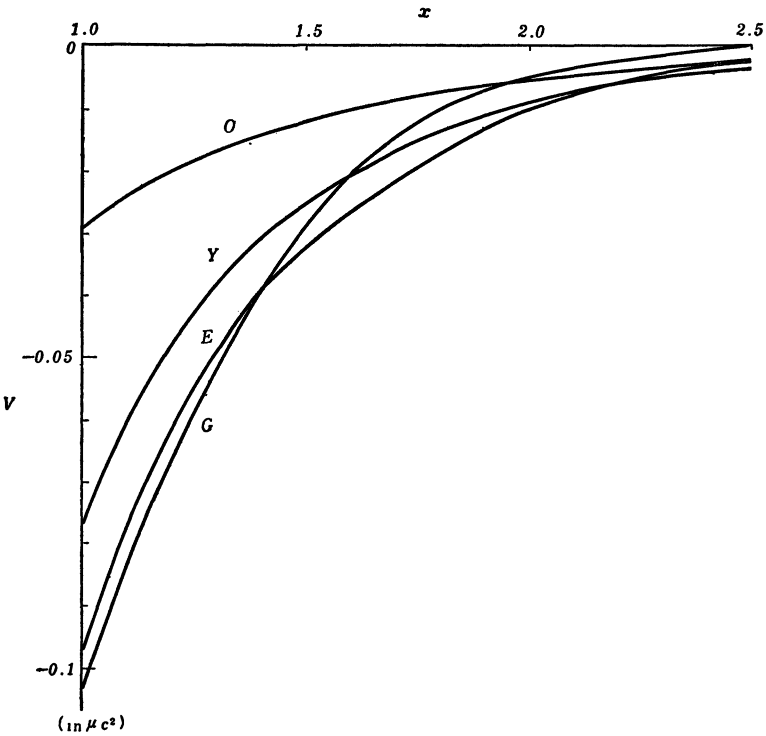

Fig. 3. The outer part of potentials in the singlet even state. The potentials can reproduce the low energy experimental data that $^1a=-20\times10^{-13}$ cm and $^1r_e=2.5\times10^{-13}$ cm.

O: the one-pion-exchange potential with $g_e^2/4\pi=0.08$,

Y: the Yukawa potential,

E: the exponential potential,

G: the Gauss potential.

Notice that the one-pion-exchange potential is weaker in the outer region than the long-tailed phenomenological potentials Y and E.

Here it is worth-while to emphasize the conclusion previously obtained with respect to the D-wave phase shift ${}^1\delta_2$.[6] ${}^1\delta_2$ is almost completely determined by the one-pion-exchange potential and is rather small, since the one-pion-exchange potential with $g_e^2/4\pi = 0.080 \pm 0.010$ is weaker than usual phenomenological potentials in the outer region (see Fig. 3). The small ${}^1\delta_2$ is very suitable to reproduce the isotropic angular distribution, since otherwise the interference between the ${}^1S_0$- and ${}^1D_2$-waves gives a deep minimum at 90° leading to a serious discrepancy.

Summarizing the above discussions, we can conclude that the p-p scattering experiment at 18.2 Mev supports the pion theoretical potential in the following points:

(1) The one-pion-exchange potential gives the small ${}^1D_2$-wave phase shift.

(2) An attractive central potential is to be added in the inner region to the one-pion-exchange potential in the triplet odd state. This attractive potential is consistent with the prediction of the two-pion-exchange potential.

(3) A striking agreement is obtained for the depths V_{OC} of the above inner central potential estimated by two analysis of the intermediate and low energy p-p scattering experiments.

An example of the phase shifts due to the pion theoretical potential is given below. It gives a precise fit to the experimental data:

$$\begin{array}{lll} {}^1\delta_0 = 48.3,^\circ & & {}^1\delta_2 = 0.25,^\circ \\ {}^3\delta_0^1 = 16.7,^\circ & {}^3\delta_1^1 = -3.3^\circ, & {}^3\delta_2^1 = 1.4^\circ, \end{array} \tag{4}$$

where $g_e^2/4\pi$ of the one-pion-exchange potential is 0.08; the depths of the inner potential of Eq. (1) are assumed as $V_{OC} = -50$ Mev and $V_{OT} = 0.08(7/3e)\mu c^2$ (the straight cut-off at $x=1$); ${}^1\delta_0$ is adjusted to reproduce the cross section at 90°.

Though we could determine the upper and lower bounds of the singlet S-wave phase shift ${}^1\delta_0$, we would not enter into its detailed investigation. It will be clear that we can not get meaningful results from such investigation at the present stage, since the singlet S-wave shifted by ${}^1\delta_0$ in Table I has its maximum just around the pion range and is strongly affected by the inner potential. If the exact shape of the two-pion-exchange potential were established, we could obtain from ${}^1\delta_0$ some knowledge of the interaction at small distances, for example, by solving the Schrödinger equation with the one- plus two-pion-exchange potential starting from outside with the boundary condition that the wave is shifted by ${}^1\delta_0$.

Martin and Verlet[8] also calculated the angular distribution using Lévy's potential[12]. Their good fit is not surprising, as Lévy's potential with $G^2/4\pi = 10.36$ is nothing but the one-pion-exchange potential with $g_e^2/4\pi = 0.057$ plus an attractive central potential of a short range ($\sim \exp(-2x)$).

Recently, there was an attempt to determine the phase shifts phenomenologically from the experimental data by Beretta, Clementel and Villi[9]. It seems to us, however, that there might be too large ambiguity in the result if one takes into account the experimental error. At such an energy, we think, an analysis on the standpoint of the pion theory may be much more superior than the purely phenomenological phase shift analysis.

However, the following fact may be worth noticing: A simplified phase shift analysis for the present experimental data assuming only a central force gives the result that ${}^1\delta_0 = 54.1°$, ${}^3\delta^1$ (${}^3P$-wave phase shift due to a central force) $= +1.0°$ and ${}^1\delta_2 = +0.4°$.[4] At low energies ($E < 5$ Mev), ${}^3\delta^1$ is negative, which is the consequence of the repulsive character of the central one-pion-exchange potential.[3] Thus it is found that ${}^3\delta^1$ changes its sign as the energy goes higher. This is just what is expected from the pion theory. As the energy goes higher, the attractive central force of the two-pion-exchange potential begins to influence ${}^3\delta^1$. The strong tensor potential also tends to make ${}^3\delta^1$ positive as if an attractive potential were added to the central potential. Therefore when a simplified phase shift analysis neglecting the tensor force is applied to the experimental data of *p-p* scattering, the resultant ${}^3\delta^1$ is expected from the pion theory to change its sign around 10 Mev.

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