1) Noise and Hearing

Task 1.

a) there are 12 semitones in an octave.

b) The semitones are given as $f_i = 440 \cdot 2^{i/12}$

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Semitone nr.0 -- 440.0 Hz
Semitone nr.1 -- 466.2 Hz
Semitone nr.2 -- 493.9 Hz
Semitone nr.3 -- 523.3 Hz
Semitone nr.4 -- 554.4 Hz
Semitone nr.5 -- 587.3 Hz
Semitone nr.6 -- 622.3 Hz
Semitone nr.7 -- 659.3 Hz
Semitone nr.8 -- 698.5 Hz
Semitone nr.9 -- 740.0 Hz
Semitone nr.10 -- 784.0 Hz
Semitone nr.11 -- 830.6 Hz
Semitone nr.12 -- 880.0 Hz
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/ Python

c)

$$\frac{f_{c(i+1)}}{f_{ci}} = r \Rightarrow \frac{\cancel{440} \cdot 2^{\frac{i+1}{12}}}{\cancel{440} \cdot 2^{i/12}} = 2^{\left(\frac{i+1}{12} - \frac{i}{12}\right)} = 2^{\frac{i+1-i}{12}}$$

$$= 2^{1/12}$$

$$f_{c(i+1)} = f_{ci} \cdot r \quad \text{or} \quad f_{c(i-1)} = f_{ci}/r$$

$$\frac{f_{ci} \cdot r}{f_{ci}} = \frac{f_{ci}}{f_{ci}/r} = r$$

Task 2

a)

Filter	f_c	f_u	V	P	Intensity	SPL
1	100	200	7.1 mV	0.14 Pa	$4.8 \cdot 10^{-5}$	77 dB
2	200	400	6.3	0.13 Pa	$3.8 \cdot 10^{-5}$	76 dB
3	400	800	11.2	0.22 Pa	$1.19 \cdot 10^{-4}$	81 dB
4	800	1600	8.9	0.18 Pa	$7.5 \cdot 10^{-5}$	79 dB
5	1600	3200	11.2	0.22 Pa	$1.19 \cdot 10^{-4}$	81 dB
6	3200	6400	7.9	0.16 Pa	$5.9 \cdot 10^{-5}$	78 dB

P_{rms}^2

b) $I = \frac{\quad}{\rho_0 c}$ $\rho_0 c = 420.175$

c) ref $20\,\mu Pa$ $SPL = 10\log\left(\frac{p_{rms}^2}{p_{ref}^2}\right)$

d) $SPL = PSL + 10\log \Delta f \Rightarrow PSL = SPL - 10\log(\Delta f)$

Filter	PSL
1	57
2	53
3	55
4	50
5	49
6	42,9

e) $I_{total} = \frac{p_{rms,total}^2}{\rho_0 c} = \frac{(0.14 + 0.13 + 0.22 + 0.18 + 0.22 + 0.16)^2}{\rho_0 c}$

$= 2.62 \cdot 10^{-3}$

f) $SPL_{total} = 10\log\left(\sum_i 10^{\frac{SPL_i}{\quad}}\right) = 86.9\,dB$

Task 3

a)

Octave	SPL
500	81.1 dB
1000	78.8 dB

b) overall SPL = 83.1 dB

c) A-weighted overall = 81.2 dB

Task 4

$\Delta L = 6dB \Rightarrow L_B = L_{train} - 6dB$

$L_{sum} = 10\log\left(\sum 10^{\frac{2L_{train} - 6}{10}}\right)$

$L_{sum} = 10\log\left(10^{\frac{L}{10}} + 10^{\frac{L - \Delta L}{10}}\right)$

$$L_{sum} = 10\log\left(10^{\frac{\;}{10}} + 10^{\frac{\;}{10}}\, 10^{\frac{\;}{10}}\right)$$

$$= 10\log\left(10^{\frac{L}{10}}\left(1 + 10^{\frac{-\Delta L}{10}}\right)\right)$$

$$= L + \underbrace{10\log\left(1 + 10^{\frac{-\Delta L}{10}}\right)}_{\text{margin of error}}$$

$\Delta L = 6dB = 0.97dB \approx 1dB$
$\Delta L = 10dB = 0.41dB \approx 0.4dB$
$\Delta L = 20dB = 0.04dB$

Task 5

a) $$\langle KE \rangle_t = \frac{1}{T}\int_0^T KE(t)\,dt$$

$KE = \frac{1}{2} m \dot{x}^2(t)$ → eq. 20 in 2.2 of L1

$$\frac{1}{T}\int_0^T \frac{1}{2} m \dot{x}^2(t)\,dt =$$

$x(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$
$\dot{x}(t) = \omega_0(A\sin(\omega_0 t) - B\cos(\omega_0 t))$
$T = 2\pi/\omega_0$

$$\frac{m}{2}\frac{1}{T}\int_0^T \omega_0^2 (A\sin(\omega_0 t) + B\cos(\omega_0 t))^2\,dt$$

$$\frac{m\omega_0^2}{2T}\int_0^T A^2\sin^2(\omega_0 t) - 2AB\sin(\omega_0 t)\cos(\omega_0 t) + B^2\cos^2(\omega_0 t)\,dt$$

u-substitution gives $u = \omega_0 t \Rightarrow \frac{du}{dt} = \omega_0$
$\Rightarrow u = \omega_0 T = \cancel{\omega_0}\frac{2\pi}{\cancel{\omega_0}}$

$$\frac{m\omega_0^2}{2T}\int_0^{u=2\pi} A^2\sin^2(u) - 2AB\sin(u)\cos(u) + B^2\cos^2(u)\,du$$

using Wolframalpha
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$$\frac{m\omega_0^2}{2T}\left(\pi(A^2 + B^2)\right)$$

b) $$\langle PE \rangle_t = \frac{1}{T}\int_0^T PE(t)\,dt$$

$PE = \frac{1}{2}kx^2(t)$ → eq. 21 in L1

$\omega_0^2 = \frac{k}{m} \Rightarrow k = \omega_0^2 m$

$$= \frac{1}{T}\int_0^T \frac{1}{2}kx^2(t)\,dt$$

$$= \frac{1}{T}\frac{\omega_0^2 m}{2}\int_0^T (A\cos(\omega_0 t) + B\sin(\omega_0 t))^2\,dt$$

$$= \frac{\omega_0^2 m}{2T} \int_0^T A^2 \cos^2(\omega_0 t) + 2AB \sin(\omega_0 t)\cos(\omega_0 t) + B^2 \sin^2(\omega_0 t)\, dt$$

Substitution as in (a)

$$\frac{\omega_0^2 m}{2T} \int_0^{u=2\pi} A^2 \cos^2(u) + 2AB \sin(u)\cos(u) + B^2 \sin^2(u)\, du$$

using Wolframalpha

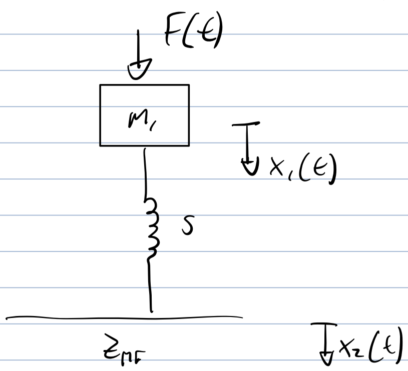
$$= \underline{\underline{\frac{\omega_0^2 m}{2T} \left(\pi (A^2 + B^2)\right)}} \Rightarrow \text{Same as in (5a)}$$

Task 6

$$Z_{m,mass1} Z_{m,F} + Z_{m,spring} (Z_{m,F} + Z_{m,mass1}) = 0$$

a) $Z_{m,F} = \infty$, $\omega_0 = \sqrt{s/m}$

$$Z_m(t) \equiv \frac{F}{v} = R_m + j\left(\omega m - \frac{s}{\omega}\right)$$



$Z_{m,mass} = j\omega m$

$Z_{m,spring} = s/j\omega$

a) $$\frac{Z_{m,mass1} \cdot \cancel{Z_{m,F}}}{\cancel{Z_{m,F}}} + \frac{Z_{m,spring}}{Z_{m,F}} \left(\cancel{Z_{m,F}} + Z_{m,mass1} \right) = 0$$

$$Z_{m,mass1} + Z_{m,spring}\left(1 + \underbrace{\frac{\cancel{Z_{m,mass1}}}{\cancel{Z_{m,F}}}}_{\infty}\right) = 0$$

$$Z_{m,mass} + Z_{m,spring} = 0$$

$$j\omega m + \frac{s}{j\omega} = 0 \quad | \cdot j\omega$$

$$-\omega^2 m + S = 0$$

$$\omega^2 = \frac{S}{m} \Rightarrow \underline{\underline{\omega_0 = \sqrt{\frac{S}{m}}}}$$

b) $Z_{m,F} = j\omega m_2$

$$j\omega m_1 \cdot j\omega m_2 + \frac{S}{j\omega}(j\omega m_2 + j\omega m_1) = 0$$

$$\frac{S}{j\omega_0}(j\omega_0 m_2 + j\omega_0 m_1) = \omega_0^2 m_1 m_2$$

$$\omega_0^2 = \frac{S}{\cancel{j\omega_0}} \frac{\cancel{j\omega_0} m_2 + \cancel{j\omega_0} m_1}{m_1 m_2}$$

$$\omega_0 = \sqrt{S \frac{m_2 + m_1}{m_1 m_2}}$$

if $m_2 >> m_1$ then $\sqrt{S \frac{m_2 + \cancel{m_1}}{m_1 m_2}} = \sqrt{S \frac{\cancel{m_2}}{m_1 \cancel{m_2}}}$

$$\omega_0 = \sqrt{\frac{S}{m_1}} \Rightarrow$$

c) $$\underset{\overset{\parallel}{0}}{Z_{m,mass1} Z_{m,F}} + Z_{m,spring}(\underset{\overset{\parallel}{0}}{Z_{m,F}} + Z_{m,mass1}) = 0$$

$$\Rightarrow Z_{m,spring} \cdot Z_{m,mass1} = 0$$

$$\Rightarrow \cancel{j\omega} m \cdot \frac{S}{\cancel{j\omega}} = 0$$

$m \cdot S = 0 \Rightarrow$ either mass or spring stiffness has to be non-existing $(=0)$, which is not possible.

d) $m_1 = 0$

$$\underset{\overset{\parallel}{0}}{j\omega m_1} \cdot j\omega m_2 + \frac{S}{j\omega}(j\omega m_2 + \underset{\overset{\parallel}{0}}{j\omega m_1}) = 0$$

$$\Rightarrow \frac{S}{\cancel{j\omega}} \cancel{j\omega} m_2 = 0$$

$\Rightarrow S m_2 = 0 \Rightarrow$ Same as (c).

e) $$Z_{m,mass1} \underset{\overset{\parallel}{Z_{m,mass2}}}{Z_{m,F}} + Z_{m,spring}(\underset{\overset{\parallel}{Z_{m,mass2}}}{Z_{m,F}} + Z_{m,mass1}) = 0$$

$$j\omega m_1 \cdot j\omega m_2 + \frac{S}{j\omega}(j\omega m_2 + j\omega m_1) = 0$$ (using eq in b)

$$\omega_0 = \sqrt{s\,\frac{m_1+m_2}{m_1 m_2}} \qquad (m_1 = m_2 = m)$$

$$\underline{\omega_0 = \sqrt{s\,\frac{2\cancel{m}}{m^{\cancel{2}}}} = \sqrt{s\,\frac{2}{m}}}$$