Task 1

a) Acoustic density

$\rho_1(x,t)$

$$p_1(x,t) = Re[\hat{p}\cos(kx)e^{j\omega t}]$$

L4 eq. (24) = $\rho = \frac{\partial \rho}{\partial p} p + \dots$

L4 eq. (25) $\frac{1}{c^2} = \frac{\partial \rho}{\partial p}$

$$\Rightarrow \underline{\underline{\rho = \frac{1}{c^2} p \cos(kx) e^{j\omega t}}}$$

b) Acoustic particle velocity $v_1(x,t)$

Euler equation gives

$$\frac{\partial v(x,t)}{\partial t} + \frac{1}{\rho}\frac{\partial P(x,t)}{\partial x} = 0$$

$$\Rightarrow \frac{\partial v(x,t)}{\partial t} = -\frac{1}{\rho}\frac{\partial P(x,t)}{\partial x}$$

$$v(x,t) = v(x)e^{j\omega t} \Rightarrow \frac{\partial v(x,t)}{\partial t} = j\omega\, v(x,t)$$

$$j\omega v = \frac{-1}{\rho}\left(-p\sin(kx)e^{j\omega t}\right)$$

$$v = Re\left\{\frac{pk}{j\omega\rho}\sin(kx)e^{j\omega t}\right\}, \qquad \frac{\omega}{k} = c$$

$$v = \frac{p}{\rho c}e^{j(\omega t - kx)}$$

c) Kinetic energy density (KE)

$$\frac{1}{2}mv^2 = \text{Kinetic energy} = \frac{1}{2}\rho\, Re\{v^2\}$$

$$= \frac{1}{2}\rho\left(\frac{p}{\rho c}e^{j(\omega t - kx)}\right)^2$$

$$= \frac{p^2}{2\rho c^2}\left(e^{j(\omega t - kx)}\right)^2 = \frac{p^2}{2\rho c^2}e^{2j(\omega t - kx)}$$

$\sin^2(\omega t)\sin^2(kx)$

$\cancel{PE = mk\,p^{2}}$

$\cancel{PE = \frac{1}{2}\frac{\omega}{c}\mathrm{Re}\{\hat{p}\}^{2}}$

$\cancel{= \frac{\rho}{2}\left(P\cos(kx)\,e^{j\omega t}\right)^{2}}$

$\cancel{= \frac{\rho^2\rho}{2}\,e^{2j(\omega t-kx)}}$

d) Potential energy density (PE)

$PE + KE = \text{konst} \Rightarrow$

$PE_{max} \Rightarrow KE = 0$, $KE_{max} \Rightarrow PE = 0$

They are $90°$ out of phase giving

$$\underline{PE = \frac{\rho^2}{2\rho c^2}\, e^{2j(\omega t - kx - \frac{\pi}{2})}}$$

$\Rightarrow$ KE and PE could be written in terms of $\cos^2$ and $\sin^2$ to show the phase shift

$\cos^2 + \sin^2 = 1$

e) Time-averaged intensity $\langle I \rangle_T$

$$I = \frac{p^2}{\rho c} = \frac{p^2 e^{2j(\omega t - kx)}}{\rho c}$$

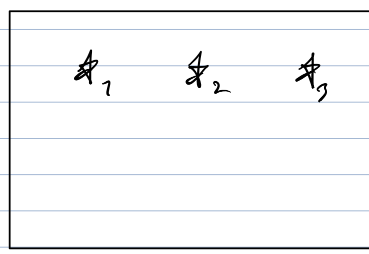
$$\langle I \rangle_T = v(t)\cdot p(t) = \int_0^T \mathrm{Re}\{v(t)\}\,\mathrm{Re}\{p(t)\}$$

$$= \int_0^T \frac{p}{\rho c}\, e^{j(\omega t - kx)}\, \hat{p}\cos(kx)\, e^{j\omega t} = \int_0^T \frac{p}{\rho c}\, e^{j(\omega t - kx)} \cdot p\, e^{j(\omega t - kx)}\, dt$$

$$= \int_0^T \frac{p^2}{\rho c}\, e^{2j(\omega t - kx)} = \frac{p^2}{\rho c}\left[\frac{1}{2j\omega}\, e^{2j(\omega t - kx)}\right]_0^T = \left[\frac{p^2}{\rho c 2j\omega}\, e^{2j(\omega t - kx)}\right]_0^T, \quad T = \frac{2\pi}{\omega}$$

$$= \frac{p^2}{\rho c 2j\omega}\, e^{-2jkx}\left(\underbrace{e^{2j(2\pi)} - e^{2j(0)}}_{\cos(2\pi) - \cos(0) = 0}\right) = 0$$

Task 2



1 = 90 dB

2 = 93 dB

3 = 95 dB

$$L_{p,tot} = 10\log\left(10^{\frac{90}{10}} + 10^{\frac{93}{10}} + 10^{\frac{95}{10}}\right)$$

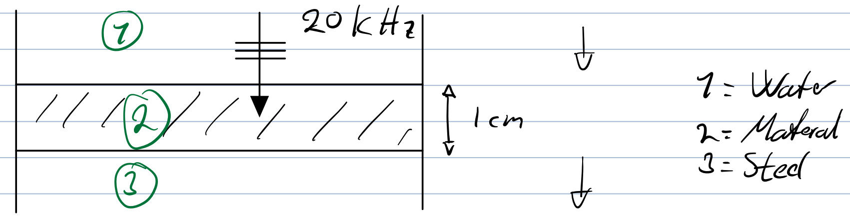
$$= 97.9\ \text{dB}$$

Task 3

$R_{ref}(H_2O) = 1\,\mu Pa_{rms}$, $\rho_{water} = 1000\ \text{kg/m}^3$, $c_{water} = 1500\ \text{m/s}$

$$I_{ref} = \frac{(R_{ref})^2}{\rho c} = \frac{(1\cdot 10^{-6})^2}{1000\cdot 1500} = 6.66\cdot 10^{-19}$$

Task 4



$\rho_{Water} = \rho_{Wa} = 1000\,kg/m^3$, $c_{Water} = c_{Wa} = 1500\,m/s$
$\rho_{Fe} = 7700\,kg/m^3$, $c_{Fe} = 6100\,m/s$

$$z = \rho \cdot c \quad , \quad z = \frac{P(t)}{u(t)}$$

We have that the mysterious layer should have impedance matching properties.

$$k_2 l = (n - 1/2)\pi, \quad n \in \mathbb{N}$$

From the reflection formula in LN 5 (29) granted by $\sin(k_2 l) = (-1)^n$ and $\cos(k_2 l) = 0$

$$(29) \qquad R = \frac{(z_2)^2 - z_1 z_3}{(z_2)^2 + z_1 z_3}$$

We want $R = 0$ for perfect transmission, meaning

$$z_2 = \sqrt{z_1 z_3} \Rightarrow \rho_2 c_2 = \sqrt{\rho_1 c_1 \cdot \rho_3 c_3}$$
$$= \sqrt{1000\,kg/m^3 \cdot 1500\,m/s \cdot 7700\,kg/m^3 \cdot 6100\,m/s}$$
$$(1) \qquad = 8.34 \cdot 10^6\,Pa/s$$

We have quarter wavelength giving us

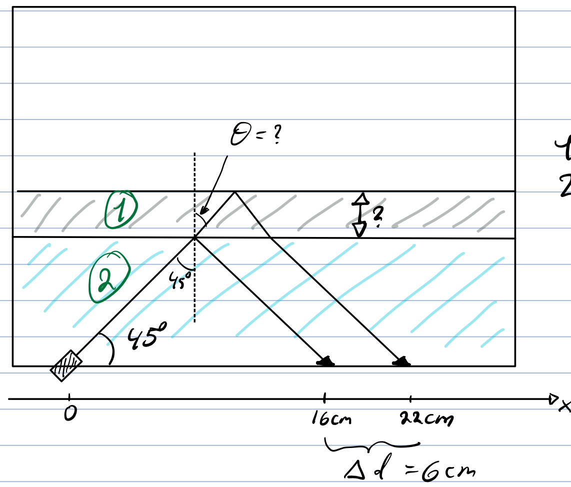
$$\lambda_2 = 1\,cm = \frac{c_2}{f} = 0.01\,m \cdot 20000\,Hz = 200\,m/s$$
$$c_2 = 4 \cdot 200\,m/s = 800\,m/s$$

Inserting into (1):

$$8.34 \cdot 10^6\,Pa/s = 800\,m/s \cdot \rho_2$$

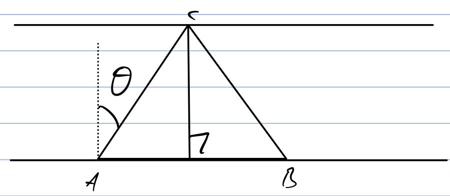
$\underline{\rho_2 = 10\,425\,kg/m^3}$ and $\underline{c_2 = 800\,m/s}$

Task 5



1: $C = 1150\ m/s$, $\rho = 790\ kg/m^3$
2: $C = 1500\ m/s$, $\rho = 1000\ kg/m^3$

We enlarge the part in ethanol



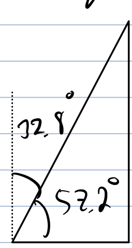
$AB = 6\ cm$
$\theta = ?$

$$\frac{\sin \theta_i}{C_i} = \frac{\sin \theta_t}{C_t}$$

$$\frac{\sin(45)}{1500} = \frac{\sin(\theta_t)}{1150} \Rightarrow \theta_t = 0{,}573\ rad$$

$$\theta_t = 32.83^\circ$$

Following trigonometry:



$$\Rightarrow \tan(57.2) = \frac{x}{3}$$

$$\underline{\underline{x = 4.65\ cm}}$$