TTT4250 - Acoustical Measurement Techniques

Underwater source directivity

performed by

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Report by

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Summary

All radiating speakers and optical devices have what they call a directivity pattern. This means that for certain angles, the radiating power is larger than that of other angles. This radiation-, or directivity, pattern can be measured in the desired medium using an omnidirectional receiver and the transmitter of interest. This report will guide you through the necessary theory to understand how directivity is formed, as well as what the directivity, and directivity index, tell us. The report will guide you through two methods, one to calculate the frequency response and another to calculate the directivity pattern. The frequency range of interest is set to 8-16KHz, where the highest directivity will be found to be for the 16KHz signal, while the directivity pattern, as well as the directivity value, is rather similar for both the 8- and 12KHz signals.

Contents

1 Introduction

This report will show how to find the directivity pattern, as well as the frequency response, for the SonoTube 008/D13-DT underwater transducer. This will be done in a water-filled enclosure and measured with the use of an omnidirectional hydrophone. The report will show in detail how to set up the equipment, as well as give the necessary theory and equation as to how to calculate the directivity, and directivity index, and plot the results in a polar plot. This report will enhance the understanding of how much the directivity change with the change in frequency. The results will also be compared with a mathematical model for a plane circular piston transducer.

2 Theory

This section will cover the necessary theory to understand the main principles for how an underwater transducer work, as well as cover the necessary theoretical aspects of the lab, such as directivity and directivity patterns.

2.1 Transducer

A transducer is something that converts energy from one form to another. The type of transducer used for this lab is an underwater transducer. Such a transducer converts the mechanical force, given by the sound waves in the water hitting the transducer membrane, into electrical signals. It can also convert electrical signals into mechanical movement at the transducer membrane. This makes the transducer perfect for both uses as a transmitter and as a receiver. Transducers can easily be modulated and constructed for use at low and high frequencies, so to perfectly fit the desired need. When the transducer is excited by an electrical signal, an acoustical output is produced. This acoustic pressure will propagate into the surrounding medium and form an intensity field in the far-field area. Since the transducer is not omnidirectional with an isotropic distribution of power for all directions, a directivity pattern will be visible. This pattern determines the directivity of the transducer with its main lobe, the direction with the highest power, and side lobes, the local maxima in directions that are not the main lobe. For a transducer with a plane circular membrane, the acoustic field shape will be like that of a flashlight beam. By measuring the pressure waves coming from the transducer at different angles, a directivity pattern can be calculated and visualized in a polar plot. Such a plot shows the intensity as a function of angle. Usually, the directivity patterns for a transducer are equal both when transmitting and receiving, but the sensitivity differs for both settings. The sensitivity of the transducer is determined as the relation between the electrical and mechanical force and is in most cases not the same when transmitting as when receiving.

The transducer used for this lab is adapted to the environment of being underwater, meaning the static density surrounding the transducer is a thousand times higher than that of air[1]. If operated above water the resistance on the membrane surface is significantly less than that underwater and could easily take permanent damage. Therefore, the transducer should not be operated as a transmitter while above water.

The receiving transducer must be placed at such a distance from the transmitting transducer so that the receiver is in the far-field area of the transmitter. The minimum distance, or the area where the near field ends and the far-field begin, is called the Rayleigh distance. The Rayleigh distance for a plane circular disk is defined as

$$R_R = \frac{\pi a^2}{\lambda} \tag{2.1}$$

where a is the radius of the circular disk, and λ is the wavelength for the selected frequency. When doing measurements in a specific frequency range, it's always the lowest frequency that determines the minimum distance between the receiver and transmitter.

2.2 Directivity

Directivity is a parameter that determines to what degree the radiation emitted by an antenna or optical system, is concentrated in a single direction. It is the ratio between the intensity in a specific direction related to the average intensity for all directions, which is called an isotropic source. The directivity can be measured by rotating the transmitter about the vertical or horizontal axis while measuring the intensity with a receiver at a fixed distance away from the transmitter. The distribution of intensity for all angles can then be plotted in a polar plot to better visualize the directivity of the transmitter. The polar plot shows the power as a function of angle. To calculate the directivity factor, D, the normalized peak-to-peak amplitude value, $B(\theta)$, at angle θ has to be calculated. The directivity is calculated as

$$D = \frac{2}{\int_0^{\pi/2} B(\theta)^2 \cdot \sin(\theta)\, d\theta} \tag{2.2}$$

and the directivity index DI as

$$DI = 10 \cdot \log_{10}(D) \quad \text{dB} \tag{2.3}$$

For isotropic source in free field, the directivity is 1 since it is equally spread in all directions. If the source is then placed near a plane surface, the radiated power from the plane will interfere with the radiation pattern towards the open space, and thus create a stronger signal in those directions. With one single plane near the source, the directivity value is 2, with the directivity index then being 3dB. If then the source is placed in the junction of two perpendicular planes, the directivity is then 4, with the directivity index being 6dB. The mathematical expression for the normalized directivity pattern for a plane circular piston transducer is given as

$$B(\theta) = \left| \frac{2J_1(ka \cdot \sin(\theta))}{ka \cdot \sin(\theta)} \right| \tag{2.4}$$

where J_1 is the first order Bessel function[2], k is the wave number for the given frequency and a is the radius of the transducer.

3 Method and Equipment

This section will describe the process of how the lab was performed as well as a figurative description of the setup with detailed positions, and an equipment list with all the equipment used for this lab.

The equipment used for this lab is listed in Table 3.1 and the setup for this lab is illustrated in Figure 3.1.

Table 3.1: Equipment list.

Equipment	Model number/type	Serial number
Oscilloscope	Tektronix TDS3012B	
Arbitrary Waveform Generator (AWG)	Keysight 33500B Series	
Amplifier for Transducer	Lab.gruppen FP 14000	00193200 V11/13
Transducer	SonoTube 008/D13-DT	
Hydrophone	Bruel&Kjær 8105	
Battery	9V, Duracell	

3.1 Setup of equipment

The measurements were performed in a water tank located in room B132 at NTNU, Trondheim, Gløshaugen, in the Elektro D+B2 building. The tank had dimensions as seen in Table 3.2.

Table 3.2: Equipment list.

	value	unit
w	1.5	m
l	3.0	m
h	1.6	m
$l_{x_0-x_1}$	0.957	m
$l_{x_1-x_2}$	0.953	m
$l_{x_2-x_3}$	1.090	m

The temperature in the water was set to 15 degrees Celsius and the water was regular tap water. The transducer was secured to a platform that allow for quick adjustment of both the positioning of the transducer in the water, as well as its azimuth angle. The transducer was then placed along the symmetry line for both the depth and width of the tank. This allowed for movement of the transducer only in the l axis of the tank. The same was then done for the hydrophone, meaning they had the same height- and width positioning in the tank. The exact positioning along the l axis for both the transducer and hydrophone is listed in Table 3.2. Between the transducer and the arbitrary waveform generator(AWG) is an amplifier that will amplify the pulse signals generated by the AWG. The pulse signal is also fed into the oscilloscope as a reference for the measurements. The hydrophone is connected to the oscilloscope via a 9V battery so to amplify the signals received from the hydrophone. To ensure that the positions never changed during adjustments, the tape was used to mark all the necessary positions.

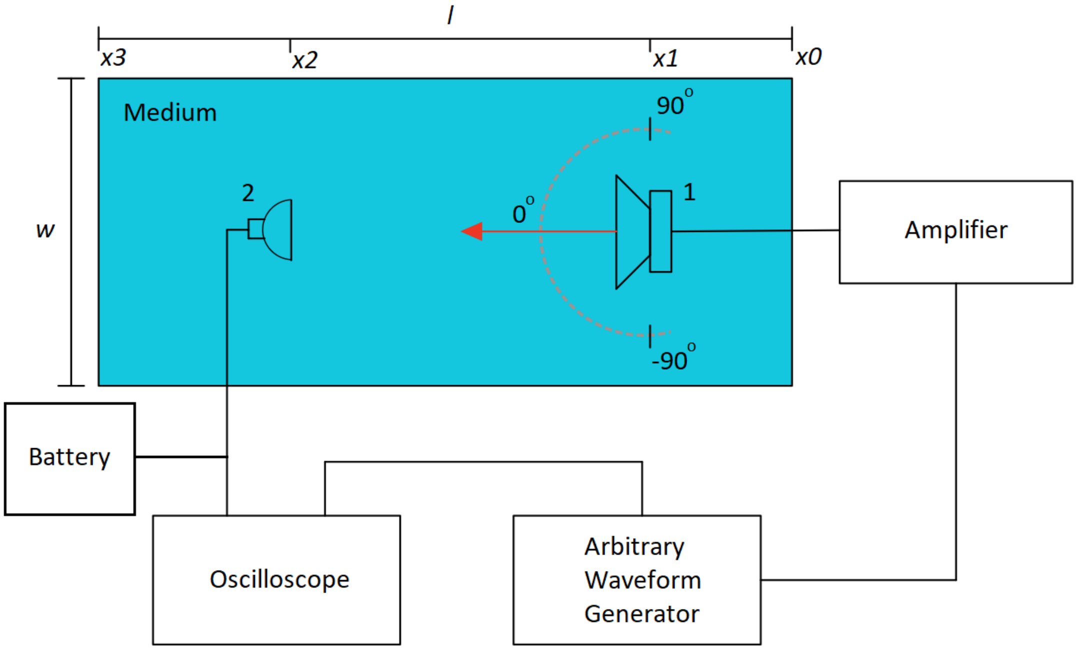


Figure 3.1: Illustration of the setup used. The blue box marks the water-filled enclosure where the transducer, 1, and hydrophone, 2, are placed inside. The transducer is fixed in such a manner that it can rotate around the vertical axis, with the angle θ, in degrees, marked in the figure around the transducer. The transducer is driven by an AWG(arbitrary waveform generator), which has its signal amplified by an amplifier. The same signal is also fed to the oscilloscope. The hydrophone is connected to the oscilloscope with a 9V battery amplifying the received signal. The enclosure has the length, l, and width, w. The enclosure is a 3D box, but since this is a 2D illustration, the depth, h, is not depicted in the figure.

3.2 Measuring Frequency at Three Distinct Angles

The object of these measurements is to calculate the frequency response at three distinct angles. The measured angles will cover the right quadrant of the transducer at 0^o, 45^o, and 90^o degrees. For each distinct angle, the measurements were done from $8 - 16$ KHz with 1KHz intervals. The AWG is then adjusted so to send the pulses with the desired frequency and the oscilloscope is set to read. Afterward, the transducer is excited with the pulse signals, and the measured sound waves from the hydrophone are then read on the oscilloscope, as well as the direct feedback from the AWG. The signal of interest is the largest and second-largest peak-to-peak amplitude value for the received signal. This value is manually found with the use of the cursor function on the oscilloscope. The procedure is then repeated for the entire frequency range of interest for all three angles.

3.3 Measuring the Beam Pattern-and Directivity at Three Distinct Frequencies

The object of these measurements is to calculate the beam pattern and directivity for the transducer at three different frequencies. The three frequencies of interest are 8-, 12- and 16 KHz. The transducer is then set to 90^o, and excited with the three distinct frequencies. The angle for the transducer is then incremented with -15^o, and the measurements are repeated. The measurements are repeated for all angles between 90^o and -90^o at -15^o increments. The values are also read on the oscilloscope in the same matter as for the previous method.

4 Results

This section will present the results of the post processing done in matlab[5] in the form of tables and plots, as well as the results from the mathematical model described in Equation (2.4).

4.1 Frequency Response for Three Distinct Angles

The distance between the transducer and hydrophone was found by using Equation (2.1), and showed that a minimum distance for the distance between the transducer and hydrophone had to be 18.3cm. The distance in-between them was set to be significantly higher than the minimum distance required, as can be seen in Table 3.2. The most important thing is to prevent reflections from the enclosure, and top of the tank, to interfere with the measured data. each of the data-sets for the three angles was normalised using the largest value in the respective data-set. The values used to normalise the data-sets are seen in Table 4.1. then, the data-sets was converted into dB values. The frequency response for the three angles was then plotted using the dB values, as seen in the right-most plot in Figure 4.1.

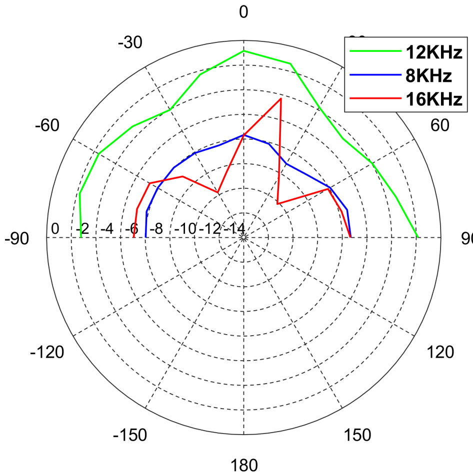
Table 4.1: Max values for the given data-sets.

data-set	value
Frequency response at 0 Degrees	5.36
Frequency response at 45 Degrees	3.90
Frequency response at 90 Degrees	5.44
Beam pattern at 8KHz	2.90
Beam pattern at 12KHz	6.36
Beam pattern at 16KHz	4.58

4.2 Directivity Pattern for Three Distinct Frequencies

Compared to the previous method where each individual data-set was normalised using the biggest value for the respective data-set, all three data-sets were normalised using the biggest value from all the three data-sets. The normalisation value was from the Beam pattern at 12KHz data-set, and had a value of 6.36.

The values was then converted into decibels and plotted by the use of the *polardb.m* script as found in Appendix A. The plot of the directivity pattern can be seen in the left-most plot in Figure 4.1. To calculate the directivity, each of the data sets were normalised using the biggest value in the respective data set, as done in the first method. The directivity was then calculated using Equation (2.2) for the range $-90^o - 90^o$, where the left quadrant is between -90^o to 0^o, and the right quadrant is from 0^o to 90^o. The directivity values, as well as the directivity index values, for the two quadrants can be seen in Table 4.2.



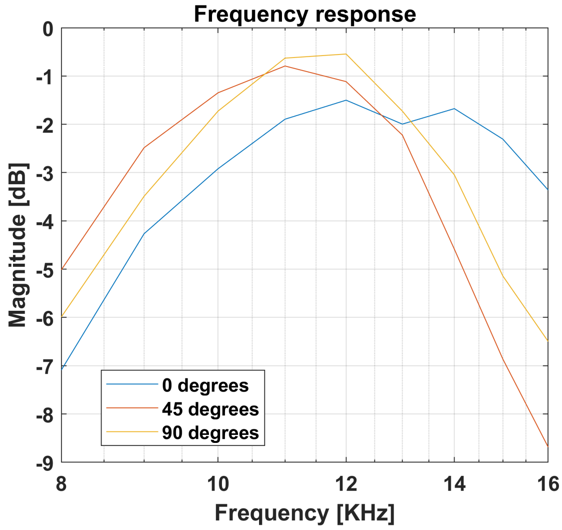


Figure 4.1: The left-most plot show the directivity pattern as a function of the angle, θ. Each ring indicate the intensity value in decibel, with the outer-most ring representing the maximum relative value at 0dB, and origo marks -15dB. The right-most plot show the frequency response for the three distinct angles, seen in the legend.

Table 4.2: Directivity, D, and directivity index, DI, for the three distinct frequencies at the left- and right quadrant.

Frequency	D_{left}	D_{right}	DI_{left}	DI_{right}
8KHz	0.61	0.59	-2.17	-2.27
12KHz	0.87	0.95	-0.59	-0.22
16KHz	1.48	1.56	1.71	1.93

4.3 The mathematical expression for the normalized directivity pattern of the plane circular piston transducer

For comparison, the mathematical model described in Equation (2.4) was used with the same parameters as in the measurements for the directivity pattern. By inserting the values for 8-,12- and 16KHz with the radius of the transducer, the normalized directivity pattern was found. Further, the polar plot for the three frequencies was then plotted as seen in Figure 4.2.

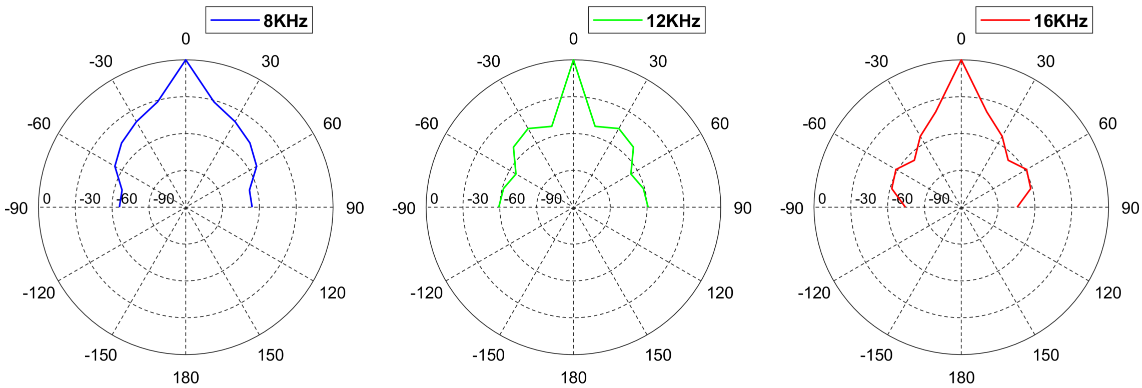


Figure 4.2: Polar plot for the mathematical model described in Equation (2.4). From left to right, the three plots mark the directivity pattern for 8-, 12- and 16KHz. The inner rings mark the damping in dB relative to the max value at 0^o which is equal to 1.

5 Discussions

This section will discuss the results from Section 4 as well as factors that could produce errors in the results.

From the plots in Figure 4.1 it is clear that at 12KHz, the transducer has the highest power output, which is verified by looking at the frequency response where all angles have a maximum at around 12KHz. With the increase in frequency, the transducer becomes more directional, as is seen by the three distinct frequencies in the polar plot. The directivity factor is highest at 16KHz, while 8- and 12KHz have quite similar D and DI values, as can be explained by the rather similar curve in the polar plot. The distance between the transducer and hydrophone was significantly higher than what was necessary. To further improve the plot and measurements, a smaller distance could have been used so to ensure better measurements. The transducer used for this lab has an effective range from 7-16KHz, so measurements in the most extreme regions should be weaker and more inaccurate than that of the middle region of the effective frequency range. This is seen in Figure 4.1, showing clear signs that the 12KHz signal is higher in power. The hydrophone used has an effective frequency range from 0.1Hz - 100KHz, covering the measured frequency range with a vast amount. This should then remove any possible errors from the hydrophone for the tests.

To achieve better results in the polar plot, a higher resolution could have been used when rotating the transducer by decreasing the increment value in-between the measurements. The measurements done were only for the azimuth angle, but since the transducer is circular, the directivity pattern can be rotated for the horizontal axis without showing signs of change. This is due to the symmetry of a circular membrane.

By comparing the measured results with the results granted by the mathematical expression, the general shape for the respective frequencies is rather similar. The dB values from the measured and mathematical values differ by a vast amount.

For this lab, there were a lot of potential error margins. When positioning the transducer and the hydrophone in the tank, all positions were done with the use of a ruler and eyesight. This is rather hard due to the refraction of light at the water surface, making it rather difficult to get exact measurements. When changing the main directivity of the transducer in-between measurements, it was also done manually which gave room for minor errors in positioning. The room where the measurements were done is surrounded by a lot of equipment with much electromagnetic noise which could affect the signals traveling through the excessive number of cables. When measuring the peak-to-peak values on the oscilloscope, it was done with the use of the cursor function and the desired resolution. The resolution varied with the amount of zoom used on the scaling for the y axis, and this gave a varying room for error, meaning for the biggest values, where no zooming where done, the step size for the cursor was larger than that for the measurements where zooming was used.

6 Conclusions

Through measurements and calculations the Directivity, and directivity index, for the two quadrants used where found to be The directivity values clearly state that for 16KHz, the directivity

Table 6.1: Directivity, D, and directivity index, DI, for the three distinct frequencies at the left- and right quadrant.

Frequency	D_{left}	D_{right}	DI_{left}	DI_{right}
8KHz	0.61	0.59	-2.17	-2.27
12KHz	0.87	0.95	-0.59	-0.22
16KHz	1.48	1.56	1.71	1.93

was the highest. This was clearly evident in the polar plot in Figure 4.1. The measured data could have been better by using a higher resolution for the angles and reducing the distance between the transducer and hydrophone.

Bibliography

[1] Comparing the density of air to water, Meteorologist Jeff Haby, https://www.theweatherprediction.com/habyhints/216/

[2] Jim Lambers, MAT 415/515, Fall Semester, 2013-14, Lecture 12 and 13 Notes, https://www.math.usm.edu/lambers/mat415/lecture12.pdf

[3] TTT4250, Acoustical Measurement Techniques, Laboratory Exercise Compendium.

[4] Lab 3: Underwater transducer frequency response and beam pattern measurement, Frequency Response, Beam Pattern and Directivity of an underwater acoustic transmitter in a water tank. Lecture notes on underwater source directivity.

[5] Matlab R2021b, The MathWorks, Inc. 1994-2022.

[6] files used for the lab, https://github.com/Erlenkb/TTT4250_Lab3/tree/main/Files

A Appendix

polardb.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% polardb.m
% modified from Matlab's polar.m by K. Bell
% Last updated 8/30/00 by K. Bell
% downloaed from http://gunston.gmu.edu/demt/oap/contents.htm
% changed 2/17/2012 by Bo Peng
% updated 2/25/2012 by bo peng
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function hpol=polardb(theta,rho,lim,NN,line_style)
% polardb(theta,rho,lim,linestyle)
% Polardb is a modified version of the regular 'polar' function.
% Plotting range is -180 to 180 deg with zero at top.
% Theta increases in clockwise manner.
% Inputs:
%   theta - angles in degrees ( axes labeled in degrees)
%   rho   - plot value in dB
%   lim   - lower limit for plot in dB (e.g. -10)
%   NN    - resolution in magnitude (unit dB)
%   line_style - string indicating line style, e.g. '-g' (optional)
%
%   Example polardb(theta,beampatt,-10,3,'*r')
%
%POLAR  Polar coordinate plot.
%       POLAR(THETA, RHO) makes a plot using polar coordinates of
%       the angle THETA, in radians, versus the radius RHO.
%       POLAR(THETA,RHO,S) uses the linestyle specified in string S.
%       See PLOT for a description of legal linestyles.
%       See also PLOT, LOGLOG, SEMILOGX, SEMILOGY.
%       Copyright (c) 1984-94 by The MathWorks, Inc.

const=1;
theta=theta/180*pi;
if nargin < 4
        error('Requires 4 or 5 input arguments.')
elseif nargin == 4
        [n,n1]=size(theta);
        if isstr(rho)
                line_style = rho;
                rho = theta;
                [mr,nr] = size(rho);
                if mr == 1
                        theta = 1:nr;
```

```
            else
                th = (1:mr)';
                theta = th(:,ones(1,nr));
            end
        else
            line_style = 'auto';
        end
elseif nargin == 3
        line_style = 'auto';
        rho = theta;
        [mr,nr] = size(rho);
        if mr == 1
            theta = 1:nr;
        else
            th = (1:mr)';
            theta = th(:,ones(1,nr));
        end
end
if isstr(theta) | isstr(rho)
        error('Input arguments must be numeric.');
end
if any(size(theta) ~= size(rho))
        error('THETA and RHO must be the same size.');
end
% NN=1;
nr = size(rho,2);
tck = -floor(lim/NN);
lim = -tck*NN;
I=find(rho<lim);
ni=size(I,2);
rho(I)=lim*ones(1,ni);
rho = rho/NN+tck*ones(1,nr);

% get hold state
cax = newplot;
next = lower(get(cax,'NextPlot'));
hold_state = ishold;

% get x-axis text color so grid is in same color
tc = get(cax,'xcolor');

% Hold on to current Text defaults, reset them to the
% Axes' font attributes so tick marks use them.
fAngle  = get(cax, 'DefaultTextFontAngle');
fName   = get(cax, 'DefaultTextFontName');
fSize   = get(cax, 'DefaultTextFontSize');
fWeight = get(cax, 'DefaultTextFontWeight');
set(cax, 'DefaultTextFontAngle',  get(cax, 'FontAngle'), ...
        'DefaultTextFontName',   get(cax, 'FontName'), ...
        'DefaultTextFontSize',   get(cax, 'FontSize'), ...
        'DefaultTextFontWeight', get(cax, 'FontWeight') )

% only do grids if hold is off
```

```
if ~hold_state

% make a radial grid
        hold on;
        hhh=plot([0 max(theta(:))],[0 max(abs(rho(:)))]);
        v = [get(cax,'xlim') get(cax,'ylim')];
        ticks = length(get(cax,'ytick'));
        delete(hhh);
% check radial limits and ticks
        rmin = 0; rmax = v(4); rticks = ticks-1;
        if rticks > 5   % see if we can reduce the number
                if rem(rticks,2) == 0
                        rticks = rticks/2;
                elseif rem(rticks,3) == 0
                        rticks = rticks/3;
                end
        end

% define a circle
        th = 0:pi/50:2*pi;
        xunit = cos(th);
        yunit = sin(th);
% now really force points on x/y axes to lie on them exactly
    inds = [1:(length(th)-1)/4:length(th)];
    xunits(inds(2:2:4)) = zeros(2,1);
    yunits(inds(1:2:5)) = zeros(3,1);
    if ~isstr(get(cax,'color')),
       patch('xdata',xunit*rmax,'ydata',yunit*rmax, ...
             'edgecolor',tc,'facecolor',get(gca,'color'),...
             'handlevisibility','off');
    end
%       rinc = (rmax-rmin)/rticks;
        rinc = const;
%       for i=(rmin+rinc):rinc:rmax
        for i=[1:1:tck-1]
%       for i=rmax:rmax
                plot(const*xunit*i,const*yunit*i,'--','color',tc,'
                    linewidth',0.5,'handlevisibility','off');
%               text(0,i+rinc/20,['  ' num2str(10*(i-tck))],'
    verticalalignment','bottom' );
                text(const*(-i+rinc/(1000)),0,[' ' num2str(NN*(i-tck))
                    ],'verticalalignment','bottom','fontsize',9 );
        end
plot(const*xunit*tck,const*yunit*tck,'-','color',tc,'linewidth',0.5,'
    handlevisibility','off');
text(const*(-tck+rinc/(1000)),0,[' ' num2str(NN*(tck-tck))],'
    verticalalignment','bottom' ,'fontsize',9);
% plot spokes
        th = (1:6)*2*pi/12;
%       th = (1:2)*2*pi/4;
        cst = cos(th); snt = sin(th);
        cs = [-cst; cst];
        sn = [-snt; snt];
```

```
        plot(const*rmax*cs,const*rmax*sn,'--','color',tc,'linewidth',0.5);

% annotate spokes in degrees
%       rt = 1.1*rmax;
        rt = 1.15*rmax;
        for i = 1:max(size(th))
                text(const*rt*snt(i),const*rt*cst(i),int2str(i*30),'horizontalalignment','center' );

                loc = int2str(i*30-180);
%               if i == max(size(th))
%                       loc = int2str(0);
%               end
                text(-const*rt*snt(i),-const*rt*cst(i),loc,'horizontalalignment','center' );
        end

% set viewto 2-D
        view(0,90);
% set axis limits
        axis(rmax*[-1 1 -1.1 1.1]);
end

% Reset defaults.
set(cax, 'DefaultTextFontAngle', fAngle , ...
        'DefaultTextFontName',   fName , ...
        'DefaultTextFontSize',   fSize, ...
        'DefaultTextFontWeight', fWeight );

% transform data to Cartesian coordinates.
yy = const*rho.*cos(theta);
xx = const*rho.*sin(theta);

% plot data on top of grid
if strcmp(line_style,'auto')
        q = plot(xx,yy);
else
        q = plot(xx,yy,line_style);
end
set(q,'LineWidth',1.0);
if nargout > 0
        hpol = q;
end
if ~hold_state
        axis('equal');axis('off');
end

% reset hold state
if ~hold_state, set(cax,'NextPlot',next); end
```

B Appendix

Lab3.m

```
file1 = "Freq_Response.txt";
file2 = "Beam_Pattern.txt";

%% Global paramaters
Beam_Pattern = importdata(file2 , "\t", 1);
Freq_Response = importdata(file1 , "\t", 1);

Degrees = [90 75 60 45 30 15 0 -15 -30 -45 -60 -75 -90];
Degrees_rad = Degrees * (pi / 180);

freq = [8 9 10 11 12 13 14 15 16];

%% Fetch the data
Freq_8KHz = Beam_Pattern.data(1,2:27);
Freq_12KHz = Beam_Pattern.data(2,2:27);
Freq_16KHz = Beam_Pattern.data(3,2:27);

deg_0 = Freq_Response.data(1,1:18);
deg_45 = Freq_Response.data(2,1:18);
deg_90 = Freq_Response.data(3,1:18);

%% Scalar values used to normalise the data set
scalar1 = max([deg_0 deg_45 deg_90]);
scalar2 = max([Freq_16KHz Freq_12KHz Freq_8KHz]);
scalar_0  = max(deg_0);
scalar_45 = max(deg_45);
scalar_90 = max(deg_90);
scalar_8 = max(Freq_8KHz);
scalar_12 = max(Freq_12KHz);
scalar_16 = max(Freq_16KHz);


%% Generate two temporary vectors to store the values in
Freq_8KHz_mean = zeros(1,13);
Freq_12KHz_mean = zeros(1,13);
Freq_16KHz_mean = zeros(1,13);

deg_0_mean = zeros(1,8);
deg_45_mean = zeros(1,8);
deg_90_mean = zeros(1,8);

%% Average the data and remove the zeros
for i=[1:2:size(Freq_8KHz,2)]
```

```
    Freq_8KHz_mean(i) = mean([Freq_8KHz(i) Freq_8KHz(i+1)]);
    Freq_12KHz_mean(i) = mean([Freq_12KHz(i) Freq_12KHz(i+1)]);
    Freq_16KHz_mean(i) = mean([Freq_16KHz(i) Freq_16KHz(i+1)]);
end
for i=[1:2:size(deg_0,2)]
    deg_0_mean(i) = mean([deg_0(i) deg_0(i+1)]);
    deg_45_mean(i) = mean([deg_45(i) deg_45(i+1)]);
    deg_90_mean(i) = mean([deg_90(i) deg_90(i+1)]);
end

%% Scale values for plot
Freq_8KHz_mean = Freq_8KHz_mean(Freq_8KHz_mean~=0) ./ scalar2;
Freq_12KHz_mean = Freq_12KHz_mean(Freq_12KHz_mean~=0) ./ scalar2;
Freq_16KHz_mean = Freq_16KHz_mean(Freq_16KHz_mean~=0) ./scalar2;

%% Scale values for D


%% Normalize the two sets by the biggest value for both sets sepertely
Freq_8KHz_mean_db = 20*log10(Freq_8KHz_mean);
Freq_12KHz_mean_db = 20*log10(Freq_12KHz_mean);
Freq_16KHz_mean_db = 20*log10(Freq_16KHz_mean);

deg_0_mean = 20*log10(deg_0_mean(deg_0_mean~=0) ./ scalar_0);
deg_45_mean = 20*log10(deg_45_mean(deg_45_mean~=0) ./ scalar_45);
deg_90_mean = 20*log10(deg_90_mean(deg_90_mean~=0) ./ scalar_90);

%% Plot beam patter
figure(1)
subplot(1,2,1)
First = polardb(Degrees, Freq_12KHz_mean_db, -15, 2,"-g")
hold on
second = polardb(Degrees, Freq_8KHz_mean_db, -15, 2, "-b")
third = polardb(Degrees, Freq_16KHz_mean_db, -15, 2, "-r")
hold off
legend("", "", "", "","","","12KHz", "8KHz", "16KHz", "location", "best
    ");
set(gca,'fontsize',12,'fontweight','bold');
%set(gcf,'units','centimeters','position',[2,1,29.7,11.0])

%% Plot frequency response for the three distinct angles
figure(1)
subplot(1,2,2)
semilogx(freq, deg_0_mean);
hold on
semilogx(freq, deg_45_mean);
semilogx(freq, deg_90_mean);

hold off
grid on
xlabel("Frequency [KHz]");
ylabel("Magnitude [dB]");
title("Frequency response");
```

```
legend("0 degrees", "45 degrees", "90 degrees","location","best");
set(gca,'fontsize',12,'fontweight','bold');
set(gcf,'units','centimeters','position',[2,1,29.7,11.0])

exportgraphics(figure(1), ['Lab3.png'],'Resolution',450)


%% Calculate the directivity D
%First_Quadrant
theta_1 = flip(Degrees(1:7),2);
rho_8khz_1 =  flip(Freq_8KHz_mean(1:7)  ,2);
rho_12khz_1 = flip(Freq_12KHz_mean(1:7),2);
rho_16khz_1 = flip(Freq_16KHz_mean(1:7),2);

%% D
D_8khz_1 = 2 / (sum((rho_8khz_1.^2) .* sind(theta_1)));
D_12khz_1 = 2 / (sum((rho_12khz_1.^2) .* sind(theta_1)));
D_16khz_1 = 2 / (sum((rho_16khz_1.^2) .* sind(theta_1)));
%% DI
DI_8kh_1 = 10*log10(D_8khz_1)
DI_12khz_1 = 10*log10(D_12khz_1);
DI_16khz_1 = 10*log10(D_16khz_1);

%Second_Quadrant
theta_2      = Degrees(7:13);
rho_8khz_2   = Freq_8KHz_mean(7:13);
rho_12khz_2 = Freq_12KHz_mean(7:13);
rho_16khz_2 = Freq_16KHz_mean(7:13);

%% D
D_8khz_2 = 2 / (sum((rho_8khz_2.^2) .* sind(theta_1)));
D_12khz_2 = 2 / (sum((rho_12khz_2.^2) .* sind(theta_1)));
D_16khz_2 = 2 / (sum((rho_16khz_2.^2) .* sind(theta_1)));
%% DI
DI_8kh_2 = 10*log10(D_8khz_2)
DI_12khz_2 = 10*log10(D_12khz_2);
DI_16khz_2 = 10*log10(D_16khz_2);


%% Bessel function
a = 0.054;
c = 1464;
%lambda_8khz =
ka_8khz = (2 * pi * 8000 / c) * (abs(Degrees_rad));
ka_12khz = (2 * pi * 12000 / c) * (abs(Degrees_rad));
ka_16khz = (2 * pi * 16000 / c) * (abs(Degrees_rad));

Bessel_8 = besselj(1, ka_8khz);
Bessel_12 = besselj(1, ka_12khz);
Bessel_16 = besselj(1, ka_16khz);

B_8 = abs(Bessel_8 ./ ka_8khz);
B_12 = abs(Bessel_12 ./ ka_12khz);
```

```
B_16 = abs(Bessel_16 ./ ka_16khz);

B_8(7) = 1;
B_12(7) = 1;
B_16(7) = 1;

%% Scale

Max_all = max([B_8 B_12 B_16]);

B_8_s = 20*log10(B_8 / max(B_8));
B_12_s = 20*log10(B_12 / max(B_12));
B_16_s = 20*log10(B_16 / max(B_16));

B_8 = 20*log10(B_8 / Max_all);
B_12 = 20*log10(B_12 / Max_all);
B_16 = 20*log10(B_16 / Max_all);

%B_8 = B_8 / Max_all;
%B_12 = B_12 / Max_all;
%B_16 = B_16 / Max_all;
N = 120;
step = 30;

figure(5)
subplot(1,3,1)
First = polardb(Degrees, B_8, -N, step,"-b");
legend("", "", "", "","","","8KHz", "location", "best");
set(gcf,'units','centimeters','position',[3,1,29.7,11.0]);
set(gca,'fontsize',12,'fontweight','bold');
figure(5)
subplot(1,3,2)
second = polardb(Degrees, B_12, -N, step, "-g");
legend("", "", "", "","","","12KHz", "location", "best");
set(gcf,'units','centimeters','position',[3,1,29.7,11.0])
set(gca,'fontsize',12,'fontweight','bold');
figure(5)
subplot(1,3,3)
third = polardb(Degrees, B_16, -N, step, "-r");
legend("", "", "", "","","","16KHz", "location", "best");
set(gcf,'units','centimeters','position',[3,1,29.7,11.0]);

%legend("", "", "", "","","","12KHz", "8KHz", "16KHz", "location", "
    best");
set(gca,'fontsize',12,'fontweight','bold');
exportgraphics(figure(5), ['Math_Directivity.png'],'Resolution',450)
```

C Appendix

Due to size and layout for the excel sheet containing the raw data, the original files can be found on github[6].