Computation Theory (COMP 170), Fall 2020
Assignment 03

Answer each problem below to the best of your ability. Submit all parts by 9:00 AM on Monday, October 5. List your collaborators. Late homework is accepted within 24 hours for half credit. After 24 hours no credit is given. The first late assignment (up to 24 hours) per student incurs no penalty. **Make sure that your submission follows the formatting guidelines given at the end of this document.**

Reading: Sipser Chapter 1.3

[1] (*6 pts.*)   **Getting Closure**

Use closure properties to prove that none of the following languages is regular.

(i) $A_1 = \{a^n b^{n+1000} \mid n \geq 0\}$.

(ii) $A_2 = \{a^{2n+1} b^{2n+1} \mid n \geq 0\}$.

(iii) $A_3 = \{a^n b^m c^k \mid n, m, k \geq 0 \text{ and } n + m = k\}$.

[2] (*8 pts.*)   **Pump It Up**

Use the pumping lemma to prove that neither of the following languages is regular.

(i) $B_1 = \{a^{2^n} \mid n \geq 0\}$.

(ii) $B_2 = \{a^n b^m \mid n, m \geq 0 \text{ and } m \text{ is a multiple of } n\}$.

[3] (*6 pts.*) Regular Expressions

Give regular expressions for the following languages. Both are defined over $\Sigma = \{a, b\}$. Simplify as much as you can. Simpler is better, and by better, I mean worth more points. No proofs required but an explanation is nice.

(i) $C_1 = \{x \mid x \text{ does not contain the substring } aaa\}$.

(ii) $C_2 = \{x \mid x \text{ has an even number of } a\text{'s and an even number of } b\text{'s}\}$.

Format requirements: work for COMP 170 should correspond to the following guidelines:

- Work must be in type-written format, with any diagrams rendered using software to produce professional-looking results. No hand-written or hand-drawn work will be graded.

- Work must be submitted in PDF format to Gradescope.

- Each answer should start on a new page of the document. When possible, try to limit answers to a single page each. (Thus, the answers to this homework must be no less than three pages, and preferably no more.)

You can find links to information about using LaTeX to produce type-written mathematical work,[1] and to a handy web-based tool for drawing finite-state diagrams, on the Piazza class site:

https://piazza.com/tufts/fall2020/comp170/resources

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[1]LaTeX was used to produce this document.