COMP160: Algorithms, **Homework 3**

- To obtain full credit, you must justify your answers. When describing an algorithm, do not forget to **state any assumptions that you make**, analyze its running time and explain why the algorithm is correct.
- Although not specifically stated, You can assume that we look for algorithms that are as fast as possible, and bounds as tight as possible.
- You may discuss these problems with others, but remember to write the answers on your own. In case of doubt, cite any source you used to do the assignment.
- Remember to submit each question in a separate page.

1. Recall the fake coin problem from this week's recitation. Can you give a lower bound on the number of weighings needed for any strategy? In other words, is the strategy described in the solution of recitation the best possible? If so, why?

   Note: normally, we look for asymptotic lower bounds (something of the form $\Omega(n^{34})$). For this exercise you will need to find the exact lower bound (say, we need to do at least $36n^{34} - 11n^{12}$ weighings). Don't worry. The real answer will not be as ugly as this example.

2. You scan through a list of n elements, keeping track of the max. Every time you encounter a new max, you update the associated variable. How many times do you expect to make such an update?
   Define the answer as a random variable, break it into indicator random variables, use linearity of expectation, evaluate each IRV, and compute the overall expectation. Give your answer first as an exact expression and then give a close asymptotic bound.

3. Recall the viking problem from recitation: A group of k Vikings independently set out to make a new home. Each Viking has a copy of the same map, showing n islands. Each Viking decides to set sail for some random island. If two or more Vikings land on the same island, they have a battle. (No matter how many Vikings land on that island, it counts as one battle.)

   (a) How many battles do we expect will occur?

   Hint: Fix a single island, what is the probability of no viking ever landing there? What about exactly one viking reaching this island? What is the relationship of these events and there being a fight in the island?

   (b) You should have obtained a closed formula that depends on n and k (similar to the one obtained in recitation). For both formulas (the one obtained in recitation and the one obtained above), consider the cases in which there is only one island on the map. Do your solutions confirm the intuitive answer for this case? What if there's only one Viking? What answers do you get for 400 Vikings and 100 islands? (use a calculator for this last part)