COMP160: Algorithms, **Homework 1**

- To obtain full credit, you must justify your answers. When describing an algorithm, do not forget to **state any assumptions that you make**, analyze its running time and explain why the algorithm is correct.
- Although not specifically stated, you can assume that we look for algorithms that are as fast as possible, and bounds as tight as possible.
- You may discuss these problems with others, but remember to write the answers on your own. In case of doubt, cite any source you used to do the assignment.
- Remember to submit each question in a separate page.

0. Read the *How to write proofs* document (available in the course's webpage)

1. We have two algorithms A_1 and A_2 for solving the same problem. Let R_1 and R_2 be their runtimes, respectively. For each of the cases below, say which algorithm is faster (if able). Justify your answers with 1 sentence.
   (a) $R_1 = O(n)$ and $R_2 = O(n^2)$
   (b) $R_1 = O(n)$ and $R_2 = \Omega(n^2)$
   (c) $R_1 = \Omega(n)$ and $R_2 = O(n^2)$
   (d) $R_1 = O(n)$ and $R_2 = \Theta(n^2)$

2. Consider function $f(n) = 3n^2 + 10n + 729$.
   (a) prove that $f(n) = O(n^2)$
   (b) prove that $f(n) = O(n^3)$
   (c) prove that $f(n) = \Omega(n)$
   (d) prove that $f(n) = \Omega(n^2)$
   (e) We have shown two different upper bounds and two different lower bounds. Which is best for each? Why?

3. Let $H(n) = H(\frac{n}{2}) + \log n$. Give bounds for $H(n)$ with the following techniques:
   (a) An upper bound using a recursion tree
   (b) Give a lower bound by substitution
   (c) Give both upper and lower bounds using the master theorem.
   (d) Do the bounds match? Which of the three methods you prefer?
   Note: the last question is thought-provoking. There is no wrong answer (just write 1-2 sentences with your opinion).