Computation Theory (COMP 170), Fall 2020
Recitation 05

The pumping lemma for context-free languages states that if A is context-free then

- $\exists\ p \geq 1$ such that
- $\forall\ w \in A$ with $w \in A$ and $|w| \geq p$
- $\exists\ x, y, u, v, z$ where $w = xyuvz$, $yv \neq \varepsilon$, and $|yuv| \leq p$, such that
- $\forall\ i \geq 0$, $xy^i uv^i z \in A$.

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[1] Context-Free

Consider the language $L = \{0^m 1^n \mid m < n\}$.

a. Show that this language is context-free, and give an informal explanation of your solution.

b. Since this language is context-free, whatever the pumping length p is, we know that the following string must satisfy it: $0^p 1^{p+1}$.

Given that fact, what do we know about the segment yuv of the string, containing the two pumpable portions? Where can it occur? Where must it not occur?

[2] Not Context-Free

Let $\Sigma = \{a, b, \$\}$. Consider the following two languages:

$$A = \{t\$t^R \mid t \in \{a, b\}^\star\}$$
$$B = \{t\$t^R\$t \mid t \in \{a, b\}^\star\}$$

where $t^R = rev(t)$ is the reverse of string t.

a. The first of these, A, is context-free. Give a grammar for that language and explain your solution (informally).

b. The second language, B, is not context-free. Show this, using the pumping lemma.

[3] Pushdown Automata

Let $\Sigma = \{a, b\}$, and for any string $w \in \Sigma^*$, let $rev(w)$ denote the reverse of w, and let $inv(w)$ denote the result of turning all a's into b's, and all b's into a's. E.g. if $w = babb$ then $rev(w) = bbab$ and $inv(w) = abaa$. Define

$$A = \{w \mid rev(w) = inv(w)\}.$$

So the string $baabba \in A$, but $baab$ is not. Specify a PDA that recognizes A, using a transition diagram to describe δ.