Computation Theory (COMP 170), Fall 2020
Test 2

- Answer each problem below to the best of your ability.
- Submit all parts with 72 hours of downloading the exam, and before the start of class at 9am on Monday, November 23rd.
- You may use your notes, the textbook, and materials from the Resources section of the course piazza page. **No other resources may be used. Your work must be entirely your own.**
- Make sure that your submission follows the formatting guidelines given at the end of this document.

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[1] (*8 pts.*) **Shorties**

For each of the following statements, decide whether it is true or false, and then justify your answer.

a. If $A \leq_m B$ and $\overline{B} \leq_m \overline{C}$, then $A \leq_m C$.

b. If $A \leq_m B$, then $B \leq_m A$.

c. If $A \subseteq B$ and B is context free, then A is decidable.

d. If A is context free and $\overline{A}$ is Turing-recognizable, then A is decidable.

[2] (*8 pts.*) **Context Free**

Let $C = \{w \mid w \in \{a, b\}^*, \#a(w) = 2\#b(w)\}$. Construct a context free grammar that generates C, and prove that it is correct.

For full points, your grammar should not have redundant or needlessly complex production rules. Your proof should be helpful in streamlining the set of rules you use.

In class we solved a very similar problem that may be worth revisiting.

[3] (*8 pts.*) **Not Context Free**

Consider the following language:

$$D = \{x\#y \mid x, y \in \{a, b\}^*, |x| = |y|, \#a(x) = \#a(y)\}.$$

Use the context free pumping lemma to prove that D is not context free.

[4] (*8 pts.*) **But it is Decidable**

Give an implementation-level description of a Turing machine that decides the set D from problem 3. That is, you don't need to explicitly specify the δ function for your TM, but you do need to provide an English description of how your TM operates.

[5] (*8 pts.*) **Overlapping TM's**

Define $F = \{\langle M_1, M_2 \rangle \mid M_1, M_2 \text{ are TM's and } L(M_1) \cap L(M_2) \neq \emptyset\}$. That is, F contains all pairs of Turing machines that both accept some string in common. Prove that F is Turing recognizable, but not decidable.

Format requirements: work for COMP 170 should correspond to the following guidelines:

- Work must be in type-written format, with any diagrams rendered using software to produce professional-looking results. No hand-written or hand-drawn work will be graded.
- Work must be submitted in PDF format to Gradescope.
- Each answer should start on a new page of the document. When possible, try to limit answers to a single page each.

You can find links to information about using LaTeX to produce type-written mathematical work,[1] and to a handy web-based tool for drawing finite-state diagrams, on the Piazza class site:

https://piazza.com/tufts/fall2020/comp170/resources

[1] LaTeX was used to produce this document.