Question 1: Getting Closure

In lecture, we have shown that $A = \{a^n b^b | n \geq 0\}$ is not a regular language. Now, I will show the following language are not regular based on this and the closure property of regular language.

(i) $A_1 = \{a^n b^{n+1000} | n \geq 0\}$
Let $B_1 = \{a^{1000} | n \geq 0\}$, it's easy to see that B_1 is a regular language.(since it's finite)
$A_1 \circ B_1 = A$ is not regular, therefore A_1 is not regular.

(ii) $A_2 = \{a^{2n+1} b^{2n+1} | n \geq 0\}$
Let $B_2 = \{a\}$ $B_3 = \{b\}$. Since B_2 and B_3 are finite, we know they are regular language.
If A_2 is regular, then so will be $A_2 \cup (B_2 \circ A_2 \circ B_2) = A$ due to closure of regular language.
Since A is not regular, we know that A_2 is not regular.

(iii) $A_3 = \{a^n b^m c^k | m, n, k \geq 0 \text{ and } n + m = k\}$
Let $B_4 = \{$ strings with less than or equal to 2 different characters $\}$
Clearly, B_4 is a regular language.
$A_3 \cap B_4 = A$, therefore A_3 is not regular.

Question 2: Pump It Up

(i) $B_1 = \{a^{2^n} | n \geq 0\}$

For all $p > 0$, choose let $w = a^{2^N}$ where $N = \lceil log_2 p \rceil + 1$
Since we have to divide this string into three pieces xyz, with $|y| > 0$ and $|xy| \leq p$. We must have $2^N > p \geq |y| > 0$.
If we pump y twice we will get $a^{2^N + p}$ which is not in B_1. Thus B_1 is not regular.

(ii) $B_2 = \{a^n b^m \mid n, m \geq 0$ and m is a multiple of n $\}$

For all $p > 0$ choose $w = a^p b^{kp}$ where k is an positive integer. Since we have to divide this string into three pieces xyz, with $|y| > 0$ and $|xy| \leq p$. We have $y = a^n$ with $n \leq p$
we pump it (k+1)p times, we get $a^{p+(k+1)pn}$ which is not in B_2.

regular expressions
(i) $(aa \cup a \cup \epsilon)(b \cup ba \cup baa)^*$

(ii)$((aa \cup bb)^* \cup (ab \cup ba)(aa \cup bb)^*(ab \cup ba))^*$