Question 1:   Context Free Pumping Lemma

$A_P = a^n b^m c^r | n \times m = r$

Assuming A_P is context free, then context free pumping lemma should apply to A_P

For any length $p > 0$, we can have a string $w = a^p b^p c^{p^2}$

Writing w = uvxyz where $|vxy| \leq p$ and $|vy| > 0$

Then we will have the following cases:

1) vxy are all a's or all b's or all c's. Then uv^2xy^2z adds only one of them it is not in A_P. This contradict to pumping lemma (which says it should be in A_P) 2) vxy have some a's and some b's. Then uv^2xy^2z only add a's and b's but not any c. So it is not in A_P(which says it should be in A_P).

3) vxy have some c's and some b's.

Assuming v,y has m b's and n c's. Then $uv^2xy^2z = a^p b^{p+m} c^{p^2+n}$. If $m \geq 1$, then n = p and $|vy| \geq p + 1$ contradict to our assumption $|vxy| \leq p$. If m =0, then n = 0, then $|vy| = 0$ contradicts to our assumption assumption that $|vy| > 0$.

In all three cases, we have contradiction. So our assumption that A_P is context free is not true.

Question 2:   CFG's from Regular Expressions

Regular expressions can be built inductively from 3 atomic element and 3 composite expressions. So we only need to prove we can create CFG for each one of them.

Atomic elements:

1) a, corresponding CFG: $S \rightarrow a$

2) ϵ corresponding CFG: $S \rightarrow \epsilon$

3) $\emptyset$ corresponding CFG: S

Composite elements: Assuming now R_1 and R_2 are regular languages whose corresponding CFG are A and B

1) $R_1 \cup R_2$, corresponding CFG: $S \rightarrow A|B$

2) $R_1 R_2$, corresponding CFG: $S \rightarrow AB$

3) R_1^*, corresponding CFG: $S \rightarrow AS|\epsilon$

we can use these to build a CFG for any regular expressions correspondingly.

Question 3: Pushdown Automata

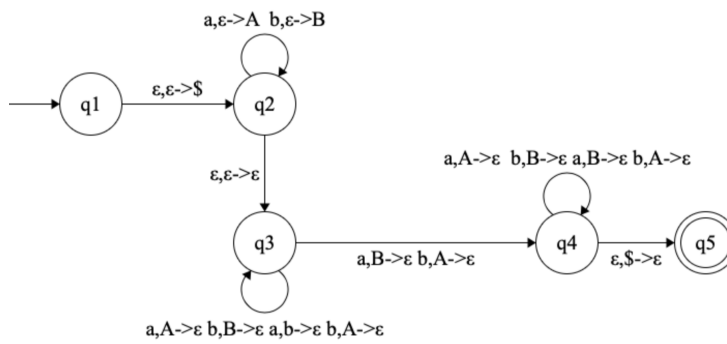Pushdown Automata $M = (Q, \Sigma, \Gamma, \delta, s, F)$
Q = q1,q2,q3,q4,q5
$\Sigma = \{a, b\}$
$\Gamma = \{A, B, \$\}$
s = q1
$F = \{q5\}$

A palindromic string of even length is symmetric from middle. Therefore, a non palindromic string would have at least one different character on each sides of middle and same distance from middle so it breaks this symmetry. Transition from q3 to q4 ensures at least one such difference.