Computation Theory (COMP 170), Fall 2020
Recitation 02

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[1] Prefix

As you know, u is a *prefix* of string w if $w = uv$ for some string v. Given a language A, define a language $Prefix(A) = \{u \mid u \text{ is a prefix of some } w \in A\}$. Prove that if A is regular, then so is $Prefix(A)$.

a. Give a precise construction of a DFA/NFA M that accepts $Prefix(A)$ given DFA M_A for A.

   Hint: Change only one element of the tuple of M_A.

   Hint: Try this on a simple example, maybe a DFA M that accepts strings $\{ab^{3n} \mid n \geq 0\}$.

b. Prove that $x \in L(M)$ if and only if $x \in Prefix(A)$.

[2] NFA Construction

(Sipser, modified) Let F be the language of all strings over $\{0, 1\}$ that contain a pair of `1`s that are separated by an odd number of symbols.

a. Find and draw a 4-state NFA that recognizes F.

b. Use the subset construction (proof of theorem 1.39 in Sipser) to construct a DFA that is equivalent to the 4-state NFA.

c. The subset construction should yield a 7-state DFA. However, this isn't a minimal DFA. Combine some states to produce an equivalent 5-state DFA.

d. This DFA should suggest an alternate, equivalent definition of F. What is it?

[3] Even Prefix

Let $OddPrefix(A)$ be the set of strings that are not only prefixes of strings in A, but also have odd length. Prove that if A is regular, $OddPrefix(A)$ is regular.