Computation Theory (COMP 170), Fall 2020
Assignment 08

Answer each problem below to the best of your ability. Submit all parts by 9:00 AM on Monday, November 16. List your collaborators. Late homework is accepted within 24 hours for half credit. After 24 hours no credit is given. The first late assignment (up to 24 hours) per student incurs no penalty. **Make sure that your submission follows the formatting guidelines given at the end of this document.**

Reading: Sipser Chapter 5

[1] (*6 pts.*) **Totally**

For this problem and the next, we define

$$TOT = \{\langle M \rangle \mid M \text{ is a TM and } M \text{ halts on all inputs}\}.$$

Let H be the halting problem, i.e.

$$H = \{\langle M, x \rangle \mid M \text{ is a TM, } x \text{ is a string, and } M \text{ halts on } x\}.$$

a. Does Rice's Theorem apply to TOT? Explain.

b. Give a direct proof that $H \leq_m TOT$. To be clear, this requires you both specify a mapping reduction and prove it works as intended. Such a result implies that TOT is not co-Turing-recognizable.

[2] (*5 pts.*) **Not Totally**

Give a direct proof that $H \leq_m \overline{TOT}$. This implies that TOT is not Turing-recognizable.

[3] (*9 pts.*) **Differences**

If X and Y are sets, the set difference $X \setminus Y$ is the set of all elements that are in X but not in Y. So you can alternately think of $X \setminus Y = X \cap \overline{Y}$.

Suppose A and A' are two arbitrary decidable languages. Suppose B is an arbitrary Turing-recognizable languages. For each of the following set differences, determine whether it (a) must be decidable, (b) may not be decidable, but must be Turing-recognizable, or (c) may not even be Turing-recognizable. Justify each of your answers.

a. $A \setminus A'$

b. $A \setminus B$

c. $B \setminus A$

Format requirements: work for COMP 170 should correspond to the following guidelines:

- Work must be in type-written format, with any diagrams rendered using software to produce professional-looking results. No hand-written or hand-drawn work will be graded.
- Work must be submitted in PDF format to Gradescope.
- Each answer should start on a new page of the document. When possible, try to limit answers to a single page each. (Thus, the answers to this homework must be no less than three pages, and preferably no more.)

You can find links to information about using LaTeX to produce type-written mathematical work,[1] and to a handy web-based tool for drawing finite-state diagrams, on the Piazza class site:

https://piazza.com/tufts/fall2020/comp170/resources

[1] LaTeX was used to produce this document.