Computation Theory (COMP 170), Fall 2020
Assignment 10

Answer each problem below to the best of your ability. Submit all parts by 9:00 AM on Friday, December 11. List your collaborators. Late homework is accepted within 24 hours for half credit. After 24 hours no credit is given. The first late assignment (up to 24 hours) per student incurs no penalty. **Make sure that your submission follows the formatting guidelines given at the end of this document.**

Reading: Sipser Chapter 7

[1] (*8 pts.*) **Scheduling**

Consider the following scheduling problem. There are n seminars that are to be offered in the evenings. Each seminar must be assigned to one day of the week. The professor running seminar i has a list of days they are willing to teach, $D_i \subseteq \{M, T, W, Th, F\}$. Furthermore, every student j has submitted a list L_j of any number of seminar/day pairs they hope will be offered. For example, $L_1 = \{(2, M), (7, F)\}$ indicates that student 1 is interested in taking seminar 2 on Monday and seminar 7 on Friday.

Given the faculty's availability (the D_i's) and the students' preferences (the L_j's), the question is whether or not there is an assignment of seminars to days that respects the faculty requests while also ensuring that every student gets at least one of their requests satisfied as well. We'll call this the SEMINAR problem. Prove that SEMINAR is NP-Complete.

[2] (*7 pts.*) **Closure Properties**

For a class of languages C, we say a language A is co-C if its **co**mplement $\overline{A}$ is in class C. For example, $\{x \in \{a\}^* \mid \#a(x) \text{ is odd}\}$ is co-regular because its complement, $\{x \in \{a\}^* \mid \#a(x) \text{ is even}\}$ is regular.

a. Argue that co-regular is equivalent to regular. That is, language A is regular if and only if A is co-regular.

b. Is context-free equal to co-context-free? If so, give a brief informal argument explaining why; if not, give a counter-example.

c. Is co-decidable equal to decidable? If so, give a brief informal argument explaining why; if not, give a counter-example.

d. Is co-P equal to P? If so, give a brief informal argument explaining why; if not, give a counter-example.

e. Prove that we don't know whether co-NP! =NP, as this would imply that P! =NP.

Format requirements: work for COMP 170 should correspond to the following guidelines:

- Work must be in type-written format, with any diagrams rendered using software to produce professional-looking results. No hand-written or hand-drawn work will be graded.
- Work must be submitted in PDF format to Gradescope.
- Each answer should start on a new page of the document. When possible, try to limit answers to a single page each. (Thus, the answers to this homework must be no less than three pages, and preferably no more.)

You can find links to information about using LaTeX to produce type-written mathematical work,[1] and to a handy web-based tool for drawing finite-state diagrams, on the Piazza class site:

https://piazza.com/tufts/fall2020/comp170/resources

[1] LaTeX was used to produce this document.