FIRST-ORDER LOGIC

ARTIFICIAL INTELLIGENCE | COMP 131

TODAY ON AI

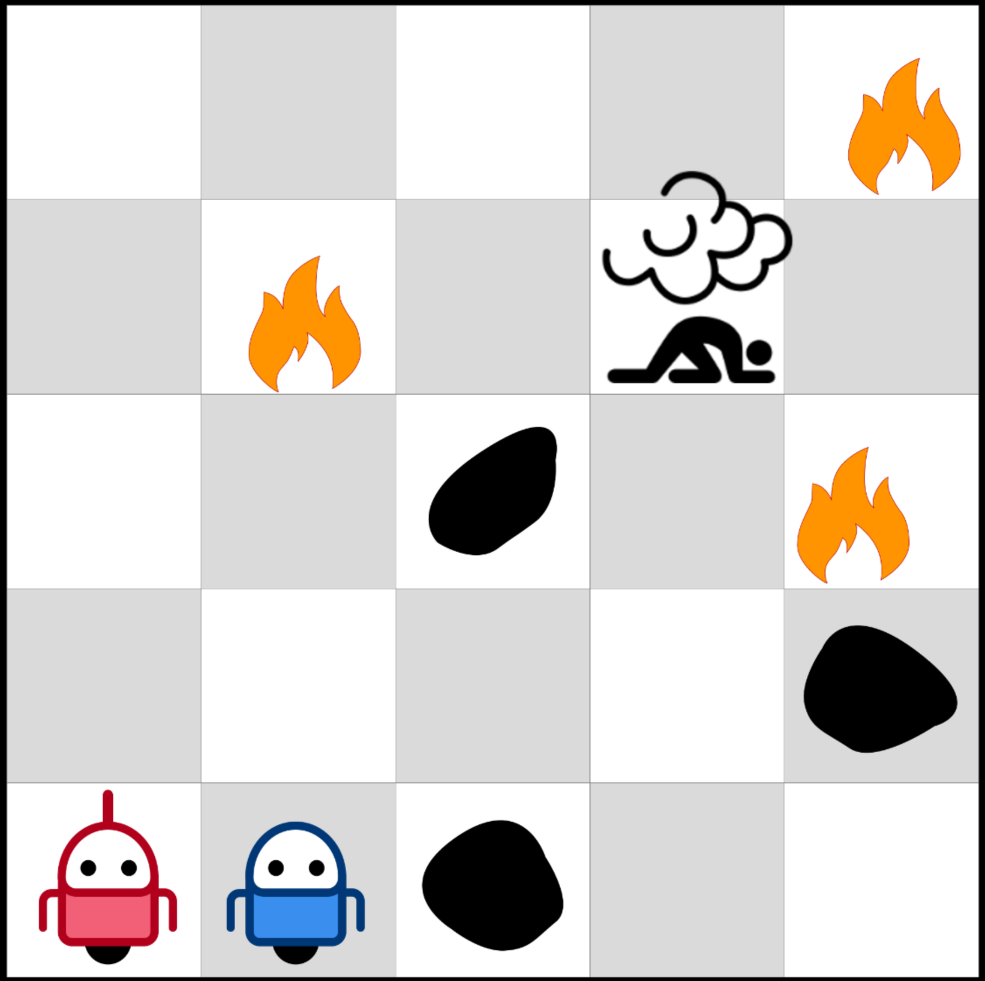
- Disaster zone
- Propositional Logic is a weak language
- First-order Logic
- Questions?

- A building of size **WIDTH** × **LENGTH**

- RR1 starts in square (1, 1)
- BR2 starts in square (2, 1)

- There is a **victim** somewhere

- Robots of type SRR have sensors that can detect **heat** and **drop-offs**
  - They reliably sense drop-offs and victims from exactly one (non-diagonal) block away
  - They are not good enough to say in what direction the victim or hole was



- Propositional Logic is **declarative**

- Propositional Logic is **compositional**: the meaning of $p \wedge q$ is derived from meaning of p and of q

- Propositional Logic is **context-independent** (unlike natural language)

- Propositional Logic assumes that the world contains **only facts**

Unfortunately, Propositional Logic has very **limited** expressive power (unlike natural language):

- Hard to identify **individuals**:
  Every pit causes breeze in adjacent squares
- Can’t directly talk about **properties** and relations between individuals:
  RR1 is red
- **Generalizations**, **patterns**, **regularities** can’t easily be represented:
  All SRR robots have a left arm

SECTION 02

First-order Logic

First-order Logic is a powerful evolution of Propositional Logic. It models the world in terms of:

- **Objects**: things with individual identities
- **Properties**: properties of objects that distinguish them from other objects
- **Relations**: relationships that hold among sets of objects
- **Functions**: a subset of relations where there is only one value for any given input

- Examples:
  - **Objects**: $Victim$, $Robot$, $RR1$, $RB2$, $Sq11$, $Sq12, \ldots$
  - **Properties**: $\text{blue}(Robot), \text{red}(Robot)$
  - **Relations**: $\text{color}(x, Red), \text{same\_type}(RR1, RB2)$
  - **Functions**: $\text{location}(Robot, Step0) = Sq11$

- **Term**: Objects, functions, or variables
- **Proposition**: Relations, or property
- **Connectives**:

$\wedge$	**AND**	Conjunction
$\vee$	**OR**	Disjunction
$\rightarrow$	**IMPLIES**	Implication / conditional
$\leftrightarrow$	**IS EQUIVALENT**	Biconditional
$\neg$	**NOT**	Negation

- **Quantifiers**:

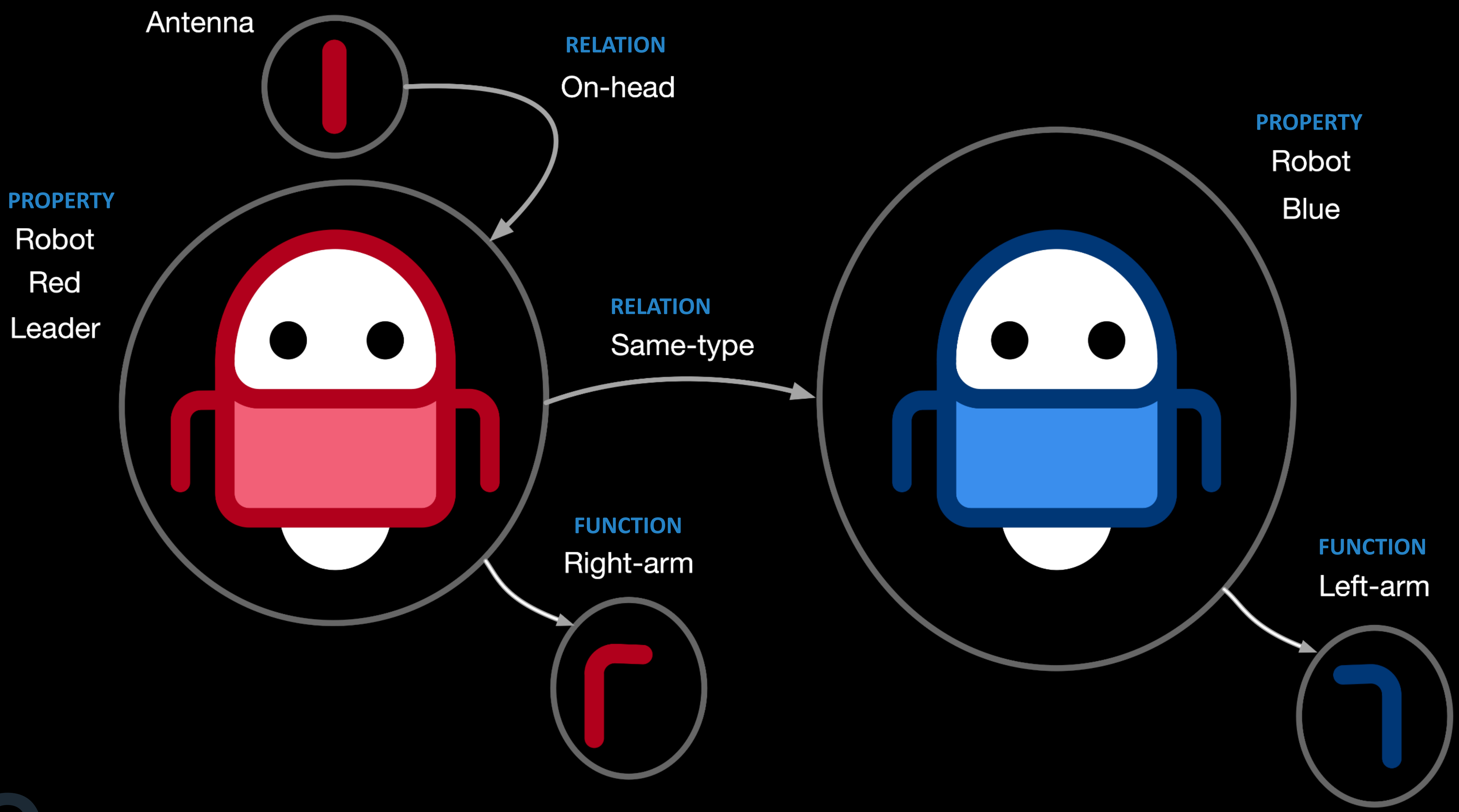
$\exists$	**EXISTS**	Existential quantifier
$\forall$	**FOR ALL**	Universal quantifier

- **Operator precedence**: $\neg = \wedge \vee \rightarrow \leftrightarrow$

- Sentences are **true** with respect to a **model** and an **interpretation**
- Model contains objects (**domain elements**) and relations among them
- Interpretation specifies referents for:

constant symbols	→	objects
predicate symbols	→	relations
function symbols	→	functional relations

- A sentence $predicate(term_1, \ldots, term_n)$ is **true** iff the **objects** referred to by $term_1, \ldots, term_n$ are in the **relation** referred to by **predicate**



Example of First-order Logic application

Existential quantification:

- $\exists \boldsymbol{x}$: $\boldsymbol{P}(\boldsymbol{x})$ **means** that P holds for some value of x in the domain associated with that variable:

  $\exists x$: $\mathbf{square}(x) \land \mathbf{unsafe}(x)$ means **There is an unsafe square**

- It permits one to make a statement about some object without naming it

- The existential quantifier is equivalent to the **disjunction** of **instantiations** of $P(x)$ for all the objects defined in the domain:

$$\begin{array}{l} \mathbf{square}(x_1) \ \land \ \mathbf{unsafe}(x_1) \ \lor \\ \mathbf{square}(x_2) \ \land \ \mathbf{unsafe}(x_2) \ \lor \\ \mathbf{square}(x_3) \ \land \ \mathbf{unsafe}(x_4) \ \lor \\ \vdots \end{array}$$

Existential quantifiers are normally used with **and** (∧) to specify a list of properties about that object:

$\exists x\colon \text{square}(x) \ \wedge\ \text{free}(x)$ means **There is an object that is square and free**

A common mistake is to use it with an implication (→):

$\exists x\colon \text{square}(x) \rightarrow \text{free}(x)$ means **There is a square object, and it is also free**

Universal quantification:

- $\forall x$: $P(x)$ means that P holds for all values of x in the domain associated with that variable:

$$\forall x\text{: adjacent}(x, \text{Sq12}) \land \text{unsafe}(x) \rightarrow \text{dropoff-detected}(x)$$

- The universal quantifier is equivalent to the **conjunction** of instantiations of $P(x)$ for all the objects defined in the domain:

$$\begin{gathered}\text{adjacent}(x_1, \text{Sq12}) \land \text{unsafe}(x_1) \rightarrow \text{dropoff−detected}(x_1) \land \\ \text{adjacent}(x_2, \text{Sq12}) \land \text{unsafe}(x_2) \rightarrow \text{dropoff−detected}(x_2) \land \\ \text{adjacent}(x_3, \text{Sq12}) \land \text{unsafe}(x_3) \rightarrow \text{dropoff−detected}(x_3) \land \\ \vdots\end{gathered}$$

Universal quantifiers are often used with "→" to form **rules**.

Universal quantification is rarely used to make blanket statements about every individual in the world:

$\forall x\colon \mathrm{unsafe}(x)$ means **Every object [in the world] is unsafe**

$\forall x\colon \mathrm{square}(x) \wedge \mathrm{unsafe}(x)$ means **Every object [in the world] is square and unsafe**

Switching the order of universal quantifiers does not change the meaning:

$$\forall x\, \forall y\colon\ P(x,y) \leftrightarrow \forall y \forall x\ P(x,y)$$

Switching the order of existential quantifiers does not change the meaning:

$$\exists x\, \exists y\colon\ P(x,y) \leftrightarrow \exists y \exists x\ P(x,y)$$

Switching the order of existential and universal quantifiers does change the meaning:

$\forall x\ \exists y\colon \mathbf{loves}(\mathrm{x}, \mathrm{y})$ or $\forall x\ (\exists y\colon \mathbf{loves}(x, y))$ means **...?**

$\exists \mathrm{y}\ \forall x\colon \mathbf{loves}(\mathrm{x}, \mathrm{y})$ or $\exists \mathrm{y}\ (\forall x\colon \mathbf{loves}(x, y))$ means **...?**

- We can relate sentences involving ∀ and ∃ using De Morgan's laws:

SENTENCE	EQUIVALENT
$\forall x:\ \neg P(x)$	$\nexists x:\ P(x)$
$\neg\forall x:\ P(x)$	$\exists x:\ \neg P(x)$
$\forall x:\ P(x)$	$\nexists x:\ \neg P(x)$
$\exists x:\ P(x)$	$\neg\forall x:\ \neg P(x)$

FOL introduces the concept of equality:

- At timestep 0, the robot was at Square (1, 1).
  $\mathbf{location}(\mathrm{Robot}, \mathrm{T0}) = \mathrm{Sq11}$

- The robot was located at different places in timestep 0 and timestep 1.
  $x = \mathbf{location}(\mathrm{Robot}, \mathrm{T0}) \land y = \mathbf{location}(\mathrm{Robot}, \mathrm{T1}) \land \neg\, (x = y)$
  $x = \mathbf{location}(\mathrm{Robot}, \mathrm{T0}) \land y = \mathbf{location}(\mathrm{Robot}, \mathrm{T1}) \land (x \neq y)$

- The robot is different from the victim.
  $\neg(\mathrm{Robot} = \mathrm{Victim})$
  $\mathrm{Robot} \neq \mathrm{Victim}$

- Every gardener likes the sun.
  $\forall x\colon\ \mathbf{gardener}(x) \rightarrow \mathbf{likes}(x, \mathbf{Sun})$

- You can fool some of the people all the time.
  $\exists x\ \forall t\colon\ (\mathbf{person}(x) \wedge \mathbf{time}(t)) \rightarrow \mathbf{can\text{-}fool}(x, t)$

- You can fool all the people some of the time.
  $\forall x\ \exists t\colon\ (\mathbf{person}(x) \wedge \mathbf{time}(t)) \rightarrow \mathbf{can\text{-}fool}(x, t)$

- All purple mushrooms are poisonous.
  $\forall x\colon\ (\mathbf{mushroom}(x) \wedge \mathbf{purple}(x)) \rightarrow \mathbf{poisonous}(x)$

- No purple mushroom is poisonous.
  $\nexists x\colon \mathbf{purple}(x) \land \mathbf{mushroom}(x) \land \mathbf{poisonous}(x)$
  $\forall x\colon\ \mathbf{mushroom}(x) \land \mathbf{purple}(x)) \rightarrow \neg\mathbf{poisonous}(x)$

- There are exactly two purple mushrooms.
  $\exists x\ \exists y\colon \mathbf{mushroom}(x) \land \mathbf{purple}(x) \land \mathbf{mushroom}(y) \land \mathbf{purple}(y) \land \neg(x = y) \land$
  $\qquad \forall z\colon (\mathbf{mushroom}(z) \land \mathbf{purple}(z)) \rightarrow ((x = z) \lor (y = z))$

- John is not tall.
  $\neg\mathbf{tall}(\mathrm{John})$

PRACTICE

Exercises from the textbook (chapter 8):
8.6, 8.10, 8.11, 8.23, 8.24

READINGS

Chapters 8

QUESTIONS ?

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VERSION 4.1