Computation Theory (COMP 170), Fall 2020
Recitation 04

---

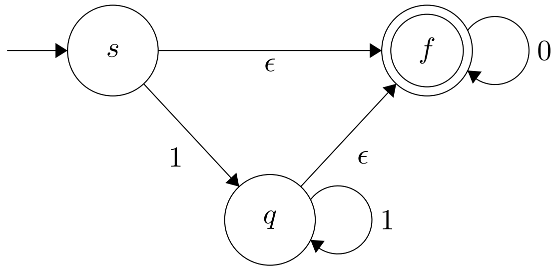
[1] NFA $\leftrightarrow$ Expressions

a. Consider the language $\{w \mid w = (01)^k \vee w = (10)^k, k \geq 0\}$.

Give a regular expression for the language.

Use the Regular-expression-to-NFA procedure (Lemma 1.55 of Sipser) to create an NFA for that language; show each of the machines you build, from simplest to most complex, as you go.

b. Consider the following NFA:



Use the NFA-to-regular-expression procedure (Lemma 1.60 of Sipser) to construct a regular expression that represents the same language; show each of the generalized finite automata that you build as you go.

[2] Which is Which?

Let A be any language over some alphabet Σ, and consider the following two functions:

$$\begin{aligned} cyc(A) &= \{x \mid x \text{ is a cyclic permutation of some } y \in A\} \\ perm(A) &= \{x \mid x \text{ is a permutation of some } y \in A\} \end{aligned}$$

For example, if $A = \{ab, abc\}$ then $cyc(A) = \{ab, ba, abc, bca, cab\}$ whereas $perm(A) = \{ab, ba, abc, acb, bac, bca, cab, cba\}$.

Suppose A is regular. Then one of $cyc(A)$ and $perm(A)$ is necessarily regular, while the other might not be regular. Which is which? Give a proof for one and a counterexample for the other.

[3] Context-Free Grammars

a. Give a context-free grammar G for the language $A = \{a^k b^n : k, n \geq 1, 2k \geq n\}$.

b. Prove by induction that $L(G) = A$. That is, show that $A \subseteq L(G)$ and $L(G) \subseteq A$. For best results, use the context-free proof paradigm resource to help structure your proofs.