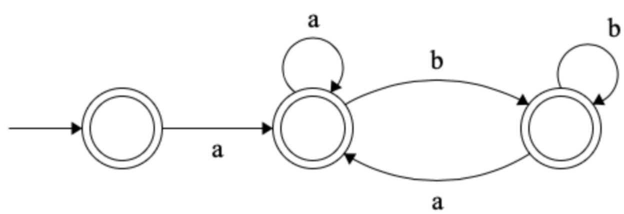
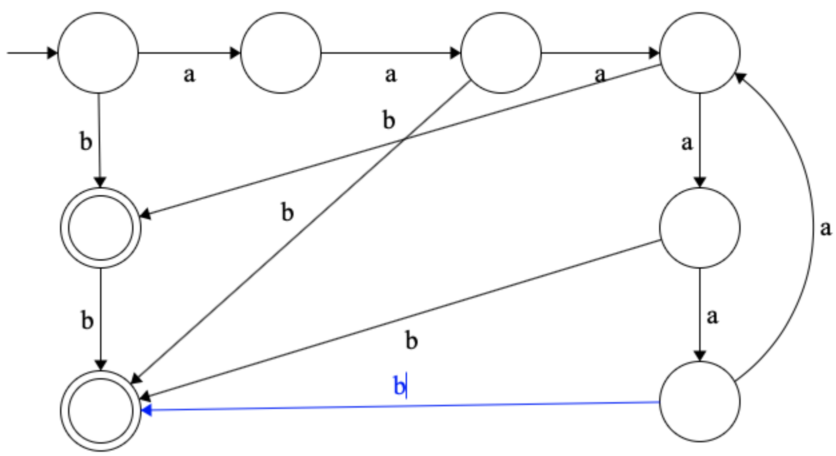
Erli Cai

Question 1: Expressions to Machines

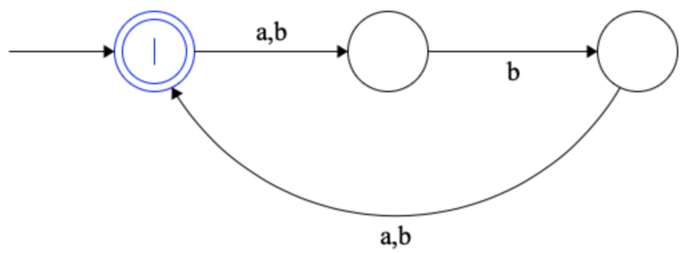
(a)



(b)



(c)



Question 2: Just One Difference

$$
\begin{aligned}
L_1 &= \{xy \mid x, y \in \{a, b\}^*, \#a(x) = \#b(y)\} \\
L_2 &= \{xcy \mid x, y \in \{a, b\}^*, \#a(x) = \#b(y)\}
\end{aligned}
$$

L_1 is regular while L_2 is not.

$L_1 = \{a, b\}^*$
Therefore, L_1 is regular

Now, prove L_2 is not regular.
For any given $p \geq 1$
let $w = a^p cb^p$
For any ways to writes w as $w = xyz$ with $|xy| \leq p$, $|y| \geq 1$,
y must be a string of a(s) and have at least one a, $y = a^k$, $1 \leq k \leq q$.
Therefore, if we pump y twice we get $a^{k+q}cb^q$, which is not an element of L_2.
In conclusion, L_2 is not regular.

Question 3: Context Free Grammars

a. $S \rightarrow aS|bS|\epsilon$

b. First proving that $L(G) \subseteq A$
Base Case:
$S \xrightarrow{1} \epsilon$ If $x = y = \epsilon$, then $\#a(x) = \#b(y)$, so $\epsilon \in A$

For the inductive step, suppose the claim is true for some $i \geq 1$. That is, suppose we know that for any $m \in \Sigma$, if $S \xrightarrow{i} m$ then $m \in A$. Now consider $n \in \Sigma$ such that $S \xrightarrow{i+1} n$;we want to show that $n \in A$. There are four cases:
1) $S \xrightarrow{1} aS \xrightarrow{s} n$. Then n = am for some $m \in \Sigma^*$. By induction hypothesis, m = xy where $\#a(x) = \#b(y)$ If we let $x' =$ a followed by x except last charter in x and $y' =$ last character in x followed by y,
either this character is a, then $\#a(x') = 1 + \#a(x) - 1 = \#a(x)$ and $\#b(y') = 0 + \#b(y) = \#b(y)$
or this character is b, then $\#a(x') = \#a(x) - 1$ and $\#b(y') = \#b(y) - 1$
Either way, we still get $\#a(x') = \#b(y')$, so $n \in A$
2) $S \xrightarrow{1} bS \xrightarrow{s} n$. Then n = bm for some $m \in \Sigma^*$. By induction hypothesis, m = xy where $\#a(x) = \#b(y)$ If we let $x' =$ a followed by x and y' =y,
then $\#a(x') = \#a(x)$ and $\#b(y') = \#b(y)$
we still get $\#a(x') = \#b(y')$, so $n \in A$
By mathematical induction, $n \in A$. Thus $L(G) \subseteq A$

Now proving that $A \subseteq L(G)$
Base case: $m = \epsilon$. Then $S \xrightarrow{1} \epsilon$ is the derivation for m. Induction Hypothesis: For the inductive step, consider a palindrome n with $|n| \geq 1$ and suppose the claim is true for all shorter n. We must have either n = am or n = bm for some m. If n= am then we can use derivation $S \xrightarrow{1} aS \xrightarrow{*} am = n$ If n = bm then we can use derivation $S \xrightarrow{1} bS \xrightarrow{*} bm = n$ By mathematical induction, $n \in L(G)$. Thus, $A \subseteq L(G)$