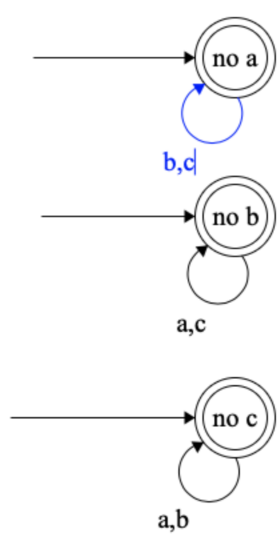
Question 1: shorties

a. If A is non-regular, then $\bar{A}$ is also non-regular
Because we know that regular language is closed under complement (if X is regular then $\bar{X}$ is also regular).
Then if $\bar{A}$ is regular, then complement of $\bar{A}$, which is just A, will also be regular. This is a contradiction thus $\bar{A}$ is non-regular

b. If A and B are Non-regular. $(A \cup B)$ could still be regular.
Example: Suppose A is non regular and B = $\bar{A}$. Then B is non-regular (as shown in part a) and $(A \cup B) = \Sigma^*$ will be regular.

c. $N = (Q, \Sigma, \Delta, S, F)$ is an NFA and $N' = (Q, \Sigma, \Delta, S, Q \backslash F)$, $L(N')$ is **not** the complement of $L(N)$
Suppose $s \in \Sigma^*$ and it end up in a set of states $\{q_1, q_2 | q_1 \in Q \backslash F, q_2 \in Q\}$. Then is accepted by both N and N'. ($s \in L(N)$ and $s \in L(N')$) . $L(N) \cap L(N') \neq \emptyset$ so they are not complement of each other.

d.



Question 2: Finite Automata

$mix(A, B) = \{mix(v, w)|v \in A, w \in B, |v| = |w|\}$
Suppose A and B are regular languages, then we have
$M_A = (Q_A, \Sigma, \delta_A, s_A, F_A)$
$M_B = (Q_B, \Sigma, \delta_B, s_B, F_B)$
where $L(M_A) = A$ and $L(M_B) = B$
To show that mix(A,B) is also regular, we construct a NFA $M = (Q, \Sigma, \delta, s, F)$ such that $L(M) = mix(A, B)$

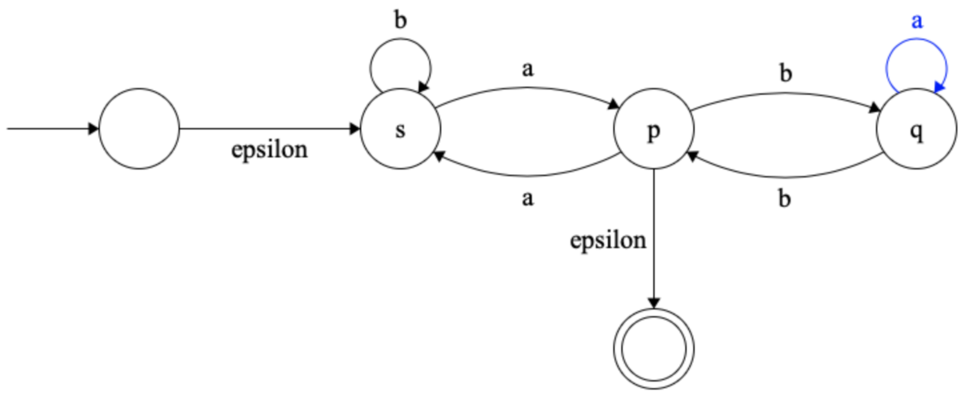
$$\begin{aligned} Q &= Q_A \times Q_B \times \{1, 0\} \\ \delta((a, b, 0), x) &= (\delta_A(a, x), b, 1) \\ \delta((a, b, 1), x) &= (a, \delta_B(b, x), 1) \\ s &= (s_A, s_B, 0) \\ F &= (f_a, f_b, 0) \quad \text{for } f_a \in F_A, f_b \in F_B \end{aligned}$$

The state of M is a 3-tuple with the first element from Q_A, second element from Q_B and third element from {0,1} indicating whether next character is from A or from B. In each transition, the third element alternate between 0 and 1, and while it equals to 0 we apply transition Q_A to first element , while it equals to 1 we apply transition Q_B to second element.

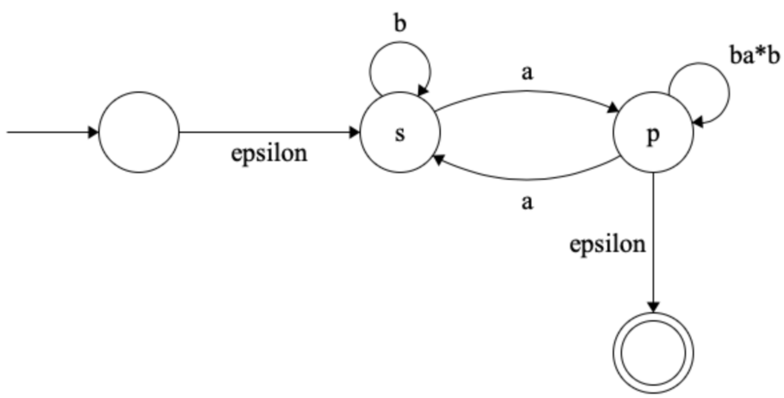
A string is accepted if first two elements are from Accepted state of A and B respectively. And third element is 0 indicating that the element if of even length.

Question 3: Machines to Expressions

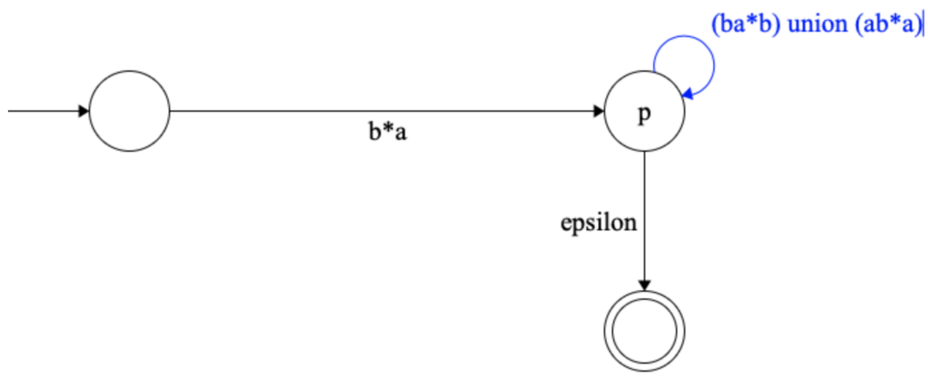
Add a super start state and super accepted state



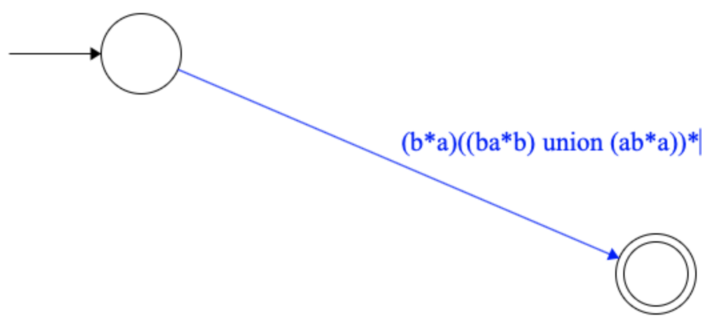
eliminate state q



eliminate state s



eliminate state p



Thus $R = (ba*)((ba*b) \cup (ab*a))*$

Question 4: Non-Regular

Let A be the set of all odd-length strings over {a, b} whose middle character is a. Assume A is regular: then pumping lemma should be true.

For any $p > 0$,
we pick $w = b^p a a^p$
we then write $w = xyz$ where $|y| > 0$ and $|xy| <= p$
Then y must be a string of b's ($y = b^k$, $p >= k > 1$)
If we pump y twice, we have $w' = b^{p+k} a a^p$.
If k is odd, then w' is of even length, so it is not in A.
If k is even, the middle character of w' should be $(p + \frac{k}{2} + 1)$-th element character which must be a b. So it is not in A
It contradicts to Pumping Lemma. Thus our assumption that A regular is false.

Question 5: CFGs

$S \to aSb|bSa|aAb|bAa$
$A \to a|b|aa|bb|aAa|bAb|aSa|bSb$

Explanation: A non-palindrome string must be asymmetric from center of the string. In my CFG, A will add symmetric part and S will add asymmetric part. Since S can't get to any terminals, we must have a at least one derivation from S to something, which means there will be some asymmetric part in the string and the string is non-palindrome