PROPOSITIONAL LOGIC 1

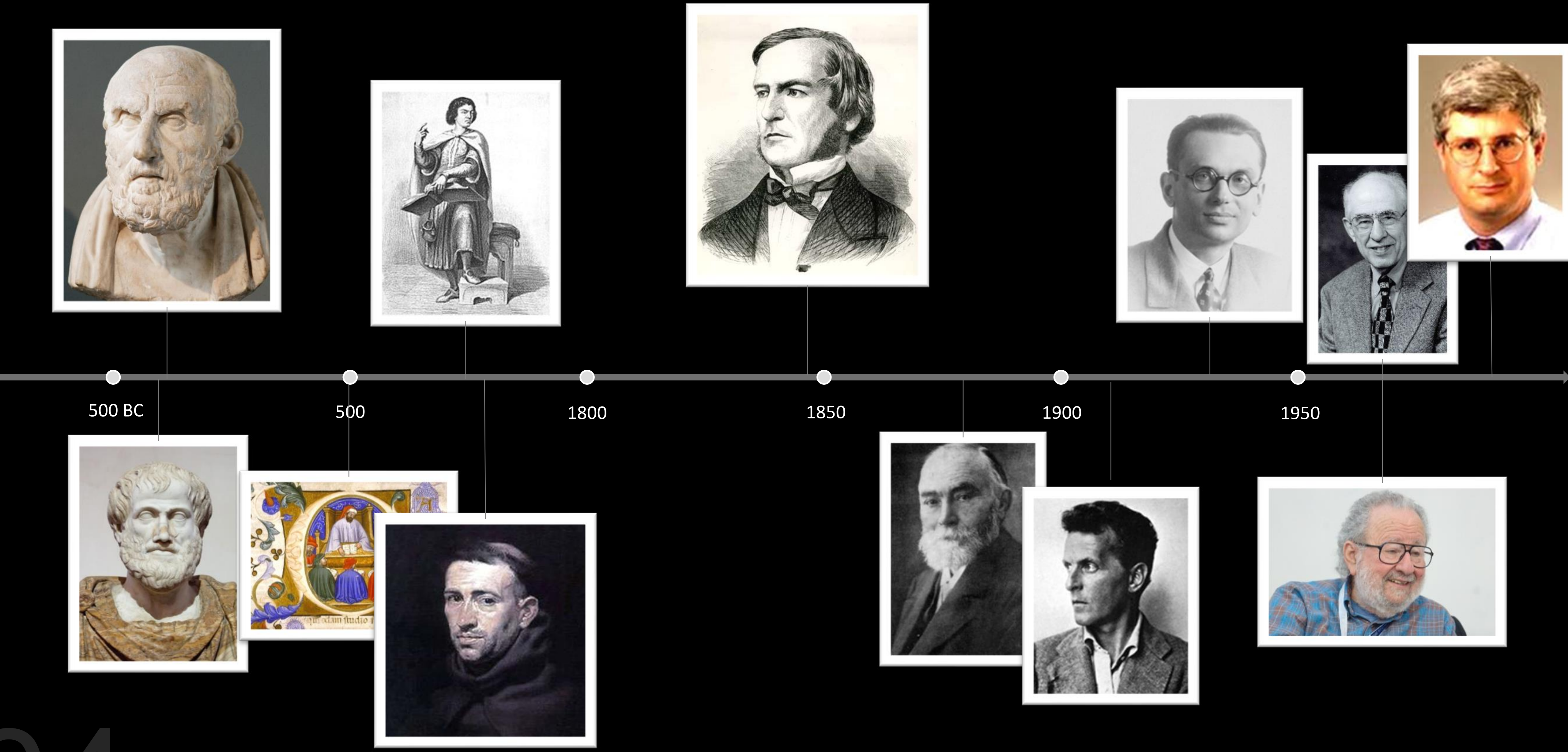
ARTIFICIAL INTELLIGENCE | COMP 131

TODAY ON AI

- What is Logic?
- Propositional Logic
- Questions?

SECTION 01

What is Logic?



04

Brief history of Logic

Logic is a way of formally representing the **state of the world** and the world's **rules of operation** so that we can make **rational decisions** and learn **new knowledge** based on our **existing one**.

It allows to:

- **Express knowledge** using a formal language
- **To carry out** reasoning in that language

There are several types of logic. Each type is increasingly complex as it captures more advanced concepts:

LANGUAGE	ONTOLOGICAL COMMITMENT	EPISTEMOLOGICAL COMMITMENT
Propositional logic	Facts	True / False / Unknown
First-order logic	Facts, Objects, Relations	True / False / Unknown
Temporal logic	Facts, Objects, Relations, Times	True / False / Unknown
Probability theory	Facts	Degree of belief $\in [0, 1]$
Fuzzy logic	Facts with degree of truth $\in [0, 1]$	Known interval value
Markov logic	Facts, Objects, Relations	Degree of belief $\in [0, 1]$

- **Programming languages**:
  - They are formal and not ambiguous
  - They lacks expressivity as they cannot accommodate partial information

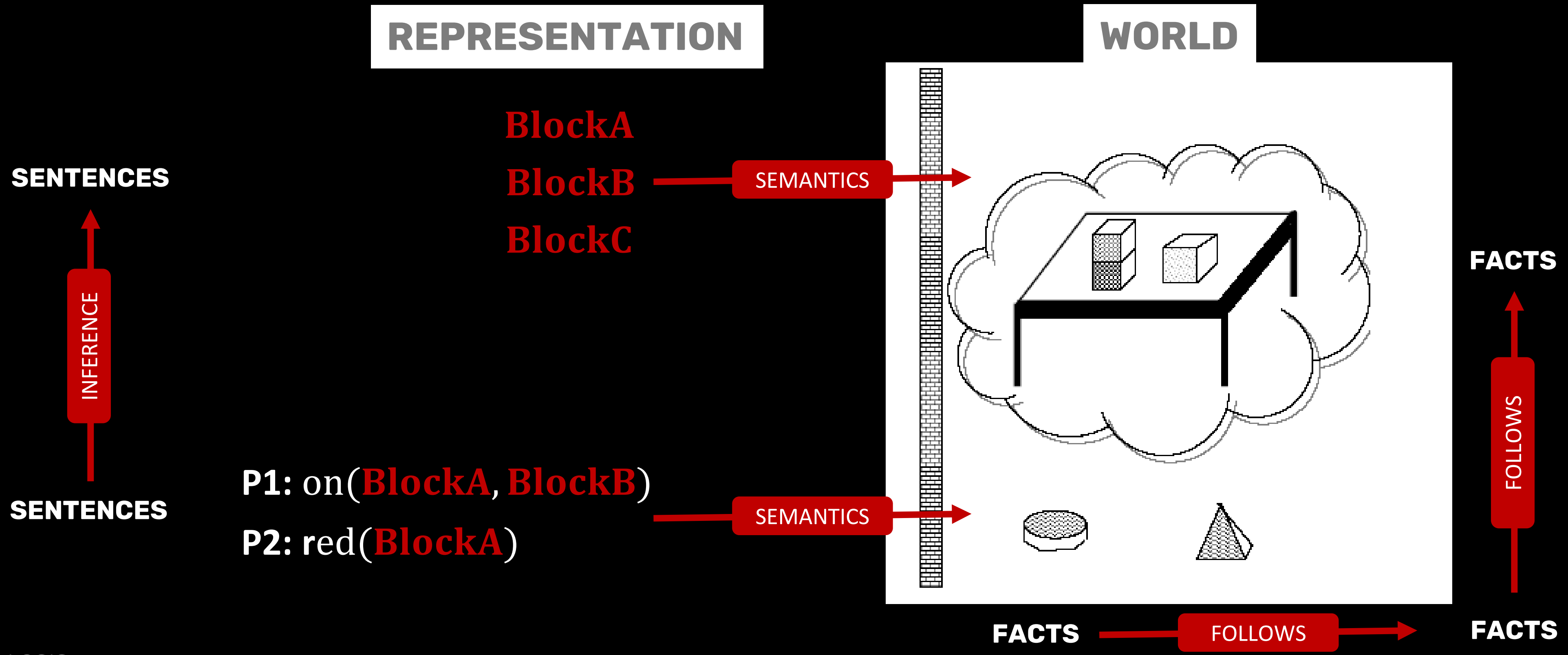
- **Natural Language**:
  - Very expressive but also ambiguous:
  - Inference possible, but hard to automate

Flying planes can be dangerous.
The teacher gave the boys an apple.

- A good **representation language** is:
  - Both formal and can express partial information
  - Can accommodate inference

The **fundamental elements** of a formal logic are:

- **Syntax**: it is the set of symbols and rules used to express knowledge
- **Semantics**: it specifies the way symbols and sentences relate to the world
- **Inference procedures**: they describe the rules for deriving new sentences (and therefore, new semantics) from existing sentences



The meaning or **semantics** of a sentence determines its **interpretation**, that is the **truth** (true, false, or unknown) value of each sentence with respect to each **possible worlds**:

- **Possible worlds** lists the scenarios in which an agent might be
- A possible **model m** for a sentence is a **possible world** in which the assignment of truth values to each propositional symbols in the sentence is **fixed** to true or false
- If a sentence s is true in model m, we say that the model m **satisfies** s or that m **is a model of s**
- A sentence is **satisfiable** is there is at least a single model m in all possible worlds

When sentences follow logically each other, we talk about of **entailment**:

- $\boldsymbol{s}$ **entails** $\boldsymbol{p}$, written $s \vDash p$, means that whenever s is **true**, so is p. In other words, all models of s are also models of p or $M(s) \subseteq M(p)$.

  p is entailed by s iff there is no logically possible world in which p is **false** while the premise s is **true**.

Example:

- The prime minister died $\vDash \text{The prime minister was assinated}$

SECTION 02

Propositional Logic: Syntax

- A **sentence** expresses a possible condition of the world

- A sentence can either **true (T)** or **false (F)**

- Most basic sentences are called **simple**, **predicates** or **atomic sentences**
  am_wet, is_raining, have_umbrella

- **Predicates**: Symbols or constants **true (T)** or **false (F)**
- **Symbols**: p, q, s, …
- Sentences are combined by **connectives** to produce other **sentences**:

$\wedge$	**AND**	Conjunction
$\vee$	**OR**	Disjunction
$\rightarrow$	**IMPLIES**	Implication / conditional
$\leftrightarrow$	**IS EQUIVALENT**	Biconditional
$\neg$	**NOT**	Negation

- o means "**It is hot**"
- h means "**It is humid**"
- r means **"It is raining"**
- $(o \wedge h) \rightarrow$ r means "**IF it is hot AND humid, THEN it is raining**"
- $q \rightarrow p$ means **"IF it is humid, THEN it is hot"**

A better way to write sentences:

- Hot: **"It is hot"**
- Humid: **"It is humid"**
- Raining: **"It is raining"**

$(o \wedge h) \rightarrow r$ $\quad$ $\text{hot} \wedge \text{humid} \rightarrow \text{raining}$

A **sentence** is a well-formed formula that can be defined recursively:

- A **symbol** is a proposition
- If s is a sentence, then $\neg s$ is a sentence
- If s is a sentence, then (s) is a sentence
- If s and t are sentences, then $(s \land t), (s \lor t), (s \rightarrow t)$ and $(s \leftrightarrow t)$ are sentences
- A sentence results from a finite number of iterations of the above rules

- **Operator precedence**: $\neg \ \land \ \lor \ \rightarrow \ \leftrightarrow$

- A **tautology** is a sentence that is **true** under all interpretations, no matter what the world is like or how the semantics are defined. Example: “It’s raining or it’s not raining.”

- A **contradiction** is a sentence that is **false** under all interpretations. The world is never like what it describes, as in “It’s raining and it’s not raining.”

p	$p \wedge \neg p$	$p \vee \neg p$
F	F	T
T	F	T

SECTION 03

Propositional Logic: Semantics

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$
F	F	T	F	F	T	T
F	T	T	F	T	T	F
T	F	F	F	T	F	F
T	T	F	T	T	T	T

- $\neg p$ is a **negation** of p: $\neg \text{am\_wet}$
- $p \wedge q$ is a **conjunction** of p **and** q: $\text{am\_wet} \wedge \neg \text{is\_raining}$
- $p \vee q$ is a **disjunction** of p **or** q: $\text{am\_wet} \vee \neg \text{is\_raining}$
- $p \rightarrow q$ is an **implication** of p (**premise**) **implies** q (**conclusion**): $\text{am\_wet} \rightarrow \text{is\_raining}$
- $p \leftrightarrow q$ is a **biconditional** of p **if-and-only-if (iff)** q: $\text{am\_wet} \leftrightarrow \text{is\_raining}$

Simple examples of knowledge base and models:

$\text{is\_raining} \land \text{am\_wet}$

model: {
$\text{is\_raining} = \boldsymbol{true}$,
$\text{am\_wet} = \boldsymbol{true}$}

$\text{is\_raining} \rightarrow \text{am\_wet}$

$\text{is\_raining} \rightarrow \text{take\_umbrella}$

model 1: {
$\text{is\_raining} = \boldsymbol{false}$,
$\text{am\_wet} = \boldsymbol{true}$,
$\text{take\_umbrella} = \boldsymbol{true}$}

model 2: {
$\text{is\_raining} = \boldsymbol{true}$,
$\text{am\_wet} = \boldsymbol{true}$,
$\text{take\_umbrella} = \boldsymbol{true}$}

$\text{is\_raining} \rightarrow \text{am\_wet}$

$\text{is\_raining} \land \text{have\_umbrella} \rightarrow \text{open\_umbrella}$

model: {
$\text{is\_raining} = \boldsymbol{true}$,
$\text{am\_wet} = \boldsymbol{true}$,
$\text{have\_umbrella} = \boldsymbol{true}$,
$\text{open\_umbrella} = \boldsymbol{true}$}

FORM	EQUIVALENCE	NAME
$p \wedge \mathbf{T},\ p \vee \mathbf{F}$	p	**Identity laws**
$p \vee \mathbf{T}$ $p \wedge \mathbf{F}$	$\mathbf{T}$ $\mathbf{F}$	**Domination laws**
$p \vee p,\ \ p \wedge p$	p	**Idempotent laws**
$\neg(\neg p)$	p	**Double negation law**
$p \vee q$ $p \wedge q$	$q \vee p$ $q \wedge p$	**Commutative laws**
$(p \vee q) \vee r$ $(p \wedge q) \wedge r$	$p \vee (q \vee r)$ $p \wedge (q \wedge r)$	**Associative laws**
$p \vee (q \wedge r)$ $p \wedge (q \vee r)$	$(p \vee q) \wedge (p \vee r)$ $(p \wedge q) \vee (p \wedge r)$	**Distributive laws**
$\neg(p \wedge q)$ $\neg(p \vee q)$	$\neg p \vee \neg q$ $\neg p \wedge \neg q$	**De Morgan's laws**
$p \to q$	$\neg q \to \neg p$	**Contrapositive equivalence**
$p \vee \neg p$	$\mathbf{T}$	**Excluded middle**
$p \wedge \neg p$	$\mathbf{F}$	**Negation creates opposite**

Evaluation seeks to **determine the truth value** of a sentence within the specified possible world:

Let's decide that:

- is_raining means **It's raining outside**
- have_umbrella means **I have an umbrella**
- am_wet means **I am wet**

We can condense propositions replacing them with symbols:

- r means: **It's raining outside**
- u means: **I have an umbrella**
- w means: **I am wet**

EVALUATION

Possible world: $\{w = \mathbf{T}, r = \mathbf{F}, u = \mathbf{T}\}$

Sentence: $(\neg w \land r) \land (\neg r \lor u)$

Result of the evaluation: **F**

Satisfaction seeks to **determine the set of models m** for the sentence.

Let's decide that:

- is_raining means **It's raining outside**
- have_umbrella means **I have an umbrella**
  am_wet means **I am wet**

We can condense propositions replacing them with symbols:

- r means: **It's raining outside**
- u means: **I have an umbrella**
- w means: **I am wet**

SATISFACTION
Sentence: $(\neg w \land r) \land (\neg r \lor u)$
Model: $\{w = ?, r = ?, u = ?\}$

Result of satisfiability:

w	r	u	S
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	T
T	F	F	F
T	F	T	F
T	T	F	F
T	T	T	F

SECTION 04

Propositional Logic: Inference procedures

A **knowledge base** is a set of sentences that lists all the rules that the agent has about the world.

A KB is **false** in models that contradict what the agent knows.

If p is a sentence and KB is a knowledge base, a fair question to ask is:
does KB entail P?

Proof by refutation is a complete inference procedure that is used to prove entailment. It tries to prove something demonstrating the opposite:

- It uses a single inference rule or **resolution**
- The knowledge base is represented in a **Conjunctive Normal Form** (or **CNF**)
- It **reduces** inference to the problem of checking satisfiability

A sentence is in **Conjunctive Normal Form** (or CNF) if it is a **conjunction of disjunction terms**:

- Examples:
  - $\boxed{q} \land \boxed{(r \lor s)}$ YES
  - $\boxed{p}$ YES
  - $\boxed{(p \lor \neg q)} \land \boxed{(r \lor s)}$ YES
  - $(p \lor q) \to r$ NO
  - $\neg(p \lor q \lor r)$ NO

- There is a way to convert a sentence into a clausal form:

$(p \lor q) \to r$ ➡ $(\neg p \lor r) \land (\neg q \lor r)$

$\neg(p \lor q \lor r)$ ➡ $\neg p \land \neg q \land \neg r$

SENTENCE	CLAUSAL FORM
$p \leftrightarrow q$	$p \to q$ $q \to p$
$p \to q$	$\neg p \lor q$
$\neg\neg p$	p
$\neg(p \lor q)$	$\neg p \land \neg q$
$\neg(p \land q)$	$\neg p \lor \neg q$

- The process is performed **conjoining** the KB with the **negation of the query**
- It's possible to say that KB entails the query iff **the truth table returns false in every row (contradiction)**

$$C_1 = (w \vee \neg u) \wedge (\neg u \wedge r \rightarrow w) \wedge \neg(r \wedge \neg w)$$

$$C_2 = (w \vee \neg u) \wedge ((\neg u \wedge r) \rightarrow w) \wedge (\neg r \vee \neg\neg w)$$

$$C_3 = (w \vee \neg u) \wedge (\neg(\neg u \wedge r) \vee w) \wedge (\neg r \vee w)$$

$$C_4 = (w \vee \neg u) \wedge (u \vee \neg r \vee w) \wedge (\neg r \vee w)$$

$$C_5 = (w \vee \neg u) \wedge (u \vee \neg r \vee w) \wedge (\neg r \vee w)$$

- Example:
  Given the following KB:
  1. $w \vee \neg u$
  2. $\neg u \wedge r \rightarrow w$

  S: $r \wedge \neg w$

  Does KB **entail** S (or $KB \vDash S$) ? **NO**

r	u	w	C_5
F	F	F	T
F	F	T	T
F	T	F	F
F	T	T	T
T	F	F	F
T	F	T	T
T	T	F	F
T	T	T	T

PRACTICE

Exercises from the textbook (chapter 7):

7.1, 7.4, 7.5, 7.7, 7.10

READINGS

Chapters 7

QUESTIONS ?

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