Computation Theory (COMP 170), Fall 2020
Assignment 09

Answer each problem below to the best of your ability. Submit all parts by 9:00 AM on Friday, December 4. List your collaborators. Late homework is accepted within 24 hours for half credit. After 24 hours no credit is given. The first late assignment (up to 24 hours) per student incurs no penalty. **Make sure that your submission follows the formatting guidelines given at the end of this document.**

Reading: Sipser Chapter 7

[1] (*8 pts.*)   **Really Independent**

Recall that given a graph $G = (V, E)$, an *independent set* $S \subseteq V$ is a set of vertices such that for all $x, y \in S$, $(x, y) \notin E$. That is, no two vertices in S are directly connected by an edge in G. This allows us to define the Independent Set problem

$$\text{IS} = \{\langle G, k \rangle \mid G \text{ has an independent set of size } k\}$$

Define a *really independent set* $S \subseteq V$ to be a set of vertices such that for all $x, y \in S$, there are no one-edge or two-edge paths from x to y in G. The Really Independent Set problem can be defined analogously:

$$\text{RIS} = \{\langle G, k \rangle \mid G \text{ has a really independent set of size } k\}$$

a. Prove that RIS $\in NP$.

b. Prove that IS $\leq_p$ RIS.

Be careful that you aren't proving the reverse reduction! See the NP-Completeness Proof Paradigm worksheet for tips.

[2] (*7 pts.*)   **Four: Slightly Bigger Than Three**

In 3-SAT instances, each clause has 3 terms (each term is either a variable x_i or its negation $\overline{x_i}$).

In a 4-SAT instance, each clause has, unsurprisingly, 4 terms. Prove that 4-SAT is NP-Complete.

Note, 4-SAT clauses may not have duplicate terms. That is, you can't just pad the clause $(x_1 \lor x_2 \lor x_3)$ with another x_3 to get $(x_1 \lor x_2 \lor x_3 \lor x_3)$. Nice try.

Format requirements: work for COMP 170 should correspond to the following guidelines:

- Work must be in type-written format, with any diagrams rendered using software to produce professional-looking results. No hand-written or hand-drawn work will be graded.

- Work must be submitted in PDF format to Gradescope.

- Each answer should start on a new page of the document. When possible, try to limit answers to a single page each. (Thus, the answers to this homework must be no less than three pages, and preferably no more.)

You can find links to information about using LaTeX to produce type-written mathematical work,[1] and to a handy web-based tool for drawing finite-state diagrams, on the Piazza class site:

https://piazza.com/tufts/fall2020/comp170/resources

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[1]LaTeX was used to produce this document.