

# Schrödinger Equation

Erlin Torres

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## 1 Introduction

In the present document is the derivation of the Schrödinger equation for harmonic oscillator and free particle in L<sup>A</sup>T<sub>E</sub>X.

## 2 Deduction

$$L = \frac{1}{2}mv^2$$

$$\psi(x, t + \varepsilon) = \int_{-\infty}^{\infty} d\zeta \frac{1}{c} e^{\frac{im}{\hbar 2\varepsilon} \zeta^2} \varphi(x + \zeta, t) \quad (1)$$

$$L = \frac{1}{2}mv^2 - V(x)$$
$$\psi(x, t + \varepsilon) = \int_{-\infty}^{\infty} d\zeta \frac{1}{c} e^{\frac{i}{\hbar} [\frac{m}{2\varepsilon} \zeta^2 - \varepsilon V(x)]} \varphi(x + \zeta, t) \quad (2)$$

$$\psi + \varepsilon \frac{\partial \psi}{\partial t} = (1 - \frac{iV}{\hbar} \varepsilon) [\int_{-\infty}^{\infty} d\zeta \frac{1}{c} e^{\frac{im}{\hbar 2\varepsilon} \zeta^2} + 0 + \int_{-\infty}^{\infty} d\zeta \frac{i}{c} e^{\frac{im}{\hbar 2\varepsilon} \zeta^2} \frac{1}{2} \zeta^2 \frac{\partial^2 \varphi}{\partial x^2}] \quad (3)$$

$$\psi + \varepsilon \frac{\partial \psi}{\partial t} = (1 - \frac{iV}{\hbar} \varepsilon) [\frac{i}{c} \sqrt{\frac{2\pi \hbar i \varepsilon}{m}} \psi + \frac{\sqrt{\pi}}{4c} (\frac{2\hbar \varepsilon i}{m})^2 \frac{\partial^2 \varphi}{\partial x^2}] \quad (4)$$

$$\psi + \varepsilon \frac{\partial \psi}{\partial t} = (1 - \frac{iV}{\hbar} \varepsilon) [1\psi + \frac{i\hbar}{2m} \varepsilon \frac{\partial^2 \varphi}{\partial x^2}] \quad (5)$$

$$\psi + \varepsilon \frac{\partial \psi}{\partial t} = \psi + \frac{i\hbar}{2m} \varepsilon \frac{\partial^2 \varphi}{\partial x^2} - \frac{iV}{\hbar} \varepsilon \psi + 0(\varepsilon^2) \quad (6)$$

$$\frac{\partial \psi}{\partial t} = \frac{i\hbar}{2m} \frac{\partial^2 \varphi}{\partial x^2} - \frac{iV}{\hbar} \psi \quad (7)$$

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \varphi}{\partial x^2} + V\psi$$