Schrödinger Equation

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1 Introduction

In the present document is the derivation of the Schrödinger equation for harmonic oscillator and free particle in LaTeX.

2 Deduction

$$L = \frac{1}{2}mv^2$$

$$\psi(x, t+\varepsilon) = \int_{-\infty}^{\infty} d\zeta \frac{1}{c} e^{\frac{im}{\hbar 2\varepsilon}\zeta^2} \varphi(x+\zeta, t) \tag{1}$$

$$L = \frac{1}{2}mv^2 - V(x)$$

$$\psi(x, t+\varepsilon) = \int_{-\infty}^{\infty} d\zeta \frac{1}{c} e^{\frac{i}{\hbar}[\frac{m}{2\varepsilon}\zeta^2 - \varepsilon V(x)]} \varphi(x+\zeta, t) \tag{2}$$

$$\psi + \varepsilon\frac{\partial\psi}{\partial t} = (1 - \frac{iV}{\hbar}\varepsilon)[\int_{-\infty}^{\infty} d\zeta \frac{1}{c} e^{\frac{im}{\hbar 2\varepsilon}\zeta^2} + 0 + \int_{-\infty}^{\infty} d\zeta \frac{i}{c} e^{\frac{im}{\hbar 2\varepsilon}\zeta^2} \frac{\partial^2\varphi}{\partial x^2}] \tag{3}$$

$$\psi + \varepsilon\frac{\partial\psi}{\partial t} = (1 - \frac{iV}{\hbar}\varepsilon)[\frac{i}{c}\sqrt{\frac{2\pi\hbar i\varepsilon}{m}}\psi + \frac{\sqrt{\pi}}{4c}(\frac{2\hbar\varepsilon i}{m})^2 \frac{\partial^2\varphi}{\partial x^2}] \tag{4}$$

$$\psi + \varepsilon\frac{\partial\psi}{\partial t} = (1 - \frac{iV}{\hbar}\varepsilon)[1\psi + \frac{i\hbar}{2m}\varepsilon\frac{\partial^2\varphi}{\partial x^2}] \tag{5}$$

$$\psi + \varepsilon\frac{\partial\psi}{\partial t} = \psi + \frac{i\hbar}{2m}\varepsilon\frac{\partial^2\varphi}{\partial x^2} - \frac{iV}{\hbar}\varepsilon\psi + 0(\varepsilon^2) \tag{6}$$

$$\frac{\partial\psi}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2\varphi}{\partial x^2} - \frac{iV}{\hbar}\psi \tag{7}$$

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2\varphi}{\partial x^2} + V\psi}$$