Наилучшая разностная схема для уравнения 2-го порядка в цилиндрических координатах.

24.

$$\begin{cases} \dfrac{1}{r}\dfrac{d}{dr}\left(r\lambda\dfrac{du}{dr}\right) + f(u) = 0 \\ r = 0 : \dfrac{du}{dr} = 0 \\ r = R : \dfrac{du}{dr} = \alpha u \end{cases}$$

Вводим поток F:

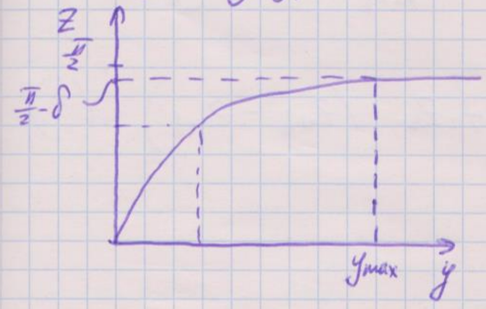
$$\begin{cases} F = -\lambda\dfrac{du}{dr} \\ -\dfrac{1}{r}\dfrac{d}{dr}(rF) + f(u) = 0 \\ r = 0 : \dfrac{du}{dr} = 0 \\ r = R : \dfrac{du}{dr} = \alpha u \end{cases}$$

Вводим $z = \dfrac{r}{R}$; $dz = \dfrac{dr}{R}$

$$\begin{cases} F = -\dfrac{\lambda}{R}\dfrac{du}{dz} \\ -\dfrac{1}{R}\cdot\dfrac{1}{z}\dfrac{d}{dz}(zF) + f(u) = 0 \\ z = 0 : \dfrac{du}{dz} = 0 \\ z = 1 : \dfrac{du}{dz} = \dfrac{\alpha}{R}u \end{cases}$$

Введем неравномерную сетку:

$z = \frac{1}{a}\,\mathrm{arctg}(y)$; $a = \mathrm{tg}(y_{max})$



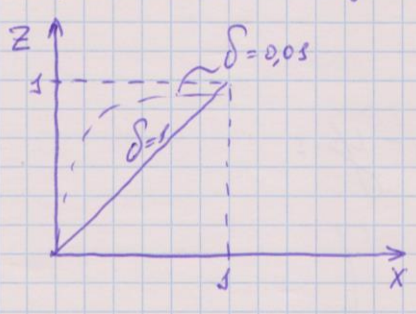
Перейдем к коорд. x:

$x = \frac{y}{y_{max}}$

$z = \frac{1}{a}\,\mathrm{arctg}(x y_{max})$

$$\frac{d}{dz} = \frac{d}{dx}\cdot\frac{dx}{dz} = \frac{d}{dx\cdot\frac{dz}{dx}} = \frac{d}{dx}\cdot\tilde{P}$$

$$\frac{dz}{dx} = \frac{y_{max}}{a}\cdot\frac{1}{1+x^2y^2_{max}}\ ; \quad \tilde{P} = \frac{a}{y_{max}}\cdot(1+x^2y^2_{max})$$



Система в неравномерной сетке:

$$\begin{cases} F = -\frac{1}{R}\cdot\lambda\cdot\frac{du}{dx}\cdot\tilde{P} & (1) \\ -\frac{1}{R}\cdot\frac{1}{z}\,\frac{d}{dx}(zF)\cdot\tilde{P} + f(u) = 0 & (2) \\ x = z = 0: \quad \frac{du}{dx} = 0 \\ x = z = 1: \quad \frac{du}{dx} = \frac{\alpha}{\tilde{P}R}\,u \end{cases}$$

Решаем интегро-интерполяционным методом:

Из (2):

$$\frac{1}{R}\int\limits_{x_{n-\frac12}}^{x_{n+\frac12}} \cancel{\frac{1}{z}}\,\frac{d}{dx}(zF)\,\cancel{\tilde{p}}\cdot \overbrace{z\,\frac{dx}{\cancel{\tilde{p}}}}^{=\,z\,dz \text{ из-за цилиндр. к-т}} = \int\limits_{x_{n-\frac12}}^{x_{n+\frac12}} f(u)\, z\, \overbrace{\frac{dx}{\tilde{p}}}^{=\,dz}$$

$$\frac{1}{R}\left(z_{n+\frac12}F_{n+\frac12} - z_{n-\frac12}F_{n-\frac12}\right) = f_n z_n\cdot\left(z_{n+\frac12} - z_{n-\frac12}\right) \qquad (3)$$

Из (1):

$$\int\limits_{x_n}^{x_{n+1}} F\,dx = -\frac{1}{R}\int\limits_{x_n}^{x_{n+1}} \lambda\frac{du}{dx}\tilde{p}\,dx$$

$$R\int\limits_{x_n}^{x_{n+1}} \frac{F}{\lambda\tilde{p}}\,dx = -\int\limits_{\lambda_n}^{x_{n+1}} du$$

$$R\,\frac{F_{n+\frac12}}{\lambda_{n+\frac12}}\cdot\left(z_{n+1} - z_n\right) = y_n - y_{n+1}$$

$$F_{n+\frac12} = \frac{y_n - y_{n+1}}{R\left(z_{n+1} - z_n\right)}\lambda_{n+\frac12}$$

Аналогично:

$$F_{n-\frac12} = \frac{y_{n-1} - y_n}{R\left(z_n - z_{n-1}\right)}\cdot\lambda_{n-\frac12}$$

(4)

Подставляем (4) в (3), группируем y_{n-1}, y_n и y_{n+1}, получаем ур-ие вида:

$$A_n y_{n-1} - B_n y_n + C_n y_n = -F_n$$

Решаем прогонкой.