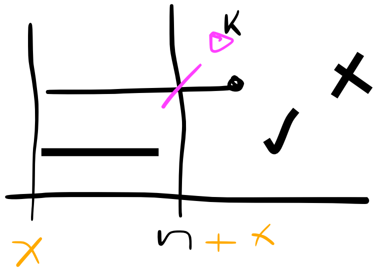
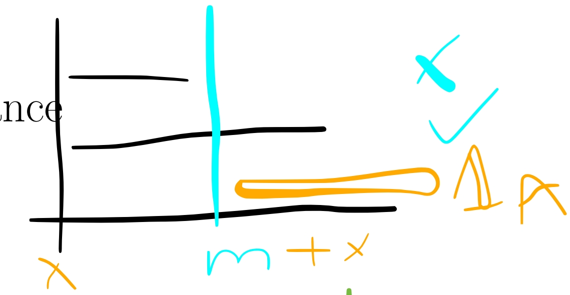


2022KapitalInsurance

October 6, 2023



$m|A_{x:n}$

Types of Insurance we want to look at

- Whole of Life A_x
- Term insurance (Temporary death benefit) $A^1_{x:n}$
- Pure Enowment ${}_nE_x$
- Mixed Endowment $A_{x:n}$
- Deferred Whole of life $m|A_x$ deffered

max n yrs

What are the respective Random Variables

- Whole of Life A_x: $Z = v^{K+1}$
- Term insurance (Temporary death benefit) $A^1_{x:n}$: $Z_1 = v^{K+1} \times \mathbb{1}_{K<n}$
- Pure Enowment ${}_nE_x$: $Z_2 = v^n \times \mathbb{1}_{K\geq n}$
- Mixed Endowment $A_{x:n}$: $Z = Z_1 + Z_2$
- Deferred Whole of life $m|A_x$: $Z = v^{K+1} \times \mathbb{1}_{K\geq n}$

What do we want to calculate for each type of insurance

- Definition
- Expected Value
- Expected Value expressed with Commutation Functions
- Recusion between different years

A_x

- Definition: $Z = v^{K+1}$
- Expected Value: $A_x = \mathbb{E}[Z] = \sum_{k=0}^{\infty} {}_kp_x q_{x+k} v^{k+1}$ $= \sum_{\omega} X(\omega) P(\omega)$

$P(K = k) =$

- Expected Value expressed with Commutation Functions

$$
\begin{aligned}
A_x &= \sum_{k=0}^{\infty} v^{k+1} {}_k p_x \, q_{x+k} && (1)\\
&= \sum_{k=0}^{\infty} v^{k+1} \frac{l_{x+k}}{l_x} \frac{d_{x+k}}{l_{x+k}} && (2)\\
&= \sum_{k=0}^{\infty} \frac{v^{x+k+1}}{v^x} \frac{l_{x+k}}{l_x} \frac{d_{x+k}}{l_{x+k}} && (3)\\
&= \sum_{k=0}^{\infty} \frac{v^{x+k+1} \times d_{x+k}}{v^x \times l_x} && (4)\\
&= \sum_{k=0}^{\infty} \frac{C_{x+k}}{D_x} && (5)\\
&= \frac{M_x}{D_x} && (6)
\end{aligned}
$$

- Recusion between different years: Need to relate A_x and A_{x+1}: You need ${}_k p_x = p_x \times {}_{k-1} p_{x+1}$

$$
\begin{aligned}
A_x &= \mathbb{E}[Z] && (7)\\
&= \sum_{k=0}^{\infty} {}_k p_x q_{x+k} v^{k+1} && (8)\\
&= (\sum_{k=0}^{0} + \sum_{k=1}^{\infty}) {}_k p_x q_{x+k} v^{k+1} && (9)\\
&= q_x \times v + \sum_{k=1}^{\infty} {}_k p_x q_{x+k} v^{k+1} && (10)\\
&= q_x \times v + p_x \times v \times \sum_{k=0}^{\infty} {}_k p_{x+1} q_{x+1+k} v^{k+1} && (11)\\
&= q_x \times v + p_x \times v \times A_{x+1} && (12)
\end{aligned}
$$

Hence

$$\boxed{A_x = q_x \times v + p_x \times v \times A_{x+1}} \qquad (13)$$

$A^1_{x:n}$

- Definition: $Z = v^{K+1} \times \mathbb{1}_{K<n}$
- Expected Value: $A_x = \mathbb{E}[Z] = \sum_{k=0}^{n-1} {}_k p_x q_{x+k} v^{k+1}$

A+n

- Expected Value expressed with Commutation Functions

$$
\begin{aligned}
A^{1}_{x:n} &= \sum_{k=0}^{n-1} v^{k+1}\,{}_{k}p_x\, q_{x+k} && (14)\\
&= \sum_{k=0}^{n-1} v^{k+1} \frac{l_{x+k}}{l_x}\frac{d_{x+k}}{l_{x+k}} && (15)\\
&= \sum_{k=0}^{n-1} \frac{v^{x+k+1}}{v^x}\frac{l_{x+k}}{l_x}\frac{d_{x+k}}{l_{x+k}} && (16)\\
&= \sum_{k=0}^{n-1} \frac{v^{x+k+1}\times d_{x+k}}{v^x \times l_x} && (17)\\
&= \sum_{k=0}^{n-1} \frac{C_{x+k}}{D_x} && (18)\\
&= \frac{M_x - M_{x+n}}{D_x} && (19)
\end{aligned}
$$

- Recusion between different years: Need to relate A_x and A_{x+1}: You need ${}_{k}p_x = p_x \times {}_{k-1}p_{x+1}$

$$
\begin{aligned}
A^{1}_{x:n} &= \mathbb{E}[Z] && (20)\\
&= \sum_{k=0}^{n-1} {}_{k}p_x q_{x+k} v^{k+1} && (21)\\
&= (\sum_{k=0}^{0} + \sum_{k=1}^{n-1}) {}_{k}p_x q_{x+k} v^{k+1} && (22)\\
&= q_x \times v + \sum_{k=1}^{n-1} {}_{k}p_x q_{x+k} v^{k+1} && (23)\\
&= q_x \times v + p_x \times v \times \sum_{k=0}^{n-3} {}_{k}p_{x+1} q_{x+1+k} v^{k+1} && (24)\\
&= q_x \times v + p_x \times v \times A_{x+1:n-1} && (25)
\end{aligned}
$$

Hence

n

$$A^{1}_{x:n} = q_x \times v + p_x \times v \times A^{1}_{x+1:n-1} \qquad (26)$$

${}_{n}E_x$ A + n

$K \equiv K_x$

- Definition: $Z = v^n \times \mathbb{1}_{K \geq n}$
- Expected Value: ${}_{n}E_x = \mathbb{E}[Z] = {}_{n}p_x v^n$
- Expected Value expressed with Commutation Functions

Hence

$${}_{n}E_x = p_x \times v \times {}_{n-1}E_{x+1} \qquad (27)$$

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