1 First one Life $T : (\Omega, \mathcal{A}, P) \to \mathbb{N}$. then we defined various Products such as A_x with $Z = v^K$

2 Used Commutation function to calculate $\mathbb{E}[Z]$, eg $A_x = \sum_{k=0}^{\infty} {}_k p_x \cdot\ q_{x+k}\ v^{k+1} = \frac{M_x}{D_x}$

3 Characteristic: one person, one decrement

4 Next two extentions, namely multiple decrements and multiple people

5 For example $\ddot{a}_{xy} = \sum_{k=0}^{\infty} {}_k p_{xy}\ v^k = \frac{N_{xy}}{D_{xy}}$

6 Characteristic: We could do it with stopping times (not in the sense of Prob Th 1) in the sense that people die at a certain point in time, and you can paint a stats-space diagram as a tree.

7 $(\star, \star) \to (\star, \dagger) \to (\dagger, \dagger)$, and $(\star, \star) \to (\dagger, \star) \to (\dagger, \dagger)$ (plus the staying in the state such as $(\star, \star) \to (\star, \star)$.

8 Note the above geometrical state-space diagrams allow the use of classical life insurance maths.

9 Next we inroduced Markov Chains and the respective modelling, including Chapman-Kolmogorov and Thiele.

10 We also embedded $1 \ldots 7$ in the Markov Model

11 Now we do disability where the tree inerpretation does not hold anymore for sure

DISABILITY INSURANCE

State space consists at least of $S = \{\star, \diamond, \dagger\}$, representing "healty", "disabled" and "death". What is different to before is that the chain $\star \to \diamond \to \star$ is possible, in consequence traditional life insurance mathematics is very tricky and difficult.

1 We call the transition $\star \to \diamond$: becoming disabled

2 We call the transition $\diamond \to \star$: reactivation

3 we call the transition $\star \to \dagger$: dying as active

4 we call the transition $\diamond \to \dagger$: dying as disabled

5 Today Disability is mostly defined in an economical sense, ie how much income is lost as aconsequence of disability. Assume for example a salary of CHF 100k pa. and the person has back pains and can therefore only work in the morning (4 instead of 8 h). This means an income of 50k or a disability ratio of 50%

6 This could mean that we split $\diamond$ into different states $\diamond_\alpha$ where α is the disability ratio. We could also consider a average disabilty ratio of say $\alpha = 0.85$

How to calculate?

1 First Question: Which states do we consider?

2 How do you consider partial disability?

3 How do you reflect waiting periods?

4 How do you consider reactivation - not at all, depending on time which you have been disabled?

Notation

1 i_x^w denote the probability of becoming disabled with a waiting period w, $p_{\star,\diamond}(x, x+1) = i_x^w$

2 q_x^a the mortality for active people $p_{\star,\dagger}(x, x+1) = q_x^a$

3 q_x^i the mortality for active people $p_{\diamond,\dagger}(x, x+1) = q_x^i$. Typically higher than q_x^a

4 r_x this is the reactivation probability as an x year old person. $p_{\diamond,\star}(x, x+1) = r_x$. We will see that reactivation probability is materially dependent on x and t the time being diabled. Hence could consider $r(x, t)$

5 If considering $r(x, t)$ we need to adjust the state space accordingly.

6 Note that for a Markov Model we have the following

$$\forall i \in S : \sum_{j \in S} p_{ij}(x, x+1) = 1$$

7 We get $p_{\star,\star}(x, x+1) = 1 - p_{\star,\diamond}(x, x+1) - p_{\star,\dagger}(x, x+1) = 1 - q_x^a - i_x^w$

8 We get $p_{\diamond,\diamond}(x, x+1) = 1 - p_{\diamond,\star}(x, x+1) - p_{\diamond,\dagger}(x, x+1) = 1 - q_x^i - r_x$

9 We get $p_{\dagger,\dagger}(x, x+1) = 1$

Calculation of Cohorts

1 Cohort of actives $l_x^a : l_0^a = 100000$

$l_{x+1}^a = l_x^a * (1 - i_x - q_x^a) + l_x^i * r_x$

2 Cohort of disabled $l_x^i : l_0^a = 0$

$l_{x+1}^i = l_x^a * i_x + l_x^i * (1 - r_x - q_x^i)$

3 Now you can form Commutation functions as per before, namely

$$\begin{aligned} D_x^a &= l_x^a \times v^x \\ D_x^i &= l_x^i \times v^x \end{aligned}$$

Ditto you can define the other commutation functions:

$$\begin{aligned} C_x^{a,t} &= d_x^{a,t} \times v^{x+1} \\ d_x^{a,t} &= l_x^a \times q_x^a \\ C_x^{a,i} &= d_x^{a,i} \times v^{x+1} \\ d_x^{a,i} &= l_x^a \times i_x \end{aligned}$$

Reaktivation

$$s_{(x,t)} = q_x + 0.008 + e^{-0.94\, t} \times (c - d \times x)$$

Question Half Time?

$$\begin{aligned} \frac{1}{2} &= e^{-0.94\, T} \\ T &= \frac{\log(\frac{1}{2})}{-0.94} \end{aligned}$$

In [1]:

```
import math
T = math.log(0.5)/-0.94

print(T)

T = math.log(0.5)/-1.25

print(T)
```

```
0.7373906176169631
0.5545177444479562
```

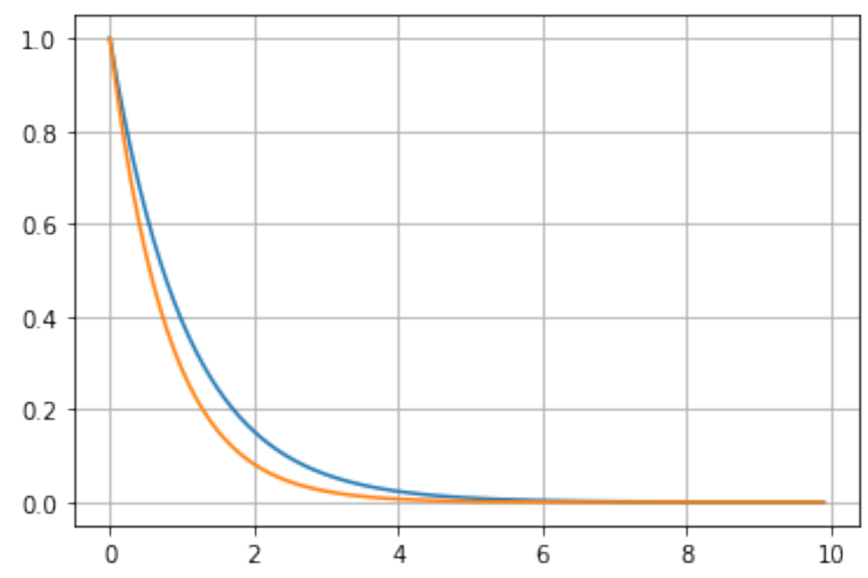
In [8]:

```
import matplotlib.pyplot as plt

x =[]
y1 =[]
y2 =[]

for i in range(100):
    x.append(0.1 * i)
    y1.append(math.exp(x[i]*(-0.94)))
    y2.append(math.exp(x[i]*(-1.25)))

plt.figure(1)
plt.plot(x,y1,x,y2)
plt.grid(True)
plt.show()
```



Different Models

$\ddot{a}_{20}^{ai}$ $\ddot{a}_{20,0}^{i}$ $\ddot{a}_{20,2}^{i}$ $\ddot{a}_{20,4}^{i}$

Markov $n = 6$ 3.014 10.565 22.112 23.799

Markov $n = 2$ 3.205 11.710 24.693 24.059

Markov N1R 2.603 7.774 7.834 8.048

KT 3.533 8.499 21.728

In [ ]: