

# Exam Feb 2021

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February 7, 2022

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## 1 NEW

Please find below the email addresses to which the presentations have to be sent (to all of them):

- michael.koller@math.ethz.ch
- florian.krach@math.ethz.ch

I have added the two teaching assistants as additional admins for the zoom call.

## 2 Aim

The aim of this document is to provide guidance for the upcoming exams and the respective preparations needs. It is expected that each student prepares the respective tasks themselves and does in particular not copy from other students. The usual plagiarism requirements are to be strictly adhered to and violations have corresponding consequences as outlined in the various guidelines of ETHZ.

The following paragraph answers the table in the student guide (section [A](#)):

**Date, time, and duration;** As per students portal on 9 and 10.2.22

**Allowed aids (books, notes, calculators, data tables, etc.;** No restrictions - open book exam. Note time limit as per exam schedule applies

**An email address** or uploading functionality to transfer an electronic copy of your written notes, if so desired:  
**The primary email address is mikoller@ethz.ch or michael.koller@bluemail.ch. The presentation for the task needs to be sent at least 24h prior to the exam. As soon as the assistant for the exam is known I will send you the respective email also, and the task is to be sent to both email addresses**

**Whether or not you** will also be required to use postal mail to send your written notes to ETH (and the address to do so): Electronic in pdf format to me and the teaching assistant is sufficient.

**Two independent ways of contacting** the examiners in case of problems, including phone/SMS: **The two emails of me and the teaching assistant plus my phone +447826943343. Note it will be difficult to phone me during an exam before the scheduled time, hence I suggest an sms, what's app etc.**

**If the exam will be recorded;** if so, the examiner will ask for your consent via e-mail and inform you who; has access to the recording for what purpose, and where and how long it is stored. **For efficiency reasons it would be good to record the exams since it gives certainty to both sides. If you DO NOT want the exam to be recorded, please send an email to me and the teaching assistant beforehand AND mention it again at the start of the exam. Note the recording will be kept solely at ETH and I will not have a copy. Storing follows the usual retention policy for oral and written exams of ETHZ.**

**And any other exam guidelines** that require preparation on your part: See below

Below again the credentials for the zoom meetings. You will be a waiting room and will be allowed to enter at the scheduled time.

mikoller@ethz.ch is inviting you to a scheduled Zoom meeting.

Topic: ETH Prüfungen

Time: Feb 9, 2022 07:00 AM Brussels

Every day, until Feb 10, 2022, 2 occurrence(s)

Feb 9, 2022 07:00 AM

Feb 10, 2022 07:00 AM

Please download and import the following iCalendar (.ics) files to your calendar system.

Daily: <https://ethz.zoom.us/join/u5Utde-rqj4iG9Q6E1W7lJBEDhQyOoQ2-c5v/ics?icsToken=98>

Join Zoom Meeting

<https://ethz.zoom.us/j/61026345810?pwd=V3ovYy9hOWpCUDgxajZnZ3lCQjJMQT09>

Meeting ID: 610 2634 5810

Password: Koller2022

Join by SIP

61026345810@fr.zmeu.us

Join by H.323

162.255.37.11 (US West)

162.255.36.11 (US East)

221.122.88.195 (China)

115.114.131.7 (India Mumbai)

115.114.115.7 (India Hyderabad)

213.19.144.110 (EMEA)

103.122.166.55 (Australia)

209.9.211.110 (Hong Kong)

64.211.144.160 (Brazil)

69.174.57.160 (Canada)

207.226.132.110 (Japan)

Meeting ID: 610 2634 5810

Password: 5413538594

Join by Skype for Business  
<https://ethz.zoom.us/j/61026345810>

### 3 Process

1. I will send you these instructions at least one week before your exam. If you have high level questions please send them to me until Tue 5.2 I will try to answer them if deemed sensible and update this paper.
2. You prepare the below allocated task and send a presentation to me and the teaching assistant at least 24h before the actual exam - ie deadline for exams hold on 9.2.2022: Mon 7.2.22 no later than 23h, For exams hold on 10.2.2022 the cut off date is Tue 8.2.22 no later than 23h. Late submission might lead to penalties.
3. On the day of exam you will present you findings and will be questioned correspondingly. Hence the exam is a mix between an expert discussion and theoretical questions according to the course. It is expected that the student shares his screen for the parts he is presenting.

#### 3.1 What is expected?

The aim of the expert discussion is to show to me your ability to apply the theory (and to prove certain parts of it) based on concrete questions. What is of utmost importance is the clarity of thought. You will be asked to define and use certain models to answer the questions below. I would follow the following grid:

- Why did I use the model I am presenting. What are the implicit and explicit assumptions I have made and which modelling risks are induced from it
- What is the basis of the model and why is the model adequate for the question
- Which parameters have I chosen and why. (Note I will give you some parameters, but you might need to tweak and amend them, If a certain parameter can not be determined exactly please make a sensible assumption. You can use all mortality tables etc from the course.
- What are the main findings of your analysis and what consequences do you draw.

Please note that your presentations should be adequately concise since we need to cover as much ground as possible - if you are not sure put a respective explanatory slide in the appendix which can be discussed if needed. Your mark will be determined based on a) the quality of the presentation b) The solution of the problem and c) The oral discussions and the theoretical questions. I expect that all the tasks are addressed in the presentation but I might focus during the oral part on one or two of them only.

### 4 Tasks for Preparation

#### 4.1 Task 1: Markov Model (Theoretical Part):

Note: If you are doing the exam for Selected Chapters I obviously expect from you also the time-continuous versions addressed.

- Definition of a Markov chain and Chapman-Kolmogorov
- Model per se, induced cash flows
- Calculation of Cash flows, including the respective calculations and proofs
- Thiele Equation

**4.2 Task 2: Reserving Strategy:**

We are given today ( $t_0 = 2022$ ) and Annuity Portfolio  $\ddot{a}_x$  as follows:

Age	Annuity (we may assume all men)
-----	
40	500'000
50	2'500'000
60	12'500'000
70	25'000'000
80	20'000'000
90	10'000'000

and the following given Mortality

$$q(x, t) = q(x, t_0) \times \exp(\lambda_x \times (t - t_0)) \quad (1)$$

with the following values (please interpolate in between ages)

Age	Male (qx)	Female (qx)
0	0.0044	0.0033
5	0.0001	0.0001
10	0.0001	0.0001
15	0.0002	0.0001
20	0.0004	0.0002
25	0.0006	0.0003
30	0.0007	0.0004
35	0.0010	0.0006
40	0.0015	0.0009
45	0.0022	0.0014
50	0.0032	0.0021
55	0.0050	0.0033
60	0.0078	0.0051
65	0.0120	0.0078
70	0.0192	0.0125
75	0.0322	0.0216
80	0.0564	0.0405
85	0.1014	0.0782
90	0.1749	0.1439
95	0.2741	0.2391
100	0.3822	0.3516
105	0.4830	0.4621
110	1.0000	1.0000

and

Age	Male (lam)	Female (lam)
0	-0.0376	-0.0359
5	-0.0373	-0.0327
10	-0.0339	-0.0279
15	-0.0274	-0.0216
20	-0.0185	-0.0160
25	-0.0096	-0.0125
30	-0.0042	-0.0109

35	-0.0037	-0.0113
40	-0.0078	-0.0132
45	-0.0143	-0.0159
50	-0.0207	-0.0184
55	-0.0250	-0.0199
60	-0.0272	-0.0206
65	-0.0278	-0.0205
70	-0.0270	-0.0198
75	-0.0245	-0.0184
80	-0.0205	-0.0164
85	-0.0153	-0.0133
90	-0.0099	-0.0095
95	-0.0053	-0.0058
100	-0.0020	-0.0028
105	0.0000	0.0000

Based on the above data we consider the following Mortality tables:

**GT:** The generation table defined by the above mortality

**PTnnnn:** The Period table induced by the above mortality using  $t = nnnn$ . Ie PT 2030 would use  $q(x, 2030)$

The aim of this task is to calculate the reserves and the profit and losses induced by different ways of reserving. We assume that the true mortality in terms of expectation is given by 1. We assume an interest rate of  $i = 2\%$ . We assume that PT is produced for the following years: 2020, 2030, 2040, etc.

What we know is the following:

- Deduce the formulae to calculate the reserves for the portfolio.
- Provide a table with the reserves for each age and the following tables with the effective as seen today GT, PT2020, PT2030, PT2040, PT2050 including the respective totals
- How would the income (in terms of expected annuities) reduce over time, using GT as effective mortality. Show in tabular form the expected annuities as per 2020, 2030, 2040 and 2050
- Compare for each of the following times 2020, 2030, 2040 and 2050 the expected reserves at this time between the GT and the respective period table
- We assume that the company uses GT2020 from 2020 until 2029, GT2030 from 2030 until 2039, etc. Calculate for each year the technical profit and loss if mortality propagates as per GT
- For each of the years 2030, 2040, 2050, the company will change the reserves. What are the respective additional reserves needed at each point of time.
- How does the PV of the former two cash flows compare to the reserve difference between PT2020 and GT (always use the interest rate  $i$  for discounting)?
- Can you prove that they are the same? Ie  $V_{GT} = V_{PT2020} + PV(PnL) + PV(\Delta V_{PTnn}, V_{PTnn+10})$ .

### 4.3 Task 3 (Life Insurance)

Discuss a Tontine and discuss the respective modelling. What would be the impact of an increase in life span. For reference <https://en.wikipedia.org/wiki/Tontine> How can you use a markov model for this? How can Gompert's Mortality law help to assess this task (See section 9.2 of the Skript)?

### 4.4 Task 3 (Selected Chapters)

Discuss the impact of policyholder behaviour on the value and the hedging of a variable annuity impact.

**A Student Guide**



Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zurich

v.8 20.08.2020

## STUDENT GUIDELINES FOR ZOOM-PROCTORED ORAL EXAMS

Due to the Corona situation, selected exams will be conducted online in individual settings. To enable oral exams, they will be conducted online via the video-conference tool Zoom. Your examiners will be ETH lecturers and teaching staff. As in face-to-face exams, a witness will be present online.

Video data may be recorded subject to prior informed consent by you; if your examiners expect to record your exam, but you do not agree, it will be left to the judgment of the examiner on how to proceed – this may include cancellation or postponement of the exam. If recordings are made, this will be clearly visible to you by a window asking you to confirm your consent prior to the start of the recording, and by a blinking red “Recording” icon in Zoom during the session. At no point in time will the video stream or the recording be made available to anybody but ETH personnel on a strictly need-to-know base. However, network connection data on bandwidth, latency, and incidents will be collected for quality assurance; these data are available to ETH staff only.

**Preparation:** your instructor will announce

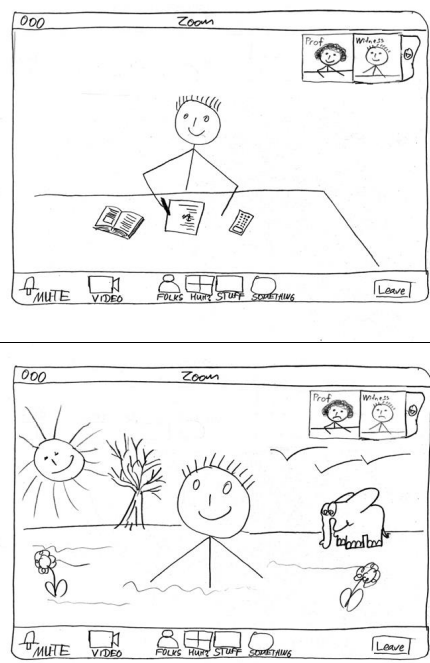
- 1) date, time, and duration;
- 2) allowed aids (books, notes, calculators, data tables, etc.);
- 3) an email address or uploading functionality to transfer an electronic copy of your written notes, if so desired;
- 4) whether or not you will also be required to use postal mail to send your written notes to ETH (and the address to do so);
- 5) two independent ways of contacting the examiners in case of problems, including phone/SMS;
- 6) if the exam will be recorded; if so, the examiner will ask for your consent via e-mail and inform you who has access to the recording for what purpose, and where and how long it is stored; and
- 7) any other exam guidelines that require preparation on your part.

In any case, **you will need**

- 8) a computer with a camera, speaker, and microphone;
- 9) a stable broadband internet connection (ideally wired);
- 10) your ETH student ID;
- 11) the latest release of the Zoom client software; and
- 12) a camera, phone or scanner to reproduce your written notes electronically and transfer them to the examiners. Scanner apps on phones are perfect, since they also provide parallax-correction.

Failure to comply with these preparation guidelines by either party will render the exam void or result in a failing grade. In case of technical difficulties during the exam, these will be documented, and it will be left to the judgment of your examiners if, how and when the exam will be continued or repeated.

Starting the exam:	
Make sure your place is well lit (no light from the back, no window) and quiet, preferably lockable. Test your setup prior to the exam using <a href="https://ethz.zoom.us/test">https://ethz.zoom.us/test</a>	
Your examiners will announce the Zoom Meeting ID for entering the exam at least 30 minutes prior to the beginning of the exam. This ID will need to be kept confidential. Please open the Zoom Client and type the meeting ID and your name as it appears on your student ID.	
You will enter your exam session at least 5 minutes prior to the announced starting time. Depending on settings, you will probably find yourself in a virtual waiting area for admission.	
Your examiners may be using your stored student photo to identify you or require you to show your ETH student ID in clear view of the camera.	
Your examiners may take a static screenshot at this point in time for the exam record.	

<p><b>Taking the exam:</b></p> <p>Your workplace should ideally be arranged such that the computer is generally out of reach, and that your face, hands, notes, aids, and background are clearly visible.</p> <p>The desk and the room should be clear of clutter. Your examiners may ask you to move the camera around the room.</p> <p>Camera, microphone, and speaker need to be turned on or under the control of the examiners (Zoom settings).</p> <p>Do not use earphones, headsets, or any earbuds (Bluetooth or otherwise, including noise-canceling devices).</p> <p>Virtual backgrounds need to be turned off; your surroundings need to be visible for the examiners. Once again, your examiners may ask you to move the camera around the room.</p>	 <p style="text-align: right;"><b>No</b></p> <p>If the examiners would like to see your work, please hold it up to the camera. Please make sure there is sufficient lighting. Clear and easy communication will contribute to your success on the exam!</p>
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<b>After the exam:</b>
While still in the video conference, photograph or scan your notes, if so required.
While still in the video conference, email or upload your notes according to the guidelines set out in 3) above, if so required.
Your examiners might delegate you to the waiting room to assess your exam; they will then bring you back to the meeting room.
<b>Do not leave the video conference</b> until your examiners tell you that it is fine to hang up.
If you are also required to mail your notes according to 4) above, please do so as soon as possible. Mail service tends to be somewhat unpredictable during Corona times, so the sooner the better.

**Very best of success!**



**B Additional Q&A re tasks**

a) Sayuli Drouard

Question:

Sent from my iPhone

On 4 Feb 2022, at 10:15, Drouard Sayouli <drouards@student.ethz.ch> wrote:

Dear Prof. Dr. Koller,

I have a question about this part of my assignment:

Provide a table with the reserves for each age and the following tables with the effective as seen today GT, PT2020, PT2030, PT2040, PT2050 including the respective totals

What does respective totals refer to here ? Is it the total reserves ?

Thank you very much in advance.

Best regards,

Sayuli Drouard

Answer:

For each age 40, 50, ... 90

You are given an annuity  $r_i$  with math reserve

$V_i$  (depends on time, table etc)

The sum is the total reserve ie  $\sum_i V_i$

b)

Question:

Sehr geehrter Professor Koller,

Ich bin ein Student und besuche Ihre Vorlesung Life Insurance. Sie haben uns heute eine Mail geschrieben, wo vier verschiedene Aufgaben als Vorbereitung drinstehen.

Leider, bin ich mir nicht ganz sicher, ob man nur eines dieser Aufgaben vorbereiten muss, oder ob man alle vier Aufgaben vorbereiten und dann an der Prüfung besprechen und diskutieren muss.

Ich entschuldige mich für meine Frage, aber da ich mir nicht sicher bin, hoffe ich, dass Sie Verständnis für meine Frage haben.

Ich wünsche Ihnen noch eine gute Woche

Freundliche Grüße

Patrik Rosa Ferreira

Answer:

Sie muessen die drei Aufgaben fuer Lebensversicherung bearbeiten

LG

Michael Koller

c) Tobias Zingg

Question:

Dear Michael Koller

There seems to be some information missing to be able to handle Task 2:

The definition of the generation table GT does not imply a reference point (e.g. the birth year of the generation considered). Therefore, I am puzzled as how GT2020, GT2030, etc. should be defined/calculated.

One of the points says "How does the PV of the former two cash flows compare to the reserve difference between PT2020 and GT (always use the interest rate  $i$  for discounting)?" . However, the previous point(s) did not specifically deal with calculating two distinctive cash flows, so there seems some context missing to which the 'two former cash flows' is referring to.

Some clarification on these points would be greatly appreciated.

Kind regards,

Tobias Zingg

Answer:

A) you are right - gt2020 should actually read pt2020

B) when you move 2030 from pt2020 to pt2030; the reserves will move; say 650m with pt2020 and 675m with pt2030. Hence you need to increase reserves by 25m, I call this  $\Delta(\text{pt2020}, \text{pt2030})$