Ray-tracing – ctd.

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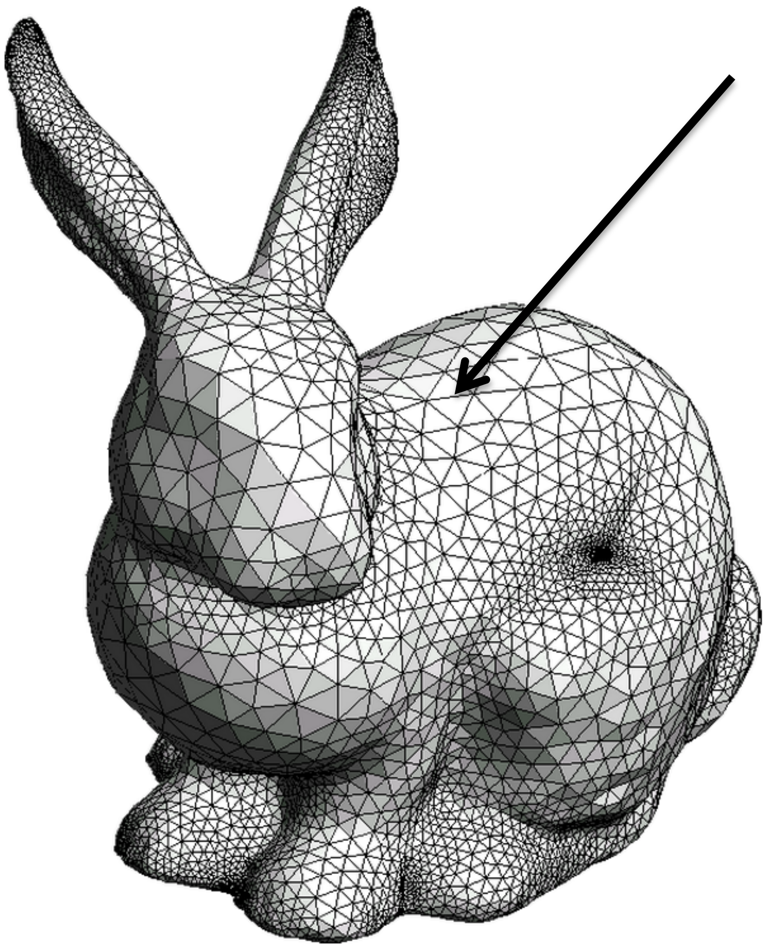
Credit: some slides from Rachel McDonnell

Ray Polygon Intersection



To test if the ray intersects the polygon:

- Assume **Planar** polygons
- First see if the ray intersects the polygon's plane
- If it does, see if the intersection point is inside or outside the polygon

Summary

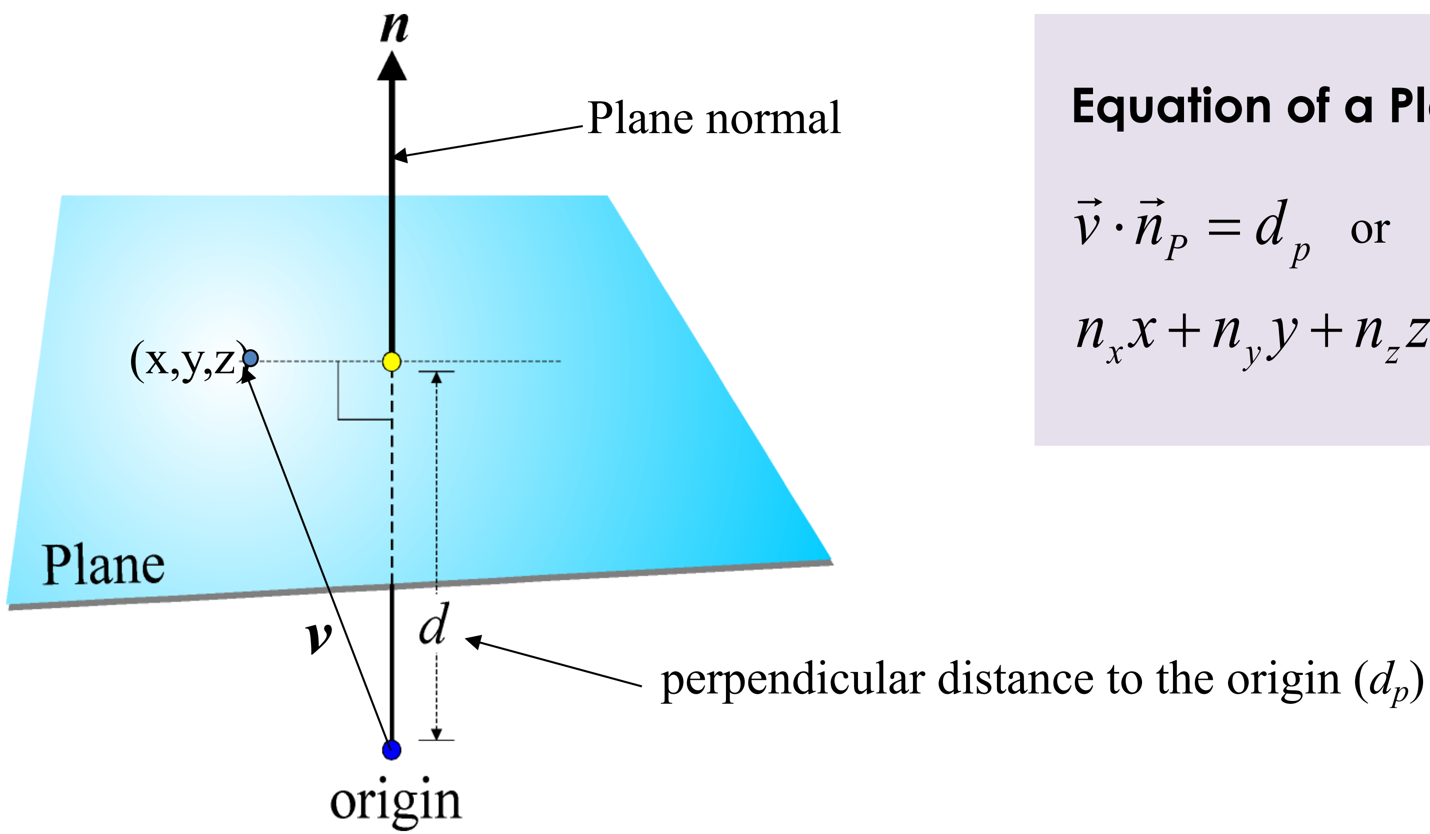
1. Check if ray is parallel or behind polygon's plane

2. If it's not, find intersection with plane
   - Substitute the ray equation into the plane equation

3. Once you have intersection point on the plane, find out if it is contained in the polygon

Plane Definition

- A *plane* may be defined by its normal and the perpendicular distance of the plane to the origin:



Equation of a Plane

$$\vec{v} \cdot \vec{n}_P = d_p \quad \text{or}$$

$$n_x x + n_y y + n_z z - d_P = 0$$

Ray Plane Intersection

1. Does ray intersect the polygon's plane?

First compute the dot product between the normalised ray and the polygon's normal (= plane's normal):

$$\vec{n}_P . \vec{d}_r$$

- If dot product == 0 it => ray & normal perpendicular
  - **No intersection!**
- If > 0 => ray and normal are in the same direction
  - **back face intersection**
- if < 0 => plane facing direction of ray
  - **There is an intersection with the polygon's plane!!**

- Note there can only be *one intersection*. The normal to the plane at each point is the same, i.e., the plane normal $\boldsymbol{n_P}$

Ray Plane Intersection

2. Now find intersection point of ray with the plane

To intersect a ray with a plane we use the same approach as with the sphere

- i.e. substitute in the ray equation and solve for t

$$\left(O_r + t d_r\right)\cdot \vec{n}_P = d_P$$

- O is origin of the ray and d_r is the direction of the ray
- Wolve for t and that will give you the point on the plane

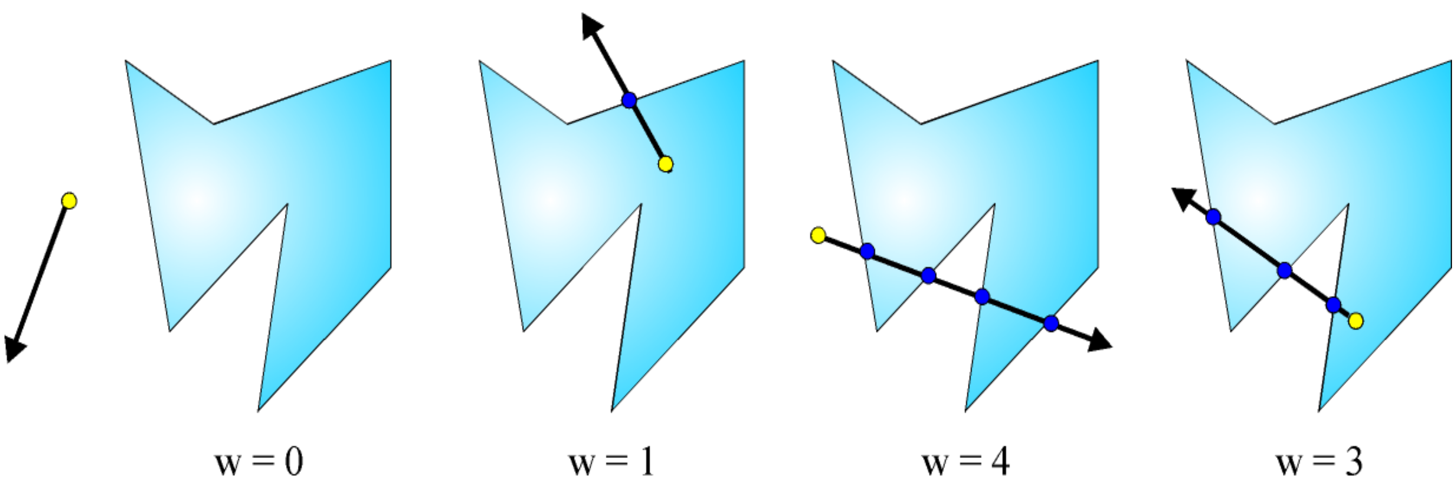
$$\Rightarrow t = \frac{d_P - \vec{n}_P \cdot O_r}{\vec{n}_P \cdot \vec{d}_r}$$

- t is substituted back into the ray equation yielding the point of intersection:

$$P_{int} = O_{ray} + t_{int} d_{ray}$$

Ray Polygon Intersections

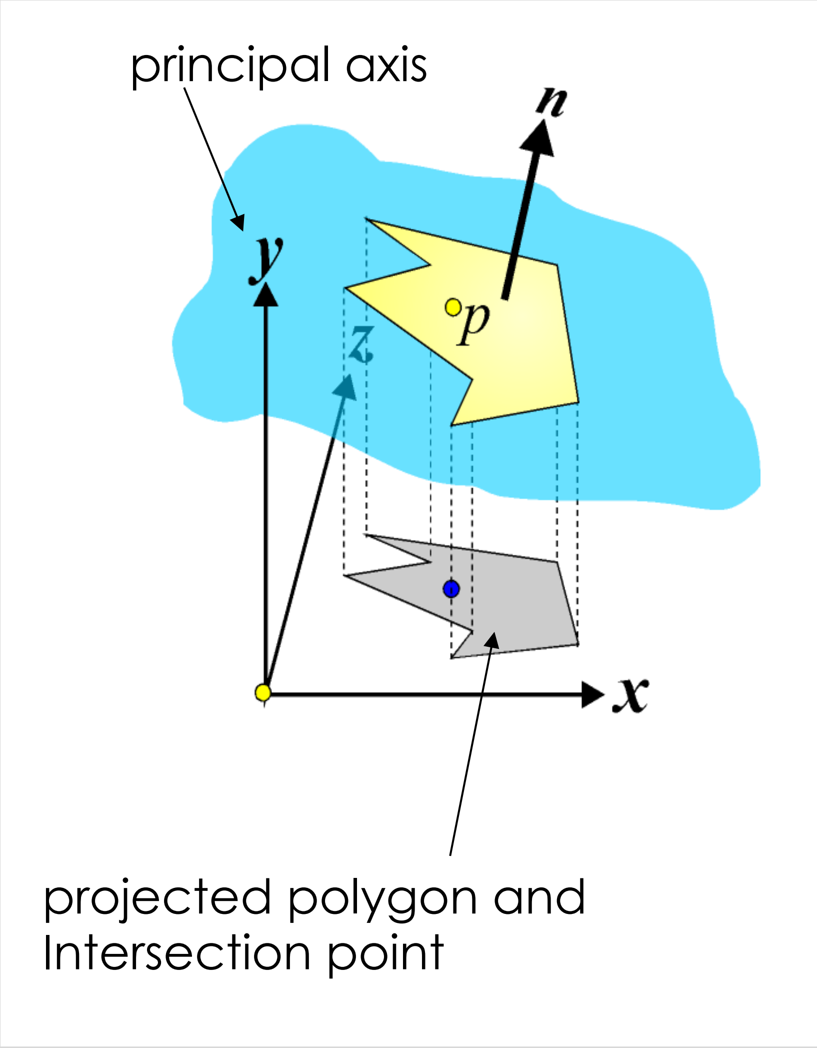
3. Next, determine if the intersection point is within the polygon interior using the *Jordan Curve Theorem*:
   - construct any ray with the intersection point as an origin
   - count the number of polygon edges this ray crosses (= *winding number*)
   - if *w* is odd then the point is in the interior



Ray Polygon Intersections

3.ctd. Note that this is essentially a 2 dimensional problem:

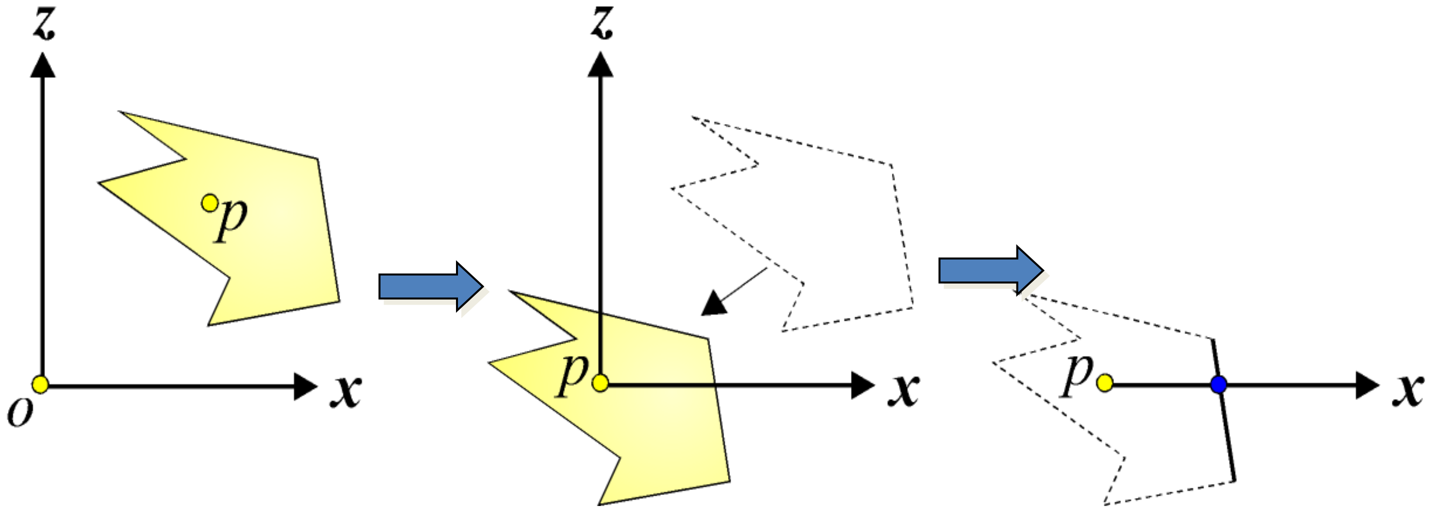
- after intersecting the ray and the plane, reduce the problem to 2D by projecting parallel to the polygon's *principal axis* onto the plane formed by the other 2 axes
- the principal axis is given by the *largest ordinate* of the polygon normal
  - *e.g.*, if ***n*** = [*0.2 0.96 0.3*] what is the principal axis?
  - What plane do we project onto?
- This projection preserves the *topology* (assuming the projection direction is not parallel to the polygon)



Ray Polygon Intersections

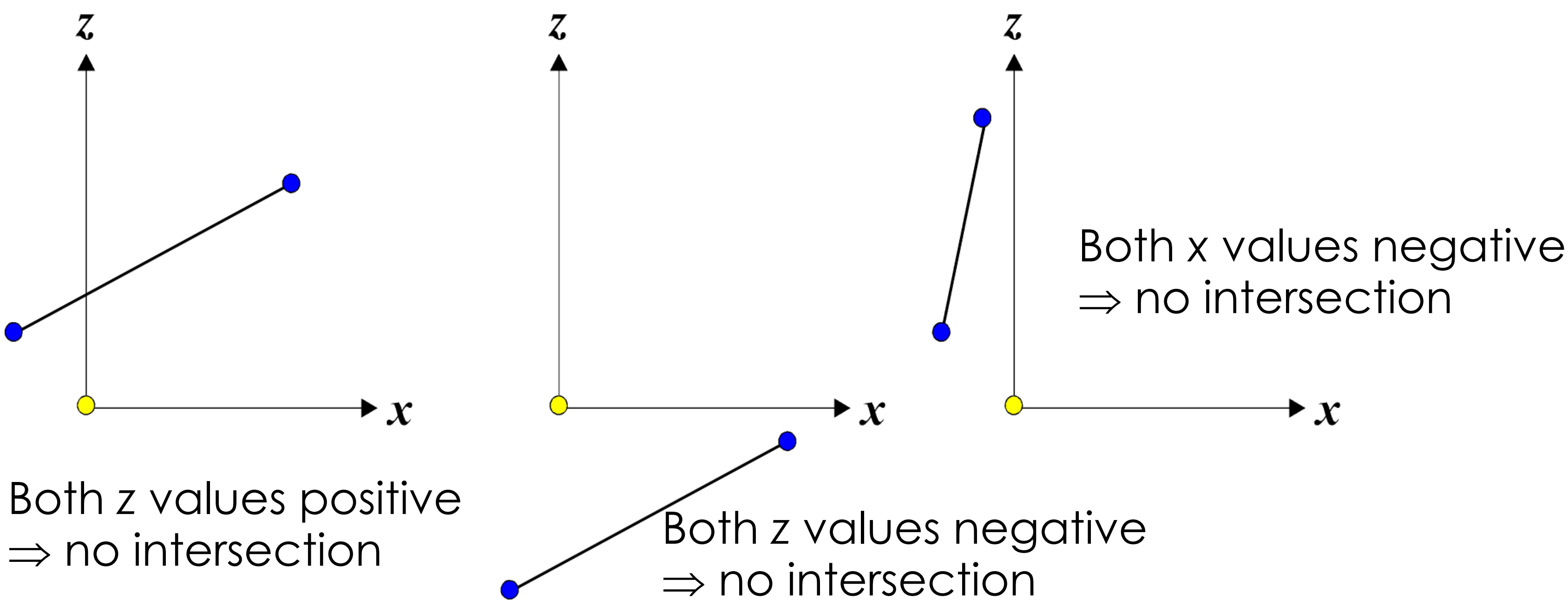
3.ctd. Note that this projection is *constant for each polygon:*

- Once projected, the *winding number* is determined.
  - translate the intersection point to the origin (apply same translation to all projected polygon vertices).
  - use an axis (not the principal axis) as the direction to try
  - determine the number of *edge crossings* along the positive *axis*.



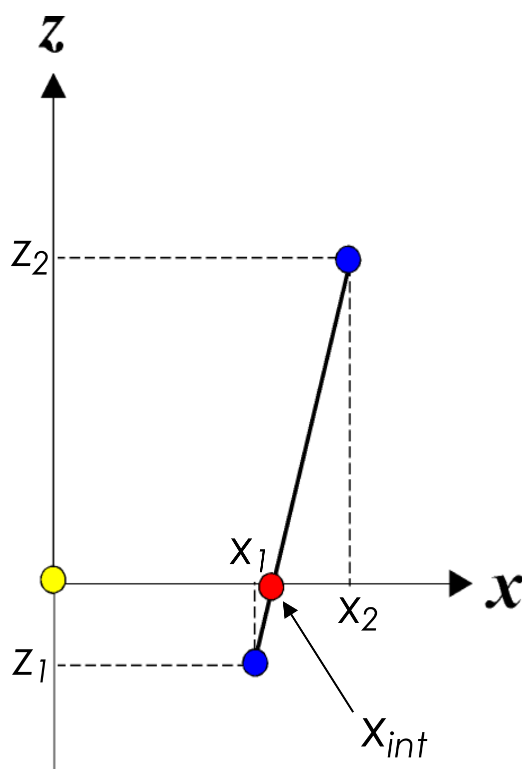
Ray Polygon Intersections

3.ctd. We can speed up the process of determining edge intersection with an axis by noting that there are 3 redundant cases of edge axis topology:



Ray Polygon Intersections

3.ctd. If none of these cases are satisfied then we must compute the intersection with the axis and determine if it lies on the positive half of the axis:



Remember the general equation of a line:

$$y = mx + \boxed{c}$$

intersection with the y axis

$$x_{int} = x_1 - z_1 \frac{(x_2 - x_1)}{(z_2 - z_1)}$$

If $x_{int} > 0$ then the edge crosses the axis and the winding number is incremented.

Summary

1. Check if ray is parallel or behind polygon's plane

2. If it's not, find intersection with plane
   – Substitute the ray equation into the plane equation

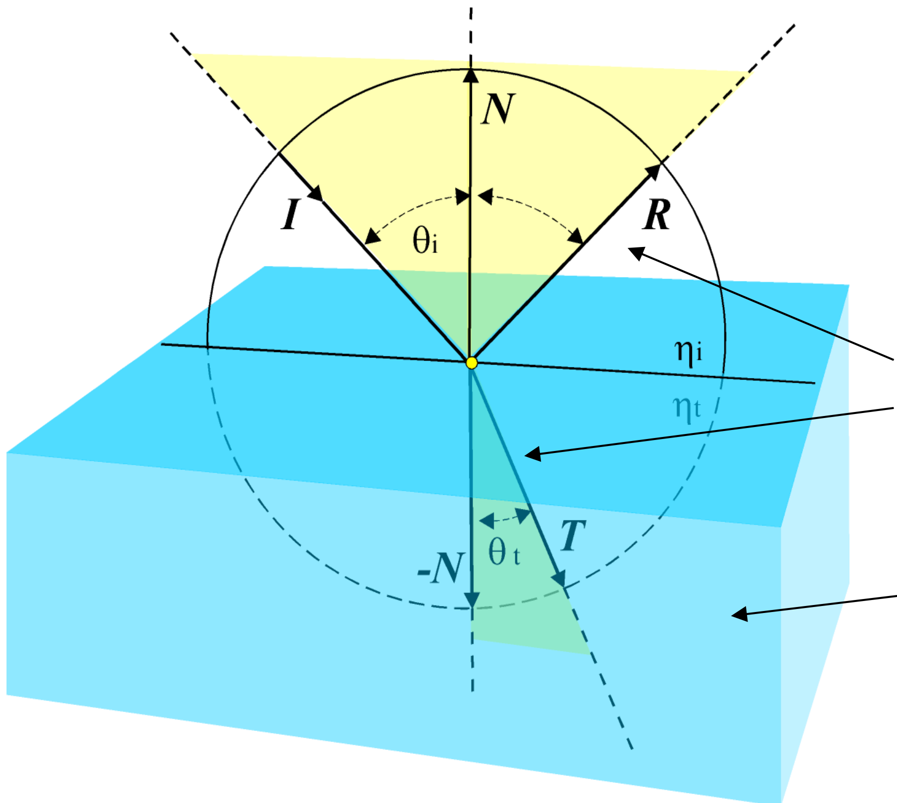
3. Once you have intersection point on the plane, find out if it is contained in the polygon

Secondary Rays

- Having determined the closest intersection point x along a ray path we then evaluate the *Whitted illumination model* at this point:
  - Compute the local illumination at x
  - Construct a *reflected* ray from x and recurse
  - Construct a *refracted* ray from x and recurse
  - Construct a shadow ray to each light source

Secondary Rays

N, ***I***, ***R*** and ***T*** all lie in the same plane

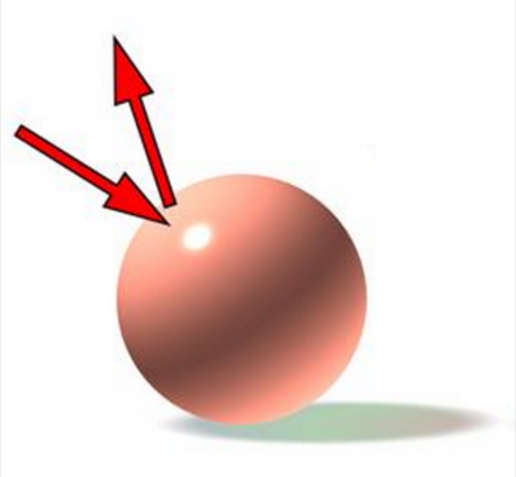


secondary rays

reflective and transparent medium (e.g., glass)

Reflection

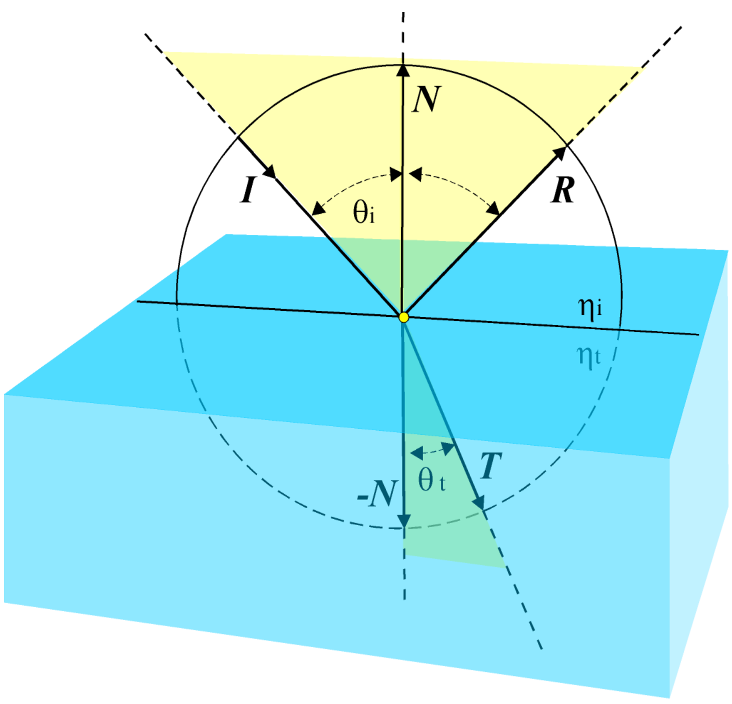
- The **law of reflection** says that for specular reflection:
  - the angle at which the wave is incident on the surface equals the angle at which it is reflected

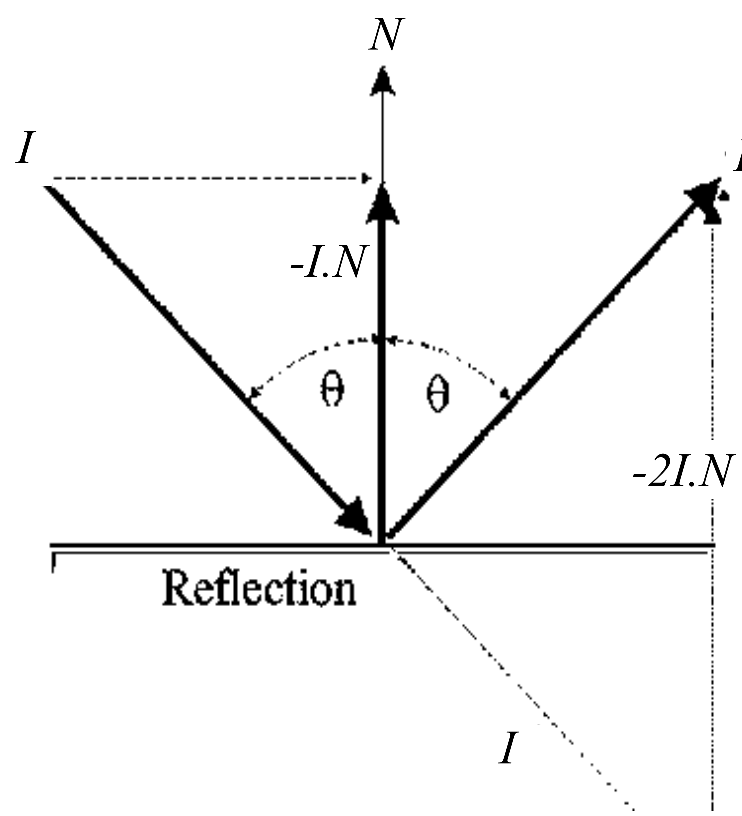


Reflected Rays

- The secondary rays will have the intersection point, **x**, as their *origin*.

The *reflected ray direction* $\boldsymbol{R}$ is computed as: $R = I - 2N(I \cdot N)$





Refraction

- Refraction is the bending of the path of a light wave as it passes from one material to another material.

http://www.andybrain.com/

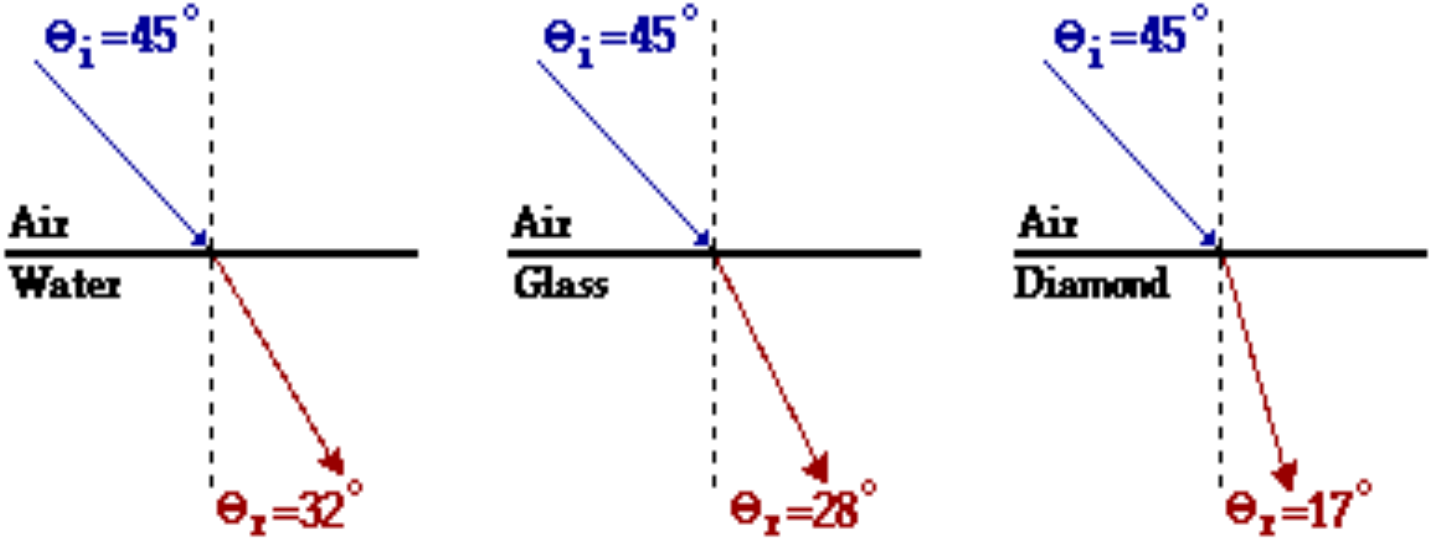
- The refraction occurs at the boundary and is caused by a change in the speed of the light wave upon crossing the boundary.
- The tendency of a ray of light to bend one direction or another is dependent upon whether the light wave speeds up or slows down upon crossing the boundary.

Snell's Law

In optics and physics, Snell's law is a formula used to describe the relationship between the angles of **incidence** and **refraction**, when referring to light or other waves passing through a boundary between two different isotropic media, such as water and glass

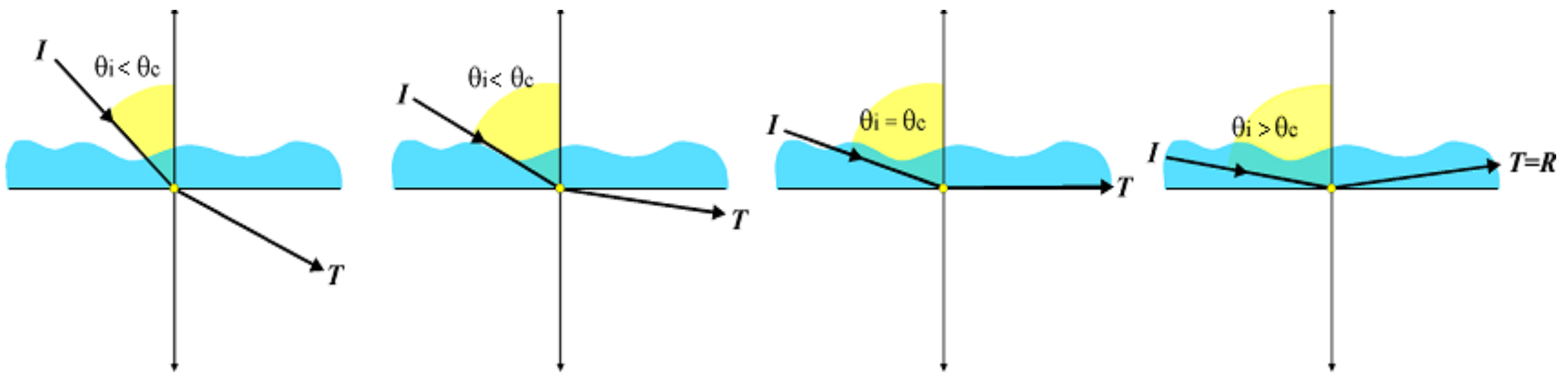
Refraction

A comparison of the angle of refraction to the angle of incidence provides a good measure of the refractive ability of any given boundary.



Total Internal Reflection

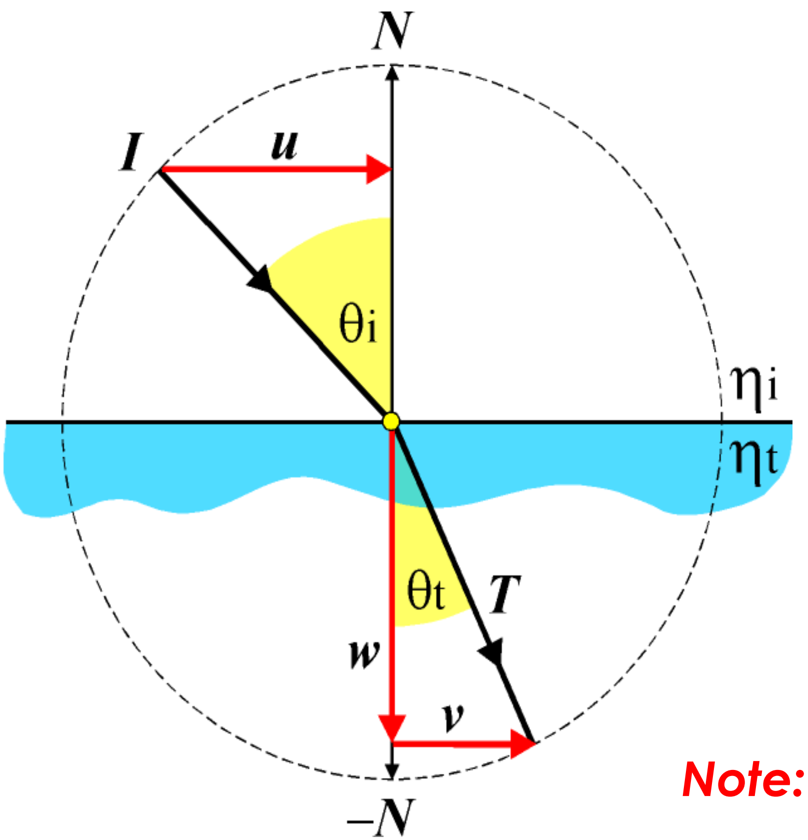
- *Total internal reflection* occurs when the incident angle exceeds the *critical angle* for the surface.
- This will only happen when passing from a *higher refractive index* to a *lower one*.
  - principle is used for fiber-optic cables



  - When TIR occurs, *there is no refracted energy*:
  - *i.e.*, all the energy is reflected ($\rho_{refl} + \rho_{trans}$)

Refracted Rays

- We use a geometric construction using *Snell's Law* to determine ***T***:



Snell's Law

$$\eta = \boxed{\frac{\eta_i}{\eta_t} = \frac{\sin\theta_t}{\sin\theta_i}} = \frac{|\vec{v}|}{|\vec{u}|}$$

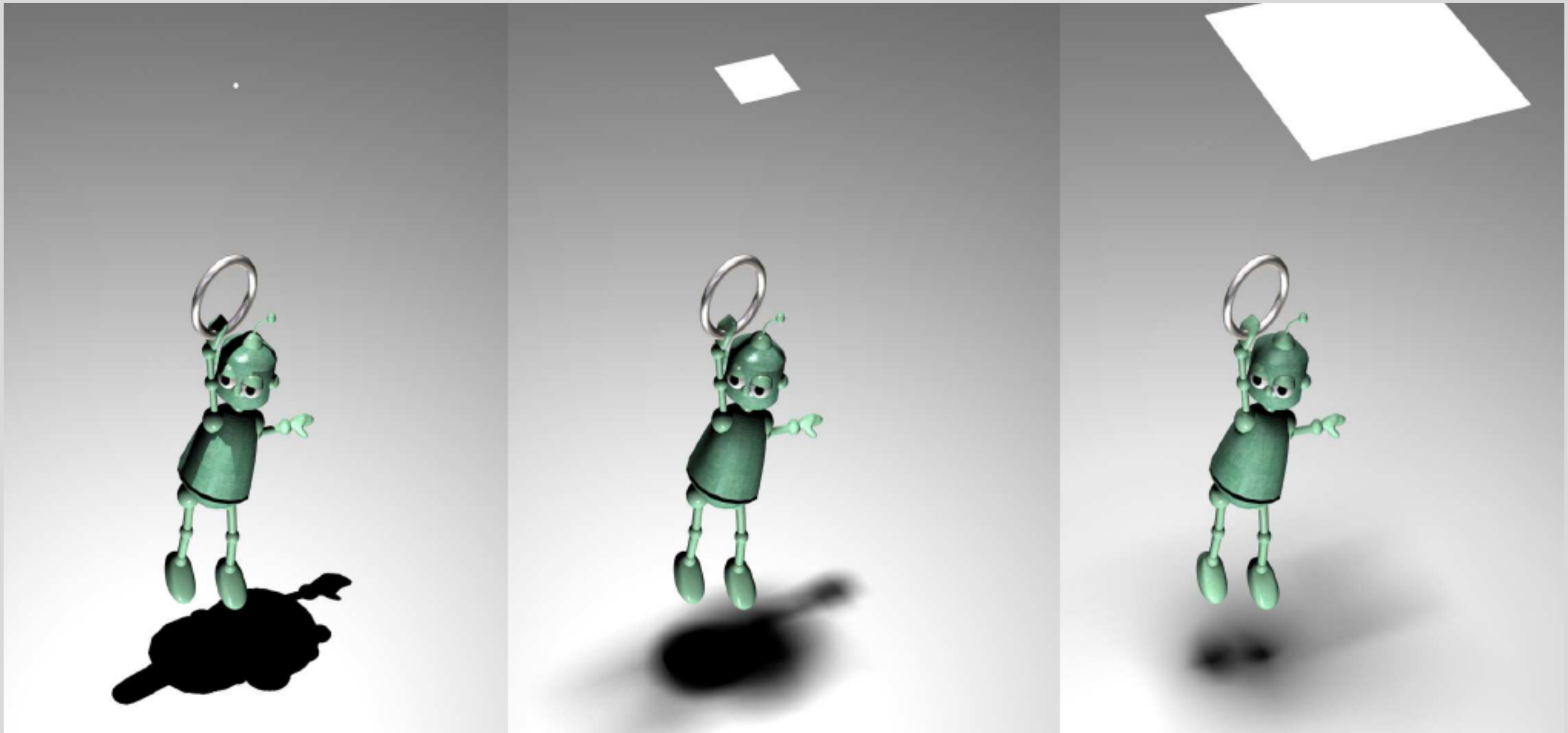
$$T = \vec{w} + \vec{v} = \vec{w} + \frac{|\vec{v}|}{|\vec{u}|}\vec{u} = \vec{w} + \eta\vec{u}$$

Note: *you don't need to know this for the quiz*

Shadows

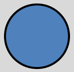
- Whether a point is in shadow or not is determined by casting a ray from each intersection point to the light source
  - Shadow feeler
- If it intersects any object then the point of interest is deemed to be in shadow
  - Easier than ray/object intersections, as we only need to know if intersection has occurred (do not need to find the nearest object)
- Shadow calculations impose a computational overhead in ray tracing that increases rapidly as the <u>number of light sources</u> increases

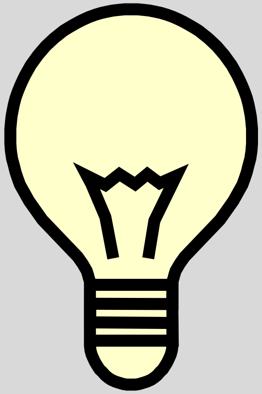
Soft Shadows



Point Light Sources

- A "point" light source is usually not truly a point light source. Considering a light source as a point is just a convenient model.
- Consider a light bulb, for example. It is not an infinitesimally small point. It has volume.
- Implication: Real light sources produce soft shadows.

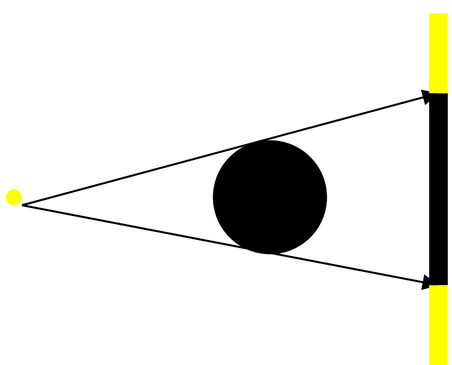




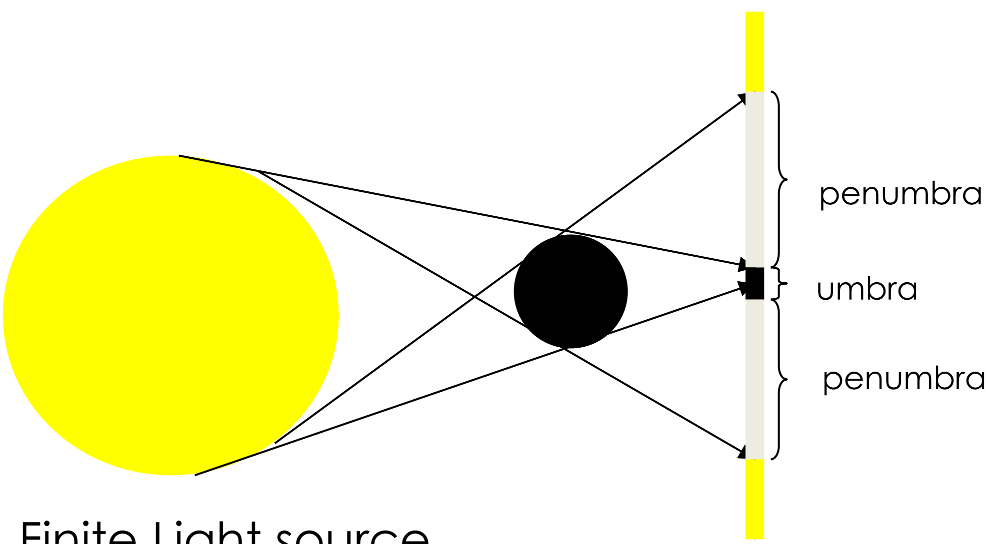
Soft Shadows

Shadows are not uniformly dark. The shadow is divided into two parts: the umbra and penumbra:

- No light at all from the light source reaches the umbra
  - It is completely dark.
- Some light from the light source reaches the penumbra
  - It is partially dark.
- For a point light source, there is no penumbra (i.e., no soft shadow).



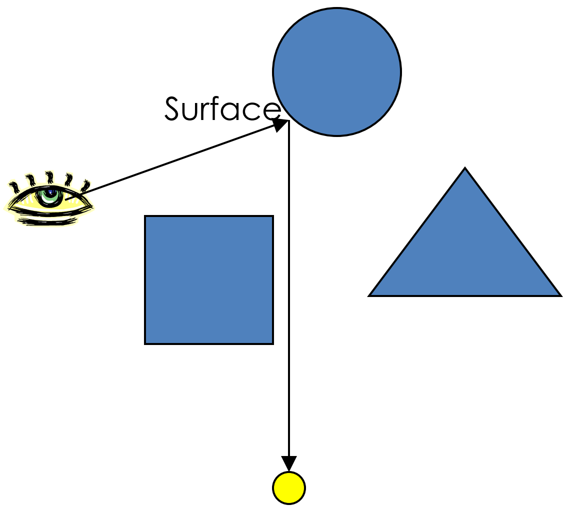
Point Light source



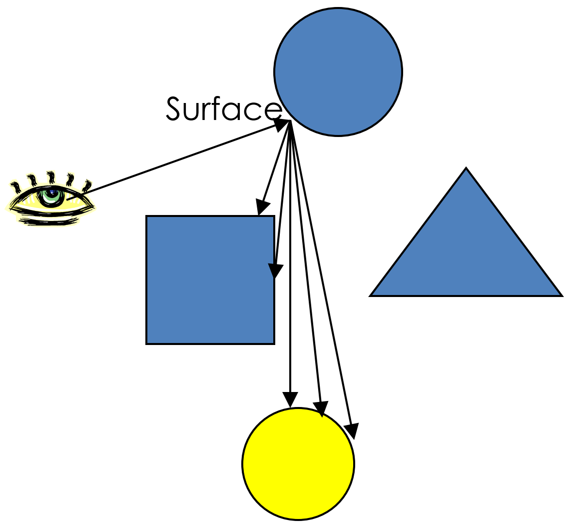
Finite Light source

Soft Shadows in Ray Tracing

- For more realistic shadows, model light sources as volumes rather than points.
  - Use a sphere to model a light source, rather than a point.
  - This is often quite realistic because many light sources in real life are spherical, e.g. light bulbs and lanterns.
  - When calculating lighting, shoot several rays to the light source, instead of just one ray.
  - Calculate lighting for each ray, and take the average.

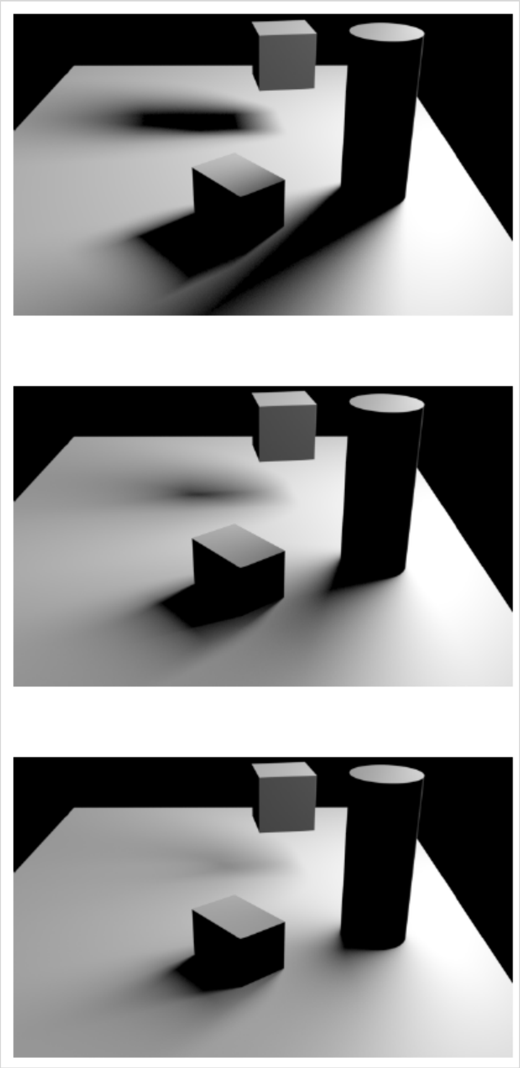


Point light source: The surface is completely lit by the light source.



Finite light source: 3/5 of the rays reach the light source. The surface is partially lighted.

Soft Shadow examples



Additional Reading

- Advanced Animation and Rendering Techniques: theory and practice, Alan Watt and Mark Watt
- Fully worked examples of intersection tests: http://www.vis.uky.edu/~ryang/teaching/cs535-2012spr/Lectures/13-RayTracing-II.pdf