



UNIVERSIDAD NACIONAL DE CÓRDOBA
FACULTAD DE CIENCIAS EXACTAS, FÍSICAS Y NATURALES
CÁTEDRA DE SISTEMAS DE CONTROL I

Ejercicios Unidad 2

Nombre: Monja Ernesto Joaquín

DNI: 43.873.728

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Problema 1:

```
pkg load symbolic;
clear all; history -c; clc;
syms R1 R2 C1 C2 s real;

% Ejercicio 1.1
G1 = 1/(s*R1*C1+1)
disp(G1);

% Ejercicio 1.2
syms Vin I1 I2 Vout real
X1 = 1/(s*C1);
X2 = 1/(s*C2);
I1 = I2*(X1+R2+X2)/X1;
I2 = Vout/X2
eq4 = Vin == Vout/X2*(X1+R2+X2)/X1*(R1+X1)-Vout/X2*X1;
S = solve(eq4,Vout);
G2 = S/Vin;
disp(G2);

% Ejercicio 1.3
G31 = 1/(s*R1*C1 + 1);
G32 = 1/(s*R2*C2 + 1);
G3 = G32 * G31;
disp(G3);
```

Problema 2:

Sistema 1:

$$y1: \frac{1643 s + 1643}{s^3 + 958 s^2 + 3491 s + 1643}$$

Sistema 2:

$$y1: \frac{1620 s + 162}{s^3 + 524.1 s^2 + 1672 s + 162}$$

Sistema 3:

$$\frac{K_p W_n^2 \sqrt{T_d T_i s^2 + T_i s + 1}}{K_p T_d T_i W_n^2 s^2 + K_p T_i W_n^2 s + K_p W_n^2 + 2 P_s T_i W_n^2 s + T_i W_n^2 s + T_i s^3}$$

Sistema 4:

$$\frac{K \cdot K_p \cdot \sqrt{T_d \cdot T_i \cdot s^2 + T_i \cdot s + 1}}{K \cdot K_p \cdot T_d \cdot T_i \cdot s^2 + K \cdot K_p \cdot T_i \cdot s + K \cdot K_p + T \cdot T_i \cdot s^2 + T_i \cdot s}$$

Código:

```
close all; clear all; clc; history -c;
pkg load control;

% Sistema 1:
G1 = zpk([-1], [-10, -100], 53);
G2 = zpk([], [0], 1);
Kp = 31;
Kd = 16;

FdS1 = feedback(G1, Kp);
FdS2 = Kp*FdS1*G2;
FdST = feedback(FdS2, 1)

% Sistema 2:
G1 = zpk([], [-200], 27);
G2 = zpk([], [-0.1], 5);
PI = zpk([-0.1], [0], 1);
P = 12;

FdS1 = feedback(G1*P, 1);
FdS2 = PI*FdS1*G2;
FdST = feedback(FdS2, 1)

% Sistema 3:
pkg load symbolic;
syms Ti Td Kp Wn Ps s real;
G1 = (Ti*Td*s^2 + Ti*s + 1)/(Ti*s);
G2 = (Wn^2)/(s^2 + 2*Ps*Wn*s + Wn^2);

GT = G1*G2*Kp;
FdS = GT/(1 + GT);
FdST = factor(FdS)

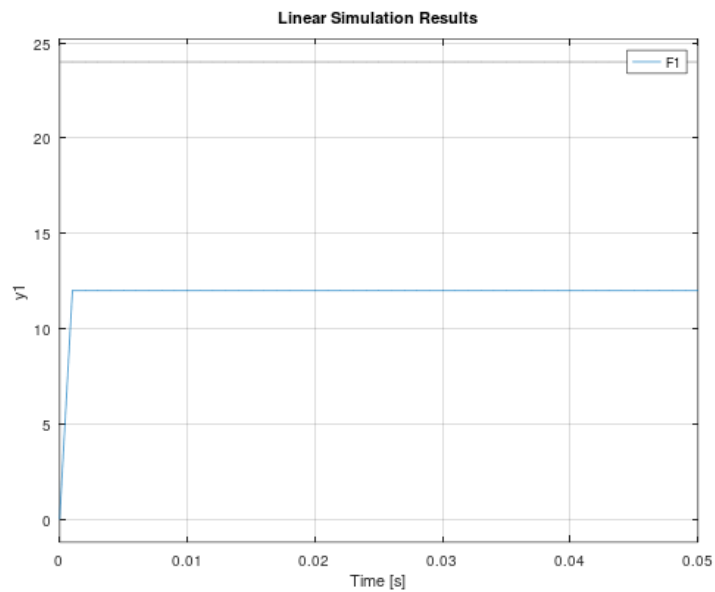
% Sistema 4:
pkg load symbolic;
syms Ti Td Kp Wn K T s real;
G1 = K/(T*s + 1);

Gadd = 1 + 1/(Ti*s) + Td*s;
F1 = Gadd*Kp*G1;
FdS = F1/(1 + F1);
FdST = factor(simplify(FdS))
```

Problema 3:

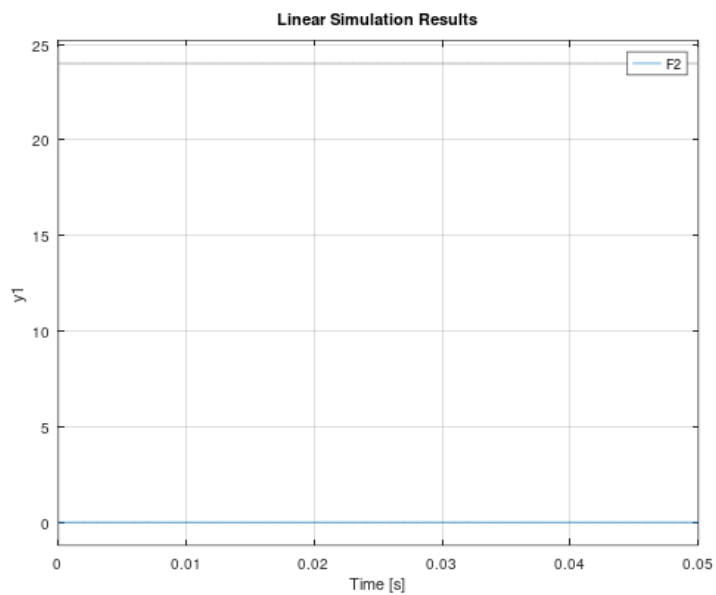
$I(s)/E(s)$:

$$y1: \frac{1e+06 s + 3e-06}{s^2 + 2e+06 s + 0.00177}$$



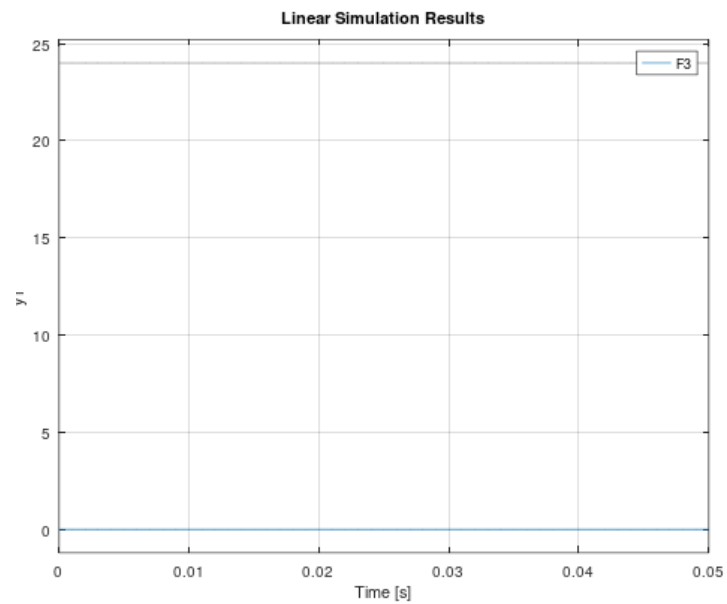
$W(s)/E(s)$:

$$y1: \frac{0.042}{s^2 + 2e+06 s + 0.00177}$$



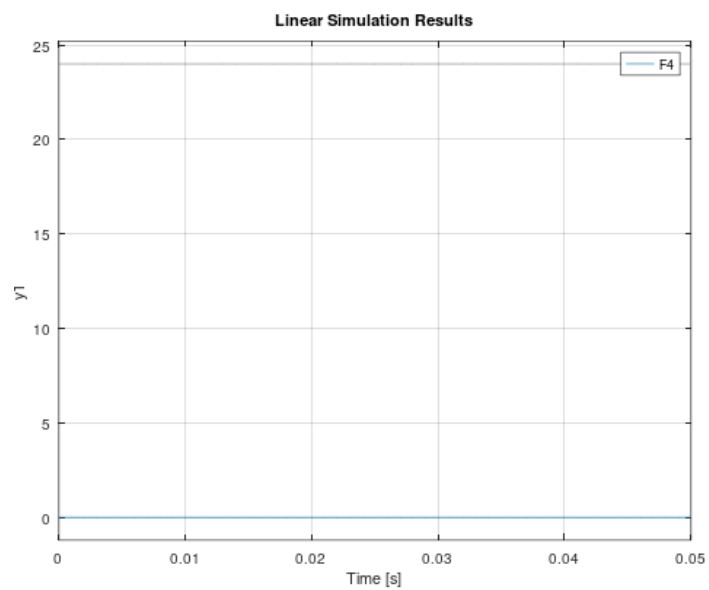
$I(s)/T(s)$:

$$y1: \frac{0.042}{s^2 + 2e+06 s - 0.001758}$$



$W(s)/T(s)$:

$$y1: \frac{1e-06 s + 2}{s^2 + 2e+06 s - 0.001758}$$



Código:

```

close all; clear all; clc; history -c;
pkg load control;

s = tf('s');
L = 1*10^-6;
R = 2;
J = 10^6;
B = 0.3*10^-5;
Kb = 0.042;
Ka = 0.042;

G1 = 1/(L*s + R);
G2 = 1/(J*s + B);

% W(s)/E(s) con Tl(s) = 0
W_E = feedback(G1*Ka*G2, Kb)

% I(s)/E(s) con Tl(s) = 0
I_E = feedback(G1, Ka*G2*Kb)

% W(s)/Tl(s) con E(s) = 0
W_Tl = -feedback(G2, -Kb*G1*Ka)

% I(s)/Tl(s) con E(s) = 0
I_Tl = -feedback(G2*(-Kb)*G1, Ka)

t = 0:0.001:0.05;
u1 = 24*ones(length(t),1);
u2 = 0.01*ones(length(t)-2,1);

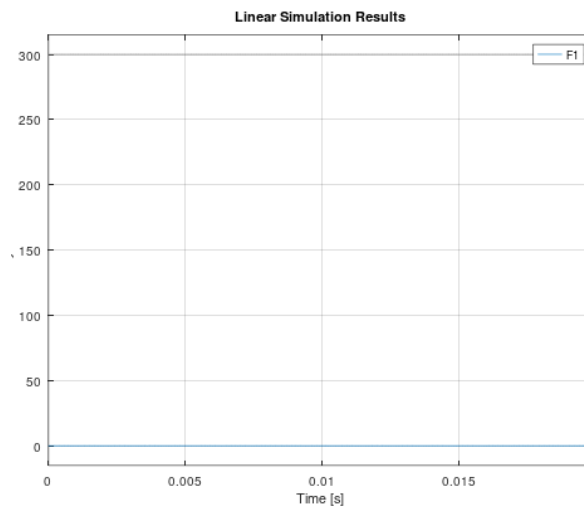
lsim(W_E, u1, t)
lsim(I_E, u1, t)
lsim(W_Tl, u2, t)
lsim(I_Tl, u2, t)

```

Problema 4:

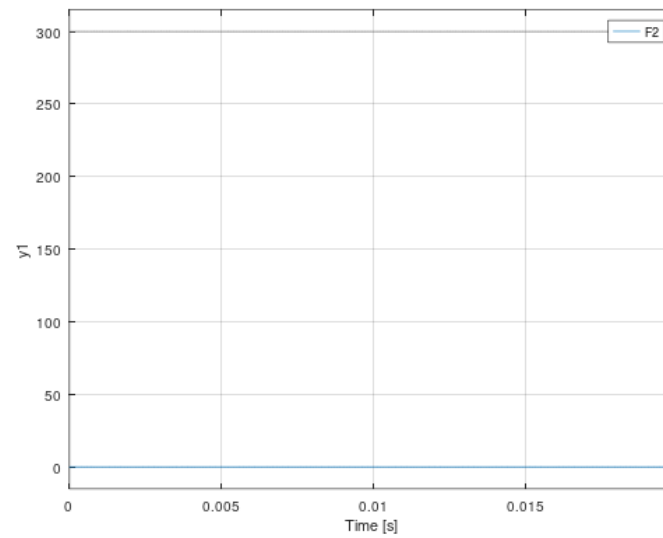
$W(s)/W_r(s)$:

$$y1: \frac{0.042}{s^2 + 2e+06 s + 0.04377}$$



$W(s)/Tl(s)$:

$$y1: \frac{-1e-06 s - 2}{4.376e+04 s + 2}$$



Código:

```
close all; clear all; history -c; clc;
pkg load control;

s = tf('s');
L = 1*10^-6;
R = 2;
J = 10^6;
B = 0.3*10^-5;
Kb = 0.042;
Ka = 0.042;
Kp = 1;

G1 = 1/(L*s+R);
G2 = 1/(J*s+B);

% F1 = W(s)/Wr(s)
F1 = feedback(feedback(G1*Ka*G2, Kb)*Kp, 1)
% F2 = W(s)/Tl(s)
F2 = -feedback(G2/(Ka*G1*(Kb + Kp)))

p = 0.0001
t = 0:p:0.02-p;
u = 300*ones(length(t),1);
lsim(F1,u,t)
lsim(F2,u,t)
```

Problema 5:

$$\frac{G1}{G1*H - G1 + 1}$$

Código:

```
close all; clear all; history -c; clc;
pkg load symbolic;
syms G1 G2 H real;

F1 = G1/(1 + G1*H);
F2 = G2/(1 + G2);
eq1 = F1 == F2;
RES = solve(eq1, G2)
```

Problema 6:

Sistema 1:

$$\frac{G}{G*H + 1}$$

Sistema 2:

$$\frac{G}{G*H + 1}$$

Código:

```
close all; clear all; history -c; clc;
pkg load symbolic;
syms G H real;

% Sistema 1
P11 = G;
L11 = -G*H;
D1 = 1 - L11;
D11 = 1;
F1 = (P11*D11)/D1

% Sistema 2
P21 = G;
L21 = -G*H;
D2 = 1 - L21;
D21 = 1;
F2 = (P21*D21)/D2
```


Problema 7:

Sistema 1:

$$\frac{G1*G2*G3}{G1*G2*G3*H1*H2*H3 + G1*G2*H1*H2 + G1*G3*H1*H3 + G1*H1 + G2*G3*H2*H3 + G2*H2 + G3*H3 + 1}$$

Sistema 2:

$$\frac{G1*G2*G3}{G1*G3*H1*H3 + G1*H1 + G2*H2 + G3*H3 + 1}$$

Código:

```
close all; clear all; clc; history -c;
pkg load symbolic;
syms G1 G2 G3 H1 H2 H3 real;

% Sistema 1:
P11 = G1*G2*G3;
L11 = -G1*H1;
L21 = -G2*H2;
L31 = -G3*H3;
Delta1 = 1 - (L11 + L21 + L31) + (L11*L21 + L21*L31 + L31*L11) - (L11*L21*L31);
D11 = 1;
M1 = P11*D11/Delta1

% Sistema 2:
P12 = G1*G2*G3;
L12 = -G1*H1;
L22 = -G2*H2;
L32 = -G3*H3;
Delta2 = 1 - (L12 + L22 + L32) + (L12*L32);
D22 = 1;
F2 = (P12*D22)/Delta2
```

Problema 8:

Sistema 1:

$$\frac{1}{(C1*R1*s + 1) * (C2*R2*s + 1) * (C3*R3*s + 1)}$$

Sistema 2:

$$\begin{aligned} \text{FdST} = & \frac{C1^2 C2^3 C3^2 R1^2 R2^2 R3^2 + C1^2 C2^2 R1^2 R2^2 s + C1^2 C2^3 C3^2 R1^2 R2^2 s + C1^2 C2^2 C3^2 R1^2 R3^2 s - C1^2 C2^3 C3^2 R1^2 s + C1^2 C2^2 R1^2 s + C1^2 C2^3 C3^2 R1^2 s}{3^2 R1^2 s + C2^2 C3^2 R1^2 R3^2 s + C2^2 C3^2 R2^2 R3^2 s + C2^2 R1^2 s + C2^2 R2^2 s + C2^2 C3^2 R1^2 s + C2^2 C3^2 R2^2 s + C2^2 C3^2 R3^2 s - C2^2 C3^2 + C2^2 + C3^2} \end{aligned}$$

Código:

```
close all; clear all; history -c; clc;
pkg load symbolic;
syms R1 R2 R3 C1 C2 C3 s real;

% Circuito 1:
F1 = 1/(s*R1*C1 + 1);
F2 = 1/(s*R2*C2 + 1);
F3 = 1/(s*R3*C3 + 1);
FdST = F1*F2*F3

% Circuito 2:
syms I1 I2 I3 Vin Vout real;
XC1 = 1/(s*C1);
XC2 = 1/(s*C2);
XC3 = 1/(s*C3);

eq1 = Vin == I1*(R1 + XC1) - I2*XC1;
eq2 = 0 == -I1*XC1 + I2*(XC1 + R2 + XC2) - I3*XC2;
eq3 = 0 == -I2*XC2 + I3*(XC2 + R3 + XC3);

S1 = solve(eq3, I2); % I2 = I3*(C2*R3*s + C2/C3 + 1)
S2 = solve(eq2, I1); % I1 = I2*(C1*R2*s + C1/C2 + 1) - C1*I3
% I1 = I3*((C2*R3*s + C2/C3 + 1)*(C1*R2*s + C1/C2 + 1) - C1)
eq4 = Vin == (I3*((C2*R3*s + C2/C3 + 1)*(C1*R2*s + C1/C2 + 1) - C1))*(R1 + XC1) - I3*(C2*R3*s + C2/C3 + 1)*XC1;
S = solve(eq4, I3);
Vout = S*XC3;
FdST = pretty(simplify(Vout/Vin))
```

Problema 9:

9.A:

$I(s)/E(s)$:

$$y1: \frac{1e-06}{s + 8.85e-10}$$

$W(s)/E(s)$:

$$y1: \frac{0.042}{s^2 + 2e+06 s + 0.00177}$$

$I(s)/T(s)$:

$$y1: \frac{-1e-06}{s + 8.85e-10}$$

$W(s)/T(s)$:

$$y1: \frac{0.042}{s^2 + 2e+06 s + 0.00177}$$

9.B:

$W(s)/Wr(s)$:

$$y1: \frac{0.042}{s^2 + 2e+06 s + 0.04377}$$

$W(s)/Tl(s)$:

$$y1: \frac{-1e-06}{s + 2.189e-08}$$

Código:

```
close all; clear all; history -c; clc;
pkg load control;
pkg load symbolic;

s = tf('s');
L = 1*10^-6;
R = 2;
J = 10^6;
B = 0.3*10^-5;
Kb = 0.042;
Ka = 0.042;
G1 = 1/(L*s+R);
G2 = 1/(J*s+B);

%% PARTE A:
% F1 = W(s)/E(s) para T=0
P1 = G1*Ka*G2;
L1 = -G1*Ka*G2*Kb;
D = 1 - L1;
D1 = 1;
F1 = minreal((P1*D1)/D)

% F2 = I(s)/E(s) para T=0
P2 = G2;
F2 = minreal((P2*D1)/D)

% F3 = W(s)/T(s) para E=0
P3 = -G2;
F3 = minreal((P3*D1)/D)

% F3 = I(s)/T(s) para E=0
P4 = -G2*(-Kb)*G1;
F4 = minreal((P4*D1)/D)

%% PARTE B:
Kp = 1;
% W(s)/Wr(s) para T=0
P5 = Kp*G1*Ka*G2;
L5 = -Kp*G1*Ka*G2;
DD = 1 - (L1 + L5);
F5 = minreal((P5*D1)/DD)

% W(s)/T(S) para Wr=0
P6 = -G2;
F6 = minreal((P6*D1)/DD)
```

Problema 10 – Sistema 1:

Y5/Y1:

$$\frac{G1*G2*G3 + G3*G4}{G1*G2*G3*H3 + G1*G3*H1*H2 + G1*H1 + G3*G4*H3 + G3*H2 + 1}$$

Y4/Y1:

$$\frac{G1*G2 + G4}{G1*G2*G3*H3 + G1*G3*H1*H2 + G1*H1 + G3*G4*H3 + G3*H2 + 1}$$

Y2/Y1:

$$\frac{G3*H2 + 1}{G1*G2*G3*H3 + G1*G3*H1*H2 + G1*H1 + G3*G4*H3 + G3*H2 + 1}$$

Código:

```
close all; clear all; history -c; clc;
pkg load symbolic;
syms G1 G2 G3 G4 H1 H2 H3 H4 real;

% Y5/Y1
P1 = 1*G1*G2*G3;
P2 = 1*G4*G3;

L1 = -G1*H1;
L2 = -G3*H2;
L3 = -G1*G2*G3*H3;
L4 = -G4*G3*H3;

D = 1 - (L1 + L2 + L3 + L4) + (L1*L2);
D1 = 1;
D2 = 1;
F1 = (P1*D1 + P2*D2)/D

% Y4/Y1
P3 = G1*G2;
P4 = G4;
D3 = 1;
D4 = 1;
F2 = (P3*D3 + P4*D4)/D

% Y2/Y1
P5 = 1;
D5 = 1 - L2;
F3 = (P5*D5)/D
```

Problema 10 – Sistema 2:

Y5/Y1:

$$\frac{G1*G2*G3 + G3*G4}{G1*G2*G3*H3 + G1*G3*H1*H2 + G1*H1*H4 + G1*H1 + G3*G4*H3 + G3*H2 + H4 + 1}$$

Y4/Y1:

$$\frac{G1*G2*(H4 + 1) + G4*(H4 + 1)}{G1*G2*G3*H3 + G1*G3*H1*H2 + G1*H1*H4 + G1*H1 + G3*G4*H3 + G3*H2 + H4 + 1}$$

Y2/Y1:

$$\frac{G3*H2 + H4 + 1}{G1*G2*G3*H3 + G1*G3*H1*H2 + G1*H1*H4 + G1*H1 + G3*G4*H3 + G3*H2 + H4 + 1}$$

Código:

```
close all; clear all; history -c; clc;
pkg load symbolic;
syms G1 G2 G3 G4 H1 H2 H3 H4 real;

% SISTEMA 2:
% Y5/Y1
P1 = 1*G1*G2*G3;
P2 = 1*G4*G3;
L1 = -G1*H1;
L2 = -G3*H2;
L3 = -G1*G2*G3*H3;
L4 = -G4*G3*H3;
L5 = -H4;
Delta = 1 - (L1 + L2 + L3 + L4 + L5) + (L1*L2 + L1*L5);
D1 = 1;
D2 = 1;
M = (P1*D1 + P2*D2)/Delta

% Y4/Y1
P3 = 1*G1*G2*1;
P4 = 1*G4*1;
D3 = 1 - L5;
D4 = 1 - L5;
M2 = (P3*D3 + P4*D4)/Delta

% Y2/Y1
P5 = 1;
D5 = 1 - (L2 + L5);
M3 = (P5*D5)/Delta
```

Problema 10 – Sistema 3:

Y5/Y1:

$$\frac{10 \sqrt{5s^2 + 1}}{2s^2(8 - 125s) + 5s + 1}$$

Y4/Y1:

$$\frac{10 \sqrt{5s^2 + 1}}{2s^2(8 - 125s) + 5s + 1}$$

Y2/Y1:

$$\frac{10s^2}{2s^2(8 - 125s) + 5s + 1}$$

Código:

```
close all; clear all; history -c; clc;
pkg load symbolic;
syms s real;

% SISTEMA 3:
% Y5/Y1
P1 = 10*(1/s)*(1/s)*1;
P2 = 10*5*1;
L1 = (1/s)*(-5);
L2 = (1/s)*(1/s)*(-1);
L3 = 5*(-1);
L4 = 5*1*(-10*s)*(-5);
L5 = (1/s)*1*(-10*s);
Delta = 1 - (L1 + L2 + L3 + L4 + L5);
D1 = 1;
D2 = 1;
M = simplify((P1*D1 + P2*D2)/Delta)

% Y4/Y1
P3 = 10*(1/s)*(1/s)*1;
P4 = 10*5*1;
D3 = 1;
D4 = 1;
M1 = simplify((P3*D3 + P4*D4)/Delta)

% Y2/Y1
P5 = 10*1;
D5 = 1;
M2 = simplify((P5*D5)/Delta)
```

Problema 10 – Sistema 4:

Y5/Y1:

$$\frac{15 \sqrt{5s^2 + 1}}{s(s(375s - 2) + 76)}$$

Y4/Y1:

$$\frac{5s^2 + 1}{2s(s(375s - 2) + 76)}$$

Y2/Y1:

$$\frac{s + 1}{2(s(375s - 2) + 76)}$$

Código:

```
close all; clear all; history -c; clc;
pkg load symbolic;
syms s real;

% SISTEMA 4:
% Y5/Y1
P1 = 1*(1/s)*(1/s)*30;
P2 = 1*5*30;
L1 = (1/s)*(-1);
L2 = (1/s)*(-1);
L3 = 5*(-1)*(-1);
L4 = 5*30*(-5*s);
L5 = (1/s)*(1/s)*30*(-5*s);
Delta = 1 - (L1 + L2 + L3 + L4 + L5);
D1 = 1;
D2 = 1;
M = simplify((P1*D1 + P2*D2)/Delta)

% Y4/Y1
P3 = 1*(1/s)*(1/s)*1;
P4 = 1*5*1;
D3 = 1;
D4 = 1;
M1 = simplify((P3*D3 + P4*D4)/Delta)

% Y2/Y1
P5 = 1*1;
D5 = 1 - L2;
M2 = simplify((P5*D5)/Delta)
```


Problema 10 – Sistema 5:

Y5/Y1:

$$\frac{5*(s + 67)}{341*s^2 + 16*s + 335}$$

Y4/Y1:

$$\frac{11 - 5*s}{341*s^2 + 16*s + 335}$$

Y2/Y1:

$$\frac{11*s*(31*s + 1)}{341*s^2 + 16*s + 335}$$

Código:

```
close all; clear all; history -c; clc;
pkg load symbolic;
syms s real;

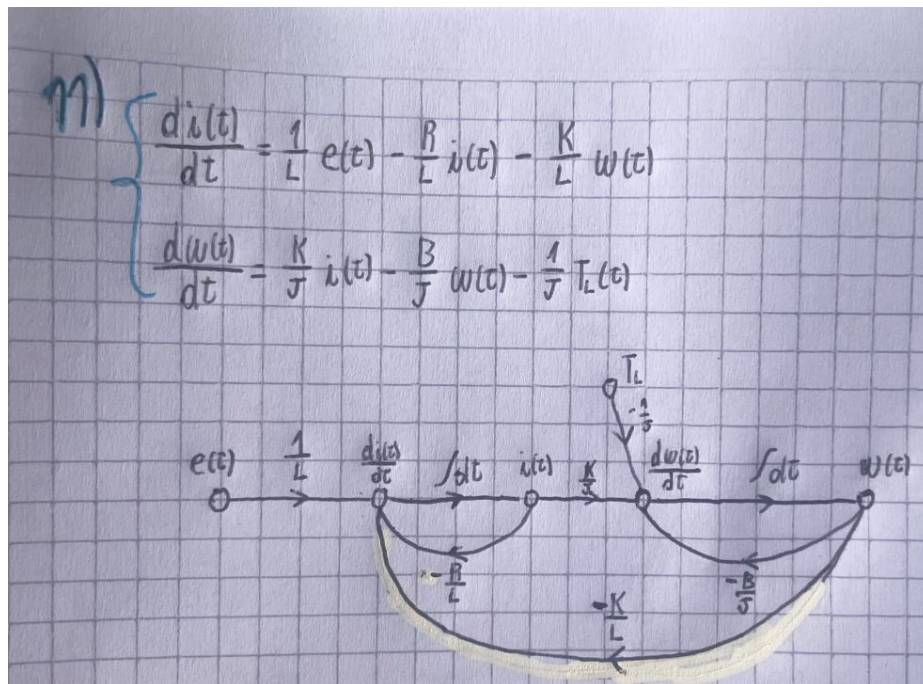
% SISTEMA 5:
% Y5/Y1
P1 = 1*(1/s)*(1/s)*30;
P2 = 1*5*(1/s);
L1 = (1/s)*(-1);
L2 = (30)*(-1);
L3 = (1/s)*(1/s)*30*(-1);
L4 = -10;
L5 = 5*(1/s)*(-1);
Delta = 1 - (L1 + L2 + L3 + L4 + L5) + (L1*L4 + L1*L5 + L2*L4 + L3*L4);
D1 = 1 - L4;
D2 = 1 - L1;
M = simplify((P1*D1 + P2*D2)/Delta)

% Y4/Y1
P3 = 1*(1/s)*(1/s)*1;
P4 = 1*5*(1/s)*(-1)*1;
D3 = 1 - L4;
D4 = 1;
M1 = simplify((P3*D3 + P4*D4)/Delta)

% Y2/Y1
P5 = 1*1;
D5 = 1 - (L1 + L2 + L4) + (L1*L4 + L2*L4);
M2 = simplify((P5*D5)/Delta)
```

Problema 11

Grafo de Fluencias:



$W(s)/E(s)$:

$$\frac{I \cdot K}{I^2 B \cdot L \cdot s^2 + I^2 B \cdot R + I^2 K - L^2 s^2 - R \cdot s}$$

Código:

```
close all; clear all; history -c; clc;
pkg load symbolic;

syms R L K H B s real;
syms V I W Tl real;

P1 = (1/L)*(1/s)*(K/J)*(1/s);

L1 = (1/s)*(-R/L);
L2 = (1/s)*(-B/J);
L3 = (1/s)*(K/J)*(1/s)*(-K/L);

Delta = 1 - (L1 + L2 + L3) + L1*L2;
D1 = 1;
M = simplify(factor((P1*D1)/Delta, 's'))
```

Problema 12

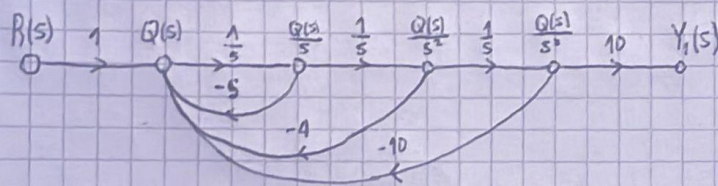
$$12) \quad G_1(s) = \frac{Y_1(s)}{R_1(s)} = \frac{10}{s^3 + 5s^2 + 4s + 10} \cdot \frac{s^{-3}}{s^{-3}} \frac{Q(s)}{Q(s)}$$

$$\frac{Y_1(s)}{R_1(s)} = \frac{10s^{-3}Q(s)}{(1+5s^{-1}+4s^{-2}+10s^{-3})Q(s)}$$

$$Y_1(s) = 10s^{-3}Q(s)$$

$$R_1(s) = (Q(s) + 5s^{-1}Q(s) + 4s^{-2}Q(s) + 10s^{-3}Q(s))$$

$$Q(s) = R_1(s) - 5s^{-1}Q(s) - 4s^{-2}Q(s) - 10s^{-3}Q(s)$$



$$G_2(s) = \frac{Y_2(s)}{R_2(s)} = \frac{6(s+1)}{s(s+2)(s+3)} = \frac{6s+6}{s^3+5s^2+6s} \cdot \frac{s^{-3}}{s^{-3}} \frac{Q(s)}{Q(s)}$$

$$\frac{Y_2(s)}{R_2(s)} = \frac{6s^{-2}Q(s) + 6s^{-3}Q(s)}{(1+5s^{-1}+6s^{-2})Q(s)}$$

$$Y_2(s) = 6s^{-2}Q(s) + 6s^{-3}Q(s)$$

$$R_2(s) = (Q(s) + 5s^{-1}Q(s) + 6s^{-2}Q(s))$$

$$Q(s) = R_2(s) - 5s^{-1}Q(s) - 6s^{-2}Q(s)$$

