EXPERIMENTAL VS THEORETICAL STUDY OF KINETICS

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Abstract

This report presents the findings of a comprehensive study on the kinetics of a particle, integrating theoretical principles with practical experimentation. The focus of the study is on analyzing the motion and impacts of a 1.88kg cart travelling down a 30-degree incline and colliding with different bumper materials; styrofoam and cardboard. The experiment leverages an accelerometer to record real-time acceleration versus time data, which were processed to deduce velocities before and after impact. Key areas of investigation included assessing velocities before impact, evaluating friction between the cart and metal bars, and calculating the coefficient of restitution for each collision. The results indicated significant discrepancies between theoretical predictions and experimental observations, primarily due to factors like friction, material deformation and data noise. Notably, the experimental accelerations for both materials were significantly lower than the theoretical calculations and the peak velocities before and after impact varied between materials, highlighting differences in energy conservation during collisions. The study concluded that while theoretical models provide a basis for understanding dynamics, real-world applications often reveal complexities not accounted for in theoretical models. It recommends more precise instruments and refined experimental setups for future studies to minimize these discrepancies.

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1 Introduction

The study of dynamics is a fundamental necessity in physics and stands as a crucial bridge connecting theoretical principles with real-world applications. Using Newtonian physics, it provides a highly sufficient understanding of how objects move and interact for most daily purposes without the need for increasingly difficult mathematics and physics. For that reason, principles of dynamics are useful across various fields, from traditional physical studies of engineering and natural sciences to more recent technological fields such as robotics and computer vision. This project acts as an introduction to the intersection between the theoretical and the applicative, by delving into the kinetic behaviors of a moving cart. It exemplifies the practical application of dynamic principles, highlighting the inevitable variances that arise when theoretical models meet the complexities of real-world scenarios.

The primary objective of this study is to analyze the kinetics of a particle, focusing specifically on a 1.88kg cart descending a 30-degree incline and colliding with either styrofoam or cardboard. Through the comparison of theoretical calculations to experimental data, it seeks to understand the impact common assumptions have on the accuracy of theoretical calculations. In this experiment, the goal is to analyze the impact of friction in the motion of the cart as it travels down the metal bars of the incline. The experiment also aims to examine the nature of collisions and momentum, particularly assessing the linear impulse and the coefficient of restitution in interactions with different bumper materials. This involves an investigation into how energy is conserved and dissipated during these collisions through an analysis of the cart's velocity. However, beyond the discrepancies between theoretical and experimental attributed to assumptions exist the natural variances of the real world. This project also serves as an introductory example to noise in data collection and measurement devices such as accelerometers, with the goal of learning how to deal with external factors and the process for cleaning data to produce accurate models.

To achieve these objectives, the experiment employs a methodical approach, using an accelerometer to firstly gather acceleration versus time data. This data is then processed in a python script with the use of different computation and plotting libraries such as *NumPy*, *Pandas*, and *Matplotlib*. Using these libraries, the acceleration data is used to create plots of velocity vs time, which are more useful for the purposes of this project. This transformation highlights the noise in the data, notably the velocity drift error, which is corrected through interpolation. Finally, the data is analyzed to determine the velocities of the cart before and after the impact, allowing conclusions to be drawn. By comparing these experimental observations with theoretical predictions, the study endeavors to provide a comprehensive understanding of the dynamics involved.

Most importantly, the project serves as a platform to engage with the practical aspects of dynamics. It allows for a hands-on experience in data collection and analysis, fostering a deeper appreciation for the intricacies involved in translating theoretical models into empirical investigations. The study not only reinforces fundamental dynamics concepts but also encourages critical thinking and problem-solving skills, essential in the field of engineering and beyond.

2 Background

In the application of dynamics, the study of motion through introductory experiments like the car-ramp project provides vital insights into physical principles. At the heart of such experiments lies the accelerometer, a device originally found in spaceships and jets, now commonly used in cars, smartphones, and gaming devices [1]. Accelerometers measure acceleration by detecting the force exerted by a mass under acceleration, meaning it is not done by calculating speed changes over time, but rather by measuring the force itself, often through mechanical or electrical means [2].

Mechanical accelerometers, for instance, function like a mass attached to a spring inside a casing, where the force of acceleration is deduced form the spring's displacement. Modern accelerometers, especially those in cellphones, utilize micro electro-mechanical systems (MEMS) to achieve this in a much smaller scale [2].

Complementing these technological advancements which provide cheap and relatively accurate measurement tools such as the accelerometer, are car-ramp experiments which serve as a practical and simple application of dynamics concepts. These experiments are often conducted in two ways: analyzing a car's motion on a horizontal track or its movement on an inclined plane. The process involves measuring variables like interval time, total time, and velocity, providing a quantitative assessment of motion. In inclined plane experiments such as the one conducted in this lab, the goal is often to study the natural motion of the car under forces of gravity and friction, followed by additional challenges such as collision testing [3].

The accessibility of technology like accelerometers and the practicality of experimental setups such as the car-ramp offers a more applicative understanding of dynamics. These experiments exemplify the transition from theoretical models to empirical understanding, shedding light on how objects move and interact under various forces and conditions.

3 Methodology

The main outcomes asked of the experiment at hand was to record experimental data of a cart sliding down a ramp in two scenarios, one where the cart impacts cardboard, the other where the cart impacts with a foam block at the bottom of the ramp. Below are diagrams to demonstrate the motion and forces acted upon the cart throughout the experiment, including a system diagram in Figure 1, a Free-Body Diagram in Figure 2, and a Kinetic Diagram in Figure 3.

Using both experimental data collected by the accelerometer mounted on the cart as well as the combination of Newton's second law for rectilinear motion and the principles of work and energy, the theoretical and experimental data can be compared to draw conclusions based on discrepancies and similarities. In the theoretical calculations, it is assumed that the cart is a point particle located at the tip/lowest point of the cart and that there is no friction between the cart and rails or the surface. Lastly, all calculations are made under the assumption that the gravitational force per unit of mass is 9.81 $\frac{m}{s^2}$.

The choice of technology is notebook on Google Colab, using Python, and Pandas, NumPy and Matplotlib libraries. The full code can be found [in this GitHub Repository](#).

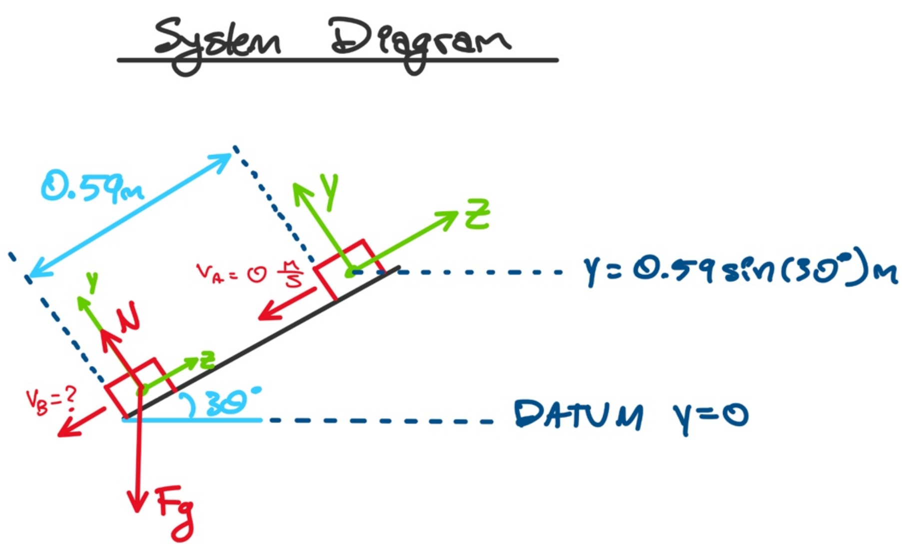

Figure 1: System Diagram of Cart Throughout Experiment

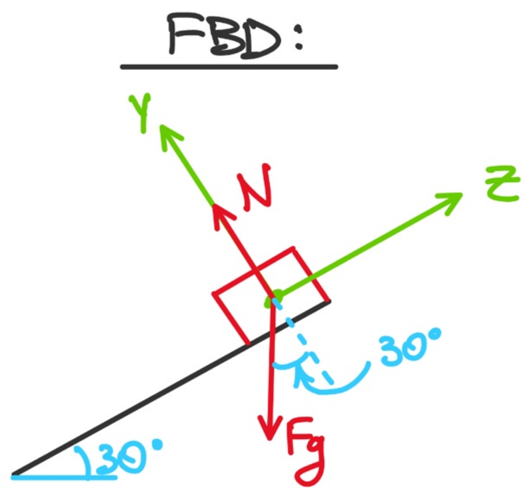

Figure 2: Free-Body Diagram of Forces Acting Upon Cart

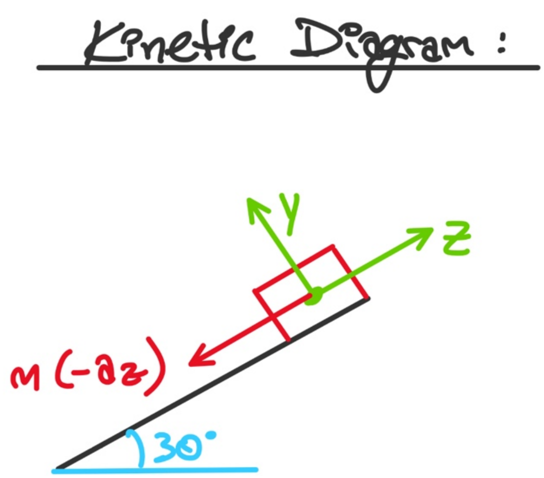

Figure 3: Kinetic Diagram of the Cart Throughout Experiment

3.1 Computing Experimental Acceleration

Figure 4 and Figure 5 below show the plot of the data collected by the accelerometer for both experiments, carboard and foam respectively.

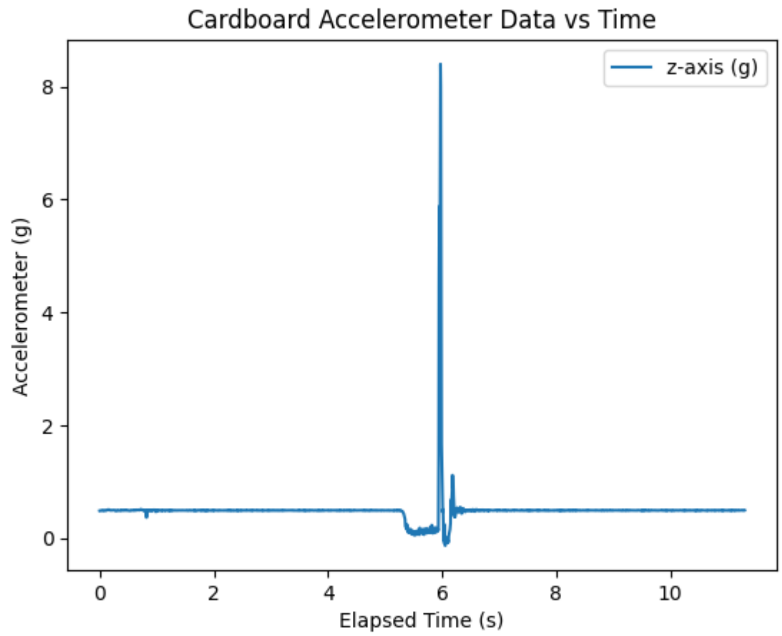

Figure 4: Carboard Accelerometer Data

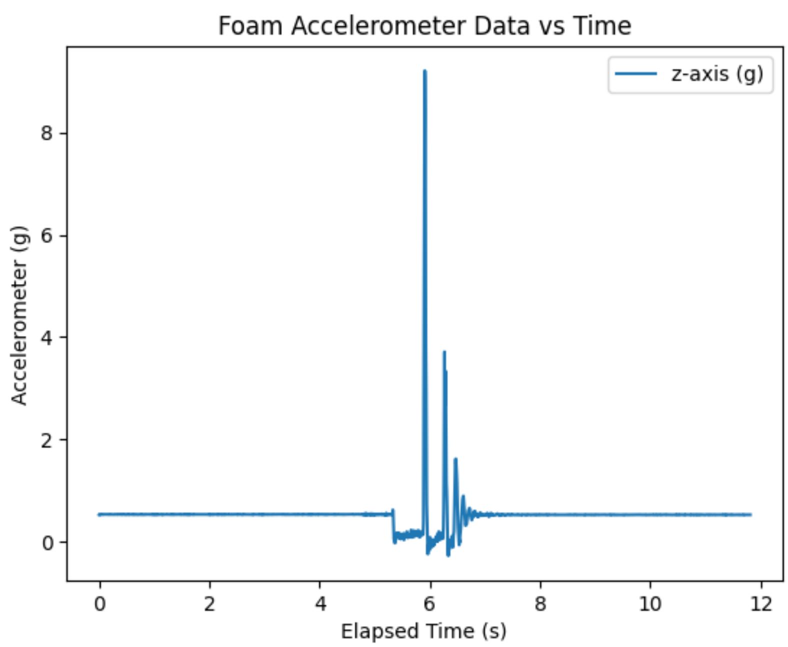

Figure 5: Foam Accelerometer Data

The accelerometer's used measured data as a function of gravity g based on Equation 1 below.

$$data = \frac{a_z}{g} + offset \tag{1}$$

To convert the accelerometer's data in terms of acceleration in the z direction, the data was manipulated according to Equation 2 below, which is the rearrangement of Equation 1 to isolate for a_z.

$$a_z = (data - offset) * g \tag{2}$$

Where the offset for the cardboard experiment was 0.486g and the offset for the foam experiment was 0.522g. The processing explained above resulted in Figure 9 and Figure 10 located in the results section, which show the plots of acceleration vs time for both the cardboard and foam experiments.

To calculate the average experimental acceleration the cart experiences throughout its travel down the ramp two data points were located. The first being the first index of data that is negative. This index represents the first point of which the cart is let go and begins its fall down the ramp. The second data point is that directly to the left of the largest positive data point collected. Since the largest peak in the graph represents the first bounce of the cart from the cardboard/foam, the point to its left signifies the final moment of free-fall down the ramp before impact. By averaging all data between these two points, the average experimental acceleration experienced by the cart, from release until just before impact is calculated.

Given the mass of the cart, distance traveled in meters, and angle of the ramp relative to the ground, the average acceleration of the cart could be determined under the assumptions that the cart is a point particle located at the tip, that friction is negligible, and that g = 9.81 $\frac{m}{s^2}$. Based on the Free-Body Diagram in Figure 2 and Kinetic Diagram in Figure 3, Newton's Second Law for Rectilinear Motion was applied to determine the acceleration in the z-direction which was dependent only on the gravitational force.

3.2 Computing Continuous Velocity from Discrete Data

Velocity, known as the change in position with respect to time, or the derivative of position with respect to time, can also be referred to as the integral of acceleration, which was gathered by processing the accelerometer data as demonstrated in Section 3.1 above. However, to derive velocity from acceleration, a continuous function for acceleration is required, which would not be possible in the experiment conducted as accelerometers collect discrete points, not a continuous function. Thus, rather than obtaining a function for velocity with respect to time through the integral of a continuous acceleration function, the velocity at every time can be calculated by applying the discrete equation of velocity at every reading, shown in Equation 3 below.

$$v_i = \sum_{j=1}^{j=i} a_j \Delta t \quad (3)$$

Utilizing Python's library Pandas and NumPy, v_i was calculated by multiplying the acceleration, shown in Section 3.1, by the difference in time between index (i) and index (i-1). Then by utilizing the cumulative summation function, the raw, drifted velocity for all the data points gathered from the accelerometer was calculated for both experiments. The data could then be plotted against time as shown in Figure 6 and Figure 7, for the cardboard and foam trials respectively.

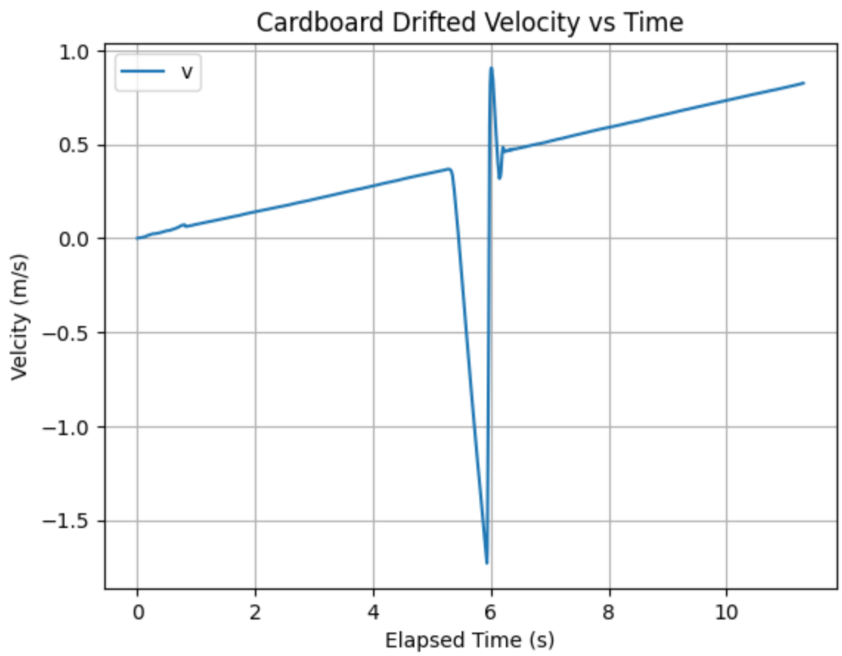

Figure 6: Drifted Velocity Plot for Cardboard Experiment

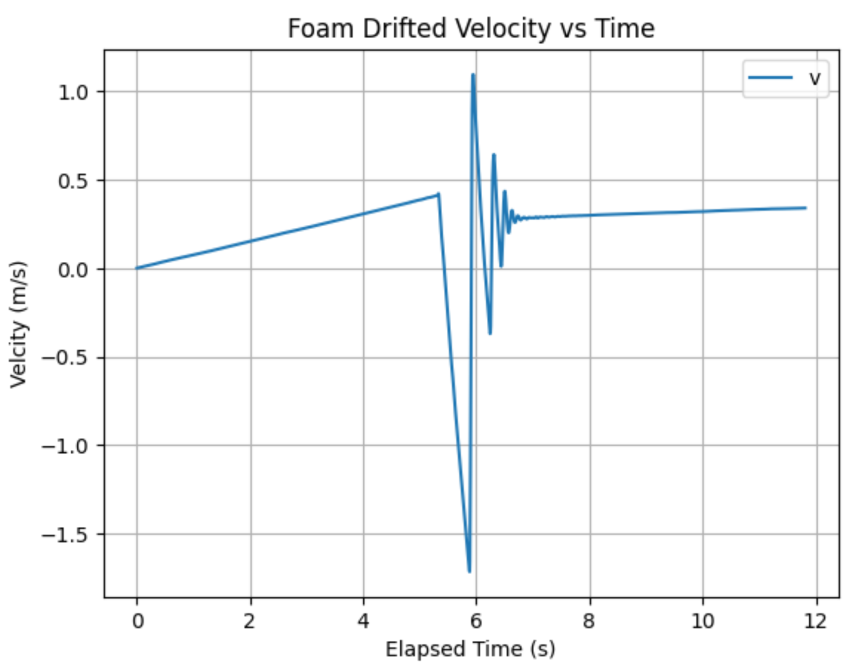

Figure 7: Drifted Velocity Plot for Foam Experiment

3.3 Velocity Drift Error

An important characteristic in both Figure 6 and Figure 7 is the significant drift shown by the velocity computed in Section 3.2 based on the accelerometer data processed in Section 3.1. In the plot of velocity vs time for the cardboard experiment, it is visible that the drift of the sensor is a linear function, as the first dip into the negative is when the cart was released and began its travel until impact. All data to the left of that point should be at zero. Similarly, on the right side of the graph, the peaks decrease in amplitude and merge into the linear drift function, which is where the cart came to a stop.

To remove this drift, 2 data points were selected, both to the left of when the cart began traveling down the ramp, to calculate the slope of the drift function. Having the function of drift with respect to time, the data could be processed to remove drift by taking the velocity at time x and subtracting the value of the drift function at time x, which is shown in Equation 4. This process would result in processing the velocity data to remove the skewed data that was created by the drift. This resulted in the graph shown in Figure 11 in the results section, which plots the relationship of velocity without drift against time for the cardboard experiment.

$$v_{i(nodrift)} = v_i - v_{drift} = v_i - mt_i \tag{4}$$

The foam experiment however, demonstrated an abnormal drift since it did not exhibit the same slope of linear drift before and after impact as shown in Figure 6. Thus, removing drift from the data would require more steps compared to the cardboard experiment. To adjust the velocity to filter out the drift of the sensors, the foam data was split into two sections, the first being all the data prior to the largest velocity impact which is at the time of impact, the second being all data point after the peak velocity. Then, Equation 4 above is applied to the two sections, each with their own slope, which was calculated by taking 2 points for each segment to calculate the slope for the respective linear drifts based on Equation 5 below.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \tag{5}$$

After doing so, the section prior to time of impact is higher than the section before time of impact. To offset this, 0.22 is subtracted from every data point after the impact.

By combining the results from Equation 4 and Equation 5, the drift can be filtered out of the velocity data of the foam experiment, shown in Figure 12 found in the results section.

3.4 Velocity Offset

During the experiment, there could be a case when the accelerometer hits its maximum allowable acceleration of 15g and happens when the material is stiff, and the impact becomes too large. When this case occurs, the offset of velocity must be accounted for. However, in the current data, this case does not occur, as seen in Figure 4 and Figure 5, the maximum accelerometer data lies around 8g to 9g, not surpassing the allowable 15g, therefore, the velocity offset is not applied.

3.5 Average Acceleration

Average acceleration was calculated 2 ways, theoretically and experimentally. For theoretical calculations of acceleration, the foam experiment and cardboard experiment will have the same value which is calculated based on the equations below and breaking up the gravitational vector into its Z and Y components.

These calculations are based off the following data:

- Given: $m = 1.88kg \quad \theta = 30°$
- Determine: $\vec{a}$
- Assumptions: Friction is negligible, cart is a point particle located at point, g = 9.81 $\frac{m}{s^2}$.
- Diagrams: Figure 1, Figure 2, Figure 3
- Newton's Second Law for Rectilinear Motion

$$\sum F_z = ma_z = -mgsin(30°)$$

$$a_z = -gsin(30°)$$

To calculate the experimental average acceleration for the two experiments, each data set was visually inspected to determine the time where the object begins its traverse down the ramp, and the time of impact with the cardboard/foam. Then the mean of acceleration, processed in Section 3.2, for all data points between the two time points resulted in the average experimental acceleration for both experiments.

3.6 Peak Velocities

The peak velocities for each experiment, were found by utilizing min and max functions which are built into both the Python Programming Language and its library Pandas. The function searched through all the velocity data to determine the minimum velocity, the velocity right before impact, and the maximum velocity which occurred right after impact. The same method was used for both the cardboard and foam experiments. These values are tabulated and demonstrated graphically in Section 4.3 in the results.

3.8 Theoretical Velocity Before Impact

To calculate the theoretical velocity the follow processes/assumptions were utilized. The calculated theoretical value for velocity before impact is the same for both cardboard and foam since the only difference between experiments is the material of the other object in the cart's collision.

- Given: $s = 0.59m \quad m = 1.88kg \quad \theta = 30° \quad \overrightarrow{V_A} = 0$
- Determine: $\overrightarrow{V_B}$
- Assumptions: $Cart\ is\ a\ point\ particle. \quad Friction\ is\ negligeble. \quad g = 9.81m/s^2$
- Diagrams: Figure 1, Figure 2, Figure 3
- Governing Principles: $Principles\ of\ Work\ and\ Energy$

$$T_A + (V_e)_A + \left(V_g\right)_A + U'_{A-B} = T_B + (V_e)_B + \left(V_g\right)_B$$

$$T_A = 0, \qquad U'_{A-B} = 0, \qquad (V_g)_B = 0$$

$$\therefore (\mathrm{V_g})_\mathrm{A} = \mathrm{T_B}$$

$$\mathrm{mgh_A} = \frac{1}{2}\mathrm{mv_B^2}$$

$$\mathrm{V_B} = \sqrt{2gh_A}$$

3.9 Friction

Using the experimental velocities before impact for both the cardboard and foam case provides the information required to calculate, word done due to friction, average friction force, and the coefficient of kinetic friction for both experiments. Figure 8 below is a Free-Body Diagram including the friction force for the cart.

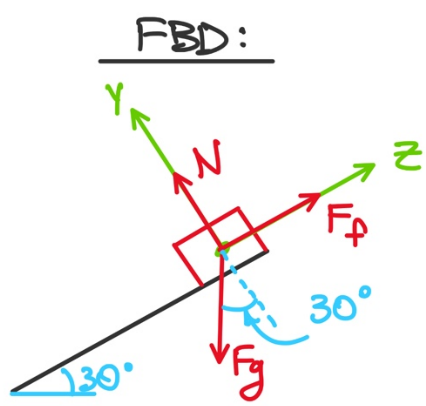

Figure 8: Free-Body Diagram of Cart Including Friction

- Given: $s = 0.59m \quad m = 1.88kg \quad \theta = 30° \quad \overrightarrow{V_A} = 0$

$$(V_{impact})_{cardboard} = -2.1557m/s \quad (V_{impact})_{Foam} = -2.1706m/s$$

- Determine: $Work\ due\ to\ friction, Average\ Frction\ Force,\ \mu_k$
- Assumptions: $Cart\ is\ a\ point\ particle\ located\ at\ the\ tip\ of\ the\ cart. \quad g = 9.81m/s^2$

- Diagrams: Figure 1, Figure 3, Figure 8
- Governing Principles:

  $Principles\ of\ Work\ and\ Energy, Newton's\ Second\ Law\ for\ Rectilinear\ Motion$

$$T_A + \left(V_g\right)_A + U_{A-B} = T_B + \left(V_g\right)_B$$

$$\sum F_z = ma_z \sum F_y = 0$$

Work due to Friction:

$$T_A + \left(V_g\right)_A + U_{A-B} = T_B + \left(V_g\right)_B$$

$$mgh + U_{1-2} = \frac{1}{2}mr^2$$

$$U_{1-2} = \frac{1}{2}mv_2^2 - mgh$$

Average Friction Force:

$$\sum F_y = 0 = N - mgcos(30°)$$

$$N = mgcos(30°)$$

$$F_f s = U_{1-2}$$

$$F_f = \frac{U_{1-2}}{s}$$

Coefficient of Kinetic Friction:

$$F_f = \mu_k N$$

$$\mu_k N = \frac{U_{1-2}}{s}$$

$$\mu_k = \frac{U_{1-2}}{sN}$$

3.10 Coefficient of Restitution

The coefficient of restitution is a kinetic property of materials in collision experiments that represents the elasticity of two materials during a collision, calculated as the ratio of final velocity to initial velocity before and after the collision. A coefficient of restitution near one represents a nearly elastic collision where minimal energy is lost to deformation, whereas a collision with a coefficient of restitution near zero represents a collision where all energy from both objects is absorbed into the collision and the objects have no velocity afterwards. Based on the velocity data a rough idea of the elasticity of the collision can be estimated. To calculate the experimental value is as follows:

- Given: $(V_{cardboard})_{before} = -2.1557696\ \frac{m}{s}, (V_{cardboard})_{after} = 0.47523\ \frac{m}{s}$
- $(V_{foam})_{before} = -2.17064\ \frac{m}{s}, \left(V_{foam}\right)_{after} = 0.810339\ \frac{m}{s}$
- Determine: $e_{cardboard}, e_{foam}$
- Assumptions: $Cart\ is\ a\ point\ particle, \quad g = 9.81m/s^2, \quad Momentum\ is\ conserved$
- Governing Principles: $Principles\ of\ Momentum\ and\ Impulse$

$$e = \frac{\left(V_{B_2}\right)_x - \left(V_{A_2}\right)_x}{\left(V_{A_1}\right)_x - \left(V_{B_1}\right)_x},\ \ where\ B\ is\ cart, A\ is\ foam/carboard$$

Since the foam/cardboard block does not move, their velocities are zero.

$$e_{cardboard} = \frac{\left(V_{B_2}\right)_C - 0}{0 - \left(V_{B_1}\right)_C}$$

$$e_{foam} = \frac{\left(V_{B_2}\right)_F - 0}{0 - \left(V_{B_1}\right)_F}$$

4 Results and discussion

4.1 Code and Graphs:

The code is submitted on Learn and can also be accessed through this GitHub repository. To find velocity at any given time, a user input is provided in the code, where the user inputs the desired time, and outputs a velocity for the respective time. For time values between discrete data points, linear interpolation is used to find the velocity.

The accelerometer data vs time graph for cardboard and foam respectively are shown in Figure 4 and Figure 5. The acceleration vs time graph for cardboard and foam respectively are shown in Figure 9 and Figure 10. And finally, the drift-less velocity vs time graph for cardboard and foam are shown in Figure 11 and Figure 12 respectively.

4.2 Average Acceleration:

Below are the processed acceleration vs time plots for both the cardboard collision and the foam collision:

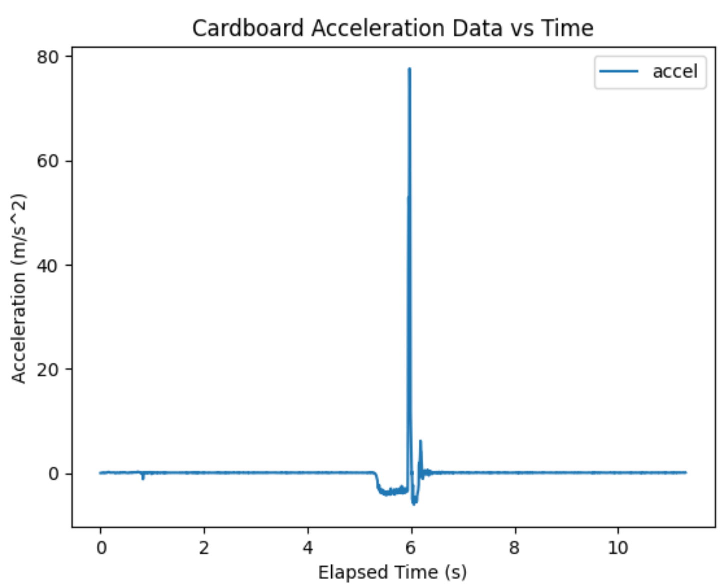

Figure 9: Acceleration vs Time for Cardboard Experiment

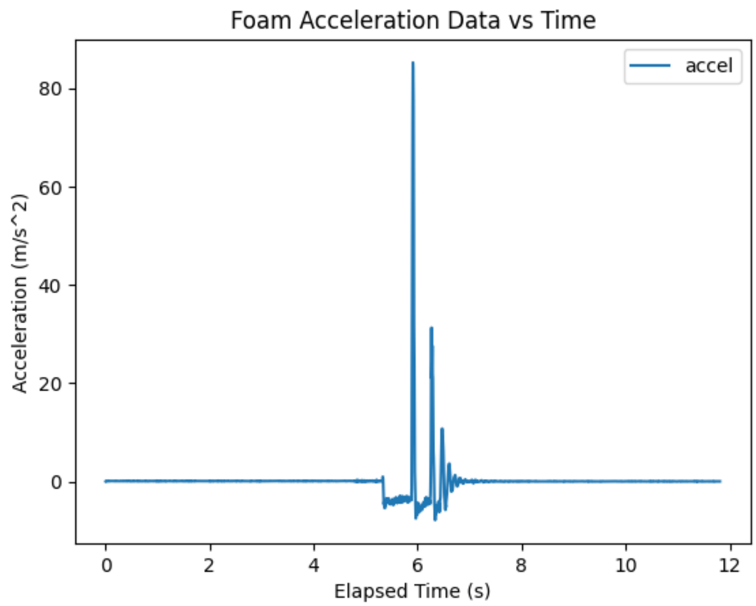

Figure 10: Acceleration vs Time for Foam Experiment

In both plots, at around the 5.5 second mark the acceleration drops where it oscillates with a roughly fixed amplitude, which is the acceleration the cart experiences as it moves down the ramp due to the force of gravity. At near the 6 second mark the acceleration jumps to a high positive value which is caused by the cart impacting the cardboard/foam, with each spike afterwards representing a bounce off the impact material, each becoming smaller as energy is lost to the deformation of the impact material and friction of the cart moving up and down the track until it reaches zero acceleration once again.

Table 1: Average Theoretical and Experimental Accelerations for Cardboard and Foam

	Average Theoretical Acceleration (m/s^2)	**Average Experimental Acceleration (m/s^2)**
Cardboard	$-4.91\,\hat{k}$	$-3.58\,\hat{k}$
Foam	$-4.91\,\hat{k}$	$-3.84\,\hat{k}$

The two experimental values are similar because the impact material is not relevant during the slide. Both values are significantly lower than the theoretical acceleration, as the theoretical calculation does not consider friction and drag, which are factors that are always present in the system in the real world. Theoretical calculations for the acceleration of both cardboard and foam can be found in Appendix A: Full Calculations for Theoretical Acceleration, while the process for calculating the average experimental acceleration can be found in Section 3.1.

4.3 Peak Velocity:

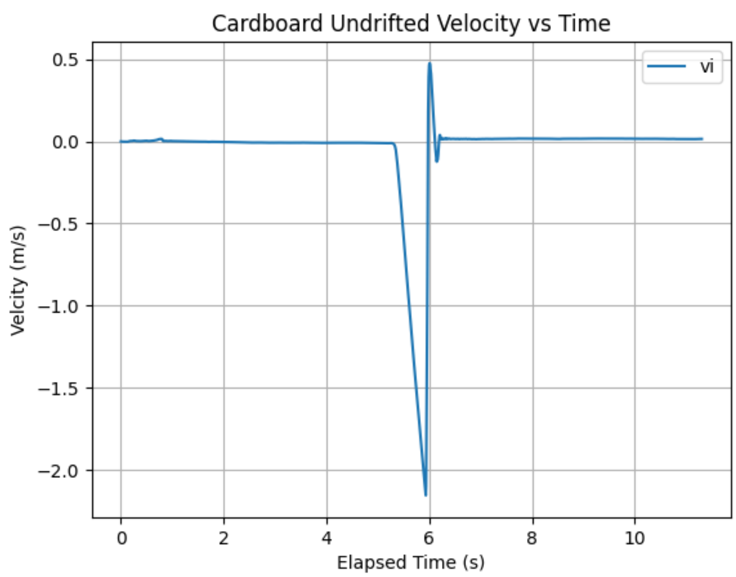

Figure 11: Drift-less Velocity vs Time Plot for Cardboard Experiment

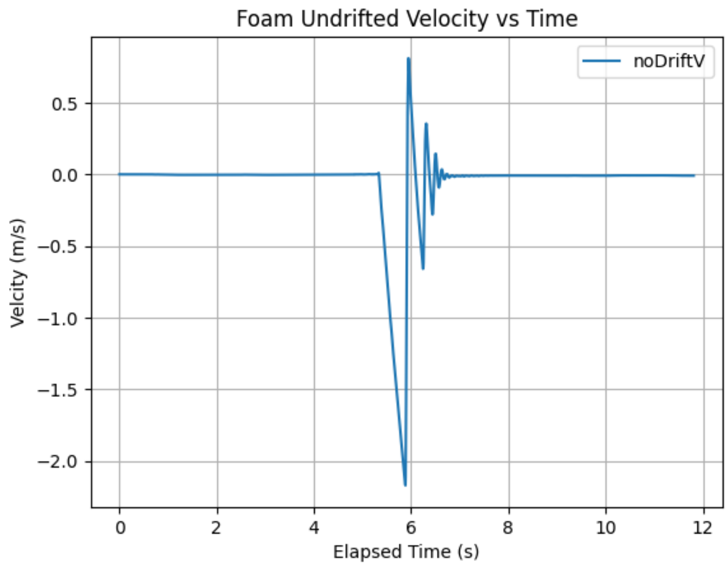

Figure 12: Drift-less Velocity vs Time Plot for Foam Experiment

The drift-less velocity vs time plots are shown in Figure 11 and Figure 12 for the cardboard and foam data respectively.

Table 2: Peak Experimental Velocities Before and After Impact with Cardboard and Foam

	Peak Velocity Before Impact (m/s)	Peak Velocity After Impact (m/s)
Cardboard	$-2.16\,\hat{k}$	$0.475\,\hat{k}$
Foam	$-2.17\,\hat{k}$	$0.810\,\hat{k}$

The peak velocities for cardboard are $-2.16\ m/s$ before impact and $0.475\ m/s$ after impact. The points are indicated graphically in Figure 13 below.

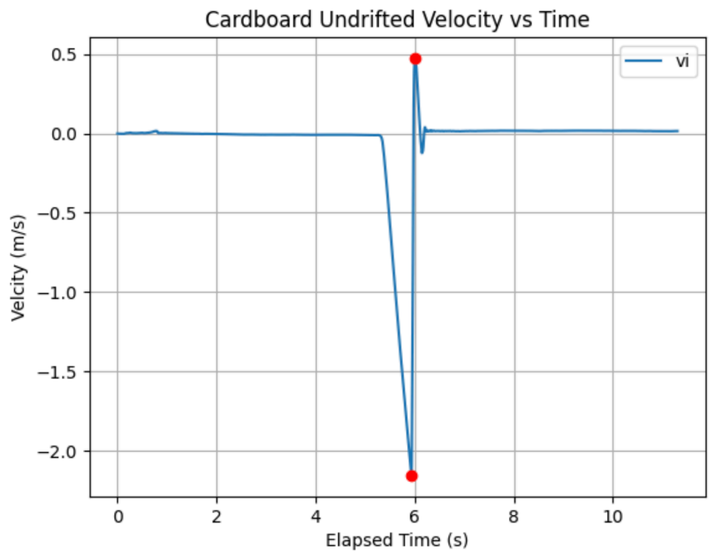

Figure 13: Peak Velocities for Cardboard Data

The peak velocities for foam are $-2.17\ m/s$ before impact and $0.810\ m/s$ after impact. The points are indicated graphically in Figure 14 below.

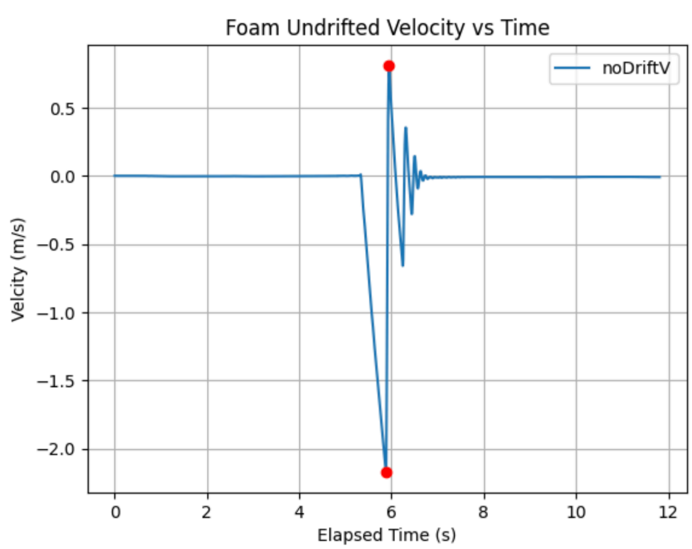

Figure 14: Peak Velocities for Foam Data

Given, that the velocities before impact of the cardboard and foam experiment were nearly equal, yet the velocity of foam is double that of carboard after impact, demonstrates the cardboard impact absorbed more energy from the cart through deformation compared to the foam impact. This relates to the coefficient of restitution, which is shown shortly.

4.4 Theoretical Velocity before Impact:

Table 3: Theoretical and Experimental Velocity Before Impact with Cardboard and Foam

	Theoretical Velocity Before Impact (m/s)	Experimental Velocity Before Impact (m/s)
Cardboard	$-2.41\,\hat{k}$	$-2.16\,\hat{k}$
Foam	$-2.41\,\hat{k}$	$-2.17\,\hat{k}$

The theoretical velocity just before impact for both impact materials is $2.41\ m/s$ in the negative z direction, calculated in Appendix B: Full Solution of Impact Velocity.

The peak velocities before impact demonstrated in the previous section represent the experimental values of the velocity before impact. For cardboard, this is 2.16 m/s in the negative k direction, and for foam, it is 2.17 m/s. These experimental values are lower than the theoretically calculated velocities. This discrepancy could be attributed to environmental factors not accounted for in the theoretical calculations, such as friction.

4.5 Work due to Friction, Average Friction Force, Coefficient of Kinetic Friction

Table 4: Work due to Friction, Average Friction Force and Coefficient of Kinetic Friction with Cardboard and Foam

	Work due to Friction [J]	Avg Friction Force [N]	Coefficient of Kinetic Friction
Cardboard	-1.07	1.82	0.114
Foam	-1.01	1.71	0.107

The work due to friction, average friction force and the coefficient of kinetic friction for cardboard are calculated in Appendix C: Full Solution for Friction Calculations and Section 3.9, resulting in values of $-1.07\ J$, $1.82\ N$ and 0.114 respectively.

The work due to friction, average friction force and the coefficient of kinetic friction for foam are $-1.01\ J$, $1.71\ N$ and 0.107 respectively.

The calculated values are similar between impact materials because they are calculated using acceleration and velocity values before collision, meaning the impact materials are irrelevant to these calculations. There are slight discrepancies between the values due to errors in measurement or slight variances in environmental factors.

4.6 Coefficient of Restitution:

Table 5: Coefficient of Restitution with Cardboard and Foam

	Coefficient of Restitution
Cardboard	0.220
Foam	0.373

Coefficient of restitution represents the elastic nature of a collision, where a coefficient of 1.00, signifies a fully elastic collision, and a coefficient of 0.00 signifies an inelastic collision where all velocity is lost in the deformation of the collision. The calculated coefficient of restitutions for the experiment, which can be found in Appendix D: Full Solution for Coefficient of Restitution, were 0.220 for cardboard and 0.373 for foam. Foam's coefficient of restitution is higher than cardboard's, which signifies a larger conservation of kinetic energy in the system throughout the collision. This is due to foam's more elastic nature, given compressed foam is more capable of restoring its original shape much like a spring. On the other hand, cardboard is less elastic and thus more susceptible to deformation, meaning more kinetic energy is lost during the collision.

5 Conclusions and Recommendations

In conclusion, the comprehensive analysis presented in this technical report highlighted the significance of including friction and other factors in any model of real-world experiments. The experimental results, while illustrating the expected behaviour of a cart travelling down a 30-degree incline and colliding with different materials, also underscored the discrepancies that often create large differences between experimental results and theoretical models. Friction, noise in data, and material deformation were identified as significant factors that could affect the measurements and outcomes. Thus, it is recommended that future studies carefully study these factors and employ measures to better control their impact on the experiment, such as using more precise instruments and refining the experimental setup - which could include testing more variables aside from collision material.

Precision of sensors is the first location where error may occur in an experiment like the one discussed in this report, as it is the first point of contact to the raw data, and as seen above, sensors can create a problematic drift in the acceleration data collection. The sensors used demonstrated uneven drift before and after the experiment, which required interpolation of sloped to correct and calibrate, reducing accuracy of the data. With the use of more accurate and reliable sensors, the drift would have the same slope before and after the impact, or ideally, have no drift which could be a costly investment.

Through processing and filtering of accelerometer data, a clean velocity plot without drift was obtained which allowed for in-depth analysis of friction, momentum, and impulse measurements such as coefficient of restitution. These factors are very important in creating theoretical models to be able to accurately predict the behaviors of the cart before and after collision. When assuming friction to be negligible, such as in the theoretical calculations shown earlier in the report, the theoretical model becomes a very weak representation of the experiment since friction has a large effect on the system, which can be seen in the discrepancies in results between the theoretical and experimental models. The discrepancies between the non-friction models and experimental drastically increase as the mass of the object of study increases, as that will directly increase the friction force, based on the setup of this experiment and the forces acting upon the cart.

Overall, the experiment was successful in demonstrating the different factors required to consider when formulating a theoretical model, as well as demonstrating the effects friction has on various kinetic measurements and properties of objects and lastly the importance of accurate sensors and measurement system.

6 Acknowledgements

This work was conducted under and administered by Prof. Dr. Azzi. The authors are also particularly grateful to Dr. Charbel Azzi for his insight into the nature of moments and impulse, as well as friction.

7 References

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Appendix A: Full Calculations for Theoretical Acceleration

2) 1 a) Given: $s = 740 - 150\,mm = 590\,mm = 0.59\,m$

$m = 1.88\,kg$

$\theta = 30°$

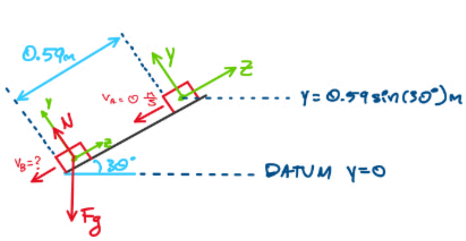
b) Determine: $\vec{a}$

c) Cart is a point particle located at the top

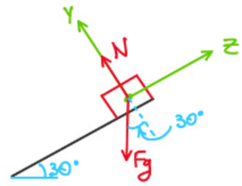
Friction is negligable

$g = 9.81\ \frac{m}{s^2}$

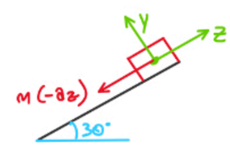
2a) System Diagram



FBD:



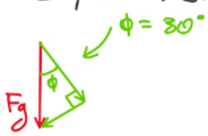
Kinetic Diagram:



b) N2L for rectilinear motion:

$\Sigma F_y = 0$, $\Sigma F_z = m a_z$

c)



$\Sigma F_z = m a_z = -mg\sin(30°)$

$a_z = -g\sin(30°)$

$a_z = -4.905\ \frac{m}{s^2}$

2d)✓ e)✓ f)✓

g) ∴ the theoretical acceleration during the slide until impact is $4.91\ \frac{m}{s^2}$ [-E] or $4.91\ \frac{m}{s^2}$ [down the ramp]

Appendix B: Full Solution of Impact Velocity

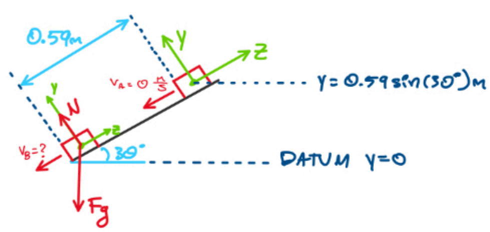
4) 1a) Given: $s = 0.74 - 0.15 = 0.59\,m$
$m = 1.88\,kg$
$\theta = 30^\circ$
$V_A = 0$

1b) Determine: $\vec{V}_B$

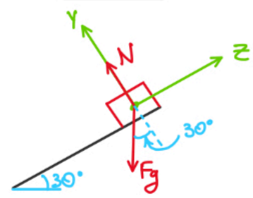
1c) Cart is a point particle located at tip
No friction between cart & metal rails
$g = 9.81\ \frac{m}{s^2}$

2a)

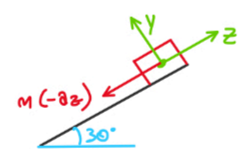
System Diagram



FBD:



Kinetic Diagram:



2b) Work-Energy Principle

$$T_A + (V_e)_A + (V_g)_A + U'_{A-B} = T_B + (V_e)_B + (V_g)_B$$

c) $\cancel{T_A} + (V_g)_A + \cancel{U'_{A-B}} = T_B + \cancel{(V_g)_B}$

0 since $V_A = 0\ \frac{m}{s}$ 0 assumed no friction 0 because of datum

$$(V_g)_A = T_B$$

$$mgh_A = \frac{1}{2} m V_B^2$$

$$V_B = \sqrt{2gh_A}$$

$$= \sqrt{(2)(9.81\ \tfrac{m}{s^2})(0.59\sin(30^\circ))}$$

$$\vec{V}_B = 2.406\ \frac{m}{s}\ [-\hat{z}]$$

2d) ✓ e) ✓ f) ✓

g) ∴ the velocity at the point of impact is $2.41\ \frac{m}{s}\ [-\hat{z}]$ or $2.41\ \frac{m}{s}$ down the ramp.

Appendix C: Full Solution for Friction Calculations

5) 1a) Given: $V_{impact\ c} = -2.1557\ [\frac{m}{s}]$

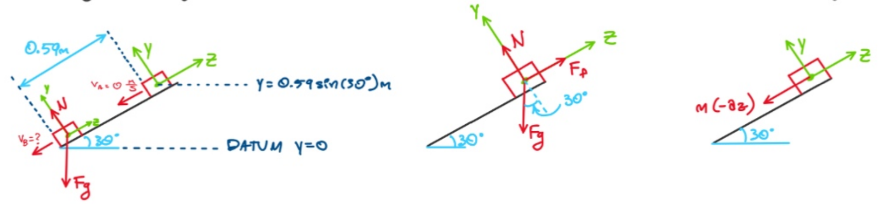
$V_{impact\ f} = -2.1706\ [\frac{m}{s}]$

b) Determine: $(U_{1-2})_c$, $(F_f)_c$, $(\mu_k)_c$

$(U_{1-2})_f$, $(F_f)_f$, $(\mu_k)_f$

c) Assume cart is point particle located at tip, $g = 9.81\ \frac{m}{s^2}$

2a) System Diagram   FBD: Kinetic Diagram:



b) Principles of Work And Energy:

$$T_1 + (V_g)_1 + U_{1-2} = T_2 + (V_g)_2$$

Newton's Second Law for Rectilinear Motion:

$$\Sigma F_z = ma_z \quad , \quad \Sigma F_y = 0$$

c) For cardboard:

$$\cancel{T_1}^{0} + (V_1)_g - U_{1-2} = T_2 + \cancel{(V_2)_g}^{0}$$

$$-U_{1-2} = \frac{1}{2}mv_2^2 - mgh$$

$$-U_{1-2} = \frac{1}{2}(1.88\ kg)(-2.1557)^2 - (1.88\ kg)(9.81\ \tfrac{m}{s^2})(0.59\sin(30°))$$

$$-U_{1-2} = -1.0724\ J$$

$$(U_{1-2})_c = 1.0724\ J = \text{work by friction}$$

$$(F_f s)_c = 1.0724\ J$$

$$F_f = \frac{1.0724\ J}{0.59\ m}$$

$$F_f = 1.817\ N$$

Using N2L to find N

$$\Sigma F_y = 0 = N - mg\cos(30°)$$

$$N = mg\cos(30°)$$

$$N = 15.972\ N$$

$$\mu_k N = 1.817\ N$$

$$(\mu_k)_c = \frac{1.817}{15.972}$$

$$(\mu_k)_c = 0.114$$

For foam: N is the same as cardboard experiment

$$\cancel{T_1}^{0} + (V_g)_1 - U_{1-2} = T_2 + \cancel{(V_g)_2}^{0}$$

$$-U_{1-2} = \frac{1}{2} m v_2^2 - mgh$$

$$-U_{1-2} = \frac{1}{2}(1.88\ kg)(-2.1706\ \tfrac{m}{s})^2 - (1.88\ kg)(9.81\ \tfrac{m}{s^2})(0.59\sin(30^\circ))$$

$$-U_{1-2} = -1.0118\ J$$

$$U_{1-2} = \text{work by friction} = 1.0118\ J$$

$$(F_f)_f\, s = 1.0118\ J$$

$$(F_f)_f = \frac{1.0118\ J}{0.59\ m}$$

$$(F_f)_f = 1.71492\ N$$

$$(\mu_k)_f\, N = 1.71492$$

$$(\mu_k)_f = \frac{1.71492\ N}{15.972\ N}$$

$$(\mu_k)_f = 0.107$$

2d) ✓ e) ✓ f) ✓

g) Cardboard experiment:

Work from friction = 1.07 J
Friction force = 1.82 N
Coefficient of kinetic friction = 0.114

Foam experiment:

Work from friction = 1.01 J
Friction force = 1.71 N
Coefficient of kinetic friction = 0.107

Appendix D: Full Solution for Coefficient of Restitution

6) 1a) Given: $(V_C)_1 = -2.15578 \left[\frac{m}{s}\right]$ $(V_A)_1 = 0 \left[\frac{m}{s}\right]$

$(V_C)_2 = 0.47523 \left[\frac{m}{s}\right]$ $(V_A)_2 = 0 \left[\frac{m}{s}\right]$

$(V_F)_1 = -2.17064 \left[\frac{m}{s}\right]$

$(V_F)_2 = 0.810339 \left[\frac{m}{s}\right]$

b) Determine: e_c, e_f

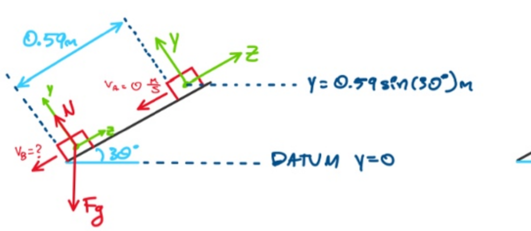
c) Assumptions: Cart is point particle at tip

$g = 9.81 \frac{m}{s^2}$

Momentum is conserved

2a)

System Diagram



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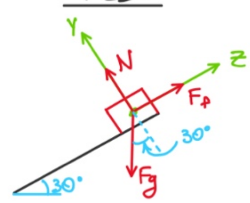
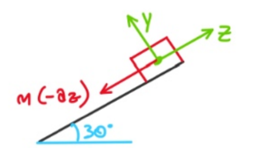


Kinetic Diagram:



b) Principles of Momentum and Impulse

$$e = \frac{(V_{B2})_x - (V_{A2})_x}{(V_{A1})_x - (V_{B1})_x}$$, V_B = speed of cart, V_A = speed of block (cardboard/foam)

c) Cardboard:

$$(e_c) = \frac{0.47523 - 0}{0 - (-2.15578)}$$

$e_c = 0.22045$

Foam:

$$(e_f) = \frac{0.810339 - 0}{0 - (-2.17064)}$$

$e_f = 0.37332$

2d)✓ e)✓ f)✓

g) ∴ coefficient of restitution for cardboard experiment is 0.220 & for foam experiment is 0.373