HW # 8 Solutions

Q1)

$$x(t) = 2^{-|t|}$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt = \int_{0}^{\infty} 2^{-t} e^{-j\omega t}\, dt + \int_{-\infty}^{0} 2^{t} e^{-j\omega t}\, dt$$

$$= \left[\frac{(2^{-1} e^{-j\omega})^t}{\ln(2^{-1} e^{-j\omega})} \right]_0^{\infty} + \left[\frac{(2 e^{-j\omega})^t}{\ln(2 e^{-j\omega})} \right]_{-\infty}^{0}$$

$$= 0 - \frac{1}{\ln(2^{-1} e^{-j\omega})} + \frac{1}{\ln(2 e^{-j\omega})} - 0$$

$$= \frac{1}{\ln 2 - j\omega} + \frac{1}{\ln 2 + j\omega} = \frac{2(\ln 2)}{(\ln 2)^2 + \omega^2}$$

Q2)

$$x(t) = u(t+2) - u(t-2)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt = \int_{-2}^{2} e^{-j\omega t}\, dt$$

$$= \left[\frac{e^{-j\omega t}}{-j\omega} \right]_{-2}^{2} = \frac{e^{-j\omega 2} - e^{+j\omega 2}}{-j\omega}$$

$$= \frac{2j \sin(2\omega)}{j\omega} = \frac{2 \sin(2\omega)}{\omega}$$

Q3)

$$x(t) = \cos(2\pi t) + \sin(4t)$$

$$= \frac{e^{j2\pi t}}{2} + \frac{e^{-j2\pi t}}{2} + \frac{e^{j4t}}{2j} - \frac{e^{-j4t}}{2j}$$

※ $x(t) = e^{j\omega_0 t} \iff X(j\omega) = 2\pi\,\delta(\omega - \omega_0)$

Why? $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\,\delta(\omega-\omega_0)\, e^{j\omega t}\, d\omega$$

$$= e^{j\omega_0 t}$$

Therefore, $X(j\omega) = 2\pi\cdot\left(\frac{1}{2}\delta(\omega-2\pi) + \frac{1}{2}\delta(\omega+2\pi) + \frac{1}{2j}\delta(\omega-4) - \frac{1}{2j}\delta(\omega+4)\right)$

Q4)

$$X(j\omega) = u(\omega+3) - u(\omega-3)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{j\omega t}\, d\omega = \frac{1}{2\pi}\int_{-3}^{3} e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega t}}{jt}\right]_{-3}^{3} = \frac{1}{2\pi}\left[\frac{e^{j3t} - e^{-j3t}}{jt}\right]$$

$$= \frac{1}{2\pi}\,\frac{2j\sin(3t)}{jt} = \frac{\sin(3t)}{\pi t}$$

Q5] P 4.21

(b) $x(t) = e^{-3|t|}\sin(2t)$

Let $x_1(t) = e^{-3|t|}$ $\quad x_2(t) = \sin(2t) = \dfrac{e^{j2t}}{2j} - \dfrac{e^{-j2t}}{2j}$

$$X_1(j\omega) = \int_{-\infty}^{\infty} x_1(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{\infty} e^{-3|t|} e^{-j\omega t}\, dt$$

$$= \int_0^{\infty} e^{-3t} e^{-j\omega t}\, dt + \int_{-\infty}^{0} e^{3t} e^{-j\omega t}\, dt$$

$$= \left[\frac{e^{(-3-j\omega)t}}{-3-j\omega}\right]_0^{\infty} + \left[\frac{e^{(3-j\omega)t}}{3-j\omega}\right]_{-\infty}^{0}$$

$$= \frac{1}{3+j\omega} + \frac{1}{3-j\omega} = \frac{6}{9+\omega^2}$$

$$X_2(j\omega) = 2\pi\left(\frac{1}{2j}\delta(\omega-2) - \frac{1}{2j}\delta(\omega+2)\right)$$

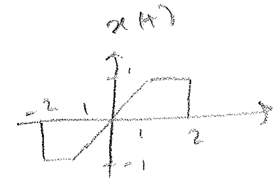
$$= \frac{\pi}{j}\delta(\omega-2) - \frac{\pi}{j}\delta(\omega+2)$$

※ $x(t) = x_1(t) \cdot x_2(t) \overset{F.T.}{\Longleftrightarrow} X(j\omega) = \frac{1}{2\pi} X_1(j\omega) * X_2(j\omega)$

$$\frac{1}{2\pi} X_1(j\omega) * X_2(j\omega) = \frac{1}{2j}\left(\delta(\omega-2) - \delta(\omega+2)\right) * \frac{6}{9+\omega^2}$$

$$= \frac{1}{2j}\left(\delta(\omega-2) * \frac{6}{9+\omega^2}\right) - \frac{1}{2j}\left(\delta(\omega+2) * \frac{6}{9+\omega^2}\right)$$

$$= \frac{1}{j}\,\frac{3}{9+(\omega-2)^2} - \frac{1}{j}\,\frac{3}{9+(\omega+2)^2}$$

(g) $X(j\omega) = \int_{-\infty}^{\infty}$  $e^{-j\omega t}\,dt$

$$= \underbrace{\int_{-2}^{-1} (-1)\, e^{-j\omega t}\,dt}_{-\int_1^2 e^{j\omega t}\,dt \text{ by change of variable}} + \int_{-1}^{1} t\, e^{-j\omega t}\,dt + \int_{1}^{2} e^{-j\omega t}\,dt$$

$$= \int_1^2 \left(-e^{j\omega t} + e^{-j\omega t}\right) dt + \int_{-1}^{1} t\, e^{-j\omega t}\,dt$$

$$= 2j\left[\frac{\cos(\omega t)}{\omega}\right]_1^2 + \left[\frac{t\,e^{-j\omega t}}{-j\omega}\right]_{-1}^{1} - \underbrace{\int_{-1}^{1} \frac{e^{-j\omega t}}{-j\omega}\,dt}_{\left[\frac{e^{-j\omega t}}{\omega^2}\right]_{-1}^{1}}$$

$$= 2j\left(\frac{\cos(2\omega) - \cos(\omega)}{\omega}\right) + \underbrace{\frac{e^{-j\omega} + e^{j\omega}}{-j\omega}}_{= 2j\frac{\cos(\omega)}{\omega}} - \underbrace{\frac{e^{-j\omega} - e^{j\omega}}{\omega^2}}_{= -2j\frac{\sin(\omega)}{\omega^2}}$$

$$= 2j\,\frac{\cos(2\omega)}{\omega} + 2j\,\frac{\sin(\omega)}{\omega^2}$$

(i) $x(t) = \begin{cases} 1-t^2 & 0<t<1 \\ 0 & \text{otherwise} \end{cases}$

$$X(j\omega) = \int_0^1 (1-t^2)\, e^{-j\omega t}\, dt = \left[\frac{(1-t^2)\, e^{-j\omega t}}{-j\omega}\right]_0^1 + \int_0^1 \frac{2t\, e^{-j\omega t}}{-j\omega}\, dt$$

$$= -\frac{1}{j\omega}(0-1) - \frac{2}{j\omega}\left[\frac{t\, e^{-j\omega t}}{-j\omega}\right]_0^1 + \frac{2}{j\omega}\underbrace{\int_0^1 \frac{e^{-j\omega t}}{-j\omega}\, dt}_{=\left[\frac{e^{-j\omega t}}{-\omega^2}\right]_0^1}$$

$$= \frac{1}{j\omega} - \frac{2}{\omega^2} e^{-j\omega} - \frac{2}{j\omega^3}\left(e^{-j\omega} - 1\right)$$

Q 6) (b) $X(j\omega) = \cos(4\omega + \pi/3) = \cos(4(\omega + \pi/12))$

※ $x(t) = \delta(t - t_0) \xleftrightarrow{F.T.} X(j\omega) = \int_{-\infty}^{\infty} \delta(t - t_0)\, e^{-j\omega t}\, dt = e^{-j\omega t_0}$

Thus, $\frac{1}{2}\delta(t+t_0) + \frac{1}{2}\delta(t-t_0)$

$$\xleftrightarrow{F.T.} \frac{1}{2} e^{jt_0\omega} + \frac{1}{2} e^{-jt_0\omega} = \cos(t_0\omega)$$

$\frac{1}{2j}\delta(t+t_0) - \frac{1}{2j}\delta(t-t_0)$

$$\xleftrightarrow{F.T.} \frac{1}{2j} e^{jt_0\omega} - \frac{1}{2j} e^{-jt_0\omega} = \sin(t_0\omega)$$

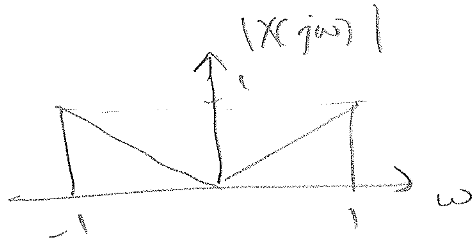
Combining Frequency-shifting above, it is eaxy to see that

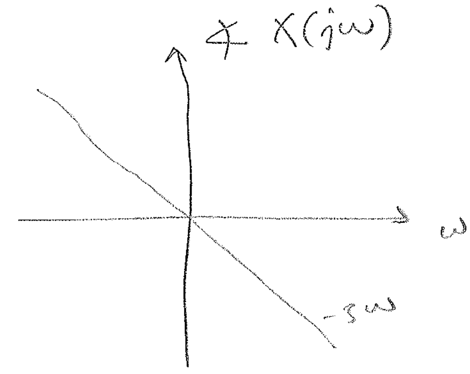
$$x(t) = \left(\frac{1}{2}\delta(t+4) + \frac{1}{2}\delta(t-4)\right) e^{-j\frac{\pi}{12}t} \xleftrightarrow{F.T.} \cos\left(4\left(\omega + \tfrac{\pi}{12}\right)\right)$$

$$= \frac{1}{2} e^{-j\frac{\pi}{12}t}\,\delta(t+4) + \frac{1}{2} e^{-j\frac{\pi}{12}t}\,\delta(t-4)$$

$$= \frac{1}{2} e^{j\frac{\pi}{3}}\,\delta(t+4) + \frac{1}{2} e^{-j\frac{\pi}{3}}\,\delta(t-4)$$ due to delta function

(c) $X(j\omega) =$





$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\Bigg[\underbrace{\int_{-1}^{0} (-\omega)\, e^{-j3\omega} e^{j\omega t}\, d\omega}_{A} + \underbrace{\int_{0}^{1} (\omega)\, e^{-j3\omega} e^{j\omega t}\, dt}_{B}\Bigg]$$

$$A = \int_{-1}^{0} (-\omega)\, e^{j\omega(t-3)}\, d\omega = \left[\frac{-\omega e^{j(t-3)\omega}}{j(t-3)}\right]_{-1}^{0} + \int_{-1}^{0} \frac{e^{j(t-3)\omega}}{j(t-3)}\, d\omega$$

$$= -\frac{e^{-j(t-3)}}{j(t-3)} - \left[\frac{e^{j(t-3)\omega}}{(t-3)^2}\right]_{-1}^{0} = -\frac{e^{-j(t-3)}}{j(t-3)} - \frac{1-e^{-j(t-3)}}{(t-3)^2}$$

$$B = \int_{0}^{1} (\omega)\, e^{j\omega(t-3)}\, d\omega = \left[\frac{\omega e^{j(t-3)\omega}}{j(t-3)}\right]_{0}^{1} - \int_{0}^{1} \frac{e^{j(t-3)\omega}}{j(t-3)}\, d\omega$$

$$= \frac{e^{j(t-3)}}{j(t-3)} + \left[\frac{e^{j(t-3)\omega}}{(t-3)^2}\right]_{0}^{1} = \frac{e^{j(t-3)}}{j(t-3)} + \frac{e^{j(t-3)} - 1}{(t-3)^2}$$

$$Ⓐ + Ⓑ = \frac{2j\sin(t-3)}{j(t-3)} + \frac{2\cos(t-3) - 2}{(t-3)^2}$$

$$\therefore \quad x(t) = \frac{1}{2\pi}\left(Ⓐ + Ⓑ\right) = \frac{\sin(t-3)}{\pi(t-3)} + \frac{\cos(t-3) - 1}{\pi(t-3)^2}$$

(d) $X(j\omega) = 2[\delta(\omega-1) - \delta(\omega+1)] + 3[\delta(\omega-2\pi) + \delta(\omega+2\pi)]$

$\Updownarrow$ F.T.

$$x(t) = \frac{2}{2\pi}e^{jt} - \frac{2}{2\pi}e^{-jt} + \frac{3}{2\pi}e^{j2\pi t} + \frac{3}{2\pi}e^{-j2\pi t}$$

$$= \frac{2j}{\pi}\sin(t) + \frac{3}{\pi}\cos(2\pi t).$$

Q7) P 4.3

(a) $\sin(2\pi t + \pi/4) = \frac{1}{2j}e^{j(2\pi t)} \cdot e^{j\pi/4} - \frac{1}{2j}e^{-j(2\pi t)}e^{-j\pi/4}$

※ $e^{j\omega_0 t} \xleftrightarrow{FT} 2\pi\,\delta(\omega-\omega_0)$

$\therefore \quad \sin(2\pi t + \pi/4) \xleftrightarrow{FT} \frac{\pi}{j}e^{j\frac{\pi}{4}}\delta(\omega-2\pi) - \frac{\pi}{j}e^{-j\frac{\pi}{4}}\delta(\omega+2\pi)$

(b) $1 + \cos(6\pi t + \pi/8) = 1 + \frac{1}{2}e^{j(6\pi t)}e^{j\frac{\pi}{8}} - \frac{1}{2}e^{-j(6\pi t)}e^{-j\frac{\pi}{8}}$

$\xleftrightarrow{FT} 2\pi\delta(\omega) + \pi e^{j\pi/8}\delta(\omega-6\pi) - \pi e^{-j\pi/8}\delta(\omega+6\pi)$

※ $1 \xleftrightarrow{FT} 2\pi\delta(\omega)$

Q8) P 4.4

(a) $X_1(j\omega) = 2\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)$

$\overset{FT}{\Longleftrightarrow} \; 1 + \frac{1}{2}e^{j4\pi t} + \frac{1}{2}e^{-j4\pi t} = 1 + \cos(4\pi t)$

(b) $X_2(j\omega) = \begin{cases} 2 & 0 \leq \omega \leq 2 \\ -2 & -2 \leq \omega < 0 \\ 0 & |\omega| > 2 \end{cases}$

$x_2(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_2(j\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_0^2 2e^{j\omega t}d\omega$

$+ \frac{1}{2\pi}\int_{-2}^{0}(-2)e^{j\omega t}d\omega$

$= \frac{1}{\pi}\left[\frac{e^{j\omega t}}{jt}\right]_0^2 - \frac{1}{\pi}\left[\frac{e^{j\omega t}}{jt}\right]_{-2}^{0}$

$= \frac{1}{\pi}\left(\frac{e^{j2t} - 1}{jt}\right) - \frac{1}{\pi}\left(\frac{1 - e^{-j2t}}{jt}\right)$

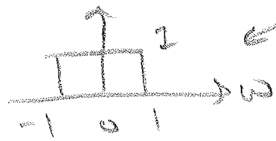
$= \frac{1}{\pi jt}\left(e^{j2t} + e^{-j2t}\right) - \frac{2}{\pi jt} = \frac{2\cos(2t) - 2}{\pi jt}$

$= \frac{-4\sin^2(t)}{\pi jt}$

Q9) P 4.10

(a) $x(t) = t\left(\frac{\sin t}{\pi t}\right)^2 \overset{FT}{\Longleftrightarrow} X(j\omega)$

① $\frac{\sin t}{\pi t} \overset{FT}{\Longleftrightarrow}$



$\leftarrow$ call it $\hat{X}(j\omega)$

② $\left(\frac{\sin t}{\pi t}\right)^2 \overset{FT}{\Longleftrightarrow} \frac{1}{2\pi}\left(\hat{X}(j\omega) * \hat{X}(j\omega)\right)$

by multiplication property

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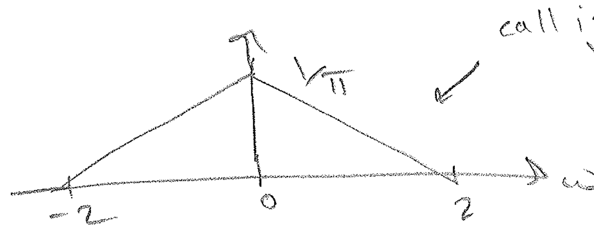


call it $Y(j\omega)$

③ $t\left(\frac{\sin t}{\pi t}\right)^2 \overset{FT}{\Longleftrightarrow} j\frac{d}{d\omega}Y(j\omega) = \begin{cases} j/2\pi & -2<\omega<0 \\ -j/2\pi & 0<\omega<2 \\ 0 & \text{otherwise} \\ & \text{except } -2,0,2 \end{cases}$

Differentiation in freq. property

(b) Parseval's Relation $= \int_{-\infty}^{\infty}|x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(j\omega)|^2 d\omega$

from (a)

$\Rightarrow \quad A = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(j\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-2}^{0}\frac{1}{4\pi^2}d\omega + \frac{1}{2\pi}\int_{0}^{2}\frac{1}{4\pi^2}d\omega$

$= \frac{1}{2\pi}\left(\frac{4}{4\pi^2}\right) = \frac{1}{2\pi^3}$

Q10) P4.12 (a): $e^{-|t|} \overset{FT}{\Longleftrightarrow} \frac{2}{1+\omega^2}$

$te^{-|t|} \overset{FT}{\Longleftrightarrow} j\frac{d}{d\omega}\left(\frac{2}{1+\omega^2}\right) = -j\frac{2}{(1+\omega^2)^2}\cdot 2\omega = \frac{-4j\omega}{(1+\omega^2)^2}$

Differentiation in frequency

HW# 9 Solutions

1. P 4.13 $X(j\omega) = \delta(\omega) + \delta(\omega - \pi) + \delta(\omega - 5)$

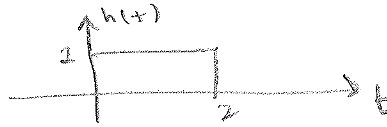
(a) By intuition,

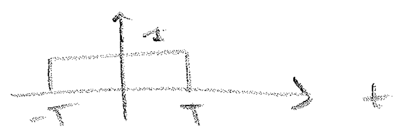
$$x(t) = \frac{1}{2\pi} + \underbrace{\frac{1}{2\pi} e^{j\pi t}}_{\text{periodic with } 2} + \underbrace{\frac{1}{2\pi} e^{j5t}}_{\text{periodic with } 2\pi/5}$$

$LCM(2, 2\pi/5)$ does not exist

$\therefore$ $x(t)$ is not periodic.

(b) $x(t) * h(t) \overset{F.T.}{\Longleftrightarrow} X(j\omega) H(j\omega)$

$h(t) = u(t) - u(t-2) =$  $\overset{F.T.}{\Longleftrightarrow} H(j\omega)$?

※  $\overset{F.T.}{\Longleftrightarrow} \dfrac{2 \sin \omega T}{\omega}$

$\delta(t - t_0) \overset{F.T.}{\Longleftrightarrow} e^{-j\omega t_0}$

$\therefore$ $H(j\omega) = e^{-j\omega} \cdot \dfrac{2\sin\omega}{\omega}$ ※ $\left.\dfrac{\sin\omega}{\omega}\right|_{\omega=0} = 1$

$$X(j\omega) \cdot H(j\omega) = e^{-j\omega} \cdot \frac{2\sin\omega}{\omega} \cdot \delta(\omega) + e^{-j\omega} \frac{2\sin\omega}{\omega} \cdot \delta(\omega - \pi)$$
$$+ e^{-j\omega} \frac{2\sin\omega}{\omega} \cdot \delta(\omega - 5)$$
$$= 2 \cdot \delta(\omega) + 0 \cdot \delta(\omega - \pi) + e^{-j5} \frac{2\sin 5}{5} \cdot \delta(\omega - 5)$$

Let $y(t) \overset{F.T.}{\Longleftrightarrow} X(j\omega) H(j\omega)$

Then, $y(t) = \dfrac{1}{\pi} + \dfrac{1}{2\pi} \cdot e^{-j5} \dfrac{2\sin 5}{5} e^{j5t}$ $\Rightarrow$ periodic

(c) Yes

2. $h(t) = \dfrac{\sin(2t)}{\pi t} \overset{F.T.}{\Longleftrightarrow} H(j\omega) = \begin{cases} 1 & |\omega| < 2 \\ 0 & |\omega| > 2 \end{cases}$

- cut-off frequency is 2 rad/sec
- $x(t) = \cos(t) + \sin(3t)$

$$\overset{F.T.}{\Longleftrightarrow} X(j\omega) = \pi\delta(\omega-1) + \pi\delta(\omega+1) + \frac{\pi}{j}\delta(\omega-3) - \frac{\pi}{j}\delta(\omega+3)$$

$$\Rightarrow X(j\omega)\,H(j\omega) = \pi\delta(\omega-1) + \pi\delta(\omega+1)$$

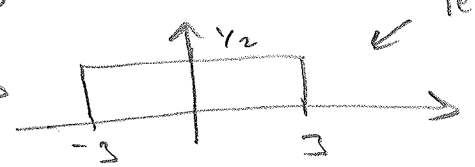
Let $y(t) \Longleftrightarrow X(j\omega)\,H(j\omega)$

$$\therefore \underline{y(t) = \cos(t)}$$

3. P 4.18

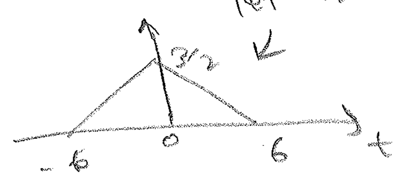
$$H(j\omega) = \frac{(\sin^2(3\omega))\cos\omega}{\omega^2}$$

① $\dfrac{\sin(3\omega)}{\omega} \overset{F.T.}{\Longleftrightarrow}$



let $x_1(t)$

② $\dfrac{\sin^2(3\omega)}{\omega^2} \overset{F.T.}{\Longleftrightarrow} x_1(t) * x_1(t) =$

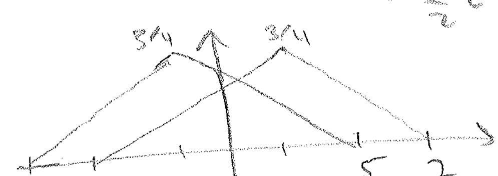


let $x_2(t)$

③ $\dfrac{\sin^2(3\omega)}{\omega^2}\cos(\omega) = \dfrac{\sin^2(3\omega)}{\omega^2}\cdot\dfrac{1}{2}e^{j\omega} + \dfrac{\sin^2(3\omega)}{\omega^2}\cdot\dfrac{1}{2}e^{-j\omega}$

By time-shifting

$\overset{F.T.}{\Longleftrightarrow} \frac{1}{2}x_2(t+1) + \frac{1}{2}x_2(t-1) =$



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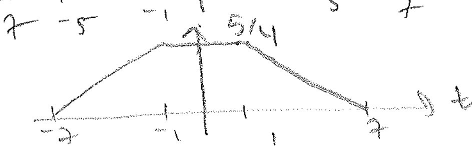


4. P 4.26 (a)

(i) $x(t) = te^{-2t}u(T) \overset{F.T.}{\Longleftrightarrow} X(j\omega) = j\frac{d}{d\omega}\left(\frac{1}{2+j\omega}\right)$

$= j \cdot (-1) \cdot (2+j\omega)^{-2} \cdot j$

$= \frac{1}{(2+j\omega)^2}$

$h(t) = e^{-4t}u(t) \overset{FT}{\Longleftrightarrow} H(j\omega) = \frac{1}{4+j\omega}$

$y(t) = x(t) * h(t) \overset{F.T.}{\Longleftrightarrow} Y(j\omega) = X(j\omega)\, H(j\omega)$

$Y(j\omega) = \frac{1}{(2+j\omega)^2}\frac{1}{(4+j\omega)} = \frac{A}{(2+j\omega)^2} + \frac{B}{2+j\omega} + \frac{C}{4+j\omega}$

$\Rightarrow$ $A = 1/2$, $C = 1/4$ $\qquad 1 = A(4+j\omega) + B(2+j\omega)(4+j\omega) + C(2+j\omega)^2$

$\Rightarrow 1 = 4A + 8B + 4C$ at $j\omega = 0$

$\therefore B = -1/4$

$\therefore y(t) = 1/2\, te^{-2t}u(t) - \frac{1}{4}e^{-2t}u(t) + \frac{1}{4}e^{-4t}u(t)$

(ii) $x(t) = te^{-2t}u(t) \overset{FT}{\Longleftrightarrow} X(j\omega) = \frac{1}{(2+j\omega)^2}$

$h(t) = te^{-4t}u(t) \overset{FT}{\Longleftrightarrow} H(j\omega) = \frac{1}{(4+j\omega)^2}$

$\Rightarrow Y(j\omega) = \frac{1}{(2+j\omega)^2(4+j\omega)^2} = \frac{A}{(2+j\omega)^2} + \frac{B}{(2+j\omega)} + \frac{C}{(4+j\omega)^2} + \frac{D}{4+j\omega}$

$A = 1/4$, $C = 1/4$ $\qquad 1/9 = A + B + C/9 + D/3$ at $j\omega = -1$

$1 = A - B + C + D$ at $j\omega = -3$

$\therefore 1 = 9A + 9B + C + 3D$

$1 = A - B + C + D$

$\Rightarrow \begin{cases} -1/2 = 3B + D \\ 1/2 = -B + D \end{cases}$

$\therefore\ B = -1/4\ ,\ D = 3/4$

$\therefore\ y(t) = \frac{1}{4} t e^{-2t} u(t) - \frac{1}{4} e^{-2t} u(t) + \frac{1}{4} t e^{-4t} u(t) + 3/4\ e^{-4t} u(t)$

(iii) $x(t) = e^{-t} u(t) \overset{FT}{\Longleftrightarrow} X(j\omega) = \frac{1}{1+j\omega}$

$h(t) = e^{t} u(-t) \overset{FT}{\Longleftrightarrow} H(j\omega) = X(-j\omega) = \frac{1}{1-j\omega}$

by time-reversal

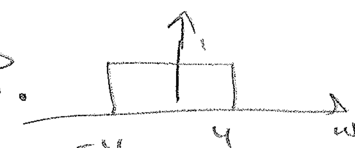
$Y(j\omega) = X(j\omega) H(j\omega) = \frac{1}{1+j\omega} \cdot \frac{1}{1-j\omega} = \frac{A}{1+j\omega} + \frac{B}{1-j\omega}$

$\begin{cases} A = 1 \\ B = 1 \end{cases}$

$\therefore\ y(t) = e^{-t} u(t) + e^{t} u(-t)$

5. P 4.32 (a,b)

$h(t) = \frac{\sin(4(t-1))}{\pi(t-1)}$ time shifting $\overset{FT}{\Longleftrightarrow}$ $e^{-j\omega} \cdot$



(a) $x_1(t) = \cos(6t + \pi/2) \overset{FT}{\Longleftrightarrow} X_1(j\omega) = \pi e^{j\pi/2} \delta(\omega - 6) + \pi e^{-j\pi/2} \delta(\omega + 6)$

$\therefore\ X_1(j\omega) H(j\omega) = 0$

$\therefore\ y_1(t) = x_1(t) * h(t) = 0$

(b) $x_2(t) = \sum_{k=0}^{\infty} (\frac{1}{2})^k \sin(3kt) \overset{FT}{\Longleftrightarrow} X_2(j\omega) = \sum_{k=0}^{\infty} (\frac{1}{2})^k \cdot \left(\frac{\pi}{j} \delta(\omega - 3k) - \frac{\pi}{j} \delta(\omega + 3k) \right)$

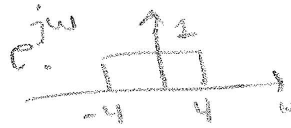
when $K > 1$, the signal $x_2(t)$ is nulled by $h(t)$

$\Rightarrow X_2(j\omega) H(j\omega) = \sum_{k=0}^{1} (\frac{1}{2})^k \cdot \left(\frac{\pi}{j} \delta(\omega - 3k) - \frac{\pi}{j} \delta(\omega + 3k) \right)$

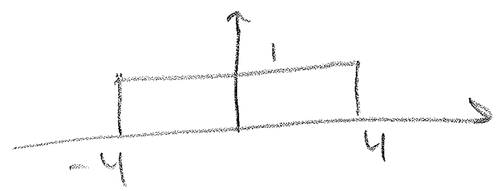
$\therefore\ y_2(t) = x_2(t) * h(t) = \sum_{k=1}^{1} \left(\frac{1}{2}\right)^k \sin(3kt)$

$= \frac{1}{2}\sin(3t)$

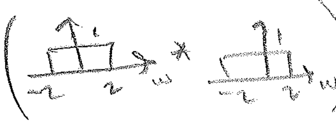
6. P 4.32 (c,d)

(c) $x_3(t) = \dfrac{\sin(4(t-1))}{\pi(t+1)} \overset{FT}{\longleftrightarrow} X_3(j\omega) = e^{j\omega} \cdot$ 

$X_3(j\omega) \cdot H(j\omega) =$

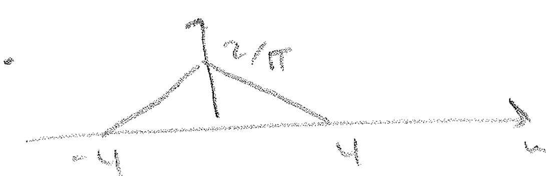


$\therefore\ y_3(t) = x_3(t) * h(t) = \dfrac{\sin(4t)}{\pi t}$

(d) $x_4(t) = \left(\dfrac{\sin 2t}{\pi t}\right)^2 \overset{FT}{\longleftrightarrow} X_4(j\omega) = \dfrac{1}{2\pi} \cdot \Big($  $\Big)$

$=$ 

$X_4(j\omega)\, H(j\omega) = e^{-j\omega} \cdot$



$\therefore\ y_4(t) = \left(\dfrac{\sin 2t}{\pi t}\right)^2 * \delta(t-1) = \left(\dfrac{\sin(2(t-1))}{\pi(t-1)}\right)^2$

7. P 4.33 (a)

LTI system: $\frac{d^2 y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$

$x(t) \xrightarrow[h(t)]{LTI} y(t) = x(t) * h(t) \iff X(j\omega) \xrightarrow[H(j\omega)]{LTI} Y(j\omega) = X(j\omega)H(j\omega)$

Thus, $(j\omega)^2 Y(j\omega) + 6(j\omega)Y(j\omega) + 8Y(j\omega) = 2X(j\omega)$

$\Rightarrow \frac{Y(j\omega)}{X(j\omega)} = H(j\omega) = \frac{2}{8 + 6j\omega + (j\omega)^2} = \frac{2}{(4+j\omega)(2+j\omega)}$

$= \frac{A}{4+j\omega} + \frac{B}{2+j\omega}$ $\therefore A = -1, B = 1$

$\therefore h(t) = -e^{-4t}u(t) + e^{-2t}u(t)$

If $x(t) = e^{-3t}u(t) \iff Y(j\omega) = \frac{1}{3+j\omega}$

$\Rightarrow X(j\omega)H(j\omega) = \frac{2}{(4+j\omega)(3+j\omega)(2+j\omega)}$

$= \frac{A}{4+j\omega} + \frac{B}{3+j\omega} + \frac{C}{2+j\omega}$

$\left(\begin{array}{lll} A = 1 & \text{at} & j\omega = -4 \\ B = 2 & \text{at} & j\omega = -3 \\ C = 1 & \text{at} & j\omega = -1 \end{array} \right.$

$\therefore y(t) = x(t) * h(t) = e^{-4t}u(t) + 2e^{-3t}u(t) + e^{-2t}u(t)$

8. P 4.35 (b)

$$x(t) = \cos\left(t/\sqrt{3}\right) + \cos t + \cos\left(\sqrt{3}t\right)$$

$$\xrightarrow[H(j\omega) = \frac{1-j\omega}{1+j\omega}]{LTI} \quad y(t) = ?$$

$$|H(j\omega)| = \left|\frac{1-j\omega}{1+j\omega}\right| = \frac{|1-j\omega|}{|1+j\omega|} = \frac{\sqrt{1+\omega^2}}{\sqrt{1+\omega^2}} = 1 \;\forall \omega$$

$$\measuredangle H(j\omega) = \measuredangle\left(\frac{1-j\omega}{1+j\omega}\right) = \measuredangle(1-j\omega) - \measuredangle(1+j\omega)$$

$$= \tan^{-1}(-\omega) - \tan^{-1}(\omega)$$

$$= -2\tan^{-1}(\omega)$$

Recall that, $x(t) = e^{j\omega_0 t} \xrightarrow[H(j\omega)]{LTI} y(t) = H(j\omega)\Big|_{\omega=\omega_0} e^{j\omega_0 t}$

Thus, in the end,

$$y(t) = \cos\left(\frac{1}{\sqrt{3}}\left(t - \underbrace{2\tan^{-1}\left(1/\sqrt{3}\right)}_{\pi/3}\right)\right)$$

$$+ \cos\left(t - \underbrace{2\tan^{-1}(1)}_{\pi/2}\right)$$

$$+ \cos\left(\sqrt{3}\left(t - \underbrace{2\tan^{-1}\left(\sqrt{3}\right)}_{2\pi/3}\right)\right)$$

9. P 4.36 (a)

(a) $x(t) = [e^{-t} + e^{-3t}]\,u(t) \overset{FT}{\longleftrightarrow} X(j\omega) = \dfrac{1}{1+j\omega} + \dfrac{1}{3+j\omega}$

$y(t) = [2e^{-t} - 2e^{-4t}]\,u(t) \overset{F.T.}{\longleftrightarrow} Y(j\omega) = \dfrac{2}{1+j\omega} - \dfrac{2}{4+j\omega}$

$\therefore \quad X(j\omega) = \dfrac{2\,(2+j\omega)}{(1+j\omega)\,(3+j\omega)} \qquad Y(j\omega) = \dfrac{6}{(1+j\omega)\,(4+j\omega)}$

$\therefore \quad H(j\omega) = \dfrac{Y(j\omega)}{X(j\omega)} = \dfrac{3\,(3+j\omega)}{(2+j\omega)(4+j\omega)} = \dfrac{A}{2+j\omega} + \dfrac{B}{4+j\omega}$

$\Rightarrow \quad 9 + 3j\omega = A(4+j\omega) + B(2+j\omega)$

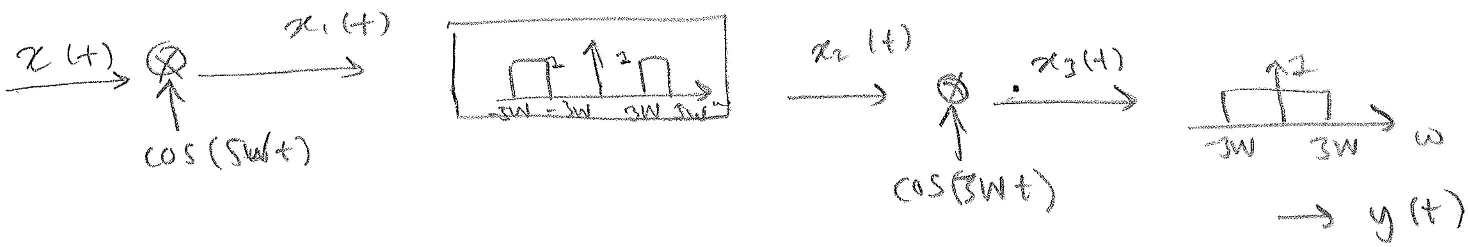
$\qquad = (4A + 2B) + (A+B)\,j\omega$

$\Rightarrow \quad \begin{aligned} 4A + 2B &= 9 \\ A + B &= 3 \end{aligned} \quad \Rightarrow \quad A = 3/2, \; B = 3/2$

$\therefore \quad H(j\omega) = \dfrac{3/2}{2+j\omega} + \dfrac{3/2}{4+j\omega}$

(b) $h(t) = 3/2\, e^{-2t} u(t) + 3/2\, e^{-4t} u(t)$

10. P 8.22



$$\cos(5Wt) = \frac{1}{2} e^{j5Wt} + \frac{1}{2} e^{-j5Wt}$$

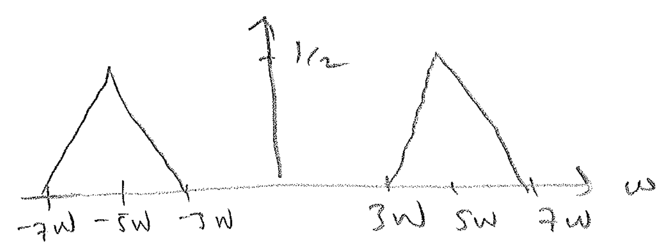
$$\xleftrightarrow{FT} \pi\delta(\omega - 5W) + \pi\delta(\omega + 5W)$$

$$x_1(t) = x(t) \times \cos(5Wt) \xleftrightarrow{FT} \frac{1}{2\pi} X(j\omega) * \left(\pi\delta(\omega - 5W) + \pi\delta(\omega + 5W)\right)$$

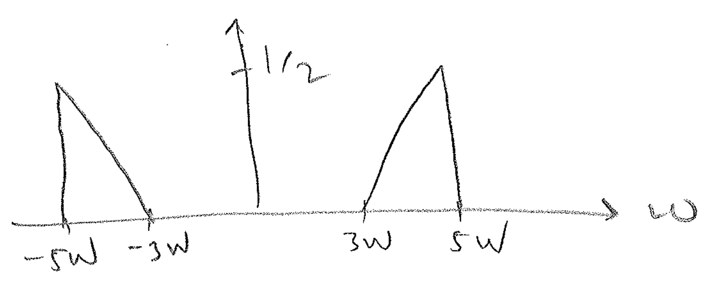
$$= \frac{1}{2} X(j(\omega - 5W)) + \frac{1}{2} X(j(\omega - 5W))$$

$$= X_1(j\omega)$$

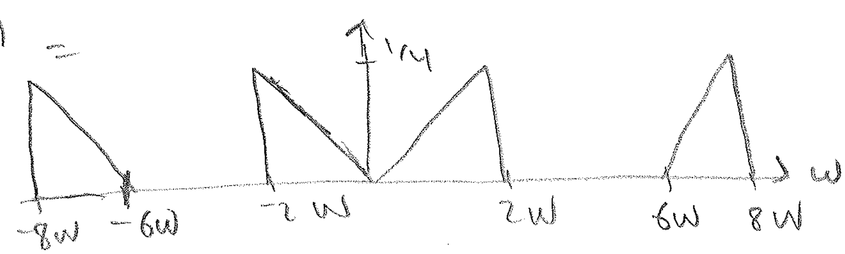
$\therefore$ $X_1(j\omega) =$



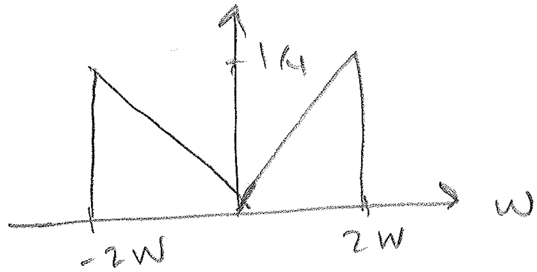
$X_2(j\omega) =$



$X_3(j\omega) =$

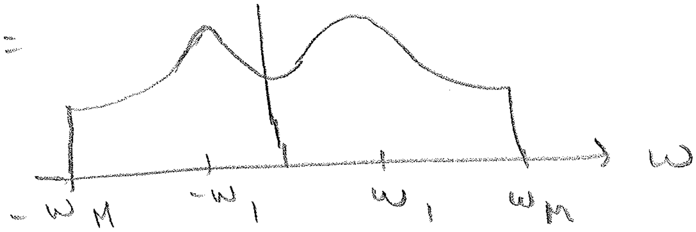


$Y(j\omega) =$

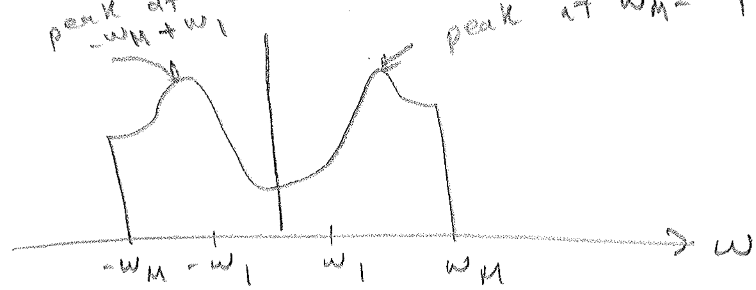


11. P8.25

$X(j\omega) =$

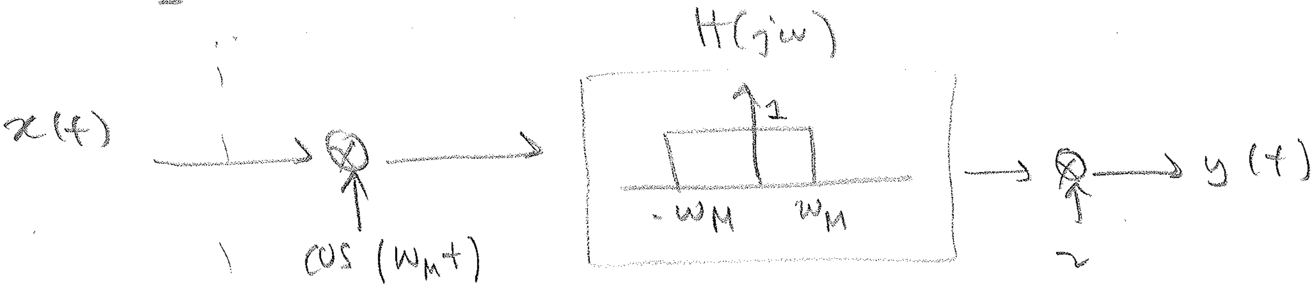


(a) $Y(j\omega) =$



note that these are just examples, you can build your own ones.

(b) Ideal scrambler



(c) Ideal unscrambler

[Optional]

let t defined as Task 2/2 or Team Project.

```
% Ideal Scrambler

h = sin(wM * t)./(wM * t);

y = 2 * ececonv301(x.* cos(wMt), h);

% Ideal Unscrambler is the same

x = 2 * ececonv301(y.* cos(wMt), h);
```