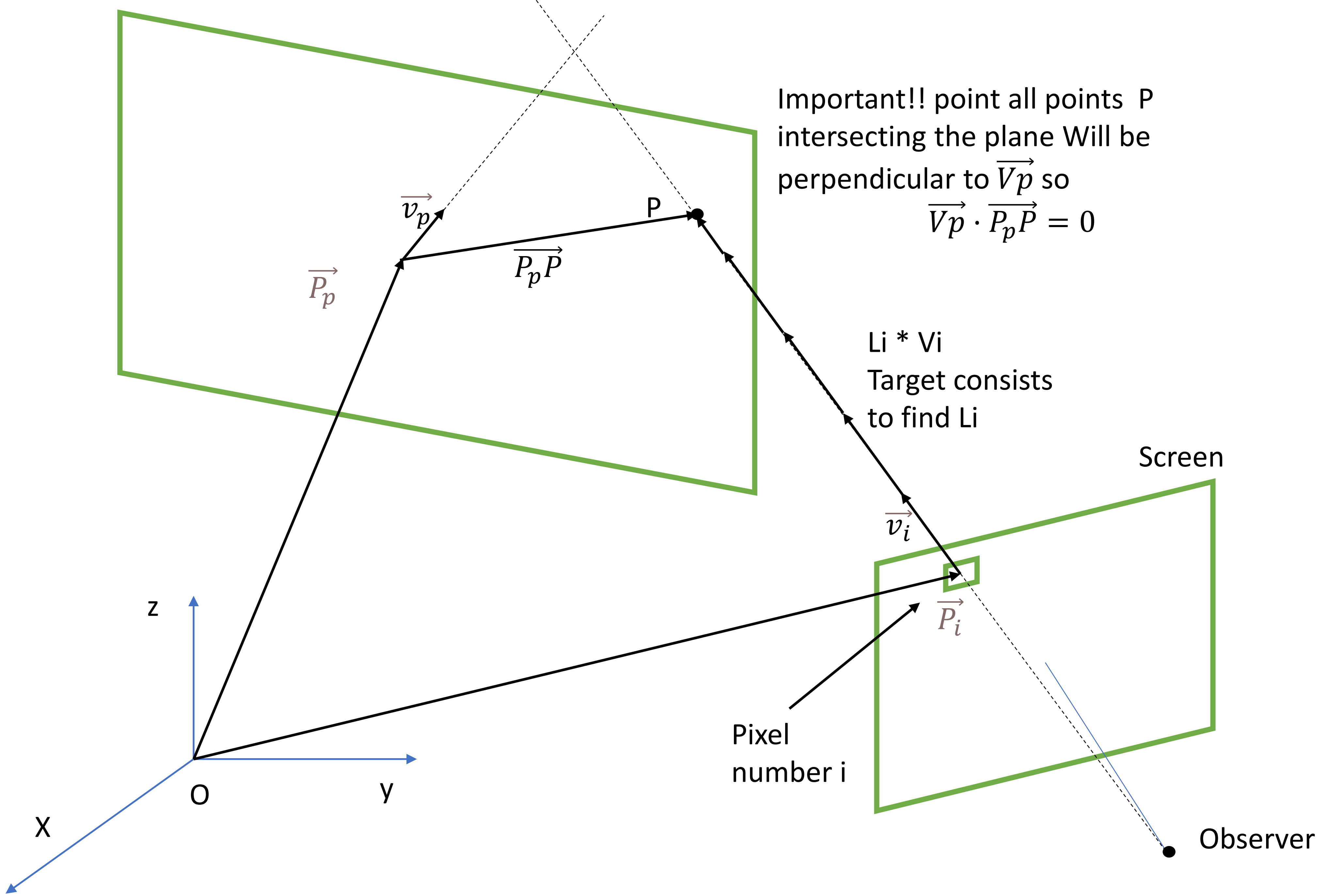

Important!! point all points P intersecting the plane Will be perpendicular to $\overrightarrow{Vp}$ so
$\overrightarrow{Vp} \cdot \overrightarrow{P_pP} = 0$
$\overrightarrow{v_p}$
P
$\overrightarrow{P_p}$
$\overrightarrow{P_pP}$
Li * Vi
Target consists to find Li
Screen
$\overrightarrow{v_i}$
$\overrightarrow{P_i}$
Pixel number i
Observer
z
O
y
X

P = point of interference.
$\overrightarrow{P_i}$ = Position in space of the pixel.
$\overrightarrow{v_i}$ = vector of sight direction of the pixel i.
$\overrightarrow{P_p}$ = Point in the plane
$\overrightarrow{v_p}$ = Direction of the perpendicular direction of the plane.

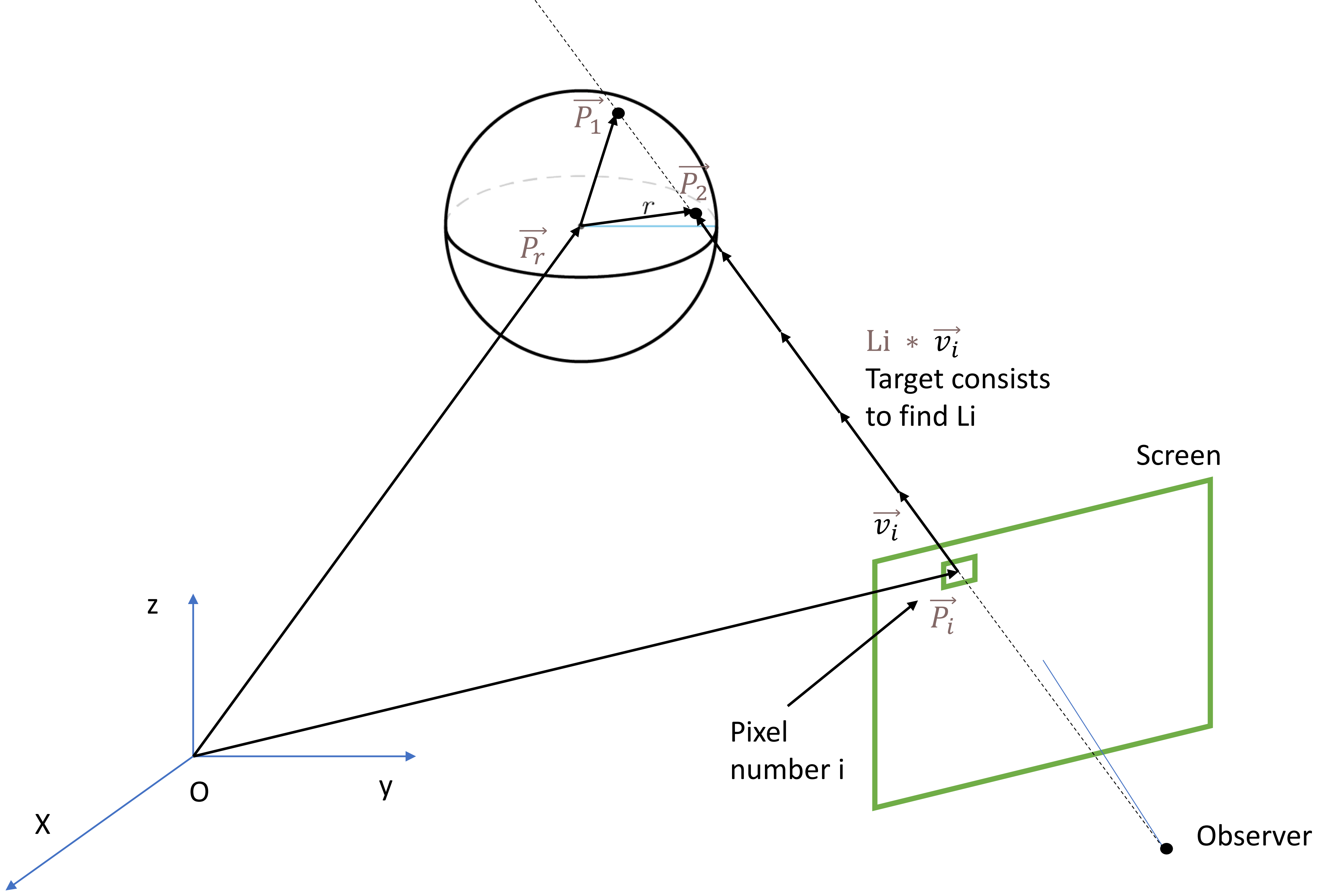
$\vec{P} = \overrightarrow{P_i} + \text{Li} * \overrightarrow{v_i}$

$(\vec{P} - \overrightarrow{P_p}) \cdot \overrightarrow{v_p} = 0$

$\Rightarrow$

$(\overrightarrow{P_i} + \text{Li} * \overrightarrow{v_i} - \overrightarrow{P_p}) \cdot \overrightarrow{v_p} = 0$

$$\boxed{Li = \frac{\overrightarrow{P_p} \cdot \overrightarrow{v_p} - \overrightarrow{P_i} \cdot \overrightarrow{v_p}}{\overrightarrow{v_i} \cdot \overrightarrow{v_p}}}$$



$\overrightarrow{P_1}$
$\overrightarrow{P_2}$
r
$\overrightarrow{P_r}$
$\mathrm{Li} * \overrightarrow{v_i}$
Target consists
to find Li
Screen
$\overrightarrow{v_i}$
$\overrightarrow{P_i}$
z
Pixel
number i
O
y
X
Observer

P = point of interference.
$\overrightarrow{P_i}$ = Position in space of the pixel.
$\overrightarrow{v_i}$ = vector of sight direction of the pixel i.
$\overrightarrow{P_r}$ = Center of the sphere
R = Radius of the sphere.

$$\overrightarrow{P_n} = \overrightarrow{P_i} + \text{Li} * \overrightarrow{v_i}$$

$$\left\|\left(\overrightarrow{P_n} - \overrightarrow{P_r}\right)\right\|^2 = r^2$$

$$\Rightarrow \left\|\left(\overrightarrow{P_i} + \text{Li} * \overrightarrow{v_i} - \overrightarrow{P_r}\right)\right\|^2 = r^2$$

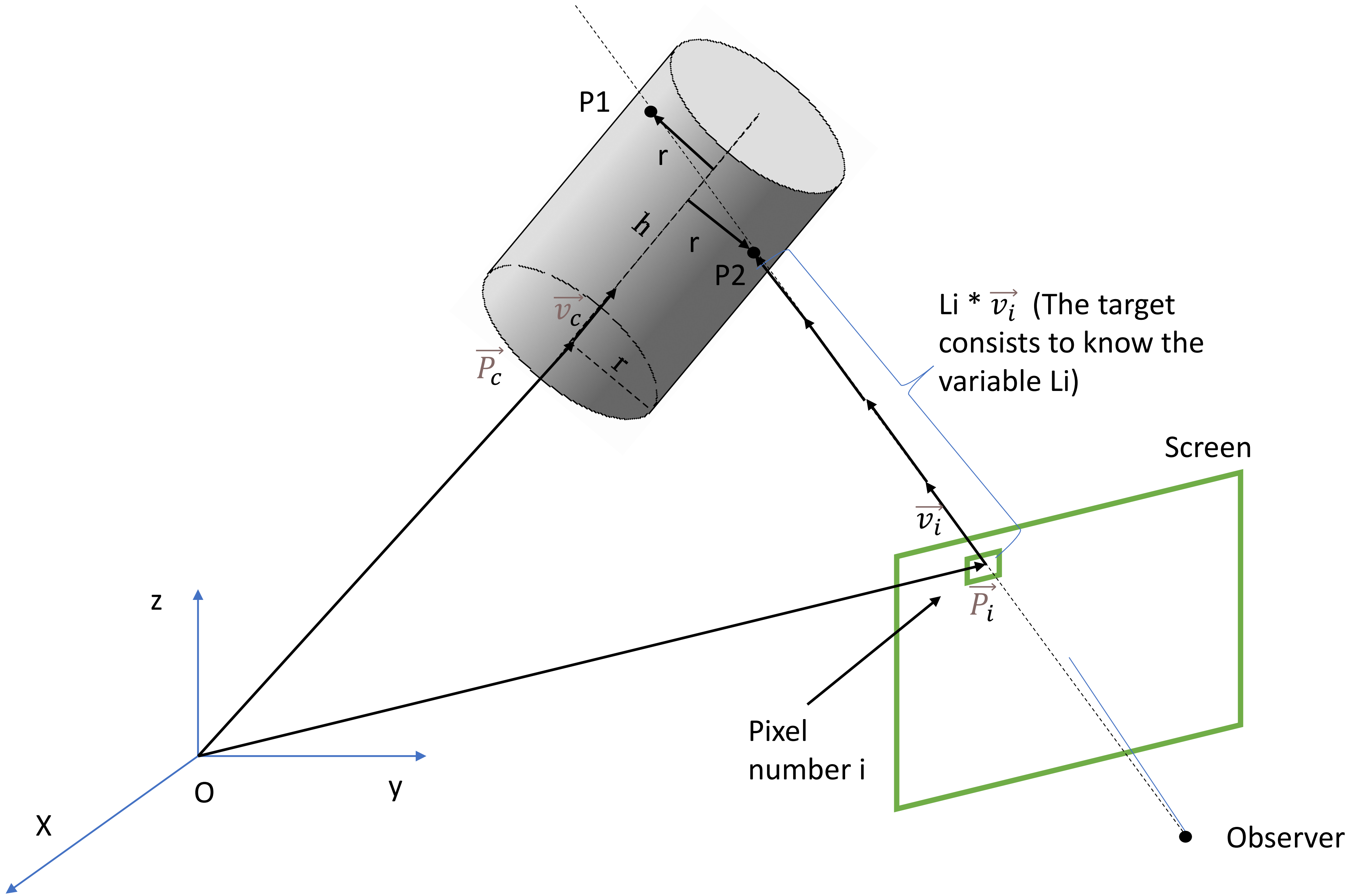
$$\left\|\left(\underbrace{\overrightarrow{P_i} - \overrightarrow{P_r}}_{\overrightarrow{P_{ri}}} + \text{Li} * \overrightarrow{v_i}\right)\right\|^2 = r^2$$

$$\left\|\left(\overrightarrow{P_{ri}} + \text{Li} * \overrightarrow{v_i}\right)\right\|^2 = r^2$$

$$\overrightarrow{P_{ri}} \cdot \overrightarrow{P_{ri}} + Li^2 * (\overrightarrow{v_i} \cdot \overrightarrow{v_i}) + Li * 2 * \left(\overrightarrow{P_{ri}} \cdot \overrightarrow{v_i}\right) = r^2$$

$$Li^2 * (\overrightarrow{v_i} \cdot \overrightarrow{v_i}) + Li * 2 * \left(\overrightarrow{P_{ri}} \cdot \overrightarrow{v_i}\right) + \overrightarrow{P_{ri}} \cdot \overrightarrow{P_{ri}} - r^2 = 0$$

$$\boxed{Li = \frac{-2 * \left(\overrightarrow{P_{ri}} \cdot \overrightarrow{v_i}\right) \pm \sqrt{4 * \left(\overrightarrow{P_{ri}} \cdot \overrightarrow{v_i}\right)^2 - 4 * (\overrightarrow{v_i} \cdot \overrightarrow{v_i}) * \left(\overrightarrow{P_{ri}} \cdot \overrightarrow{P_{ri}} - r^2\right)}}{2 * (\overrightarrow{v_i} \cdot \overrightarrow{v_i})}}$$


P1
r
h
r
P2
$\vec{v_c}$
$\vec{P_c}$
r
Li * $\vec{v_i}$ (The target consists to know the variable Li)
Screen
$\vec{v_i}$
$\vec{P_i}$
z
O
y
X
Pixel number i
Observer

Pn = points of interference. P1 and P2
$\overrightarrow{P_i}$ = Position in space of the pixel.
$\overrightarrow{v_i}$ = vector of sight direction of the pixel i.
$\overrightarrow{P_c}$ = Start point of Cylinder
$\overrightarrow{v_c}$ vector of cylinder direction
R = Radius of the cylinder

$$\overrightarrow{P_n} = \overrightarrow{P_i} + \mathrm{Li} * \overrightarrow{v_i}$$
$$\overrightarrow{P_n} = \overrightarrow{P_c} + \mathrm{Lc} * \overrightarrow{v_c} + \overrightarrow{r_n}$$
$$\Longrightarrow \quad \overrightarrow{P_c} + \mathrm{Lc} * \overrightarrow{v_c} + \overrightarrow{r_n} = \overrightarrow{P_i} + \mathrm{Li} \cdot \overrightarrow{v_i}$$

$$\overrightarrow{r_n} = \overrightarrow{P_i} + \mathrm{Li} * \overrightarrow{v_i} - \overrightarrow{P_c} - \mathrm{Lc} \cdot \overrightarrow{v_c}$$

$\overrightarrow{r_n} \cdot \overrightarrow{v_c} = 0$ (Vector $\overrightarrow{v_c}$ and vector radius are perpendicular)

$$0 = (\overrightarrow{P_i} + \mathrm{Li} * \overrightarrow{v_i} - \overrightarrow{P_c} - \mathrm{Lc} * \overrightarrow{v_c}) \cdot \overrightarrow{v_c}$$

$$\mathrm{Lc} = \frac{\overrightarrow{(P_i - P_c)} \cdot \overrightarrow{v_c} + Li * (\overrightarrow{v_i} \cdot \overrightarrow{v_c})}{\overrightarrow{v_c} \cdot \overrightarrow{v_c}} \qquad \mathrm{Li} = \frac{Lc * (\overrightarrow{v_c} \cdot \overrightarrow{v_c}) - \overrightarrow{(P_i - P_c)} \cdot \overrightarrow{v_c}}{(\overrightarrow{v_i} \,.\, \overrightarrow{v_c})}$$

$$\overrightarrow{r_n} = \overrightarrow{P_i} + \mathrm{Li} * \overrightarrow{v_i} - \overrightarrow{P_c} - \mathrm{Lc} \cdot \overrightarrow{v_c}$$

$$\mathrm{Lc} = \frac{\overrightarrow{(P_i - P_c)} * \overrightarrow{v_c} + Li * (\overrightarrow{v_i} \cdot \overrightarrow{v_c})}{\overrightarrow{v_c} \cdot \overrightarrow{v_c}}$$

$$\Longrightarrow$$

$$\overrightarrow{r_n} = \overrightarrow{P_i} + \mathrm{Li} * \overrightarrow{v_i} - \overrightarrow{P_c} - \frac{\overrightarrow{(P_i - P_c)} \cdot \overrightarrow{v_c} + Li * (\overrightarrow{v_i} \cdot \overrightarrow{v_c})}{\overrightarrow{v_c} \cdot \overrightarrow{v_c}} \cdot \overrightarrow{v_c}$$

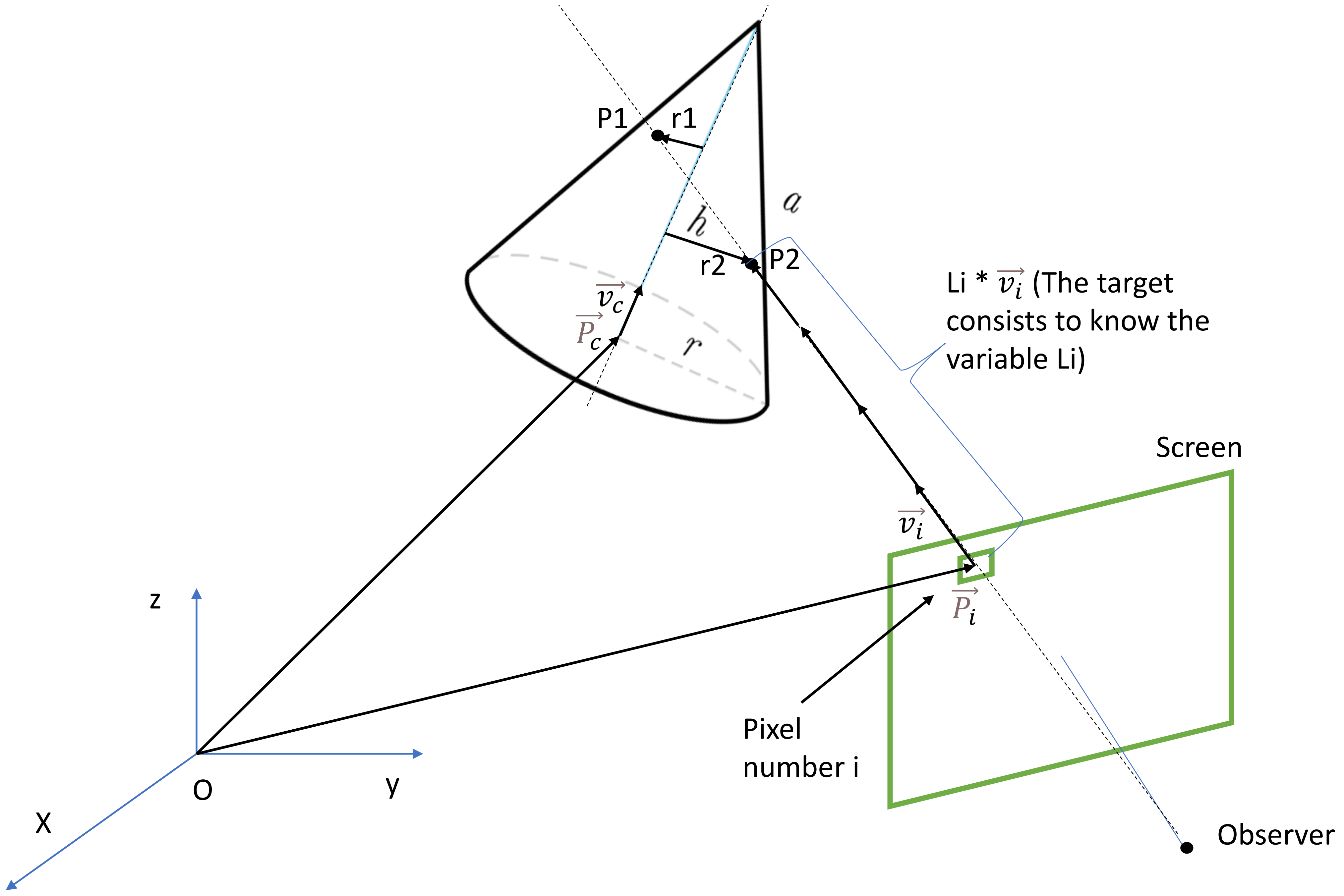
$$\overrightarrow{r_n} = \underbrace{\overrightarrow{(P_i - P_c)} - \frac{\overrightarrow{(P_i - P_c)} \cdot \overrightarrow{v_c}}{\overrightarrow{v_c} \cdot \overrightarrow{v_c}} \cdot \overrightarrow{v_c}}_{\overrightarrow{v_1}} + \mathrm{Li} * \underbrace{\overrightarrow{v_i} - \left[\frac{(\overrightarrow{v_i} \cdot \overrightarrow{v_c})}{\overrightarrow{v_c} \cdot \overrightarrow{v_c}}\right] \cdot \overrightarrow{v_c}}_{\overrightarrow{v_2}}$$

$$\overrightarrow{r_n} = \overrightarrow{v_1} + \mathrm{Li} * \overrightarrow{v_2} \quad \Longrightarrow \quad \overrightarrow{r_n} \cdot \overrightarrow{r_n} = (\overrightarrow{v_1} + \mathrm{Li} * \overrightarrow{v_2}) \cdot (\overrightarrow{v_1} + \mathrm{Li} * \overrightarrow{v_2})$$

$$r^2 = \|\overrightarrow{r_n}\|^2 = \|\overrightarrow{v_1}\|^2 + \mathrm{Li}^2 * \|\overrightarrow{v_2}\|^2 + 2 * \mathrm{Li} * (\overrightarrow{v_1} \cdot \overrightarrow{v_2})$$

$$0 = \mathrm{Li}^2 * \|\overrightarrow{v_2}\|^2 + \mathrm{Li} * 2 * (\overrightarrow{v_1} \cdot \overrightarrow{v_2}) + \|\overrightarrow{v_1}\|^2 - r^2$$

$$\boxed{Li = \frac{-2 * (\overrightarrow{v_1} \cdot \overrightarrow{v_2}) \pm \sqrt{4 * (\overrightarrow{v_1} \cdot \overrightarrow{v_2})^2 - 4 * \|\overrightarrow{v_2}\|^2 * (\|\overrightarrow{v_1}\|^2 - r^2)}}{2a}}$$


P1
r1
a
h
r2
P2
$\vec{v_c}$
$\vec{P_c}$
r
Li * $\vec{v_i}$ (The target consists to know the variable Li)
Screen
$\vec{v_i}$
$\vec{P_i}$
Pixel number i
z
O
y
X
Observer

Pn = points of interference. P1 and P2
$\overrightarrow{P_i}$ = Position in space of the pixel.
$\overrightarrow{v_i}$ = vector of sight direction of the pixel i.
$\overrightarrow{P_c}$ = Start point of conus
$\overrightarrow{v_c}$ = vector of cylinder conus
R = Radius of the conus
h = High of the conus

$$\overrightarrow{P_n} = \overrightarrow{P_i} + \text{Li} * \overrightarrow{v_i}$$
$$\overrightarrow{P_n} = \overrightarrow{P_c} + \text{Lc} * \overrightarrow{v_c} + \overrightarrow{r_n}$$

$$\Rightarrow \quad \overrightarrow{P_c} + \text{Lc} * \overrightarrow{v_c} + \overrightarrow{r_n} = \overrightarrow{P_i} + \text{Li} * \overrightarrow{v_i}$$

$$\overrightarrow{r_n} = \overrightarrow{P_i} + \text{Li} * \overrightarrow{v_i} - \overrightarrow{P_c} - \text{Lc} * \overrightarrow{v_c}$$

$\overrightarrow{r_n} \cdot \overrightarrow{v_c} = 0$ (Vector Vc and vector radius are perpendicular)

$$0 = (\overrightarrow{P_i} + \text{Li} * \overrightarrow{v_i} - \overrightarrow{P_c} - \text{Lc} * \overrightarrow{v_c}) \cdot \overrightarrow{v_c}$$

$$\text{Lc} = \frac{(\overrightarrow{P_i} - \overrightarrow{P_c}) \cdot \overrightarrow{v_c} + Li * (\overrightarrow{v_i} \cdot \overrightarrow{v_c})}{\overrightarrow{v_c} \cdot \overrightarrow{v_c}} \qquad \text{L}i = \frac{Lc * (\overrightarrow{v_c} \cdot \overrightarrow{v_c}) - (\overrightarrow{P_i} - \overrightarrow{P_c}) \cdot \overrightarrow{v_c}}{(\overrightarrow{v_i} \,.\, \overrightarrow{v_c})}$$

$$\overrightarrow{r_n} = \overrightarrow{P_i} + \text{Li} * \overrightarrow{v_i} - \overrightarrow{P_c} - \text{Lc} * \overrightarrow{v_c}$$
$$\text{Li} = \frac{Lc * (\overrightarrow{v_c} \cdot \overrightarrow{v_c}) - (\overrightarrow{P_i} - \overrightarrow{P_c}) \cdot \overrightarrow{v_c}}{(\overrightarrow{v_i} \,.\, \overrightarrow{v_c})}$$

$$\Rightarrow \quad \overrightarrow{r_n} = \overrightarrow{P_i} + \frac{Lc * (\overrightarrow{v_c} \cdot \overrightarrow{v_c}) - (\overrightarrow{P_i} - \overrightarrow{P_c}) \cdot \overrightarrow{v_c}}{(\overrightarrow{v_i} \,.\, \overrightarrow{v_c})} \cdot \overrightarrow{v_i} - \overrightarrow{P_c} - \text{Lc} * \overrightarrow{v_c}$$

$$\overrightarrow{r_n} = \underbrace{(\overrightarrow{P_i} - \overrightarrow{P_c}) - \frac{(\overrightarrow{P_i} - \overrightarrow{P_c}) \cdot \overrightarrow{v_c}}{(\overrightarrow{v_i} \,.\, \overrightarrow{v_c})} \cdot \overrightarrow{v_i}}_{\overrightarrow{v_1}} + \text{Lc} * \underbrace{\left(\left[\frac{(\overrightarrow{v_c} \cdot \overrightarrow{v_c})}{(\overrightarrow{v_i} \,.\, \overrightarrow{v_c})} \right] \cdot \overrightarrow{v_i} - \overrightarrow{v_c} \right)}_{\overrightarrow{v_2}} = \overrightarrow{v_1} + \text{Lc}\, \overrightarrow{v_2}$$

$$r = r_{max} \left(1 - \frac{Lc}{h} \right)$$
$$\overrightarrow{r_n} = \overrightarrow{v_1} + \text{Lc}\, \overrightarrow{v_2}$$

$$\Rightarrow \quad \overrightarrow{r_n} \cdot \overrightarrow{r_n} = (\overrightarrow{v_1} + \text{Lc} \cdot \overrightarrow{v_2}) \cdot (\overrightarrow{v_1} + \text{Lc} \cdot \overrightarrow{v_2})$$

$$r^2 = \|\overrightarrow{r_n}\|^2 = \left(r_{max} \left(1 - \frac{Lc}{h} \right) \right)^2 = \|\overrightarrow{v_1}\|^2 + \text{L}c^2 * \|\overrightarrow{v_2}\|^2 + 2 * \text{Lc} * (\overrightarrow{v_1} \cdot \overrightarrow{v_2})$$

$$0 = \underbrace{\mathrm{Lc}^2 * \left(\|\overrightarrow{v_2}\|^2 - \left(\frac{r_{max}}{h}\right)^2\right)}_{a} + \underbrace{\mathrm{Lc} * 2 * \left((\overrightarrow{v_1} \cdot \overrightarrow{v_2}) + \frac{r_{max}{}^2}{h}\right)}_{b} + \underbrace{\|\overrightarrow{v_1}\|^2 - r_{max}{}^2}_{c}$$

2nd order ecuation to solve Lc

$$Lc = \frac{-2 * \left((\overrightarrow{v_1} \cdot \overrightarrow{v_2}) + \frac{r_{max}{}^2}{h}\right) \pm \sqrt{4 * \left((\overrightarrow{v_1} \cdot \overrightarrow{v_2}) + \frac{r_{max}{}^2}{h}\right)^2 - 4 * \left(\|\overrightarrow{v_2}\|^2 - \left(\frac{r_{max}}{h}\right)^2\right) (\|\overrightarrow{v_1}\|^2 - r_{max}{}^2)}}{2 * \left(\|\overrightarrow{v_2}\|^2 - \left(\frac{r_{max}}{h}\right)^2\right)}$$

$$\mathrm{L}i = \frac{Lc\ (\overrightarrow{v_c} \cdot \overrightarrow{v_c}) - \overrightarrow{(P_i - P_c)} \cdot \overrightarrow{v_c}}{(\overrightarrow{v_i} \cdot \overrightarrow{v_c})}$$