Electromagnetismo

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Bibliografia:
- → Electromagnetismo - Jaime E. Villate, McGraw-Hill
- → Serway

__Avaliação__
- * 2 Testes
- * Problemas (> 50% - 0,5 val / > 75% - 1 val)

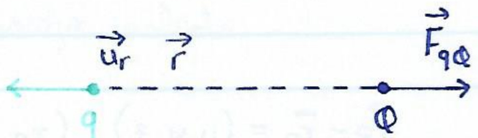
__Aula 1__ - 18.02.2015

Resinosa - Negativa
Vitrea - Positiva

__Lei de Coulomb__

$$|\vec{F}| = K \frac{q_1 q_2}{r^2}$$

A força é diretamente proporcional à carga e inversamente proporcional à distância



$\varepsilon_0 K = \frac{1}{4\pi}$

$\varepsilon_0 = \frac{1}{4\pi K}$

$$\vec{F} = K \frac{qQ}{r^2} \vec{u}_r \qquad K = 9 \times 10^9 \ NC^{-2} m^2 = \frac{1}{4\pi \varepsilon_0}$$

ε_0 → Permitividade eléctrica do vazio

Se as cargas forem iguais têm o sentido de $\vec{u}_r$, caso contrário, têm o sentido de $-\vec{u}_r$

A interacção eléctrica sobrepõe-se à interacção gravítica (devido às ordens de grandeza)

__Campo Eléctrico__

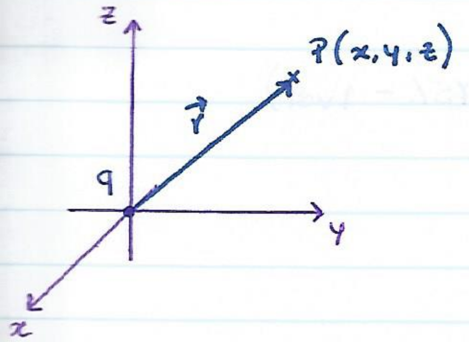
$$\vec{E} = \frac{\vec{F}}{q}$$

Força por unidade de carga

Aula 2 - 20.02.2015

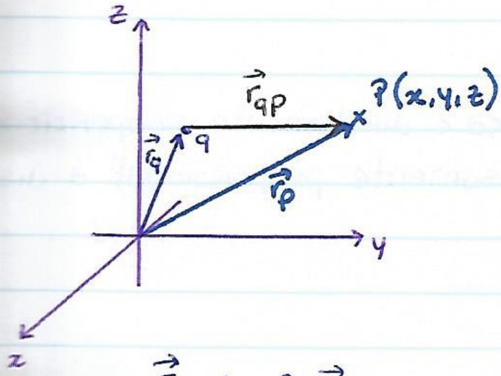
Independentemente da carga colocada no campo temos uma grandeza que só depende da carga geradora:

$$\frac{\vec{F}}{Q} = K \frac{q}{d^2} \vec{u}_r \rightarrow \text{Campo Eléctrico}$$



$$\vec{r} = x\vec{u}_x + y\vec{u}_y + z\vec{u}_z$$

$$\vec{u}_r = \frac{\vec{r}}{|\vec{r}|}$$



$$\vec{r}_{qP} = \vec{r}_P - \vec{r}_q$$

$$\vec{E} = K_e \frac{q}{d^2} \vec{u}_{r_{qP}}$$

$$\vec{E} = K_e \frac{q}{|\vec{r}_{qP}|^2} \frac{\vec{r}_{qP}}{|\vec{r}_{qP}|}$$

$$\vec{r}_P - \vec{r}_q = (u, y, z) - (x_0, y_0, z_0)$$

$$\vec{r}_{qP} = (x - x_0, y - y_0, z - z_0)$$

$$|\vec{r}_{qP}| = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$$

Exemplo

$q = 1C$ $\quad P_q = (1, 2, 3)$ $\quad K \sim 9 \times 10^9\ NC^{-2} m^2$

$P(0, 0, 5)$

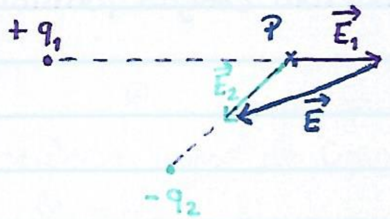
$$\vec{E} = K \frac{q}{d^2} \vec{u}_r$$

$$\vec{r} = P - P_q = (-1, -2, 2)$$

$$|\vec{r}| = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

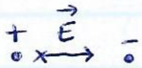
$$\vec{E} = 9 \times 10^9 \times \frac{1}{9} \frac{-1\vec{u}_x - 2\vec{u}_y + 2\vec{u}_z}{3}$$

$$= 10^9 \left(-\frac{1}{3}\vec{u}_x - \frac{2}{3}\vec{u}_y + \frac{2}{3}\vec{u}_z\right) N/C$$



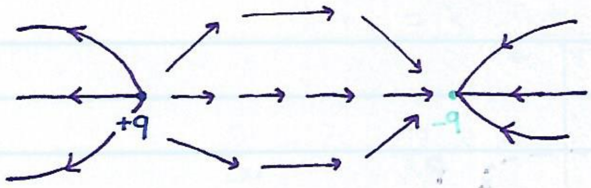
Se houver uma carga muito superior a outras cargas, o campo eléctrico final é praticamente o mesmo que o campo criado por essa carga

Dipolo Eléctrico - carga positiva e outra carga negativa

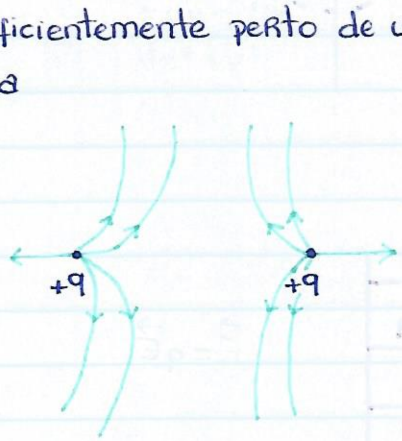


$|q_1| = |q_2|$

Condutor - electrões têm alguma mobilidade entre o material



Numa vizinhança suficientemente perto de uma carga há um campo semelhante a uma carga isolada



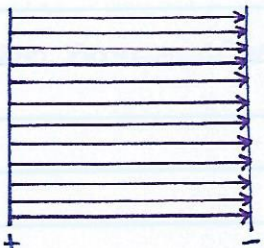
Caso de duas cargas iguais

$|\vec{F}| = 5\ N$

$1\ \mu C = 10^{-6}\ C$

$||\vec{E}|| = \frac{||\vec{F}||}{q'} = \frac{5}{10^{-6}}\ N/C = 5 \times 10^{6}\ N/C$

O campo é mais intenso onde houver uma maior densidade de linhas de campo.

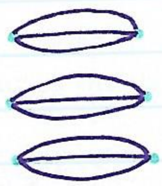


Campo Uniforme

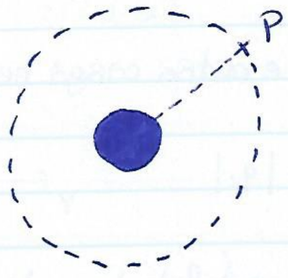
Simetria de Translacção Vertical

Por isso basta calcular para uma região que o resto será igual

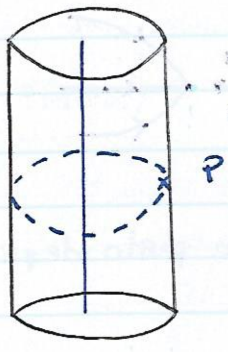
Com uma distribuição semelhante mas com cargas pontuais



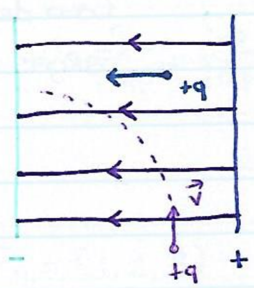
Simetria Esférica



Simetria Cilíndrica



Aula 3 - 23.02.2015

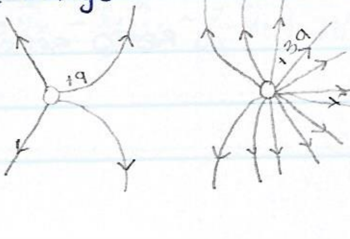


$$\vec{F} = q\vec{E}$$

Os campos uniformes servem para acelerar cargas nos aceleradores lineares.

$$\vec{E} = \sum_i \vec{E}_i$$

O número de linhas de carga associadas a uma carga é proporcional à intensidade da carga.



$|e| = 1{,}6 \times 10^{-19}\ C$

Densidade Superficial de Carga

$$\frac{Q}{S} = \sigma$$

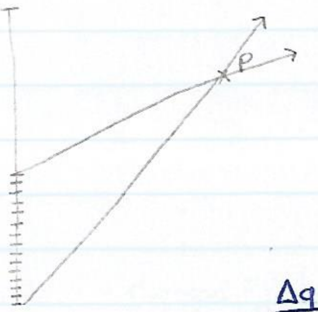
$$\overline{\quad Q \quad}$$

Densidade Linear de Carga

$$\frac{Q}{L} = \lambda$$

Densidade Volúmica de Carga

$$\frac{Q}{V} = \rho_Q$$



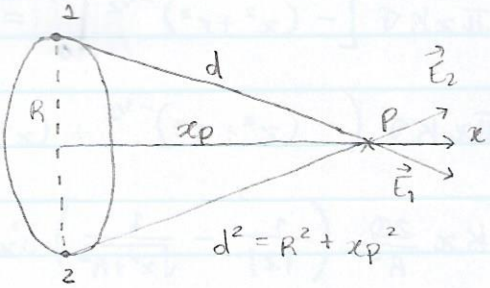
$$\vec{E} = \sum_i \vec{E_i} = \sum_i K \frac{\Delta q_i}{d_i^2}\vec{u_i} = K\int \frac{dq}{d_q^2}\vec{u_q}$$

$$= K\int \frac{\lambda\, dy}{x_q^2 + y^2}\vec{u_i}$$

$$\frac{\Delta q}{\Delta y} = \lambda \qquad \frac{dq}{dy} = \lambda \qquad dq = \lambda\, dy$$

$$\vec{u} = \frac{\vec{r}}{|\vec{r}|} = \frac{x\vec{u_x} + y\vec{u_y}}{\sqrt{x^2+y^2}}$$

Exemplo



$$d^2 = R^2 + x_P^2$$

$$\vec{E} = \vec{E_x} = \int K \frac{\lambda\, dl}{R^2 + x_P^2}\left(\frac{x}{d}\right)$$

$$(\vec{u_r})_x = \frac{x - x_q}{|\vec{r} - \vec{r_q}|} = \frac{x_P}{d} \qquad \vec{u_r} = \frac{\vec{r} - \vec{r_q}}{|\vec{r} - \vec{r_q}|}$$

$$\vec{E_x} = K\lambda\int \frac{dl}{d^2}\frac{x_P}{d} = K\lambda \frac{x_P}{d^3}\int dl = K\lambda \frac{x}{(x^2+R^2)^{3/2}}\, 2\pi R = \frac{2\pi R \lambda K x}{(x^2+R^2)^{3/2}}$$

$$* \quad \vec{E_x} = K\int \frac{\lambda\, dl}{x^2+R^2}\overbrace{\frac{x}{\underbrace{\sqrt{x^2+R^2}}_{d}}}^{\frac{x}{d}} = \frac{K\lambda x}{(x^2+R^2)^{3/2}}\underbrace{\int dl}_{2\pi R}$$

Aula 4 - 25.02.2015

Aro

$$\vec{E} = K \frac{dq}{d^2} \vec{x}$$

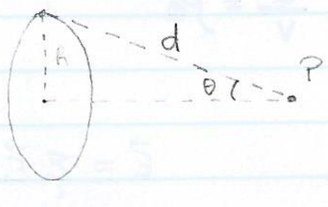
$$\vec{E}_x = \int \frac{K dq\, x_P}{(x_P^2 + R^2)^{3/2}} = \frac{\lambda K x_P}{(x_P^2+R^2)^{3/2}} \int d\ell$$

$$E_x = K \frac{dq}{x_P^2 + R^2} \frac{x_P}{d}$$

$$\frac{Q}{L} = \lambda \quad dq = \lambda d\ell$$

Disco

$$\sigma = \frac{Q}{S} = \frac{Q}{\pi R^2}$$



$$Q_{\text{aro c/ espessura infinitesimal}} = \sigma \underbrace{dR\, \ell}_{\substack{\text{superfície do aro} \\ \ell \times dR}}$$

1 aro de raio R

$$\vec{E} = \frac{K \sigma dr\, 2\pi r x\, \vec{u}_x}{(x^2+r^2)^{3/2}}$$

$$\vec{E} = \int_{\substack{\text{todos} \\ \text{os aros} \\ 0<r<R}} E_{\text{aros}} = \vec{u}_x \int \frac{Kx\, \sigma 2\pi r\, dr}{(x^2+r^2)^{3/2}}$$

$$= \vec{u}_x K \sigma 2\pi x \int_0^R \frac{2r}{2(x^2+r^2)^{3/2}} dr =$$

$q = \sigma S dr$

$dq = \sigma dr\, 2\pi r$

$$= 2\pi x K \sigma \left[-(x^2+r^2)^{-1/2}\right]_0^R =$$

$$= 2\pi x K \overset{\frac{Q}{\pi R^2}}{\sigma} \left(-(x^2+R^2)^{-1/2} + (x^2)^{-1/2}\right) =$$

$$= Kx \frac{2Q}{R^2} \left(\frac{1}{|x|} - \frac{1}{\sqrt{x^2+R^2}}\right) \vec{u}_x$$

$\boxed{x \gg R}$

$$\vec{E} = K \frac{Q}{x^2} \vec{u}_x$$

Desenvolver em potências de R em torno de R=0

$$\frac{1}{\sqrt{x_P^2+R^2}} \simeq \frac{1}{|x_P|}$$

$f(R=0)$

$$f'(R) = -\frac{1}{2} \frac{2R}{(\sqrt{x_P^2+R^2})^3} = -R\,(x_P^2+R^2)^{-3/2}$$

$$f''(R) = -(x_P^2+R^2)^{-3/2} - R\left(-\frac{3}{2}\frac{2R}{(x_P^2+R^2)^{3/2}}\right) \quad f''(R=0) = -\frac{1}{(x^2)^{3/2}}$$

$$f(R) = f(0) + f'(0)(R-0) + \frac{1}{2} f''(0)(R-0)^2 + \dots$$

$$\vec{E} = Kx \frac{2Q}{R^2}\left(\frac{1}{|x|} - \left(\frac{1}{|x|} - \frac{1}{2}\frac{R^2}{|x|^3}\right)\right) = Kx\frac{2Q}{R^2}\frac{1}{2}\frac{R^2}{|x|^3} = KQ\frac{x}{|x^3|} = \frac{KQ}{x^2}\,\mathrm{sgn}($$

$\boxed{x \ll R}$

$$\frac{1}{\sqrt{x^2+R^2}} \simeq \frac{1}{R} \qquad \vec{E} = Kx\frac{2Q}{R^2}\left(\frac{1}{|x|} - \frac{1}{R}\right)\vec{u_x} = K\frac{x}{|x|}\frac{2Q}{R^2}\vec{u_x} =$$

$$= 2K\,\mathrm{sgn}(x)\frac{Q}{R^2} = 2\pi K\sigma\left(\mathrm{sgn}(x)\vec{u_x}\right) = \frac{\sigma}{2\varepsilon_0}\left(\mathrm{sgn}(x)\vec{u_x}\right.$$

Nota: $\mathrm{sgn}(x) = \frac{x}{|x|}$
Sinal de x
(1 ou -1)

$$K = \frac{1}{4\pi\varepsilon_0}$$

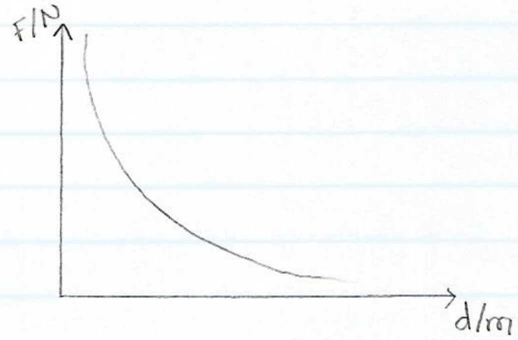
T.P. 1 – 25.02.2015

Série 1 – Campo Eléctrico

1→ a)

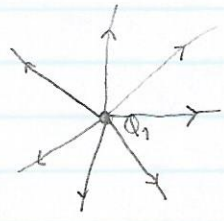
$$|\vec{F}| = \frac{KQ_1Q_2}{d^2} = 9\times10^9 \times \frac{1{,}0\times10^{-6}\times 0{,}5\times10^{-6}}{(20\times10^{-2})^2} = 1{,}13\times10^{-1}\ N \qquad \vec{F} = (1{,}1\times10^{-1}\,\vec{u_r})\ N$$

b)



$\vec{F} = \frac{KQ_1Q_2}{d^2}$ → Meter na calculadora

c)

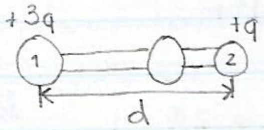


Igualmente espaçadas

d) i) A carga movimenta-se no sentido de se afastar da outra, na direcção da linha de campo (mov. Rectilíneo), com aceleração variável

ii) A carga afasta-se seguindo uma trajectória curvilínea

3 →



A carga está em equilíbrio quando $\vec{F_1} + \vec{F_2} = 0$ ou $\vec{E_1} + \vec{E_2} = 0$

$$|\vec{F_1}| = \frac{K\,3qq_0}{x^2}$$

$$|\vec{F_2}| = \frac{Kqq_0}{x^2}$$

$$E_1 + E_2 = 0 \Leftrightarrow \frac{K\,3q}{x^2} - \frac{Kq}{(d-x)^2} \Leftrightarrow Kq\left(\frac{3}{x^2} - \frac{1}{(d-x)^2}\right) = 0$$

$$\Leftrightarrow \frac{3}{x^2} - \frac{1}{(d-x)^2} = 0 \Leftrightarrow$$

$q_0 > 0$ estável

$q_0 < 0$ instável

$$\Leftrightarrow \frac{3(d-x)^2 - x^2}{x^2(d-x)^2} = 0 \Leftrightarrow$$

$$\Leftrightarrow 3d^2 - 6xd + 3x^2 - x^2 = 0 \Leftrightarrow$$

$$\Leftrightarrow 3d^2 - 6xd + 2x^2 = 0 \Leftrightarrow$$

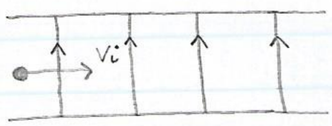
$$\Leftrightarrow 2x^2 - 6xd + 3d^2 = 0 \Leftrightarrow$$

$$\Leftrightarrow x = \frac{3d \pm \sqrt{9d^2 - 6d^2}}{2} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{3d \pm \sqrt{3}\,d}{2} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{3-\sqrt{3}}{2}\,d = 0{,}634d$$

5 → a)



$v_i = 4{,}50 \times 10^5$ m/s

$E = 9{,}60 \times 10^3$ NC^{-1}

$m_p = 1{,}67 \times 10^{-27}$ Kg

$q = 1{,}60 \times 10^{-19}$ C

b) $d = 5{,}00$ cm $= 5{,}00 \times 10^{-2}$ m

$$v = \frac{d}{\Delta t} \Leftrightarrow 4{,}50 \times 10^5 = \frac{5{,}00 \times 10^{-2}}{\Delta t} \Leftrightarrow \Delta t = 1{,}11 \times 10^{-7}\ s$$

c) $\Delta t = 1{,}11 \times 10^{-7}$ s

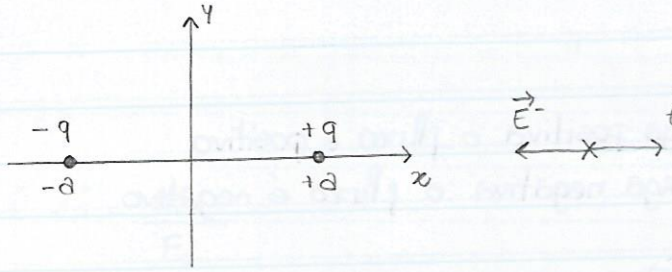
$v_{i_y} = 0$ m/s

$$|\vec{E}| = \frac{|\vec{F}|}{q} \Leftrightarrow |\vec{F}| = 9{,}60 \times 10^3 \times 1{,}60 \times 10^{-19} \Leftrightarrow |\vec{F}| = 1{,}54 \times 10^{-15}\ N$$

$$|\vec{F}| = m|\vec{a}| \Leftrightarrow 1{,}54 \times 10^{-15} = 1{,}67 \times 10^{-27} \times a \Leftrightarrow a = 9{,}22 \times 10^{11}\ m/s^2.$$

$$y_f = y_i + v_0 t + \frac{1}{2}at^2 \Leftrightarrow y_f = 5{,}68 \times 10^{-3}\ m$$

6 →



a) $\vec{E} = \vec{E}^- + \vec{E}^+$

$$E^{(-)} = -\vec{u}_x \frac{Kq}{(x+a)^2} + \vec{u}_x \frac{Kq}{(x-a)^2} = \vec{u}_x Kq \left(\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2} \right)$$

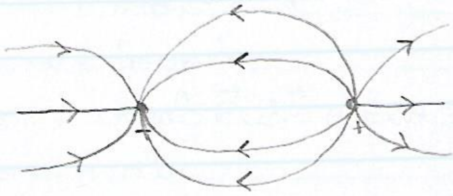
$$\left(\frac{1}{(x+a)^2} \right)' = -2 \frac{1}{(x+a)^3} \quad \text{Derivada em ordem a } a$$

$$\frac{1}{(x+a)^2} = \frac{1}{x^2} - \frac{2}{x^3} a$$

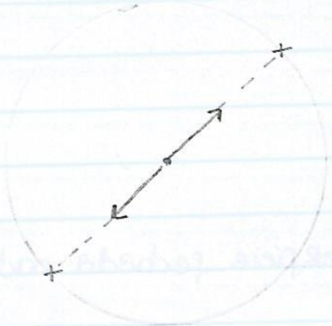
$$\vec{E} = \vec{E}^- + \vec{E}^+ = \vec{u}_x Kq \left(\frac{1}{x^2} + \frac{2}{x^3} a - \left(\frac{1}{x^2} - \frac{2}{x^3} a \right) \right) = \vec{u}_x K_e q \frac{4a}{x^3}$$

b) O campo do dipolo decai mais rapidamente que o campo de uma única carga

c)

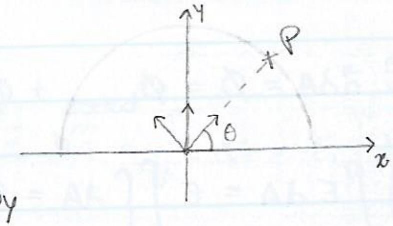


8 → a)



$\vec{E}_c = \vec{0}$

b)



$\vec{E} \parallel Oy$

i) $\frac{15 \mu C}{2} = 7,5 \mu C$

ii) $E_y = E \sin\theta = \frac{K}{R^2} \int \sin\theta \, dq$

$$\vec{E} = \int K \frac{dq}{d^2} \vec{u}_q = \frac{K}{R^2} \int dq \, \vec{u}_q$$

$$= \frac{K}{R^2} \int \sin\theta \, \lambda R \, d\theta = \frac{K}{R^2} \lambda R \int_0^{\pi} \operatorname{sen}\theta \, d\theta =$$

$$\lambda = \frac{Q}{L} \Rightarrow dq = \lambda \, d\ell = \lambda R \, d\theta$$

arco $= R\theta$
$d\ell$

$$= \frac{K}{R} \lambda \left[-\cos\theta \right]_0^{\pi} = \frac{2K\lambda}{R} = 2,164 \times 10^7 \ N/C$$

Aula 5 - 27.02.2015

No caso de uma carga positiva o fluxo é positivo.
No caso de uma carga negativa o fluxo é negativo.

$$\phi_E = \int \vec{E} \cdot d\vec{A} = \int \vec{E} \cdot \vec{n} \, dA$$

Esfera de Raio R

$$\vec{E} = K \frac{Q}{R^2} \vec{u}_r \qquad \phi = \iint K \frac{Q}{R^2} \underbrace{\vec{u}_R \cdot \vec{n}}_{1} \, dA = \frac{KQ}{R^2} \iint dA$$

área da superfície esférica

$$\phi = \frac{KQ}{\cancel{R^2}} 4\pi \cancel{R^2} = 4\pi K Q$$

$$\text{Como } K = \frac{1}{4\pi \varepsilon_0} \rightarrow \phi = \frac{Q}{\varepsilon_0}$$

No caso de uma carga pontual o fluxo é independente da forma da superfície.

Como o campo eléctrico total se pode obter somando os campos existentes, também o mesmo pode ser obtido da mesma forma.

$$\boxed{\phi_{2\,cargas} = \frac{q_1 + q_2}{\varepsilon_0}}$$

Teorema de Gauss

$$\phi_{\substack{sup.\\fechada}} = \oint\!\!\oint \vec{E} \cdot d\vec{A} = \frac{q_{int}}{\varepsilon_0}$$

Uma superfície de Gauss é uma superfície fechada onde se calcula ϕ_E.

$$\vec{E} \perp \text{fio} \qquad \iint \vec{E} \cdot \vec{n} \, dA = \phi = \underbrace{\phi_{bases}}_{0} + \phi_{\text{parede lateral}}$$

$$= \iint E \, dA = E \iint dA = E \underbrace{A_{\text{parede lateral}}}_{2\pi R h}$$

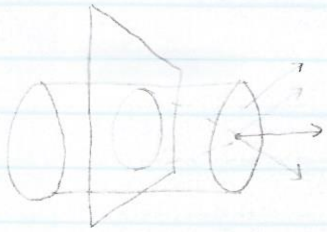
$$\text{Com } \frac{Q}{L} = \lambda \Rightarrow \phi = \frac{q_{int}}{\varepsilon_0} \Leftrightarrow 2\pi R \cancel{h} E = \frac{\lambda \cancel{h}}{\varepsilon_0}$$

Eletro!!! yeee!!!

Aula 6 - 02.03.2015

Fluxo de Campo Eléctrico

$$\phi = \oiint \vec{E}\cdot d\vec{A} = \frac{Q_{int}}{\varepsilon_0}$$



$$\int \vec{E}\cdot d\vec{A} = E\int dA$$

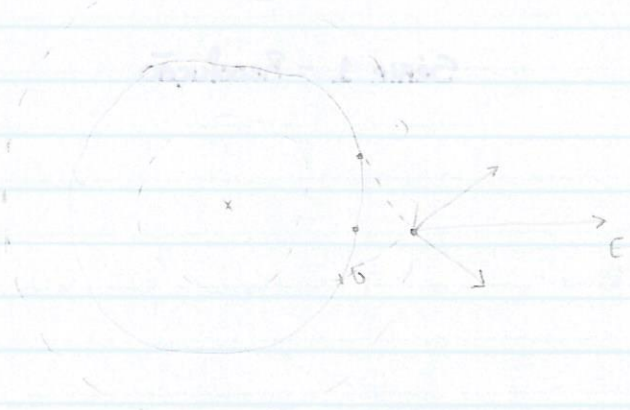
$$\phi = \phi_{bases} + \underbrace{\phi_{\substack{sup\\cilindrico\\lateral}}}_{0} \longrightarrow \phi = 2EA_b$$

$$2EA_b = \frac{Q_{int}}{\varepsilon_0} = \frac{\sigma A_{contida}}{\varepsilon_0} \Leftrightarrow 2EA_b = \frac{\sigma A_b}{\varepsilon_0} \Leftrightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

($A_{contida} = A_b$)

→ campo não depende da distância ao plano, é uniforme

Superfície Gaussiana
- → Campo é constante
- → Campo é tangente
- → Campo normal $\vec{E}\cdot d\vec{A} = E\,dA$
- → Campo é nulo

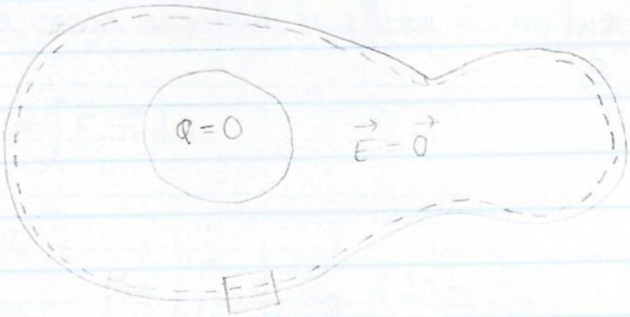


$$\phi = \int \vec{E}\cdot d\vec{A} = \int E\,dA = E\cdot\int dA = E\cdot A$$

$$\phi = E\,4\pi R^2 = \frac{Q}{\varepsilon_0} \Leftrightarrow E = \underbrace{\frac{1}{4\pi\varepsilon_0}}_{K}\frac{Q}{R^2}$$

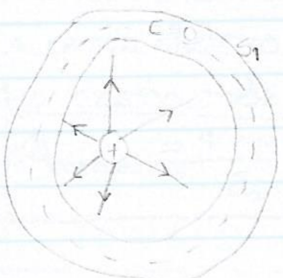
Portanto o campo num ponto exterior seria igual ao de uma pontual no centro da sup esférica

$$\phi_{int} = \int \vec{E}\cdot d\vec{A} = -E\int dA = \frac{\overset{=0}{Q_{int}}}{\varepsilon_0} \Rightarrow -EA = 0 \qquad \vec{E} = \vec{0}$$



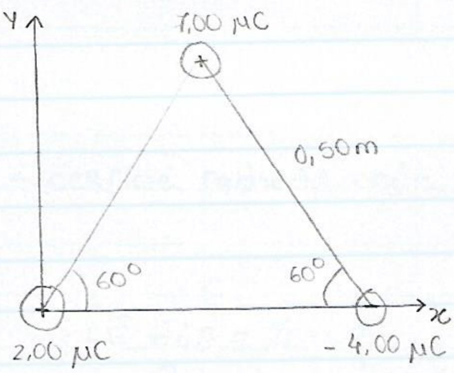
$$\phi = E_{sup}A_{base} = \frac{Q_{sup}}{\varepsilon_0} \qquad E_{sup} = \frac{Q_{sup}}{A}\,\frac{1}{\varepsilon_0} = \frac{\sigma_{sup}}{\varepsilon_0}$$

$$\boxed{\begin{array}{l} \vec{E}_{int} = \vec{0} \\ \vec{E}_{sup} = \dfrac{\sigma}{\varepsilon_0}\,\vec{u_n} \end{array}}$$



$$\phi_{S_1} = \int_{S_1} \vec{E}\cdot d\vec{A} = 0 = \frac{Q_{int}}{\varepsilon_0}$$

Série 1 - Resolução



2 →

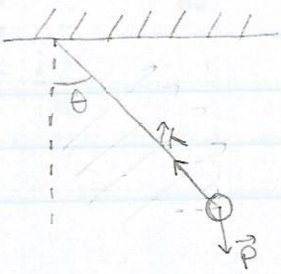
a) $|\vec{E}| = |\vec{E}_1| + |\vec{E}_2|$

$|\vec{E}_1| = K\dfrac{q_1}{d^2}$

$|\vec{E}_2| = K\dfrac{q_2}{d^2}$

$|\vec{E}| = \dfrac{K}{d^2}\left(q_1 + q_2\right) = 1,1\times10^5 \text{ N/C}$

b) $|\vec{E}| = \dfrac{|\vec{F}|}{Q} \Leftrightarrow E = \dfrac{F}{Q} \Leftrightarrow F = 1,1\times10^5 \times 2,00\times10^{-6} = 0,22 \text{ N}$



4 → $m = 1,00\,g = 1,00\times10^{-3}$ Kg

$\vec{E} = (3,00\,\vec{u}_x + 5,00\,\vec{u}_y)\times10^5$ N/C

$\theta = 37,0°$

$E = \frac{F}{Q}$

Para que a carga esteja em equilíbrio $F_{resul} = 0$

$$\vec{P} + \vec{T} + \vec{F} = 0$$

$$\begin{cases} \vec{u_x}: -T\sin\theta + F_x = 0 \\ \vec{u_y}: -mg + T\cos\theta + F_y = 0 \end{cases} \Leftrightarrow \begin{cases} -T\sin\theta + 3{,}00\times10^5 \times Q = 0 \\ -mg + T\cos\theta + 5{,}00\times10^5\times Q = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} T = 3{,}00\times10^5 \times Q \times \frac{1}{\sin\theta} \\ \_\_\_\_ \end{cases} \Leftrightarrow \begin{cases} \_\_\_\_ \\ 3{,}00\times10^5 \times Q \times \frac{\cos\theta}{\sin\theta} + 5{,}00\times10^5 Q = mg \end{cases}$$

$$\Leftrightarrow \begin{cases} \_\_\_\_ \\ Q\left(3{,}00\times10^5 \cdot \frac{\cos 37}{\sin 37} + 5{,}00\times10^5\right) = 1{,}00\times10^{-3}\times 9{,}8 \end{cases} \Leftrightarrow$$

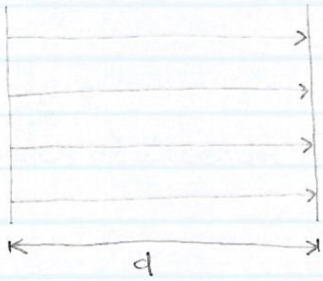
$$\Leftrightarrow \begin{cases} T = 3{,}00\times10^5 \times 1{,}09\times10^{-8} \times \frac{1}{\sin 37} \\ Q = 1{,}09\times10^{-8}\ C \end{cases} \Leftrightarrow \begin{cases} T = 5{,}43\times10^{-3}\ N \\ Q = 1{,}09\times10^{-8}\ C \end{cases}$$

7 → $|\vec{E}| = 640\ N/C$

$d = 4{,}00\times10^{-2}\ m$

$q_p = 1{,}60\times10^{-19}\ C$

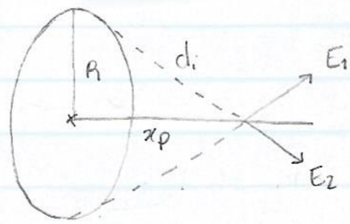
$q_e = -1{,}60\times10^{-19}\ C$



9 → $R \rightarrow$ Raio

$\ell \rightarrow$ espessura

a)



$$|\vec{E}| = E_x = \sum E_{x_i} = \sum K \frac{dq}{d_i^2}\vec{u_x}$$

$$= \int K \frac{dq}{d_i^2}\vec{u_x} = K\vec{u_x}\int\frac{dq}{d_i^2}$$

$\frac{Q}{L} = \lambda \Leftrightarrow dq = \lambda d\ell$

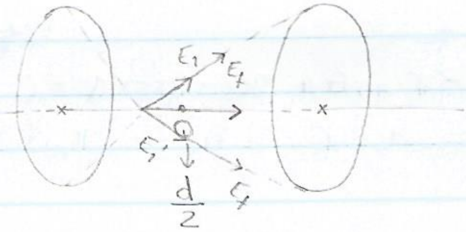
$$= K\frac{x_p}{d}\times\frac{1}{d_i^2}\int\lambda d\ell = K\frac{x_p}{d^3}\lambda\, 2\pi R =$$

$\vec{u_x} = \frac{x_p}{d}$

$$= K\lambda\, 2\pi R\, x_p\,(R^2 + x_p^2)^{3/2}$$

b) Relativamente ao caso de um único aro o campo eléctrico vai aumentar na região entre os aros e diminui nas outras regiões.

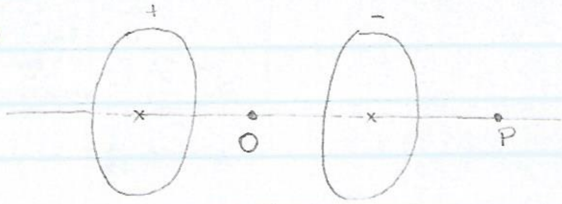
c) i)



$$|\vec{E}| = \frac{K\lambda 2\pi R \left|\frac{d}{2} - x_p\right|}{\left(R^2 + \left(\frac{d}{2} - x_p\right)^2\right)^{3/2}} + \frac{K\lambda 2\pi R \left|x_p + \frac{d}{2}\right|}{\left(R^2 + \left(x_p + \frac{d}{2}\right)^2\right)^{3/2}} =$$

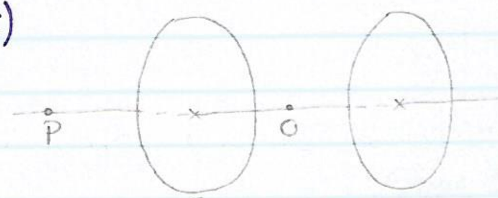
$$= K\lambda 2\pi R \left[\frac{\frac{d}{2} - |x_p|}{\left(\left(\frac{d}{2} - x_p\right)^2 + R^2\right)^{3/2}} + \frac{|x_p| + \frac{d}{2}}{\left(R^2 + \left(x_p + \frac{d}{2}\right)^2\right)^{3/2}}\right]$$

ii)



$$|\vec{E}| = K\lambda 2\pi R \left[\frac{\frac{d}{2} + |x_p|}{\left(\left(\frac{d}{2} + x_p\right)^2 + R^2\right)^{3/2}} - \frac{|x_p| - \frac{d}{2}}{\left(\left(x_p - \frac{d}{2}\right) + R^2\right)^{3/2}}\right]$$

iii)

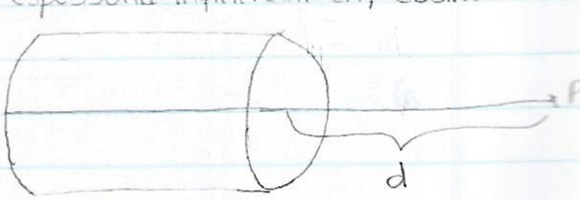


$$|\vec{E}| = K\lambda 2\pi R \left[\frac{\frac{d}{2} + |x_p|}{\left(\left(\frac{d}{2} + |x_p|\right)^2 + R^2\right)^{3/2}} - \frac{|x_p| - \frac{d}{2}}{\left(\left(|x_p| - \frac{d}{2}\right)^2 + R^2\right)^{3/2}}\right]$$

10 → $R \to$ Raio
$h \to$ comprimento

O cilindro sem bases é como se fosse um grande conjunto de aros de espessura infinitesimal, assim

$$|\vec{E}| = \int_0^h \frac{2\pi K R\, dx\, \sigma\, (x_p - x)}{\left[(x_p - x)^2 + R^2\right]^{3/2}}$$



$$\vec{E}_{anel} = K Q \frac{x_p}{(x_p^2 + R^2)^{3/2}} \vec{u}_x$$

Considerando cada anel

$$Q_{anel} = 2\pi R\, dx\, \sigma$$

$w = x_p - x$
$x = x_p - w$
$dx = -dw$
$x = 0 \to w = x_p$
$x = h \to w = x_p - h$

$$\vec{E} = \frac{K 2\pi R\, dx\, \sigma\, (x_p - x)}{\left[(x_p - x)^2 + R^2\right]^{3/2}}$$

Aula 7 - 04.03.2015

Num condutor o excesso de carga colocada à superfície leva a que a carga se separe o mais possível. Quando se atinge o equilíbrio o campo no interior do condutor é nulo e como tal a força eléctrica que actua nas cargas também é nula.

Para um condutor com uma carga

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\vec{u}_r \qquad \vec{E}_2 = 0$$

$$\phi_{S_2} = \frac{Q_{int_2}}{\varepsilon_0} \qquad \int \vec{E}_2\, d\vec{A} = \frac{Q_{int_2}}{\varepsilon_0} = \frac{Q_{sup\,int} + Q}{\varepsilon_0}$$

$$Q_{int\ sup\ cond} = -(+Q)$$

Dentro do condutor o fluxo e o campo continuam a ser nulos porque há conpensação de cargas

O fluxo de uma grandeza vetorial depende da fonte dessa grandeza vectorial.

T.P. 2 - 04.03.2015

Conclusão 10 →

$$= \sigma 2\pi K R \underbrace{\int_{x_p}^{x_p-h} \frac{-2w\, dw}{2\left[w^2+R^2\right]^{3/2}}}_{-\frac{1}{2}\, 2w\,(w^2+R^2)^{-3/2}\ *} = \vec{u}_x\, \sigma 2\pi K R \left(-\frac{1}{2}\right)\left[\frac{(w^2+R^2)^{-1/2}}{-1/2}\right]$$

$$*\ -\frac{1}{2}\int 2w\,(w^2+R^2)^{-3/2}\, dw \qquad E_x = 2\pi K R\,\sigma\left[\frac{1}{((x_p-h)^2+R^2)^{1/2}} - \frac{1}{(x_p^2+R^2)^{1/2}}\right]$$

$$\sigma = \frac{Q}{2\pi R h} \qquad = 2\pi R\, K\frac{Q}{2\pi R h}\left(\frac{1}{(d^2+R^2)^{1/2}} - \frac{1}{((h+d)^2+R^2)^{1/2}}\right)$$

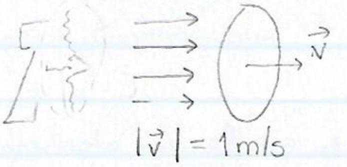
Série 2 - Resolução

1 → $d = 20$ cm

$v = 1$ m/s

$R = 5$ cm

a)



$|\vec{v}| = 1$ m/s

b) $\phi = \int \vec{v}\cdot d\vec{A} = \int v \times dA = v \times A = \pi R^2$

c) $\phi = \int \vec{v}\cdot d\vec{A} = 0 \quad (\vec{v} \perp \vec{n})$

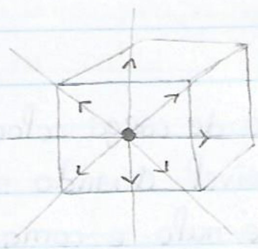
d) $\phi = \int \vec{v}\, d\vec{A} - \int v\, dA \cos(45) = \frac{\sqrt{2}}{2} v\, A = \frac{\sqrt{2}}{2}\pi R^2 = 5,6 \times 10^{-3}$

e) $\phi_{sai} = \phi_{entra}$, logo o fluxo total através do cilindro é nulo

3 → $Q = 170\ \mu C$

$d = 80\ cm$

$A_e = 4\pi R^2$



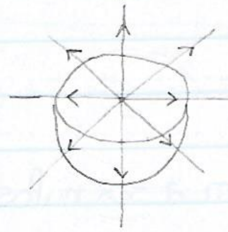
$\varepsilon_0 = 8{,}854 \times 10^{-12}$

a) $\phi_{\substack{sup\\esf}} = \dfrac{Q_{int}}{\varepsilon_0} = \phi_{\substack{sup\\cubo}} = 6\,\phi_{1\,face}$

b) $\phi_{1\,face} = \dfrac{1}{6}\dfrac{Q}{\varepsilon_0}$

c) O fluxo total mantém-se o mesmo, no entanto o fluxo de cada face vai variar devido à proximidade / afastamento à carga (O fluxo é maior para as faces mais próximas

4 →



a) $\phi_{\substack{sup\\curv}} = \int \vec{E} \cdot d\vec{A} = \int E\,dA = E \int dA = EA$

$= E\,2\pi R^2 = \dfrac{KQ}{\cancel{R^2}}\,2\pi \cancel{R^2} = 2\pi KQ$

b) $\phi_{sup\ plana} + \phi_{curva} = 0 \Rightarrow \phi_{sup\ plana} = -\phi_{curva} = -2\pi KQ$

5 → $R = 40{,}0\ cm$

$Q = 26{,}0\ \mu C$

$\rho = \dfrac{Q}{V}$

a) $d = 0\ cm$ Como para uma esfera com a carga uniformemente distribuída é equivalente a um campo criado por uma carga pontual localizado no centro e no ponto da carga o campo é nulo

$\vec{E} = \vec{0}$

$\vec{E}_{centro} = \sum \vec{E}_i = \vec{0}$

b) $d = 10\ cm$

→ sup gaussiana - sup esférica de raio 10 cm centrado no centro da esfera

$\vec{E}$ é radial e tem o mesmo módulo em todos os pontos

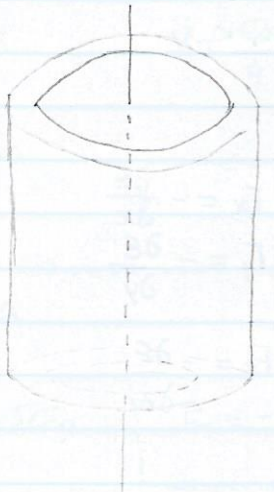
$\oint_{\substack{sup\\gauss}} \vec{E}\,d\vec{A} = \dfrac{Q_{int}}{\varepsilon_0} \Leftrightarrow E\,A = \dfrac{\rho\,V_{contido}}{\varepsilon_0}$

$E\,4\pi r^2 = \rho\,\dfrac{4/3\,\pi r^3}{\varepsilon_0} \Rightarrow E = \rho\,\dfrac{\frac{\cancel{4}}{3}\cancel{\pi} r^{\cancel{3}}}{\cancel{4\pi} \cancel{r^2}\,\varepsilon_0} = \dfrac{\rho r}{3\varepsilon_0} = 3{,}65 \times 10^5\ N/C$

c) $EA = \dfrac{Q}{\varepsilon_0} \Rightarrow E = \dfrac{Q}{4\pi\varepsilon_0 R^2} = 1{,}46 \times 10^6\ N/C$

d) $E = K\frac{Q}{r^2} \Leftrightarrow E = 6,5\times10^5\ N/C$

8 ->



$\rightarrow \frac{Q_{fio}}{L} = \lambda$

$\rightarrow \frac{Q_{cil}}{L} = 2\lambda$

Se escolhermos uma superfície gaussiana contida nas paredes do condutor o fluxo eléctrico será igual a 0.

$$\phi_{\substack{sup\ gaussiana \\ sup\ laterais \\ do\ cilindro}} = 0 = \frac{Q_{int}}{\varepsilon_0}$$

$Q_{int} = Q_{fio} + Q_{sup\ int} = 0 \qquad Q_{sup\ int\ cilindro} = Q_{fio}$

$Q_{\substack{sup \\ interior}} = -\lambda L \qquad Q_{sup\ ext} + \underbrace{Q_{sup\ int}}_{-\lambda L} = 2\lambda L \rightarrow Q_{\substack{sup \\ ext}} = 2\lambda L + \lambda L = 3\lambda L$

Aula 8 - 06.03.2015

Trabalho da Força Eléctrica de Uma carga Pontual

$$W = \int_{r_A}^{r_B} K\frac{Qq}{R^2}\,dr \Leftrightarrow W = KQq\int_{r_A}^{r_B}\frac{dr}{r^2} \Leftrightarrow W = KQq\left[-\frac{1}{r}\right]_{r_A}^{r_B} \Leftrightarrow$$

ao longo de uma linha de campo

$$W = KQq\left(\left(-\frac{1}{r_B}\right) - \left(-\frac{1}{r_A}\right)\right)$$

Só depende da posição inicial e a posição final $\Rightarrow$ não depende da trajectória

$\Downarrow$

Força Conservativa

$E_{P_B} = K\frac{Qq}{r} + E_{P_0} \leftarrow$ Esta constante não altera a variação da energia mas afecta o seu módulo

$$\lim_{r \to \infty} Ep_B = Ep_0$$

$$\boxed{E_p = K \frac{Qq}{r}}$$

contém definição do ZERO

Gradiente transforma um escalar em vetor

$$\nabla \mathcal{E} = \frac{\partial \mathcal{E}(r)}{\partial r} \vec{u}_r \qquad \vec{F} = -\nabla \mathcal{E} = \text{grad}(\mathcal{E}) \begin{cases} F_x = -\frac{\partial \mathcal{E}}{\partial x} \\ F_y = -\frac{\partial \mathcal{E}}{\partial y} \\ F_z = -\frac{\partial \mathcal{E}}{\partial z} \end{cases}$$

$$\mathcal{E} = -\int F_r \, dr \qquad \frac{\partial \mathcal{E}}{\partial r} = -F_r \qquad \mathcal{E} = K \frac{Qq}{r}$$

$$\vec{F}_r = -\frac{\partial \mathcal{E}}{\partial r} \vec{u}_r \qquad \vec{F}_r = -\left(-K \frac{Qq}{r^2}\right) \vec{u}_r$$

Potencial Eléctrico (V)

$$V = \frac{\mathcal{E}}{q}$$

$$\boxed{\begin{array}{l} \vec{F} = -\nabla \mathcal{E} \\ \vec{E} = -\nabla V \end{array}}$$

O campo eléctrico é sempre perpendicular às equipotenciais

$$\begin{aligned} W_{A \to B} &= -\Delta E_p \\ &= -(Ep_B - Ep_A) \\ &= -q\,(V_B - V_A) \\ &= -q\,(-10 + 2)\ V = +8\ V.C = +8 \end{aligned}$$

$$E_x = -\frac{dV}{dx}$$

$$E_x = -\frac{\Delta V}{\Delta x} = -\frac{\Delta V_{pPa}}{d}$$

Aula 9 - 09.03.2015

$E_p = K\frac{Qq}{r}$ $\qquad V = \frac{E_p}{q} = K\frac{Q}{r}$

Definida para o par de cargas pontuais

$E_p(r=\infty) = 0$

$\underset{Q}{\xrightarrow{\vec{v_1}}} \qquad \underset{q}{\xrightarrow{\vec{v_2}}} \Rightarrow E = E_{c_1} + E_{c_2} + E_{p_{12}}$

$\vec{F} = -\nabla E_p$

Para forças conservativas

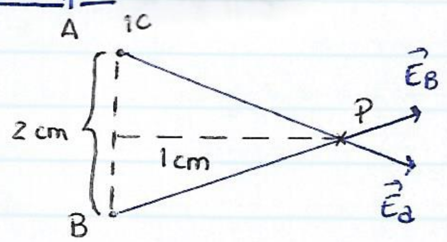
$$\begin{cases} F_x = -\frac{\partial E_p}{\partial x} \\ F_y = -\frac{\partial E_p}{\partial y} \\ F_z = -\frac{\partial E_p}{\partial z} \end{cases}$$

$$W = \int \vec{F}.d\vec{l} = -\Delta E_p$$

$$\frac{\vec{F_{ei}}}{q} = -\frac{\nabla \mathcal{E}_{pei}}{q} \Rightarrow \vec{E} = -\nabla V$$

$$\mathcal{E}_p = \sum_{i=1}^{N} \frac{q_i q}{r_i} K \quad e \quad V_p = \sum_{i=1}^{N} \frac{q_i}{r_{ip}} K$$

Exemplo:



$x_p = 1$ $\quad P(1,0)$ $\quad \vec{u}_r = \frac{\vec{r}}{|\vec{r}|}$

$y_p = 0$ $\quad A(0,1)$

$B(0,-1)$

$\vec{r}_A = (P-A) = (1,0)-(0,1) = (1,-1)$ $\qquad \vec{u}_{\vec{r}_A} = \frac{\vec{u}_x - \vec{u}_y}{\sqrt{2}}$ $\qquad \vec{u}_{\vec{r}_B} = \frac{\vec{u}_x + \vec{u}_y}{\sqrt{2}}$

$\vec{r}_B = (P-B) = (1,1)$

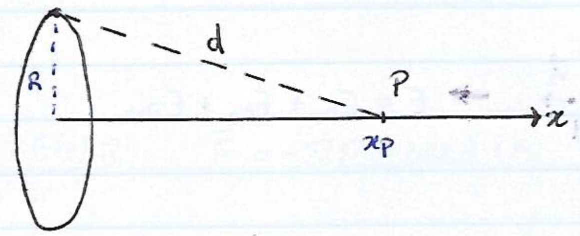
$$\vec{E} = \vec{E}_A + \vec{E}_B = K\frac{1}{2}\vec{u}_{\vec{r}_A} + K\frac{1}{2}\vec{u}_{\vec{r}_B}$$

$$= \frac{K}{2}\left[\left(\frac{\vec{u}_x - \vec{u}_y}{\sqrt{2}}\right) + \frac{\vec{u}_x + \vec{u}_y}{\sqrt{2}}\right] = \frac{K}{\sqrt{2}}\vec{u}_x \ (SI)$$

$$V = V_A + V_B = K\frac{1}{\sqrt{2}} + K\frac{1}{\sqrt{2}} \Leftrightarrow V = \frac{2K}{\sqrt{2}}$$

$$V = K \sum_{i=1}^{N} \frac{Q_i}{r_{iP}} \longrightarrow V = k\int \frac{dQ}{r}$$

Exemplo 2:



$$\frac{Q}{L} = \lambda \qquad \frac{dQ}{d\ell} = \lambda \Leftrightarrow dQ = \lambda d\ell \qquad d = \sqrt{R^2 + x_P^2}$$

$$V_{aro} = \int \frac{K\lambda d\ell}{\sqrt{R^2 + x_P^2}} = \frac{K}{\sqrt{R^2 + x_P^2}} \underbrace{\int \lambda d\ell}_{Q} = \frac{KQ}{\sqrt{R^2 + x_P^2}}$$

$$\frac{Q}{A_{disco}} = \sigma$$

1 aro [espessura dR

↓

$$\text{área} = \underbrace{2\pi R}_{\text{Perímetro}} \underbrace{dR}_{\text{espessura}}$$

$$V_{disco} = \int \frac{K 2\pi r dr \sigma}{\sqrt{r^2 + x_P^2}} = K2\pi\sigma \int_0^R \frac{r dr}{\sqrt{r^2 + x_P^2}} = K2\pi\sigma \left[(r^2 + x_P^2)^{1/2} \right]_0^R$$

$$= K2\pi\sigma \left[(R^2 + x_P^2)^{1/2} - (x_P^2)^{1/2} \right] = K 2\pi \frac{Q}{\pi R^2} \left[(R^2 + x_P^2)^{1/2} - |x_P| \right]$$

$$\vec{E} = -\nabla V$$

$$E_x = -\frac{\partial V}{\partial x} = -K\frac{2Q}{R^2}\left[\frac{1}{2} \frac{2x}{(R^2 + x_P^2)^{1/2}} - \operatorname{sgn}(x) \right]$$

$$V = \int v(dq) = \text{soma de potenciais cargas pontuais}$$

$$\vec{E} = -\nabla V \qquad \Delta V = -\int_{in}^{fim} \vec{E} \cdot d\vec{\ell}$$

$$\vec{E} = K\frac{Q}{r^2}\vec{u}_r$$

$$\Delta V = -\int_\infty^r \vec{E} \cdot d\vec{r} \qquad V_P - V_\infty = -\int_\infty^r \frac{KQ}{r^2} \underbrace{\vec{u}_r \cdot d\vec{r}}_{dr} = -KQ\int \frac{1}{r^2} dr =$$

$$= -KQ\left[-\frac{1}{r} \right]_\infty^r = \frac{KQ}{r}$$

Série 2 - Resolução

2 → $V = 10\ \ell/min$

a) i) Pode-se dizer que o fluxo é de 10 ℓ/min.

ii) O fluxo é de ainda 10 ℓ/min pois continua a ser atravessado pelo mesmo número de linhas de campo.

iii) O fluxo de água é de 0 ℓ/min uma vez que a quantidade de água que entra é igual à que sai.

b) Esse fluxo é nulo.

7 →

R - Raio do cilindro

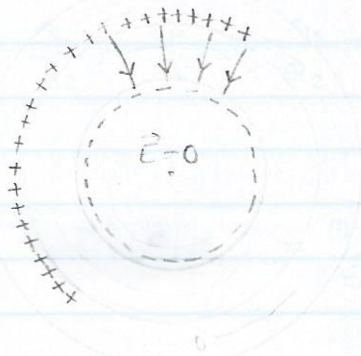
r - distância ao eixo

$$\rho = \frac{Q}{V} \Leftrightarrow Q = \rho V_{contido}$$

$$\phi_{sup\ esf} = \int \vec{E} \cdot d\vec{A} \Leftrightarrow EA = \frac{Q_{int}}{\varepsilon_0} \Leftrightarrow EA = \frac{\rho V}{\varepsilon_0} \Leftrightarrow EA = \frac{\frac{4}{3}\pi R^3 \rho}{\varepsilon_0} \Leftrightarrow$$

$$\Leftrightarrow E = \frac{\frac{4}{3}\pi r^3 \rho}{\varepsilon_0} \times \frac{1}{4r^2\pi} \Leftrightarrow E = \frac{r\rho}{3\varepsilon_0}$$

9 →



$-Q$ → carga da esfera interior

$+3Q$ → carga da camada exterior

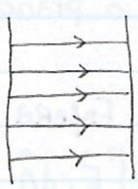
a) A carga na superfície externa da esfera é $-Q$.

b) A carga na superfície interna da esfera é $+Q$.

c) $|\vec{E}| = k\frac{Q}{r^2}\vec{u_r}$ (sendo r a distância ao centro da esfera e o sentido a apontar para o mesmo)

10 → a) $\vec{E} = 0$

b) $\phi = \frac{Q_{int}}{\varepsilon_0} \Leftrightarrow EA = \frac{\sigma A}{\varepsilon_0} \Leftrightarrow E = \frac{\sigma}{\varepsilon_0}$

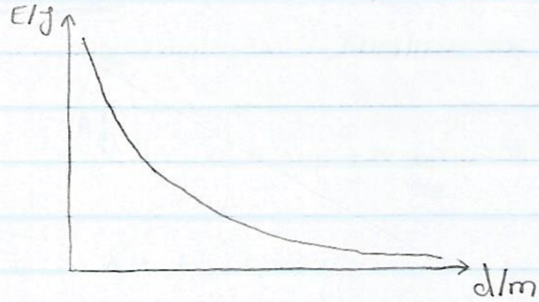


c) $\vec{E} = 0$

Série 3 - Resolução

1→ a) $E = K \frac{Q_1 Q_2}{d} = 9,0 \times 10^9 \times \frac{1,0 \times 10^{-6} \times 0,50 \times 10^{-6}}{20 \times 10^{-2}} = 2,25 \times 10^{-2}$ J

b)



c) $V = \frac{\mathcal{E}}{q_2} \Leftrightarrow V = \frac{2,25 \times 10^{-2}}{0,5 \times 10^{-6}} \Leftrightarrow V = 4,5 \times 10^4$

d)

e) $E_i = E_{P_{Q_2}} = K \frac{Q_1 Q_2}{d} = 2,25 \times 10^{-2}$ J

$E_f = E_{c_{Q_2}} + E_{P_{Q_2}} = E_{c_{Q_2}} + K \frac{Q_1 Q_2}{1,2} = E_{c_{Q_2}} + 3,75 \times 10^{-3}$

$E_{c_{Q_2}} = 2,25 \times 10^{-2} - 3,75 \times 10^{-3} = 1,86 \times 10^{-2}$ J

f) $E_p + E_{c_2} + E_{c_1} = E_T$

$E_{p_0} = E_{p_1} + E_{c_2} + E_{c_1}$

g) Deve ser considerada do par, uma vez que resulta da interacção de ambas as cargas.

Aula 10 - 11.03.2015

O potencial eléctrico num ponto do exterior da esfera é equivalente ao potencial que resultaria da carga concentrada no centro da esfera, como acontece com o plano. campo.

Interior da Esfera

$$\frac{Q_{int}}{\varepsilon_0} = \int \vec{E} \cdot d\vec{A} \Leftrightarrow \frac{\rho \, \frac{4}{3} \pi r^3}{\varepsilon_0} = E_r A \Leftrightarrow \frac{\rho \, \frac{4}{3} \pi r^3}{\varepsilon_0} = E_r \, 4 \pi r^2$$

$$\Leftrightarrow \boxed{E_r = \frac{\rho}{3 \varepsilon_0} r}$$

$$\Delta V = \int \vec{E} \cdot d\vec{r}$$

$$\Delta V_{12} = \int_1^2 \frac{\rho}{3\varepsilon_0} r \underbrace{\vec{u}_r \cdot d\vec{r}}_{dr} = \frac{\rho}{3\varepsilon_0} \int_1^2 r\,dr = \frac{\rho}{3\varepsilon_0} \left[\frac{r^2}{2}\right]_2^1$$

$$V(R) = K \frac{Q}{R}$$

$$V_2 - V_1 = \frac{-\rho}{3\varepsilon_0} \left(\frac{r_2^2}{2} - \frac{r_1^2}{2}\right)$$

$V(r) - V(R)$

$$V(r) = -\frac{\rho}{6\varepsilon_0}\left(r^2 - R^2\right) + V(R) = \frac{\rho}{6\varepsilon_0}\left(R^2 - r^2\right) + \frac{Q}{R} K$$

$$V(r) = \frac{Q}{8\pi R^3 \varepsilon_0}\left(R^2 - r^2\right) + \frac{Q}{R} K_e$$

$$K_e = \frac{1}{4\pi\varepsilon_0}$$

$$= \frac{Q}{2R^3} K_e \left(R^2 - r^2 + 2R^2\right) =$$

$$= K_e \frac{Q}{2R}\left(3 - \frac{r^2}{R^2}\right)$$

Um condutor é um bloco equipotencial.

Duas esferas condutoras ligadas por um fio condutor

$$V_1 = K\frac{Q_1}{R_1} \quad e \quad V_2 = K\frac{Q_2}{R_2}$$

$$\frac{Q_2}{R_2} = \frac{Q_1}{R_1} \Leftrightarrow \frac{Q_1}{Q_2} = \frac{R_1}{R_2}$$

$$\sigma_1 = \frac{Q_1}{S_1} \qquad \sigma_2 = \frac{Q_2}{S_2}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{Q_1/4\pi R_1^2}{Q_2/4\pi R_2^2} \rightarrow \frac{\sigma_1}{\sigma_2} = \frac{Q_1 R_2^2}{Q_2 R_1^2} = \frac{R_2}{R_1}$$

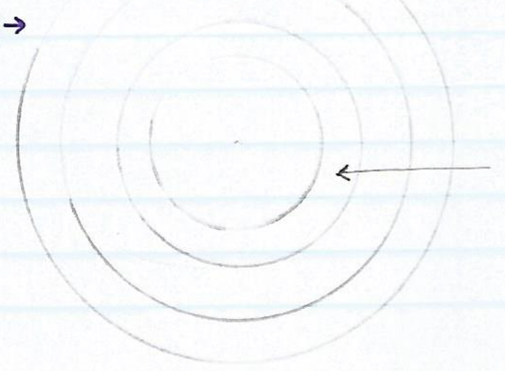
$$\int \vec{E} \cdot d\vec{A} = E \cdot A_{base} = \frac{Q_{int}}{\varepsilon_0} \Leftrightarrow EA = \frac{\sigma A}{\varepsilon_0} \Leftrightarrow E = \frac{\sigma}{\varepsilon_0}$$

Só uma base porque a base interior não tem fluxo ($\vec{E} = 0$)

T.P. 3 - 11.03.2015

Série 2 - Correcção

9 →



$\vec{E} = \vec{0}$

$Q_1 = -Q$

$Q_2 = +3Q$

$$\oint_E = \int \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0} = 0$$ → Logo não há carga na sup interna da esfera

$$Q_{1\,Total} = Q_{sup\ int} + Q_{sup\ ext}$$

a) $Q_{1ext} = -Q$

b) $\phi_{\substack{sup\ Gauss \\ dentro\ da \\ camada}} = 0 = \dfrac{Q_{contida} \to 0}{\varepsilon_0}$

$$\underset{esfera}{Q_{sup\ ext}} + \underset{camada}{Q_{sup\ int}} = 0 \Leftrightarrow \underset{camada}{Q_{sup\ int}} = +Q$$

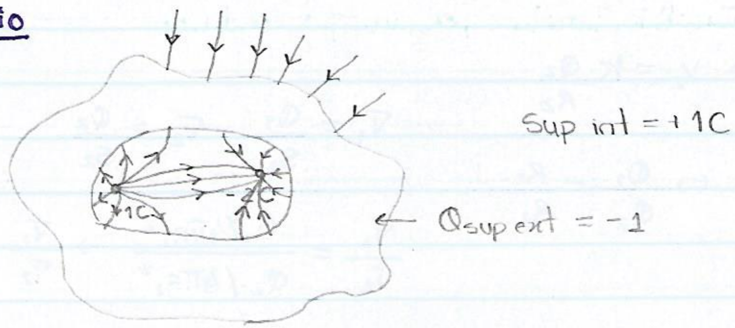
Camada

$$Q_{sup\ int} + Q_{sup\ ext} = +3Q \Leftrightarrow Q_{sup\ ext} = +2Q$$

c) $\int \vec{E}\, d\vec{A} = \int E_r\, dA = E_r A = -\dfrac{Q}{\varepsilon_0}$

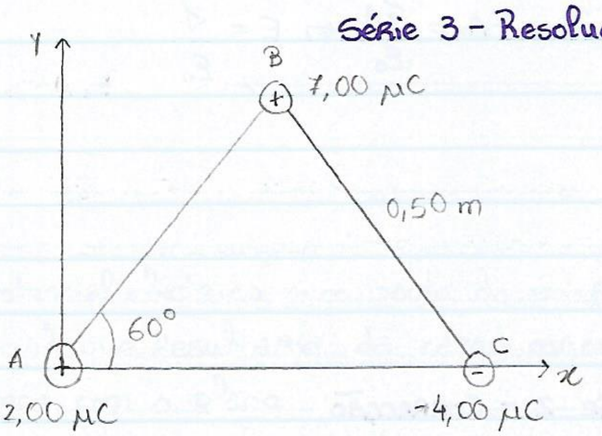
$$E_r A\, 4\pi r^2 = -\frac{1}{4\pi\varepsilon_0} \Leftrightarrow E_r = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Exercício



Série 3 - Resolução

2 →



a) $V_A = V_1 + V_2 = K\dfrac{q_1}{d_{1A}} + K\dfrac{q_2}{d_{2A}}$

$$= K\frac{7\times10^{-6}}{0,50} + \frac{9\times10^{9}\times(-4\times10^{-6})}{0,50} =$$

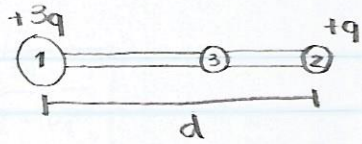
$$= +1,26\times10^{5}\ V - 7,2\times10^{4}\ V =$$

$$= 5,4\times10^{4}\ V$$

b) $\varepsilon_{q_3} = q_3 V = 2,0\times10^{-6}\times 5,4\times10^{4}$

$$= 1,08\times10^{-1}\ J$$

3 →



$$\mathcal{E}_{p_3} = \mathcal{E}_{p_1} + \mathcal{E}_{p_2} \Leftrightarrow \mathcal{E}_{p_3} = K\frac{q_1 q_3}{x_3} + K\frac{q_1 q_3}{d-x_3} \Leftrightarrow$$

$$\Leftrightarrow \mathcal{E}_{p_3} = Kq_3\left(\frac{+3q}{x_3} + \frac{q}{d_3-x_3}\right)$$

$$\mathcal{E}_{p_3}' = -\frac{K\,3qq_3}{x_3^2} + Kqq_3\left(\frac{1}{d_3-x}\right)' = 0 \Leftrightarrow$$

$$\Leftrightarrow Kqq_3\left(-\frac{3}{x_3^2} + \frac{1}{(d_3-x)^2}\right) = 0 \Leftrightarrow$$

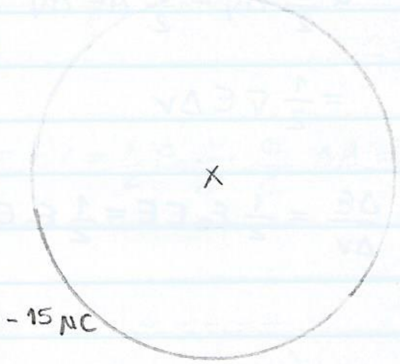
$$\Leftrightarrow \frac{1}{(d-x_3)^2} = \frac{3}{x_3^2} \Leftrightarrow x_3^2 = 3(d^2 - 2xd + x^2) \Leftrightarrow$$

$$\Leftrightarrow 2x_3^2 - 6x_3 d + 3d^2 = 0 \Leftrightarrow x_3^2 - 3x_3 d + \frac{3}{2}d^2 = 0 \Leftrightarrow$$

$$\Leftrightarrow x = \frac{3d \pm \sqrt{9d^2 - 6d^2}}{2} \Leftrightarrow x_0 = \frac{(3\pm\sqrt{2})d}{2} \Leftrightarrow x_0 = \frac{3\pm\sqrt{3}}{2}d = 0{,}634d$$

$$\mathcal{E}_p = Kqq'\left(\frac{3}{0{,}634d} + \frac{1}{0{,}366d}\right)$$

5 →



a) $V = K\int\frac{dq}{r} = \frac{K}{R}\int dq = K\frac{Q}{R} =$

$= 9{,}0\times10^{9}\times\frac{(-15\times10^{-6})}{28\times10^{-2}}\times 2\pi =$

$= -3{,}03\times10^{6}$ V

b) Teorema de Gauss

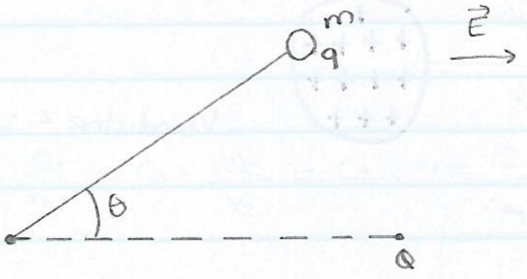
$$\int \vec{E}\, d\vec{A} = \frac{Q_{int}}{\varepsilon_0} \Leftrightarrow E_r A = 0$$

Como o campo em todo o interior do aro é 0 então $\Delta V = 0$ e todo o interior do aro é um bloco equipotencial → Verdade a 2d

c) O potencial é metade do potencial calculado em a)

d) Não conseguimos calcular e desenhar de forma simples

7 →



$m = 0{,}0100$ Kg

$q = +2{,}00$ μC

$|\vec{E}| = 300$ V/m

$\theta_i = 60°$

$$\vec{F} = q\vec{E} = qE\vec{u}_x \qquad \vec{F}\cdot d\vec{P} = F_x\,dx$$

$$W = \int \vec{F}\cdot d\vec{P} = \int F_x\,dx = qE\int dx = qE\Delta x = 4{,}50 \times 10^{-4}\ J$$

deslocamento

$$x_0 = 1{,}50 \times \cos 60 = 0{,}75\ m$$

$$x = 1{,}50\ m$$

$$W = -\Delta E_p$$

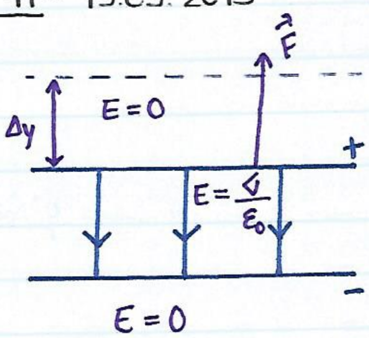
$$\Delta E_c + \Delta E_p = 0 \Leftrightarrow \Delta E_c = -\Delta E_p = 4{,}50 \times 10^{-4}\ J$$

$$E_{c_i} = 0.$$

$$E_{c_f} = \frac{1}{2}mv^2 = 4{,}50 \times 10^{-4}$$

$$\Rightarrow v = 0{,}300\ m/s$$

<u>Aula 11</u> - 13.03.2015



$$\sigma = \frac{|Q|}{A}$$

$$W = \vec{F}\cdot\vec{\Delta y} = |F||\Delta y|$$
$$= Q\frac{E}{2}\Delta y = \frac{\sigma}{2}AE\Delta y$$
$$= \frac{1}{2}\sigma E\Delta V$$

$$F = QE = Q\ (\text{média}\ (E_{baixo}, E_{cima}))$$

$$\frac{\Delta \mathcal{E}}{\Delta V} = \frac{1}{2}\varepsilon_0 EE = \frac{1}{2}\varepsilon_0 E^2$$

Condutores e Condensadores

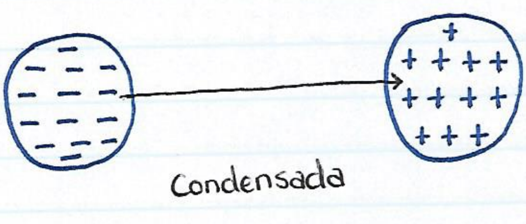
$$\boxed{C = \frac{Q}{V} = \frac{\cancel{Q}}{k\frac{\cancel{Q}}{R}}}$$

$$V = k\frac{Q}{R}$$

Capacidade do Condutor

farad (F)

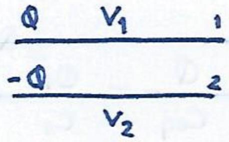
$$C = \frac{R}{k} = 4\pi\varepsilon_0 R = \frac{1}{9\times 10^9}\ 1 \sim 10^{-10}\ \boxed{C/V} = 100\ pF$$



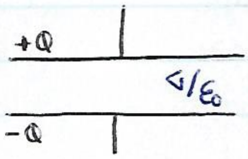
Condensada

$V_{condutor} < V_{condutor\ isolado}$

$$\boxed{C = \frac{Q}{V_2 - V_1}}$$



2 condutores afastados de 1 certa distância → condensador



$$E = \frac{\sigma}{\varepsilon_0} \qquad W = F_{el} \times \Delta y = q' \frac{\sigma}{\varepsilon_0} d = -\Delta \varepsilon_p = -q' \Delta V$$

$$q' \frac{\sigma}{\varepsilon_0} d = -q \Delta V \Rightarrow \Delta V = -\frac{\sigma}{\varepsilon_0} d$$

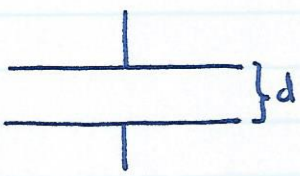
$$C = \frac{|Q|}{|\Delta V|} = \frac{Q}{\frac{\sigma}{\varepsilon_0} d} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0} d} = \varepsilon_0 \frac{A}{d}$$

$d = 0{,}01 \text{ mm} = 10^{-5} \text{ m}$

$C = 1 \mu F$

$$10^{-6} = \frac{8 \times 10^{-12} A}{10^{-5}} \qquad 10^{-11} \sim 10^{-11} A \rightarrow A = 1 m^2$$

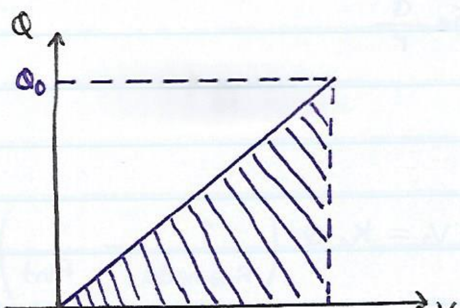
Aula 12 - 16.03.2015



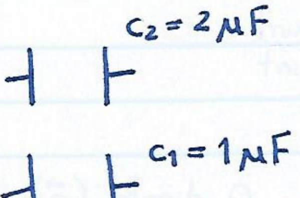
$$\frac{\varepsilon}{Vol} = \frac{1}{2} \varepsilon_0 E^2 = \frac{1}{2} \varepsilon_0 \frac{V^2}{d^2} = \frac{1}{2} \varepsilon_0 \frac{V}{d^2} \frac{Q}{C}$$

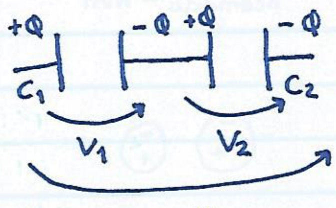
$$C = \frac{Q}{V} \rightarrow V = \frac{Q}{C}$$

$$\mathcal{E} = \varepsilon V = \frac{1}{2} \varepsilon_0 \frac{V}{d^2} \frac{Q}{C} A d = \frac{1}{2} \underbrace{\frac{\varepsilon_0 A}{d}}_{C} \frac{1}{C} QV = \frac{1}{2} QV$$



$$\mathcal{E} = \int q . \Delta V = \frac{1}{2} QV$$

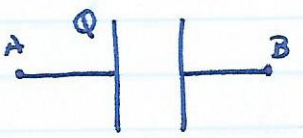




$Q_1 = Q_2 = Q$

$V_A - V_B = V_1 + V_2$

$$\frac{Q}{V} = C \quad \frac{Q_1}{V_1} = C_1 \quad \frac{Q_2}{V_2} = C_2 \ ; \ V = \frac{Q}{C_{eq}} \quad V_1 = \frac{Q_1}{C_1} \quad V_2 = \frac{Q_2}{C_2}$$

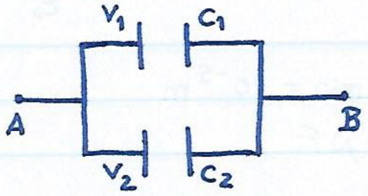


$$\frac{Q}{C_{eq}} = \frac{Q_1}{C_1}^{=Q} + \frac{Q_2}{C_2}^{=Q}$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Condensadores em Série

Conseguimos aumentar a diferença de potencial mas não a carga, logo a capacidade diminui.



$Q = Q_1 + Q_2$

$V_A - V_B = V_1 = V_2$

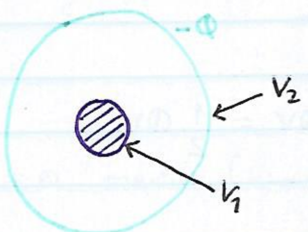
$$\frac{Q}{V} = \frac{Q_1 + Q_2}{V} = \frac{Q_1}{V} + \frac{Q_2}{V}$$

$$\boxed{C = C_1 + C_2}$$

Condensadores em Paralelo

$$\boxed{C = \varepsilon_0 \frac{A}{d}}$$

Condensador Plano



$$V = K_e \frac{Q}{R_{int}}$$

$$V = K_e \frac{Q}{R_{int\,cam}}$$

$$V = K_e \frac{Q}{r}$$

O potencial não varia descontinuamente

$$V_2 - V_1 = K_e \frac{Q}{R_{int\,cam}} - K_e \frac{Q}{R_{int}} \Leftrightarrow V_2 - V_1 = K_e Q \left(\frac{1}{R_{camada}} - \frac{1}{R_{int}} \right)$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{K_e Q \left(\frac{1}{R_{camada}} - \frac{1}{R_{int}} \right)} = \frac{1}{\frac{1}{4\pi\varepsilon_0} \cdot \frac{R_{camada} - R_{int}}{R_{camada} \times R_{int}}} =$$

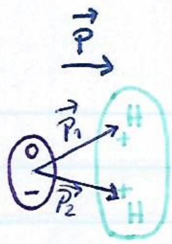
$$= 4\pi\varepsilon_0 \frac{R_{camada} \cdot R_{int}}{R_{camada} - R_{int}} = \varepsilon_0 \frac{4\pi R_{camada} R_{int}}{R_{camada} - R_{int}}$$

Dipolo Eléctrico

$|\vec{p}| = qd$

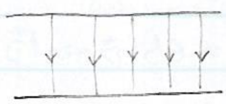
$d \begin{cases} -q \\ +q \end{cases}$

O dipolo ($\vec{p}$) é um vetor com direcção que une as duas cargas e com sentido da negativa para a positiva.



Exercícios - Série 4

1→ a) $\Delta V = \int \vec{E} \cdot \vec{d} \Leftrightarrow$



$A = 7{,}60 \text{ cm}^2$

$d = 1{,}80 \text{ mm}$

$\Delta V = 20{,}0 \text{ V}$

$\Leftrightarrow \Delta V = E \cdot d \Leftrightarrow E = \frac{\Delta V}{d} \Leftrightarrow$

$\Leftrightarrow E = \frac{20{,}0}{1{,}80 \times 10^{-3} \text{ m}} \Leftrightarrow$

$\Leftrightarrow E = 1{,}1 \times 10^{4} \text{ N/C}$

b) $E = \frac{\sigma}{\varepsilon_0} \Leftrightarrow \sigma = E \times \varepsilon_0 \Leftrightarrow \sigma = 8{,}85 \times 10^{-12} \times 1{,}1 \times 10^{4} \Leftrightarrow$

$\Leftrightarrow \sigma = 9{,}74 \times 10^{-8}$

c) $C = \varepsilon_0 \frac{A}{d} \Leftrightarrow C = 8{,}85 \times 10^{-12} \times \frac{7{,}60 \times 10^{-4}}{1{,}80 \times 10^{-3}} \Leftrightarrow C = 3{,}74 \times 10^{-12} \text{ F}$

$\frac{Q}{V_2 - V_1} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0} d}$

d) $C = \frac{Q}{V} \Leftrightarrow Q = CV \Leftrightarrow Q = 3{,}74 \times 10^{-12} \times 20{,}0 \Leftrightarrow Q = 7{,}5 \times 10^{-11} \text{ C}$

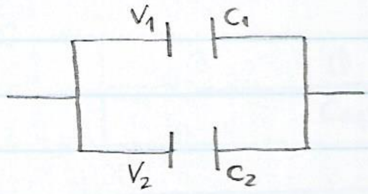
2→ $A = 25{,}0 \text{ cm}^2$

$d = 1{,}50 \text{ cm}$

$\Delta V = 250 \text{ V}$

$\varepsilon_0 = 8{,}854 \times 10^{-12}\ N^{-1} m^{-2} C^2$

3→



$C_1 = C_2 = 2\,\mu F$

$V_1 = V_2 = 20\ V$

a) $\boxed{C = \frac{Q}{V}}$ $Q_1 = C_1 V_1 \Leftrightarrow Q_1 = 4{,}0 \times 10^{-5}\ C$

$Q_2 = C_2 V_2 \Leftrightarrow Q_2 = 4{,}0 \times 10^{-5}\ C$

b) $Q_T = Q_1 + Q_2 = 8{,}0 \times 10^{-5}\ C$

c) $C = C_1 + C_2 \Leftrightarrow C = 4{,}0 \times 10^{-6}\ F$

d) $\boxed{V = \frac{\varepsilon}{Q}}$ $\varepsilon = \frac{1}{2} V Q \Leftrightarrow \varepsilon = 8{,}0 \times 10^{-4}\ J$ → Em paralelo

Para cada condensador:

$\varepsilon_T / 2 = 4{,}0 \times 10^{-4}\ J$

4→ a) $\boxed{V_A - V_B = V_1 + V_2 \Leftrightarrow \Delta V = V_1 + V_2}$

$\left.\begin{array}{l} V_1 = 10\ V \\ V_2 = 10\ V \end{array}\right\}$ Como são iguais dividem-se igualmente

$Q_1 = C_1 V_1 \Leftrightarrow Q = 2{,}0 \times 10^{-5}\ C$

$Q_2 = C_2 V_2 \Leftrightarrow Q = 2{,}0 \times 10^{-5}\ C$

b) $Q = Q_1 = Q_2 = 2{,}0 \times 10^{-5}\ C$ → Porque é o que entra num lado e sai do outro

c) $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \Leftrightarrow \frac{1}{C} = 5{,}0 \times 10^5 + 5{,}0 \times 10^5 \Leftrightarrow$

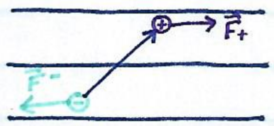
$\Leftrightarrow \frac{1}{C} = 10 \times 10^5 \Leftrightarrow C = 1{,}0 \times 10^{-6}\ F$

d) $\varepsilon_T = \frac{1}{2} Q V \Leftrightarrow \varepsilon = 2{,}0 \times 10^{-4}\ J$ → Em Série

$\frac{\varepsilon_T}{2} = 1{,}0 \times 10^{-4}\ J$ → Para cada condensador

$\boxed{\frac{1}{2} Q V = \frac{1}{2} C V^2 = \frac{1}{2} \frac{Q^2}{C}}$ → Energia

Aula 12 - 18.03.2015

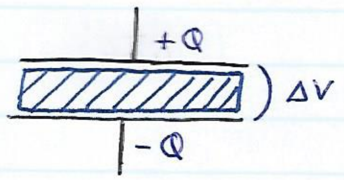


$$\vec{M} = \vec{\tau} = \vec{r}_+ \times \vec{F}_+ + \vec{r}_- \times \vec{F}_- = \vec{r}_+ \times \vec{F}_+ + \vec{r}_- \times (-\vec{F}_+) =$$
$$= (\vec{r}_+ - \vec{r}_-) \times \vec{F}_+ = \vec{d} \times q\vec{E} = \vec{p} \times \vec{E}$$

$$|\vec{p} \times \vec{E}| = |\vec{p}||\vec{E}| \sin\theta$$

$$W = \int \vec{F}\, d\vec{l} = \int |\vec{\tau}|\, d\theta = \int |\vec{p}||\vec{E}| \sin\theta\, d\theta = |\vec{p}||\vec{E}| \left[-\cos\theta\right]$$
$$= \left[-\vec{p}.\vec{E}\right]_1^2$$

A diferença entre um isolante e um condutor é que no primeiro as cargas não se podem mover ao longo do material enquanto no segundo as cargas são móveis.



$$\left.\begin{aligned} E_y &= -\frac{\Delta V}{d} \\ E_y &= -\frac{dV}{dy} \end{aligned}\right\}$$ Com a diminuição do campo eléctrico também diminui a diferença de potencial

$$C = \frac{Q}{\Delta V}$$ → Diminui a dif. de potencial aumentando a capacidade do condutor

$$\varepsilon = K\, \varepsilon_0$$

$$K = \frac{\varepsilon}{\varepsilon_0} =$$ constante dieléctrica relativa

$$C = \varepsilon \frac{S}{d}$$

permitividade eléctrica do meio = constante dieléctrica

$$\vec{D} = \varepsilon \vec{E} = \varepsilon_0 \vec{E} + \vec{P} = \underbrace{\varepsilon_0 (1 - \chi)}_{\varepsilon} \vec{E}$$

$$\vec{P} = -\chi\, \varepsilon_0 \vec{E}$$

susceptibilidade eléctrica

$$\vec{D} = \varepsilon_0 \vec{E}$$

Vector Deslocamento Eléctrico

Rigidez Dieléctrica → Campo eléctrico máximo que um condensador consegue aguentar.

T.P. 4 – 18.03.2015

Série 3 – Resolução

7 (Conclusão) →

$W_{el} = \Delta E$

$W = \int \vec{F}_{el} \cdot d\vec{l} = q \int \vec{E} \cdot d\vec{l} = q \int E\, dx = qE\Delta x$

$\vec{E}\, d\vec{l} = |\vec{E}||d\vec{l}| \cos\alpha = |E|\,|dl \text{ projectado em } E| = |d\vec{l}|\,|\vec{E} \text{ projectado em } dl$

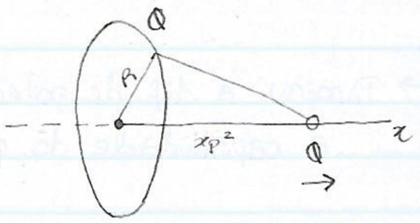
$x_1 = l \cos 60° = 1{,}50 \times \frac{1}{2} = 0{,}75 \text{ m}$

$x_2 = 1{,}50 \text{ m}$

$W = 2 \times 10^{-6} \times 300 \times (1{,}50 - 0{,}75)$
$= 4{,}50 \times 10^{-4} \text{ J}$

$W = \Delta E_c = \frac{1}{2} m v^2 - 0 \Leftrightarrow v^2 = \frac{2W}{m} = 9{,}00 \times 10^{-2}$

9 →



a) $\frac{Q}{L} = \lambda \qquad \frac{dQ}{dl} = \lambda \Leftrightarrow dQ = \lambda\, dl$

$V_{aro} = \int \frac{K\lambda\, dl}{\sqrt{R^2 + x_p^2}} = \frac{K}{\sqrt{R^2 + x_p^2}} \int \lambda\, dl = \frac{KQ}{\sqrt{R^2 + x_p^2}}$

b) $V(x=0) = K_e \frac{Q}{R} \qquad \mathcal{E} = Q\,V(x=0) = K_e \frac{Q^2}{R} \qquad \Delta\mathcal{E}_p + \Delta E_c = 0$

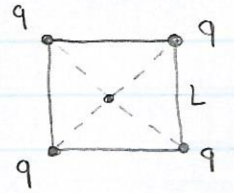
$\Delta\mathcal{E}_p = -W_{el}$

$\mathcal{E}_p(x=0) + E_c(x=0) = \mathcal{E}_p(x=\infty) + E_c(\infty)$

$\mathcal{E}_p(x=0) = E_{c_{max}}$

$K_e \frac{Q^2}{R} = \frac{1}{2} M v_{max}^2 \qquad v_{max}^2 = \frac{2K_e Q}{RM} \qquad v_{max} = \sqrt{\frac{2K_e Q^2}{RM}}$

10 →



As cargas vêm do infinito

$E_p(x=\infty)=0$

a) Trazendo as cargas uma a uma

$$0 + k\frac{qq}{a} + k\frac{qq}{a} + k\frac{qq}{\sqrt{2}a} + k\frac{qq}{a} + k\frac{qq}{\sqrt{2}a} + k\frac{qq}{a}$$

$$= k_e\frac{qq}{a}\left(4+\frac{2}{\sqrt{2}}\right) = k_e\frac{qq}{L}\left(4+\frac{2}{\sqrt{2}}\right)$$

b) $d=\frac{\sqrt{2}}{2}L$

$$\Delta E_p + \Delta E_c = 0 \Leftrightarrow \Delta E_c = -\Delta E_p =$$

$$d' = 2\frac{\sqrt{2}}{2}L = \frac{\sqrt{2}}{2}L'$$

$$= -\left(k_e\frac{qq}{L}\left(2+\frac{1}{\sqrt{2}} - \left(4+\frac{2}{\sqrt{2}}\right)\right)\right)$$

$$L' = \sqrt{2}d' = \sqrt{2}\,\frac{2\sqrt{2}}{2}L$$

$$= +k_e\frac{qq}{L}\left(2+\frac{1}{\sqrt{2}}\right) \Leftrightarrow 4\Delta E_{c_1} \simeq k_e\frac{qq}{L}\left(2+\frac{1}{\sqrt{2}}\right)$$

$$\frac{1}{2}mv^2 = \frac{1}{4}\frac{k_e qq}{L}\left(2+\frac{1}{\sqrt{2}}\right) \Leftrightarrow v^2 = \frac{1}{2m}\frac{k_e qq}{L}\left(2+\frac{1}{\sqrt{2}}\right) = \frac{k_e q^2}{mL}\left(1+\frac{1}{\sqrt{8}}\right)$$

Série 4 - Resolução

5 → $$\phi = \int \vec{E}\cdot d\vec{A} = \int_{\text{sup lat}} \vec{E}\,d\vec{A} + \int_{\text{bases}} \vec{E}\cdot d\vec{A} = 0$$

$$= \int_{\text{sup lateral}} \vec{E}d\vec{A} = E\underbrace{\int d\vec{A}}_{A} = E\,2\pi r L \qquad \phi = \frac{Q_{int}}{\varepsilon_0}$$

$$E\,2\pi r L = \frac{\lambda L}{\varepsilon_0} \qquad \vec{E} = -\nabla V$$

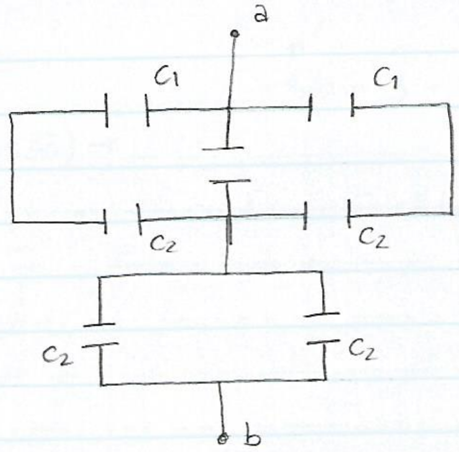
$$E = \frac{\lambda}{2\pi r\varepsilon_0} \qquad \Delta V = -\int \vec{E}\cdot d\vec{r} = \frac{\lambda}{2\pi\varepsilon_0}\int_{r_{int}}^{r_{ext}} \frac{1}{r}\,dr$$

$$V_{ext} - V_{int} = \frac{\lambda}{2\pi\varepsilon_0}\Big[\ln r\Big]_{r_{int}}^{r_{ext}} = \frac{\lambda}{2\pi\varepsilon_0}\Big[\ln r_{ext} - \ln r_{int}\Big] = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_{ext}}{r_{int}}$$

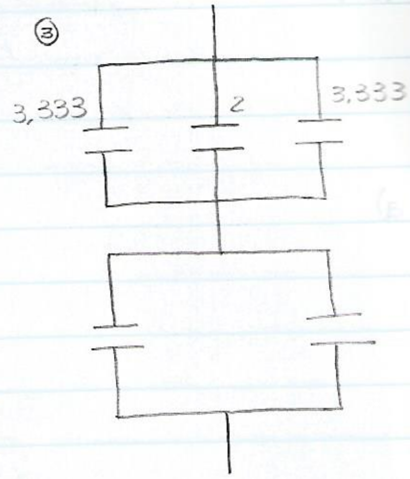
$$C_\ell = \frac{Q}{\Delta V} = \frac{\lambda\ell}{\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_{ext}}{r_{int}}} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{r_{ext}}{r_{int}}\right)}\ell \qquad \frac{C}{\ell} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{r_{ext}}{r_{int}}\right)}$$

8 →

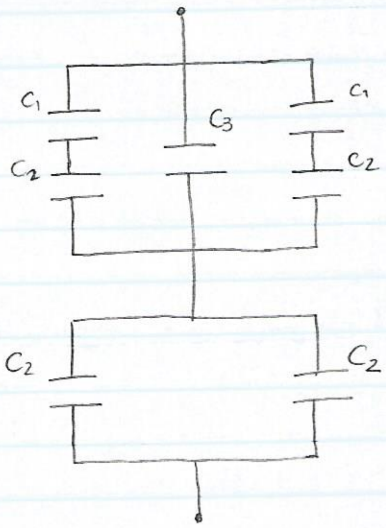
①



③



②



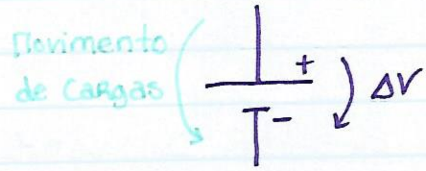
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}\ \mu F^{-1}$$

$$C_{eq} = \frac{10}{3}\ \mu F = 3{,}333\ \mu F$$

④

Aula 13 – 20.03.2015

Gerador → Elementos que convertem energia de outra forma em energia eléctrica



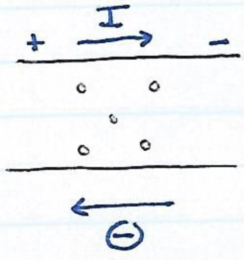
Activo

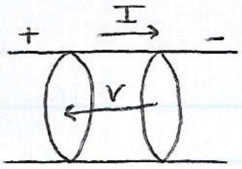
movimento ordenado de carga → Corrente Eléctrica

$[I] = C s^{-1} = A$ $\boxed{I = \frac{\Delta Q}{\Delta t}}$ → Intensidade de corrente Eléctrica

$\mathcal{E} = qV$

$\Delta V \rightarrow \Delta \mathcal{E}$





$N_1 = A v n e \qquad n_e = \frac{N_e}{V}$

$N(-e) = \frac{\Delta Q}{\Delta t}$

$I = |(-e) A v n_e| \qquad I = n e v A_{\perp v}$

Fluxo de carga → $\boxed{\frac{I}{A} = nev}$ $\qquad \vec{J} = n(-e)\vec{v}$

vetor Densidade de Corrente

$\text{Fluxo} = \int \vec{J} \cdot d\vec{A} = I$

Um condutor não está em equilíbrio eletroestático (há movimento de cargas por diferença de potencial nas pontas) → O campo no interior não é 0.

ΔV constante ⟶ I constante

I ~~aumenta no tempo~~ → perdas para o material condutor

Os electrões em movimento por acção de uma força eléctrica perdem energia por colisões com as impurezas do material.

$\boxed{\Delta V = RI}$ Lei de Ohm

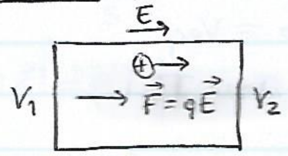
Resistência Eléctrica

Exprime-se em $\frac{V}{A} = \Omega$

$T_{cte} = R_{cte}$

material óptimo

Aula 14 - 23.03.2015



$I = \frac{dQ}{dt} \qquad I = cte = \frac{\Delta Q}{\Delta t}$

$\frac{d\vec{p}}{dt} = \vec{F} \Leftrightarrow m\frac{d\vec{v}}{dt} = \vec{F} \qquad \frac{\Delta \vec{p}}{\tau} = q\vec{E} \Leftrightarrow m\Delta\vec{v} = q\vec{E}\tau$

Velocidade

$|\vec{v}| = \frac{|e|E\tau}{m}$

velocidade de Deriva no Campo

Efeito das Colisões

→ Aquecimento quando passa corrente eléctrica num condutor

↓

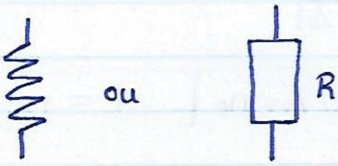
Efeito de Joule

$\frac{W}{\Delta t} = \frac{q}{\Delta t}\Delta V = I\Delta V \qquad P = VI = RI^2$

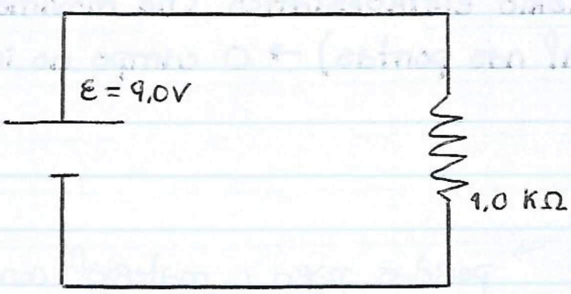
d.d.p.

$R = \frac{V}{I}$ $\quad\quad$ $\Delta \mathcal{E} = q\,\Delta V(q)$

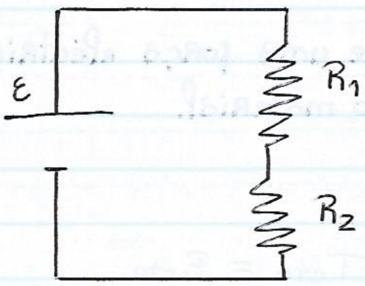
Resistência:



Gerador:



$$R = \frac{V}{I} \Leftrightarrow I = \frac{9{,}0\,V}{1{,}0\,K\Omega} = 9{,}0\,mA$$

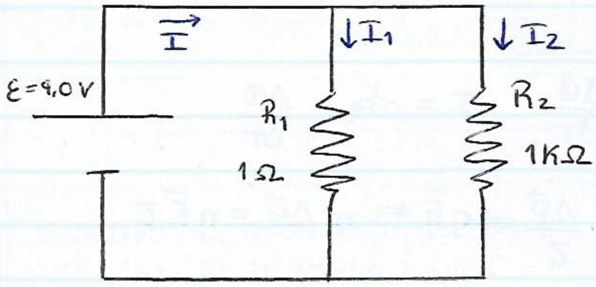


$$R_{série} = R_1 + R_2$$

$$\Delta V = \Delta V_1 + \Delta V_2$$

$$R_{eq} I = R_1 I + R_2 I$$

A resistência total aumenta com a introdução de mais resistências, diminuindo a corrente.



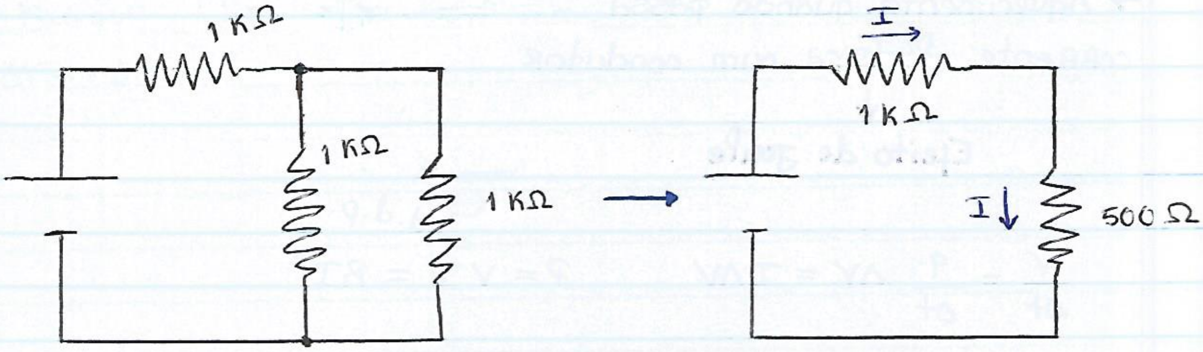
$$V_1 = V_2 = V_{eq}$$

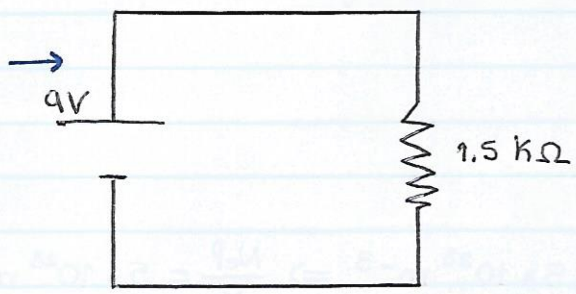
$$I_{eq} = I_1 + I_2$$

$$I_{eq} = I_1 + I_2$$

$$\frac{V_{eq}}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} \Rightarrow \frac{V_{eq}}{R_{eq}} = \frac{V_{eq}}{R_1} + \frac{V_{eq}}{R_2}$$

$$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$





$$\frac{1}{R} = \frac{1}{1k\Omega} + \frac{1}{1k\Omega}$$

$$\Leftrightarrow \frac{1}{R} = \frac{2}{1k\Omega} \Leftrightarrow$$

$$\Leftrightarrow R = \frac{1k\Omega}{2} \Leftrightarrow R = 500\,\Omega$$

$$I = \frac{9}{1{,}5\times10^3} = 6\times10^{-3}\,A$$

$V_{eq} = 500 \times 6\times10^{-3} = 3V$

$V_1 = V_2 = 3V$

$I_1 = \frac{3V}{1k\Omega} = 3\,mA$

$I_2 = \frac{3V}{1k\Omega} = 3\,mA$

Outra maneira,

$V_1 = 9V - 6\times10^{-3}\,mA \times 1k\Omega$
$= 3V$

Aula 15 - 25.03.2015

* A resistência é proporcional ao comprimento do condutor ($R \propto l$)
* A resistência é inversamente proporcional à secção do condutor ($R \propto \frac{1}{S}$)

Então $\boxed{R \propto \frac{l}{S}} \rightarrow \boxed{R = \rho \frac{l}{S}}$

ρ → Resistividade Eléctrica

$\Delta V = RI$

$\Delta V = \rho \frac{I}{S} l \Leftrightarrow \frac{\Delta V}{l} = \rho \frac{I}{S}$

$\vec{E} = -\nabla V$

$\Delta V = -\int \vec{E} \cdot d\vec{l}$

$\frac{dV}{dl} = -\rho \frac{I}{S} = E$

$|\vec{E}| = \rho |\vec{J}|$ uma vez que $\frac{\Delta V}{l} = \rho \frac{I}{S} = E$

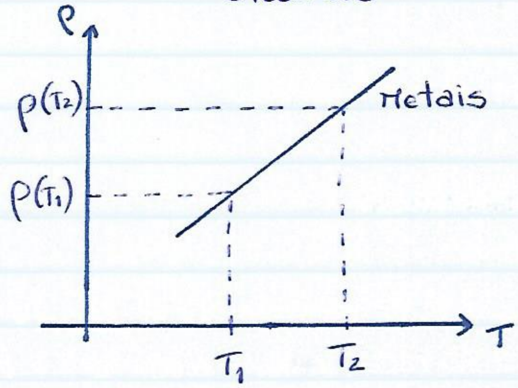
$\frac{I}{S} = |\vec{J}|$

$|\vec{J}| = \frac{1}{\rho} |\vec{E}|$

$\frac{1}{\rho}$ → condutividade Eléctrica (σ)

$\sigma = \frac{1}{\rho} = \frac{1}{\Omega m} = \Omega^{-1} m^{-1} = Sm^{-1} = \mho m^{-1}$

S → Siemens



$\frac{\Delta\rho}{\Delta T} = cte \qquad \rho(T_2) - \rho(T_1) = cte\,(T_2 - T_1)$

$$\frac{\rho(T_2) - \rho(T_1)}{\rho(T_1)} = \alpha\,(T_2 - T_1)$$

α → 3×10^{-3}

$$\frac{\rho - \rho_0}{\rho_0} = \alpha\,(T - T_0) \Leftrightarrow \boxed{\rho = \rho_0\,(1 + \alpha\,(T - T_0))}$$

$$\Rightarrow \boxed{R = R_0 \left(1 + \alpha (T - T_0)\right)}$$

Exemplo

Cu $\quad \rho = 10^{-8}\ \Omega m$

$\frac{N}{V} \sim 5 \times 10^{22}\ cm^{-3} = 5 \times 10^{28}\ m^{-3} \Rightarrow \frac{N e l}{V} = 5 \times 10^{28}\ m^{-3}$

1 electrão → 1 electrão de condução

$1\,mA = 1{,}0 \times 10^{-3}\ A$

Secção $= 5\,mm^2$
$= 5 \times 10^{-6}\ m^2$

$$J = \frac{I}{A} = \frac{10^{-3} A}{5 \times 10^{-6} m^2} = 0{,}2 \times 10^3\ A/m^2 = 2 \times 10^2\ A/m^2$$

$J = n e v$
↳ v deriva

$$2 \times 10^2 = 5 \times 10^{28} \times 1{,}6 \times 10^{-19} v \Leftrightarrow$$
$$\Leftrightarrow v = \frac{2 \times 10^2}{8 \times 10^9} = 0{,}25 \times 10^{-7}\ m/s$$

$$\frac{\Delta V}{d} = \frac{RI}{d} = \frac{0{,}2 \times 10^{-2} \times 10^{-3}}{1} = 2 \times 10^{-6}\ V/m$$

$$R = \frac{10^{-8}\ \Omega m \times 1 m}{5 \times 10^{-6}} = 0{,}2 \times 10^{-2}\ \Omega$$

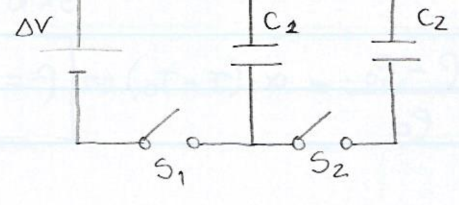
Notas Importantes:

✓ Os electrões movem-se porque há um campo eléctrico no interior do condutor (este não se encontra em equilíbrio electroestático)
✓ O campo eléctrico no condutor pode ser considerado uniforme para secções iguais
✓ A corrente é quase instantânea devido à acção colectiva das cargas no condutor
✓ A resistividade eléctrica depende do material e varia com a temperatura sendo a variação praticamente linear

T.P. 5 – 25.03.2014

Série 4 – Resolução

7 →



$V_{C_1} = 20\ V$

$Q_1^{ini} = C V_{C_1} = 6{,}00 \times 10^{-6} \times 20$
$= 120 \times 10^{-6}\ C$

$V'_{final} = V_1 = V_2 = \frac{40}{3}\ V$

$Q = Q_1 + Q_2$

$Q_{Tot} = C_1V_1 + C_2V_2$
$= (C_1 + C_2)V$
$= 80 \times 10^{-6}$

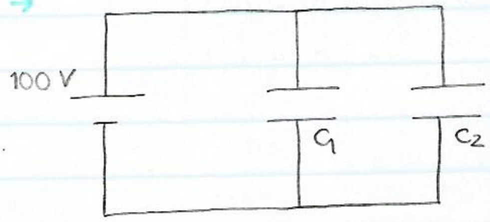
$V = \frac{Q_{Total}}{9{,}00 \times 10^{-6}} \simeq \frac{120 \times 10^{-6}}{9 \times 10^{-6}}$

$Q_1 = C_1V = 6 \times 10^{-6} \times \frac{40}{3} = 80 \times 10^{-6}\ C = 8{,}00 \times 10^{-5}\ C$

$Q_2 = C_2V = 3 \times 10^{-6} \times \frac{40}{3} = 4{,}00 \times 10^{-5}\ C$

Um condensador enche o outro até que a dif. de potencial seja igual

10 →



$\boxed{C = \frac{Q}{V}}$

$Q_1 = C_1V$
$= 25{,}0 \times 10^{-6} \times 100$
$= 2{,}5 \times 10^{-3}\ C$

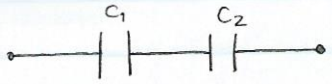
$Q_2 = C_2V$
$= 5{,}0 \times 10^{-6} \times 100$
$= 5{,}0 \times 10^{-4}\ C$

$V = \frac{E_p}{Q}$ $\boxed{E = \frac{1}{2}QV = \frac{1}{2}CV^2}$

$$\left.\begin{aligned} E_{p1} &= 100 \times 2{,}5 \times 10^{-3} \times \tfrac{1}{2} = 0{,}125\ J \\ E_{p2} &= 100 \times 5{,}0 \times 10^{-4} - 0{,}025\ J \end{aligned}\right\} 0{,}15\ J$$

Podíamos ter efectuado os cálculos utilizando um condensador equivalente.

Em série.



$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Leftrightarrow \frac{1}{C_{eq}} = \frac{1}{25 \times 10^{-6}} + \frac{1}{5 \times 10^{-6}} \Leftrightarrow \frac{1}{C_{eq}} = 40000 + 200000$

$\Leftrightarrow C_{eq} = \frac{25}{6}\ \mu F$

$E = \frac{1}{2}CV^2 \Leftrightarrow 15 \times 10^{-2} = \frac{1}{2} \times \frac{25}{6} \times 10^{-6} \times V^2 \Leftrightarrow$

$\Leftrightarrow 15 \times 10^{-2} = \frac{25}{12} \times 10^{-6} \times V^2 \Leftrightarrow V^2 = \frac{15 \times 10^{-2}}{\frac{25}{12} \times 10^{-6}} \Leftrightarrow$

$\Leftrightarrow V = 268\ V$

$V_{paralelo} = 100\ V$ // $V_{série} = 268\ V$

Série 5 - Resolução

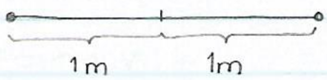
1 → $30{,}0 \times 10^{-6} \times 40{,}0 = 1200 \times 10^{-6} = 1{,}2 \times 10^{-3}\ C$

$e = 1{,}602 \times 10^{-19}\ C \Rightarrow \dfrac{1{,}2 \times 10^{-3}}{1{,}602 \times 10^{-19}} = \underbrace{7{,}49 \times 10^{5}}_{\text{nº de átomos}}$

4 → $\rho = 2{,}70\ g\,cm^{-3}$ $\qquad \rho_{Cu} = 1{,}68 \times 10^{-8}\ \Omega m$

$M_{Al} = 26{,}98\ u.m.a$ $\qquad M_{Cu} = 63{,}55\ g/mol$

a)



$\rho_{Cu} = 1{,}68 \times 10^{-8}\ \Omega m$

$S = 5{,}00 \times 10^{-6}\ m^2$

$\rho_{Al} = 2{,}82 \times 10^{-8}\ \Omega m$

$d = 0{,}100\ mm$

$A = \pi R^2 = \pi \times 0{,}050\ mm^2$

$= \pi \times (5 \times 10^{-5} m)^2$

$= 25\pi \times 10^{-10}\ m^2$

$R = \rho \dfrac{l}{S}$ $\qquad R_{Cu} = \dfrac{1{,}68 \times 10^{-8} \times 1}{5 \times 10^{-6}} = 3{,}36 \times 10^{-3}\ \Omega$

$R_{Al} = \dfrac{2{,}82 \times 10^{-8} \times 1}{25\pi \times 10^{-10}} = 0{,}036 \times 100\ \Omega = 3{,}6\ \Omega$

$R = R_1 + R_2 = 3{,}6\ \Omega + 0{,}36 \times 10^{-2}\ \Omega = 3{,}6\ \Omega$

b) $I = \dfrac{\Delta V}{R} = \dfrac{0{,}100\ V}{3{,}6\ \Omega} \simeq 0{,}03\ A$

c) $V_1 = R_1 I = 3{,}36 \times 10^{-3} \times 0{,}03 = 1 \times 10^{-4}\ V$

$V_2 = R_2 I = 3{,}6 \times 0{,}03 = 0{,}11\ V$

$E_2 = \dfrac{V_2}{1m} \simeq 0{,}10\ V/m$

$E_1 = \dfrac{V_1}{1m} \simeq 10^{-3}\ V/m$

d) $J_1 = \dfrac{I}{A} = \dfrac{0{,}03\ A}{5 \times 10^{-6}} = 0{,}06 \times 10^{6} \simeq 6 \times 10^{4}\ Am^{-2}$

$J_2 = \dfrac{I}{A} = \dfrac{0{,}03\ A}{25\pi \times 10^{-10}} = 3{,}8 \times 10^{7}\ Am^{-2}$

e) $|\vec{J}| = n|e||\vec{v}|$

$Al \rightarrow n_e = n_{at} = \dfrac{\rho}{M} = \dfrac{2{,}70}{26{,}98} \times 6{,}022 \times 10^{23}\ cm^{-3}$

$\simeq 6{,}022 \times 10^{22}\ cm^{-3}$

$\simeq 6{,}022 \times 10^{28}\ m^{-3}$

Cu $\underbrace{n_{\ell} = n_{at}}_{\text{são iguais}} = \frac{\rho}{M} = \frac{8,96}{63,55} \times 6,022 \times 10^{23}\ cm^{-3} = 0,9 \times 10^{23}\ cm^{-3}$

$\simeq 9 \times 10^{28}\ m^{-3}$

porque 1 átomo liberta
1 electrão

$$v = \frac{J_{Al}}{n_{Al}\, e} = 6,2 \times 10^{-6}\ m/s^2$$

$$v = \frac{J_{Cu}}{n_{Cu}\, e} = 2,6 \times 10^{-3}\ m/s^2$$

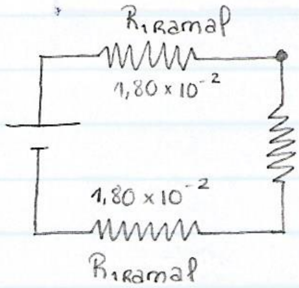
f) $P = VI = RI^2 = \frac{V^2}{R} \rightarrow R_{eq} I^2 = 3,6 \times 0,03^2$

$= 3,6 \times 9 \times 10^{-4}$

$= 3,2 \times 10^{-3}\ W$

5 → a) $\frac{1}{R_{eq}} = \frac{1}{0,36} + \frac{1}{3,6 \times 10^{-2}} \Leftrightarrow R_{eq} = 3,6 \times 10^{-2}\ \Omega$

7 →



a) $0,108\ \Omega$ —— 300 m

$x\ \Omega$ —— 50 m

$x = 1,8 \times 10^{-2}\ \Omega$

$V_{cliente} + RI = V_{fonte}$

$V_{cliente} = \mathcal{E} - V_{Ramal} =$

$= 120 - 3,60 \times 10^{-2} \times 110\ A$

$= 116\ V$

b) $P_{cliente} = V_{cliente}\, I_{cliente}$

$= 116\ V \times 110\ A$

$= 12,8\ kW$

c) $P = V_{Ramal}\, I_{Ramal}$

$= R I^2_{Ramal}$

$= 436\ W$

$\frac{W}{\Delta t} = \frac{q}{\Delta t}$ $\Delta V = I \Delta V$ $I = \frac{\Delta Q}{\Delta t} \Leftrightarrow \Delta Q = I \Delta t$

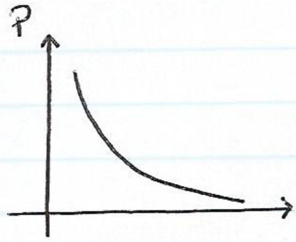
Aula 16 - 27.03.2015

Resistência Interna de Um Gerador de Tensão

$V = \mathcal{E} - rI = RI$ $I = \frac{\mathcal{E}}{r+R}$

$\mathcal{E} = (r+R) I \Leftrightarrow I = \frac{\mathcal{E}}{r+R}$

$P_{fornecida} = VI = RI^2 = \frac{R\mathcal{E}^2}{(r+R)^2}$ $I_{ideal} = \frac{\mathcal{E}}{R}$ $P_{ideal} = \frac{\mathcal{E}^2}{R}$



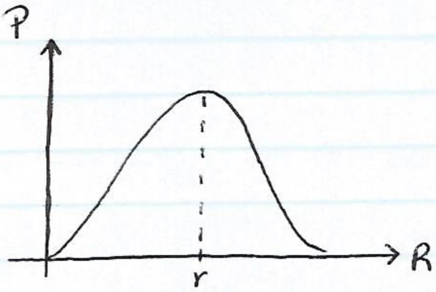
$$\frac{dP}{dR} = -\frac{\varepsilon^2}{R^2} \Leftrightarrow$$

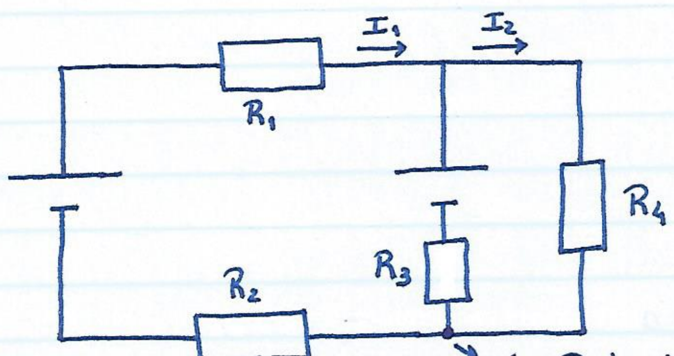
$$\Leftrightarrow dP = \varepsilon^2 \left[\frac{1(r+R)^2 - 2(r+R)R}{(r+R)^4} \right] =$$

$$= \varepsilon^2 \left[\frac{(r+R)(r+R) - 2R(r+R)}{(r+R)^4} \right] =$$

$$= \frac{\varepsilon^2 (r+R)(r+R-2R)}{(r+R)^4} = \frac{\varepsilon^2}{(r+R)^3}(r-R) = 0$$

$$\boxed{\text{Quando } R = r \text{ temos que } \frac{dP}{dR} = 0 \Rightarrow P = P_{máx}}$$





$I = I_1 + I_2$

Nó → Ponto de encontro de Ramos

Regras Para Resolução de Circuitos

1ª Há conservação da carga

$$\underbrace{\sum I_{chegam} = \sum I_{ponta}}_{1 \text{ Nó}}$$

2ª Há conservação da energia

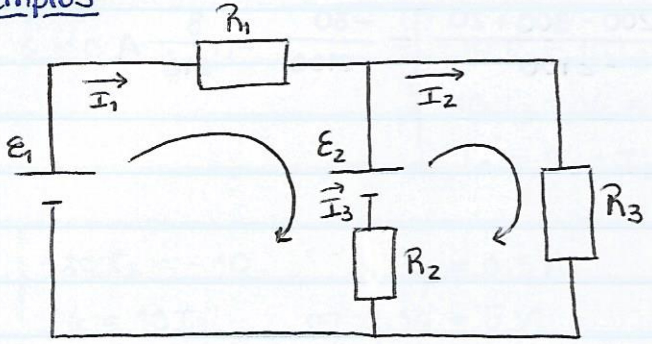
Uma malha é um caminho fechado no circuito.

$\underset{\text{todas as correntes que chegam}}{\sum I} = 0$ ➡ Lei dos Nós

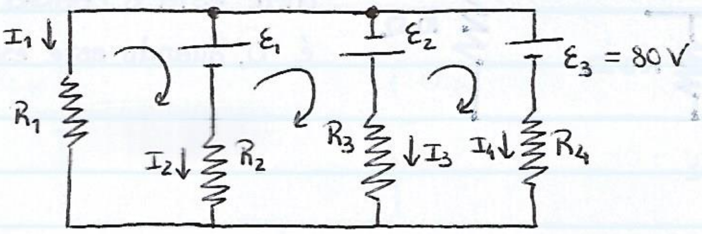
$\underset{\text{ddp malha}}{\sum V} = 0$ ➡ Lei das Malhas

LEIS DE KIRCHHOFF

Exemplos

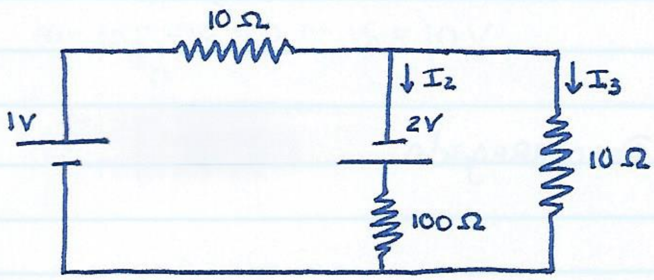


$$\begin{cases} -R_1 I_1 - \varepsilon_2 - R_2 I_3 + \varepsilon_1 = 0 & (a) \\ -R_3 I_2 + R_2 I_3 + \varepsilon_2 = 0 & (b) \\ -R_1 I_1 - R_3 I_2 + \varepsilon_1 = 0 & (c) = (a)+ \end{cases}$$



$$\begin{cases} -\varepsilon_1 - R_2 I_2 + R_1 I_1 = 0 \\ \varepsilon_2 - R_3 I_3 + R_2 I_2 + \varepsilon_1 = 0 \\ -\varepsilon_3 - R_4 I_4 + R_3 I_3 - \varepsilon_2 = 0 \\ -I_1 - I_2 - I_3 - I_4 = 0 \end{cases}$$

Aula 17 - 30.03.2015



$$\begin{cases} -10I_1 + 2 - 100I_2 + 1 = 0 \\ -10I_3 + 100I_2 - 2 = 0 \\ I_1 = I_2 + I_3 \end{cases} \Leftrightarrow \begin{cases} 10I_1 + 100I_2 = 3 \\ 100I_2 - 10I_3 = 2 \\ I_1 - I_2 - I_3 = 0 \end{cases}$$

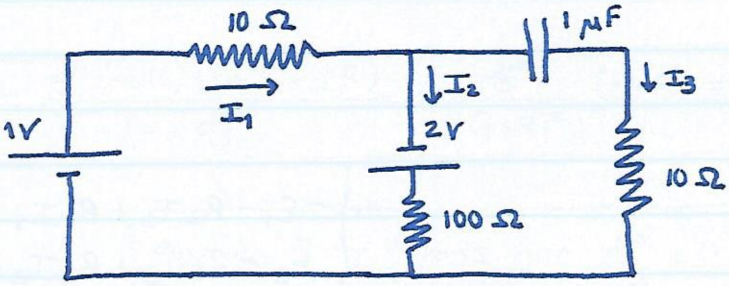
$$\Delta = \begin{vmatrix} 10 & 100 & 0 \\ 0 & 100 & -10 \\ 1 & -1 & -1 \end{vmatrix} = -1000 - 1000 - 100 = -2100$$

$$I_1 = \frac{\begin{vmatrix} 3 & 100 & 0 \\ 2 & 100 & -10 \\ 0 & -1 & -1 \end{vmatrix}}{-2100} = \frac{-300 + 200 - 30}{-2100} = \frac{-130}{-2100} = \frac{13}{210}\ A$$

$$I_2 = \frac{\begin{vmatrix} 10 & 3 & 0 \\ 0 & 2 & -10 \\ 1 & 0 & -1 \end{vmatrix}}{-2100} = \frac{-20 - 30}{-2100} = \frac{50}{2100} = \frac{5}{210}\ A$$

$$I_3 = \frac{\begin{vmatrix} 10 & 100 & 3 \\ 0 & 100 & 2 \\ 1 & -1 & 0 \end{vmatrix}}{-2100} = \frac{200 - 300 + 20}{-2100} = \frac{-80}{-2100} = \frac{8}{210}\ A$$

$$\Leftrightarrow \begin{cases} I_1 = \frac{13}{210}\ A \\ I_2 = \frac{5}{210}\ A \\ I_3 = \frac{8}{210}\ A \end{cases}$$



A corrente no Ramo onde está o condensador é 0, quando este está carregado

$$I_1 = I_2$$

$t = \infty$

$I_3 = 0$

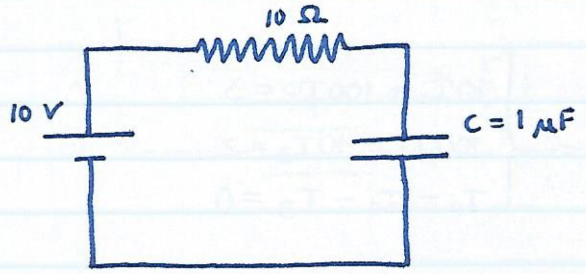
$V_c = V_{max}$

$I_{Ramo\ c} = 0$

$$\begin{cases} 1 - 10I_1 + 2 - 100I_1 = 0 \\ 3 - 110I_1 = 0 \end{cases} \Leftrightarrow \begin{cases} I_1 = \frac{3}{110}\ A \\ I_2 = \frac{3}{110}\ A \\ I_3 = 0 \end{cases}$$

$t = 0$ Condensador Descarregado

$V_c = 0$

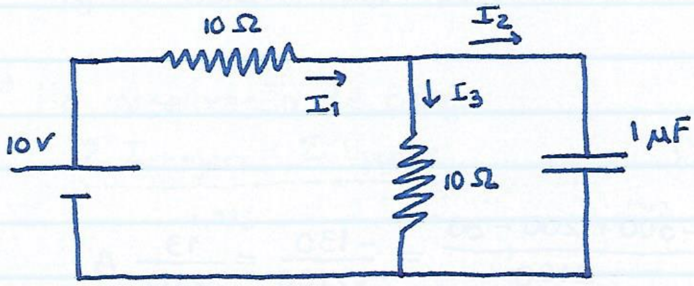


$t = \infty$

$V_c = 10\ V$

$10V - 10I_1 - V_c = 0$

$ \overset{\shortparallel}{0}$



$t = 0\,s$

$$\begin{cases} -10I_1 - 10I_3 + 10 = 0 \\ 10I_3 - V_c = 0 \end{cases}$$

$$V_c = \frac{Q}{C}$$

$$\begin{cases} -10I_1 + 10 = 0 \\ V_c = 0 \Rightarrow I_3 = 0 \end{cases} \Rightarrow \begin{cases} I_1 = 1A \\ I_2 = 1A \\ I_3 = 0\,A \end{cases}$$

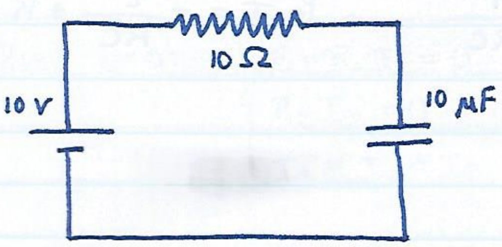
Muito Tempo Depois

$V_c \neq 0 \quad I_2 = 0$

$$\begin{cases} -10I_1 - 10I_3 + 10 = 0 \\ 10I_3 - V_c = 0 \\ I_1 = I_2 + I_3 \end{cases} \Leftrightarrow \begin{cases} -10I_1 - 10I_3 + 10 = 0 \\ 10I_3 - V_c = 0 \\ I_1 = I_3 \end{cases}$$

$$\Leftrightarrow \begin{cases} -20I_3 = -10 \\ V_c = 10I_3 \\ I_1 = I_3 \end{cases} \Leftrightarrow \begin{cases} I_3 = 0{,}5\text{ A} \\ V_c = 5\text{ V} \\ I_1 = 0{,}5\text{ A} \end{cases}$$

Exemplo - complicação



$t = 0 \quad V_c = 0$

$10 - 10I - \underbrace{V_c}_{=0} = 0 \Rightarrow I = 1\text{A}$

$t \gg \quad I = 0$

$10 - 10\underbrace{I}_{=0} - V_c = 0 \Leftrightarrow V_c = 10\text{ V}$

e outros t?

$10 - 10I - V_c = 0$

$V_c = \frac{Q}{C}$

$I = \frac{dq}{dt}$

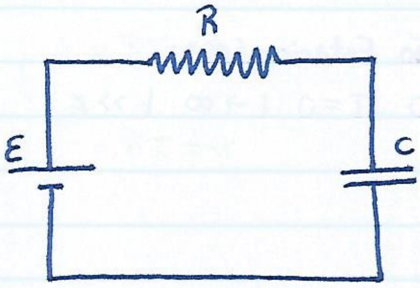
$10 - 10I - \frac{Q}{C} = 0$

$\Rightarrow 0 - 10\frac{dI}{dt} - \frac{1}{C}\underbrace{\frac{dQ}{dt}}_{=I} = 0$

$10\frac{dI}{dt} + \frac{I}{C} = 0$

$\Rightarrow \frac{1}{I}\frac{dI}{dt} = -\frac{1}{10C} \Leftrightarrow$

$\Leftrightarrow \ln I = -\frac{1}{10C}t + K$



$\varepsilon - RI - V_c = 0$

$t = 0 \quad V_c = 0 \quad I = \frac{\varepsilon}{R}$

$t \gg \quad I = 0$

$V_c = 0 \Rightarrow \varepsilon - RI = 0$

$\Leftrightarrow I = \frac{\varepsilon}{R}$

$\varepsilon - V_c = 0 \Leftrightarrow V_c = \varepsilon$

Outros t

$\mathcal{E} - RI + \frac{Q}{C} = 0$ derivo em ordem a t

$$0 - R\frac{dI}{dt} - \frac{1}{C}\frac{dQ}{dt} = 0$$

$\to I$

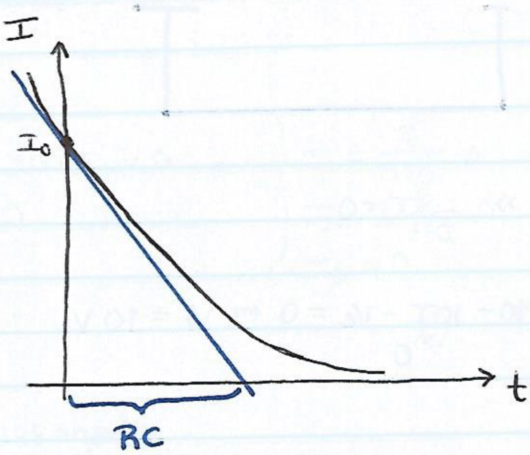
$$\to R\frac{dI}{dt} = -\frac{1}{C} I \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{I}\frac{dI}{dt} = -\frac{1}{RC}$$

$$\frac{I'}{I} = -\frac{1}{RC} \to \frac{d}{dt}(\ln I) = -\frac{1}{RC} \xrightarrow{\text{Primitivando}} \ln I = -\frac{t}{RC} + K \Leftrightarrow$$

$$\Leftrightarrow I = A e^{-t/RC}$$

$\swarrow$ e^K

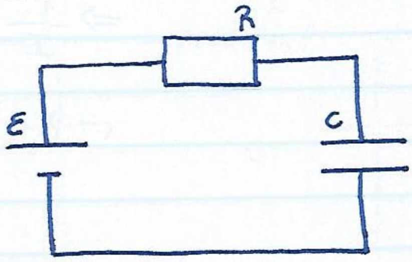


$t = 0$ $I = A$

$$I = I_0 e^{-t/RC}$$

$$V_C = \mathcal{E} - RI_0 e^{-t/RC}$$
$$= \mathcal{E}(1 - e^{-t/RC})$$

Aula 18 - 08.04.2015



Situação Descarregado:

$Q = 0$ $Q = CV = 0$ $V_C = 0$ $t = 0s$

Situação Estacionária:

$Q \neq 0$ $I = 0$ $t \to \infty$ $t \gg \tau$

$$\mathcal{E} - RI - V_C = 0 \Leftrightarrow \mathcal{E} - RI - \frac{Q}{C} = 0$$

$$-R\frac{dI}{dt} - \frac{1}{C}\frac{dQ}{dt} = 0 \Leftrightarrow -R\frac{dI}{dt} = +\frac{I}{C} \Rightarrow \frac{1}{I}\frac{dI}{dt} = -\frac{1}{RC}$$

$$\ln I = -\frac{t}{RC} + K \qquad I = I_0 e^{-t/RC} \qquad \tau = RC$$

$$\mathcal{E} - RI - V_c = 0 \Leftrightarrow V_c = \mathcal{E} - RI \Leftrightarrow V_c = \mathcal{E} - RI_0 e^{-t/RC}$$

Em $t=0$, $V_c = V_0$

$$V_c = \mathcal{E} - RI_0 = V_0$$

$$RI_0 = \mathcal{E} - V_0$$

Então, $\boxed{V_c = \mathcal{E} - (\mathcal{E} - V_0) e^{-t/RC}}$

Exemplo do PowerPoint

Para $t=0$:

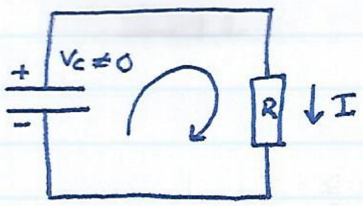
$$\begin{cases} \mathcal{E} - R_1 I_1 = 0 \\ R_2 I_2 = 0 \\ I_1 = I_2 + I_3 \end{cases} \Leftrightarrow \begin{cases} I_1 = \frac{\mathcal{E}}{RI} \\ I_2 = 0 \\ I_3 = I_1 = \mathcal{E} \end{cases}$$

Estacionária:

$I_3 = 0$

$$\begin{cases} \mathcal{E} - R_1 I_1 - R_2 I_1 = 0 \\ V_c = R_2 I_1 \\ I_1 = I_2 \end{cases} \Leftrightarrow \begin{cases} I_1 = \frac{\mathcal{E}}{R_1 + R_2} \\ V_c = \frac{R_2}{R_2 + R_1} \mathcal{E} \end{cases}$$

Descarga de um Condensador



$t = 0\,s$

$V_c = V_0 \quad I_R = \frac{V_0}{R}$

$$\begin{cases} V_c - RI = 0 \\ \frac{Q}{C} - RI = 0 \end{cases}$$

$$\frac{1}{C}\underbrace{\frac{dQ}{dt}}_{-I} - R\frac{dI}{dt} \Rightarrow -\frac{I}{C} - R\frac{dI}{dt} = 0$$

$$\frac{1}{I}\frac{dI}{dt} = -\frac{1}{RC} \rightarrow I = I_0 e^{-t/RC}$$

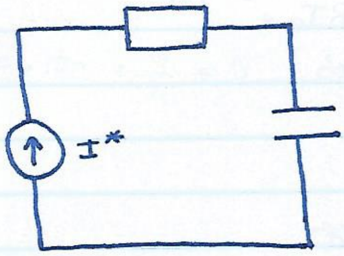
Tempo característico da Descarga

τ

$$V_c = RI = RI_0 e^{-t/RC}$$

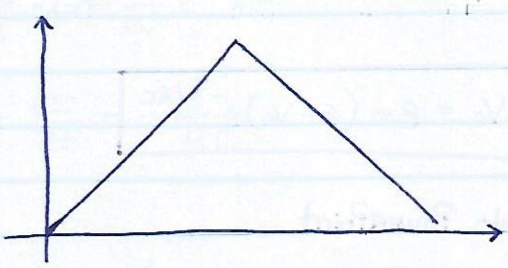
Então, $V = V_0 e^{-t/RC}$

Para Uma Fonte de Corrente



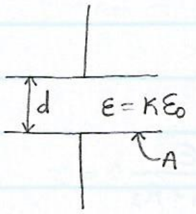
$$V_c = \frac{Q}{C} \qquad \frac{dV_c}{dt} = \frac{1}{C}\frac{dQ}{dt} = I^*$$

$$\frac{dV_c}{dt} = \frac{I^*}{C}$$



Série 5 - Resolução

8 →



$R \neq \infty$ Material Dieléctrico → Não é condutor

a) $RC = \frac{K\varepsilon_0}{\sigma} = \frac{\varepsilon}{\sigma}$

$\boxed{\text{O dieléctrico polariza o meio e diminui a d.d.p}}$

$C_0 = \frac{Q}{\Delta V_0} \qquad \Delta V = \frac{\Delta V_0}{K}$

$C = K C_0 = K \frac{Q}{\Delta V_0} = K \frac{\varepsilon_0 A}{d}$

$R = \rho \frac{d}{A} = \frac{d}{\sigma A}$

$R \times C = \frac{d}{\sigma A} \times \frac{K\varepsilon_0 A}{d} = \frac{K\varepsilon_0}{\sigma} = 1{,}79 \times 10^{15}\ \Omega$

b) $K = 3{,}78$

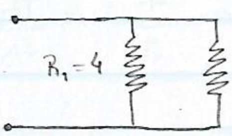
$\rho = \frac{1}{\sigma} = 75 \times 10^{6}\ \Omega m \qquad \varepsilon_0 = 8{,}854 \times 10^{-12}\ F/m \qquad C = 14{,}0\ nF$

$R = 1{,}79 \times 10^{15}\ \Omega$

9 → $R = 4\ \Omega$

60 W

fusível 4,00 A



$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$R_{eq} = 2\ \Omega$

60 W em cada canal

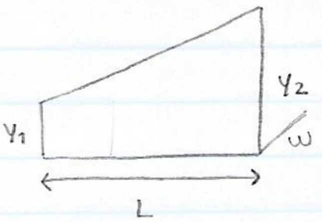
$$60 = R_1 I^2$$

O fusível serve para não deixar a corrente atingir o seu máximo

$$60 = 4I^2 \Rightarrow I = \sqrt{\frac{60}{4}} = 3{,}87\ A$$

Fusível ⇒ 4 A não é adequado porque é superior à intensidade máxima.

10 → (*)



$$R = \rho \frac{l}{S} \qquad R = \rho \int_L \frac{1}{S}\, dl$$

$$S = w\, y(l)$$

$$y(l) = mx + b \qquad y(l) = \left(\frac{y_2 - y_1}{L}\right) l + y_1$$

$$m = \frac{y_2 - y_1}{L}$$

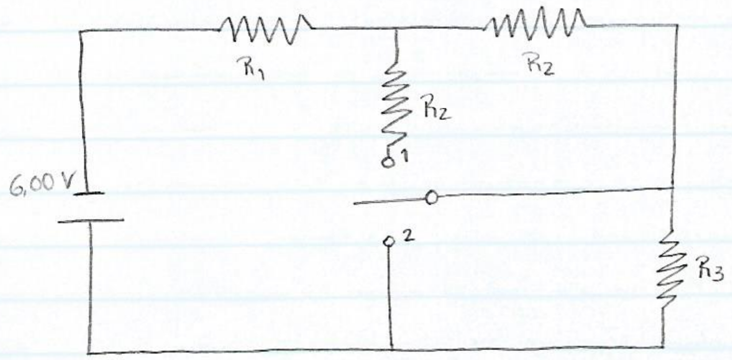
$$b = y_1$$

$$R = \rho \int_0^L \frac{1 . dl}{w\left[\left(\frac{y_2 - y_1}{L}\right) l + y_1\right]} = \frac{\rho}{w\left(\frac{y_2 - y_1}{L}\right)} \int_0^L \frac{\left(\frac{y_2 - y_1}{L}\right) dl}{\left(\frac{y_2 - y_1}{L}\right) l + y_1} = \frac{\rho L}{w(y_2 - y_1)} \left[\ln\left(\left(\frac{y_2 - y_1}{L}\right) l + y\right)\right]_0^L$$

$$= \frac{\rho L}{w(y_2 - y_1)} \left[\ln(y_2) - \ln(y_1)\right] = \frac{\rho L}{w(y_2 - y_1)} \ln\left(\frac{y_2}{y_1}\right)$$

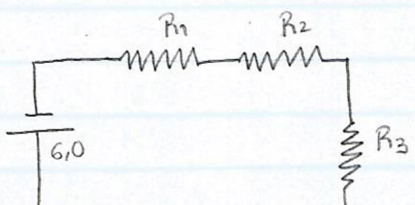
Série 6 – Resolução

3 →

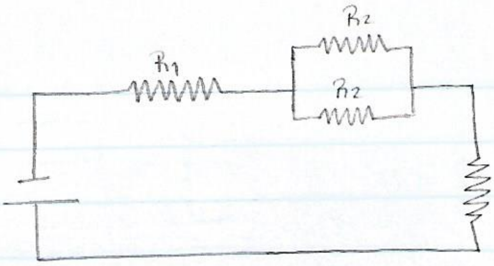


Para o caso (A)

$$I_A = \frac{6}{R_1 + R_2 + R_3} = 1{,}0\ mA$$



(B)

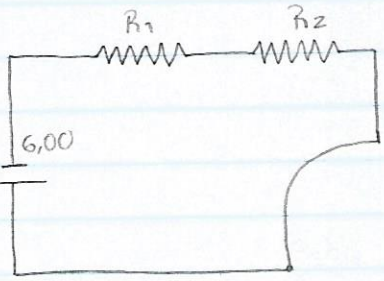


$$\frac{1}{R_{eq_{//}}} = \frac{1}{R_2} + \frac{1}{R_2} = \frac{2}{R_2}$$

$$R_{eq_{//}} = \frac{R_2}{2}$$

$$R_{eq_{(B)}} = R_1 + R_{//} + R_3$$

(C)



$$R_C = R_1 + R_2$$

$$\frac{6}{R_1 + R_2} = 2{,}00 \text{ mA}$$

$$\frac{1}{R_{//}} = \frac{1}{0} + \frac{1}{R_3}$$

$$\frac{1}{R_{//}} = \infty$$

$$R_{//} = 0$$

$$\begin{cases} 1 \times 10^{-3} = \dfrac{6}{R_1 + R_2 + R_3} \\ 1{,}2 \times 10^{-3} = \dfrac{6}{R_1 + \frac{R_2}{2} + R_3} \\ 2 \times 10^{-3} = \dfrac{6}{R_1 + R_2} \end{cases} \Leftrightarrow \begin{cases} R_1 + R_2 + R_3 = 6 \times 10^3 \\ R_1 + \dfrac{R_2}{2} + R_3 = \dfrac{6}{1{,}2} \times 10^3 \\ R_1 + R_2 = 3 \times 10^3 \end{cases} \Leftrightarrow \begin{cases} R_3 = 3 \times 10^3 \ \Omega \\ R_2 = 2 \times 10^3 \ \Omega \\ R_1 = 1 \times 10^3 \ \Omega \end{cases}$$

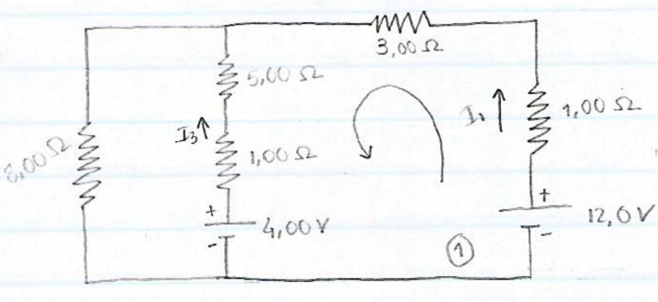
4 →

$$\begin{cases} R_1 + R_2 = \dfrac{P_1}{I^2} \\ \dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{I^2}{P} \end{cases} \Leftrightarrow \begin{cases} R_1 + R_2 = 9 \\ \dfrac{R_1 + R_2}{R_1 R_2} = \dfrac{1}{2} \end{cases} \Leftrightarrow \begin{cases} R_1 + R_2 = 9 \\ \dfrac{9}{R_1 R_2} = \dfrac{1}{2} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} R_1 + R_2 = 9 \\ \dfrac{1}{R_1 R_2} = \dfrac{1}{18} \end{cases} \Leftrightarrow \begin{cases} R_2 = 3 \ \Omega \ \vee \ R_2 = 6 \ \Omega \\ R_1 = 6 \ \Omega \ \vee \ R_1 = 3 \ \Omega \end{cases}$$

Uma vez que $RI^2 = P \Leftrightarrow R = \dfrac{P}{I^2} \Leftrightarrow \dfrac{1}{R} = \dfrac{I^2}{P}$

5 →



Malha ①

$$12 - 1I_1 - 3I_1 + 5I_2 + I_2 - 4 = 0 \Leftrightarrow$$
$$\Leftrightarrow 8 - 4I_1 + 6I_2 = 0$$

Malha ②

$$4 - I_2 - 5I_2 + 8I_3 = 0 \Leftrightarrow$$
$$\Leftrightarrow 4 - 6I_2 + 8I_3 = 0$$

Malha ③

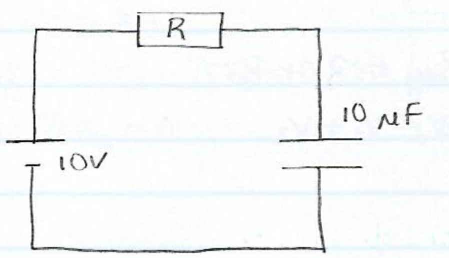
$$12 - I_1 - 3I_1 + 8I_3 = 0 \Leftrightarrow$$
$$\Leftrightarrow 12 - 4I_1 + 8I_3 = 0$$

$$\begin{cases} I_1 + I_2 + I_3 = 0 \\ 12 - 4I_1 + 8I_3 = 0 \\ 8 - 4I_1 + 6I_2 = 0 \end{cases} \Leftrightarrow \begin{cases} I_1 + I_2 + I_3 = 0 \\ -4I_1 + 8I_3 = -12 \\ -4I_1 + 6I_2 = -8 \end{cases} \Leftrightarrow \begin{cases} I_2 = -I_1 - I_3 = -3 - 2I_3 - I_3 \\ I_1 = 3 + 2I_3 \\ \underline{\qquad\qquad} \end{cases}$$

$$\Leftrightarrow \begin{cases} I_2 = -3 - 3I_3 \\ I_1 = 3 + 2I_3 \\ I_1 = 2 + \frac{3}{2}I_2 = 2 - \frac{9}{2} - \frac{9}{2}I_3 \end{cases} \Leftrightarrow \begin{cases} \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \\ I_1 = 2 - \frac{9}{2} - 4{,}5I_3 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \underline{\qquad\qquad} \\ 3 + 2I_3 = -\frac{5}{2} - 4{,}5I_3 \\ \underline{\qquad\qquad} \end{cases} \Leftrightarrow \begin{cases} \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \\ 6{,}5I_3 = -\frac{5}{2} - 3 = -\frac{11}{2} \end{cases} \Leftrightarrow \begin{cases} I_2 = -\frac{6}{11}\,A \\ I_1 = 3 - \frac{22}{13} = \frac{17}{13}\,A \\ I_3 = -\frac{11}{13}\,A \end{cases}$$

8 →

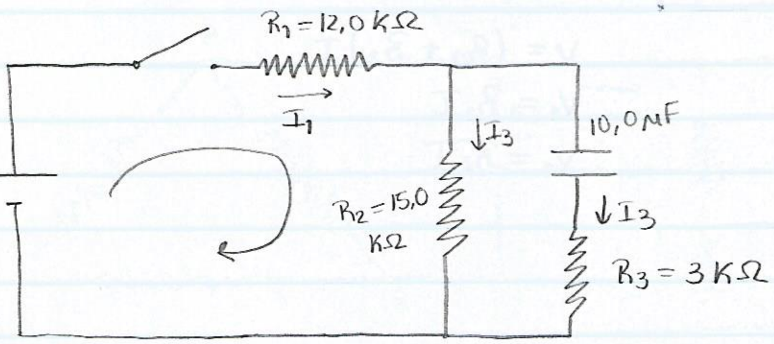


$$V = \varepsilon\left(1 - e^{-t/RC}\right)$$
$$4 = 10\left(1 - e^{-3/RC}\right)$$
$$4 = 10 - 10e^{-3/RC}$$

$$10e^{-3/RC} = 10 - 4 = 6$$

$$e^{-3/RC} = 0{,}6 \Leftrightarrow -\frac{3}{RC} = \ln(0{,}6) = -0{,}511 \rightarrow RC = 5{,}873 \Leftrightarrow R = 587\ k\Omega$$

10 →

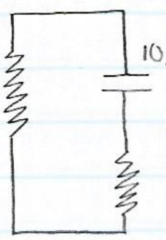


$$\begin{cases} 9 - R_1 I_1 - R_2 I_2 = 0 \\ R_2 I_2 - V_c - R_3 I_3 = 0 \\ I_1 = I_2 + I_3 \end{cases}$$

a) $t = \infty \quad I_3 = 0$

$$\begin{cases} 9 - R_1 I_1 - R_2 I_2 = 0 \\ R_2 I_2 - V_c = 0 \\ I_1 = I_2 \end{cases} \Leftrightarrow \begin{cases} I_1 = \dfrac{9}{R_1 + R_2} = \dfrac{9V}{27\ k\Omega} = \dfrac{1V}{3\ k\Omega} = 0{,}333\ mA \\ V_C = R_2 I_2 = 15\ k\Omega \times \frac{1}{3}\ mA = 5V \Rightarrow Q = CV = 10 \times 10^{-6} \times \\ \qquad = 50{,}0V \\ I_2 = 333\ \mu A \end{cases}$$

b) Quando o interruptor é aberto



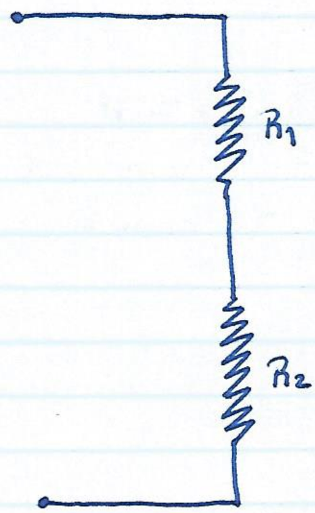
$$I = I_0 e^{-t/RC} \leftarrow R_{série} = 18\ k\Omega \Rightarrow RC = 0{,}180s$$

$$I_0 = \frac{5V}{R_2 + R_3} = 278\ \mu A$$

$$Q = CV = \underbrace{CV_0}_{Q_0} e^{-t/RC} = \frac{Q_0}{5}$$

Aula 19 - 10.04.2015

Exemplo 1:



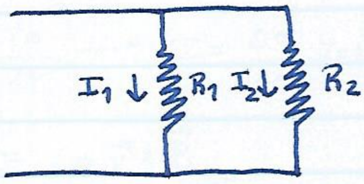
$$R_{eq} = R_1 + R_2$$
$$V = V_1 + V_2$$

Divisor de Tensões

$$\frac{V_2}{V} = \frac{R_2}{R_1 + R_2} \qquad \frac{V_1}{V} = \frac{R_1}{R_1 + R_2}$$

$$V = (R_1 + R_2) I$$
$$V_1 = R_1 I$$
$$V_2 = R_2 I$$

Exemplo 2:



$V_1 = V_2 = V$

$I = I_2 + I_1$

Divisor de correntes

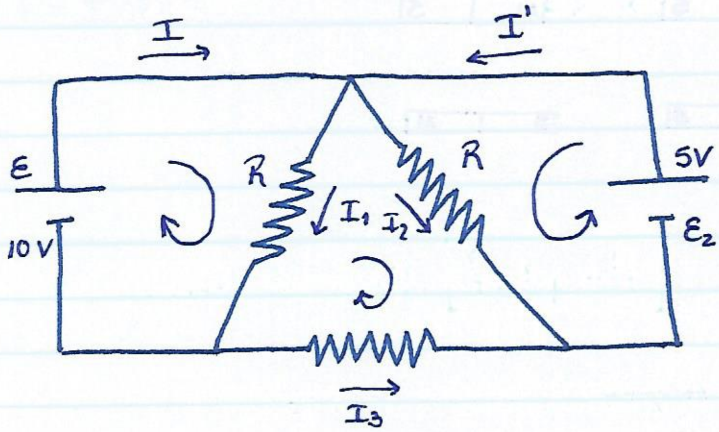
$I = \frac{V}{R_{eq}}$ $\quad I_1 = \frac{V}{R_1}$ $\quad I_2 = \frac{V}{R_2}$

$I = G_{Total} V$ $\quad I_1 = G_1 V$ $\quad I_2 = G_2 V$

Condutância → Inverso da Resistência $\quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

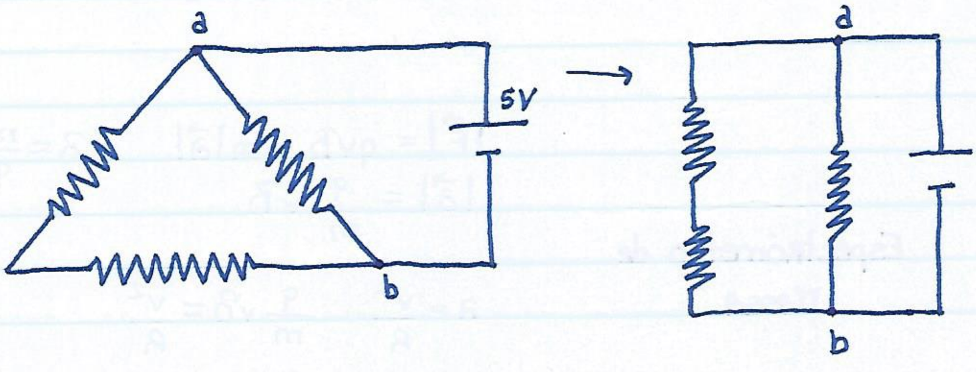
$G_{Total} = G_1 + G_2$

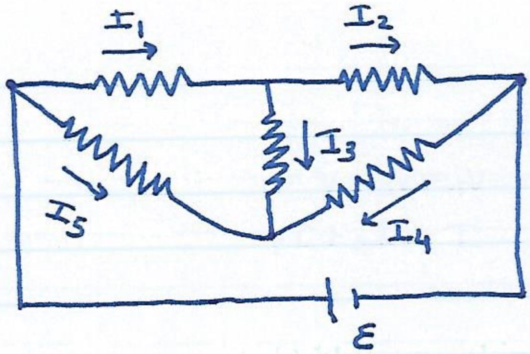
Exemplo Importante



$$\begin{cases} 10 - RI_1 = 0 \\ RI_1 - RI_2 + RI_3 = 0 \\ 5 - RI_2 = 0 \end{cases} \Leftrightarrow \begin{cases} I_1 = \frac{10}{R} = \frac{10\,V}{1\,k\Omega} = 10\,mA \\ I_3 = I_2 - I_1 = 5 - 10 = -5\,mA \\ I_2 = \frac{5}{R} = \frac{5V}{1\,k\Omega} = 5\,mA \end{cases}$$

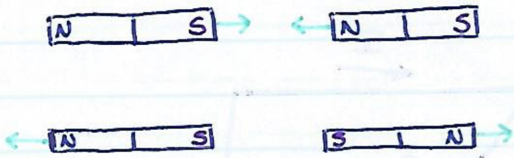
$$\begin{cases} I + I' = I_1 + I_2 \\ I' = I_2 + I_3 \end{cases} \Leftrightarrow \begin{cases} I' = I_2 + I_3 = 0 \\ I = I_1 + I_2 - I' = 15\,mA \end{cases}$$





$$\begin{cases} \varepsilon - RI_5 + RI_4 = 0 \\ RI_5 - RI_1 - RI_3 = 0 \\ RI_3 - RI_2 - RI_4 = 0 \\ I_4 + I_5 + I_3 = 0 \\ I_2 = I + I_4 \\ I_1 = I_2 + I_3 \end{cases}$$

Aula 20 – 15.04.2015



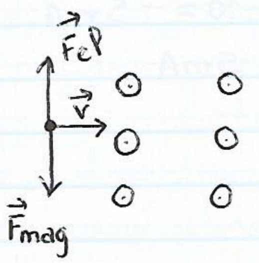
$|\vec{F}| \propto q$

$|\vec{F}| \propto |\vec{v}|$

$|\vec{F}| \propto |\vec{B}|$

$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$ Força de Lorentz

A força é sempre perpendicular à velocidade.



$$\vec{a} = \frac{\vec{F}}{m} = \frac{q}{m} \vec{v} \times \vec{B}$$

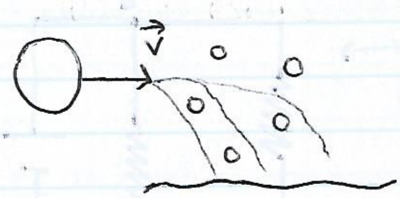
$\vec{v} \perp \vec{B}$ $\quad |\vec{F}_{el}| = |\vec{F}_{mag}|$

$\vec{F}_{el} + \vec{F}_{mag} = 0$

$|qE| = |qvB|$

$|E| = |v||B|$

$$|v| = \frac{|E|}{|B|}$$



Espectrómetro de Massa

$|\vec{F}| = qvB = m|\vec{a}|$ $\qquad R = \frac{mv}{qB}$

$|\vec{a}| = \frac{q}{m} vB$

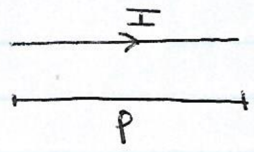
$a = \frac{v^2}{R}$ $\qquad \frac{q}{m} vB = \frac{v^2}{R}$

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$I = \frac{dq}{dt} \longrightarrow I = \frac{\Delta q}{\Delta t} \Leftrightarrow \Delta q = I\Delta t \qquad \Delta q\,\vec{v} = I\,d\vec{l}$$

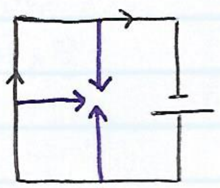
$$\vec{F} = \Delta q\,\vec{v} \times \vec{B}$$
$$= I\,\Delta t\,\vec{v} \times \vec{B}$$
$$= I\,\Delta\vec{l} \times \vec{B}$$

$$\vec{F} = I\Delta\vec{l} \times \vec{B}$$
$$= I\vec{l} \times \vec{B}$$

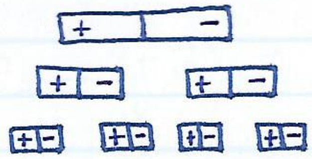


$d\vec{l}$

$$d\vec{F} = I\,d\vec{l} \times \vec{B} \qquad \vec{F} = \int d\vec{F} = \int I\,d\vec{l} \times \vec{B}$$



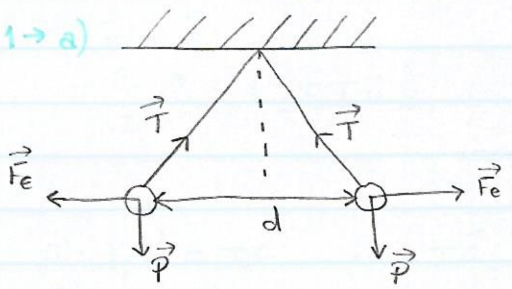
$$\vec{F} = I\Delta\vec{l} \times \vec{B}$$



T.P. 7 - 15.04.2015

Correcção do Teste

1 → a)



b) $$|\vec{F}| = K_e \frac{q^2}{d^2} = \frac{8,988 \times 10^9}{(4\times 10^{-2})^2} \times q^2$$

Direcção Horizontal
Sentido Repulsivo Relativo à Outra Esfera

c) $$\vec{F_e} + \vec{T} + \vec{P} = 0 \qquad \begin{cases} -T\cos\alpha + F_{e_1} = 0 \\ T\sin\alpha - mg = 0 \end{cases} \Leftrightarrow \quad q = 4,18 \times 10^{-9}$$

d) $$\vec{E} = \vec{E_1} + \vec{E_2}$$

$$|\vec{E_1}| = |\vec{E_2}| = K_e \frac{q}{d^2} = \frac{8,988 \times 10^9 \times 4,175 \times 10^{-8}}{(0,1)^2} = 4,129 \times 10^4 \text{ N/C}$$

$$\vec{E} = 2\vec{E}_{1y}$$
$$= 2 \times 4,129 \times 10^4 \times \cos\theta$$
$$= 8,09 \times 10^4 \text{ N/C}$$

e) $V = 2V_1 = 3,75 \times 10^4\ V$

2 → a) $\vec{E} = 0$

b) $\phi = \phi_{\substack{sup \\ lat}} + \phi_{bases} = \phi_{\substack{sup \\ lat}} = A_{lat} \times E = 2\pi R \times h$

Q contida $= \lambda h$

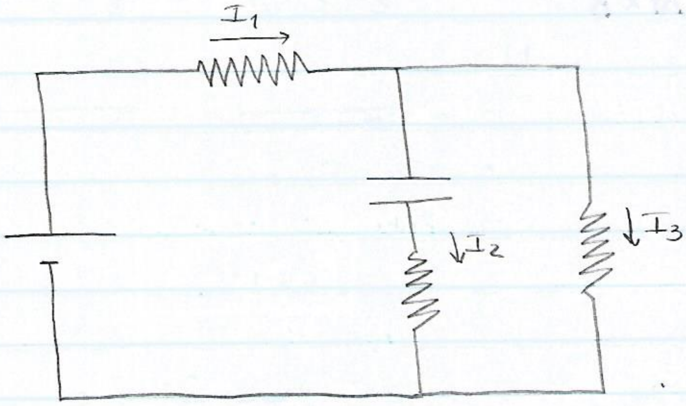
$$E\, 2\pi R h = \frac{\lambda h}{\varepsilon_0} \Leftrightarrow E = \frac{\lambda}{2\pi \varepsilon_0 R} = \frac{1}{\pi \varepsilon_0 R}$$

$$E_{max} = \frac{1}{\pi \varepsilon_0 \times 6 \times 10^{-2}} \qquad E_{min} = \frac{1}{\pi \varepsilon_0 \times 10^{-1}}$$

c) $\lambda_{\substack{sup\ int \\ C_1}} = 0$ $\qquad \lambda_{\substack{sup\ int \\ C_2}} = -\lambda$

$\lambda_{\substack{sup\ ext \\ C_1}} = \lambda$ $\qquad \lambda_{\substack{sup\ ext \\ C_2}} = 0$

3 → a)



$$\begin{cases} \varepsilon - 1000 I_1 - 100 I_2 = 0 \\ 100 I_2 - 2000 I_3 = 0 \Rightarrow I_2 = 20 I_3 \\ I_1 = I_2 + I_3 \Rightarrow I_1 = 21 I_3 \end{cases}$$

b) $t = 0$ $\quad Q = 0\,C$

$V_c = 0\,V$

$$\begin{cases} \varepsilon - 1000 I_1 - 100 I_2 = 0 \\ 100 I_2 - 2000 I_3 = 0 \Rightarrow I_2 = 20 I_3 \\ I_1 = I_2 + I_3 \Rightarrow I_1 = 21 I_3 \end{cases} \Leftrightarrow \begin{cases} I_3 = 0,652\ mA \\ I_2 = 20 I_3 = 13,0\ mA \\ I_1 = 13,7\ mA \end{cases}$$

c) i) $\begin{cases} 15 - 1000 I_1 - V_c = 0 \\ V_c - 2000 I_3 = 0 \\ I_1 = I_2 + I_3 \end{cases} \Leftrightarrow \begin{cases} 15 - 1000 I_1 - 2000 I_1 = 0 \\ V_c = 2000 I_3 \\ I_1 = I_3 \end{cases} \Leftrightarrow \begin{cases} I_1 = 5{,}0\ mA \\ V_c = 10{,}0\ V \end{cases}$

ii) $V_c = \frac{Q}{V} \Leftrightarrow Q = 10\ \mu C$

iii) $E = \frac{1}{2} Q V = 50\ \mu J$

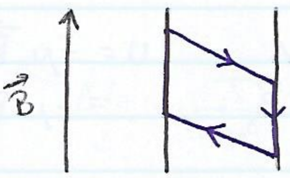
d) Como se corta a fonte de corrente, o condensador vai descarregar através das resistências

Aula 21 - 17.04.2015

$$\vec{F} = dq\, \vec{v} \times \vec{B}$$
$$\vec{F} = \int_{fio} I\, d\vec{l} \times \vec{B}$$

$\vec{B}$ uniforme

$$\vec{F} = I \left(\int d\vec{l} \right) \times \vec{B} \Leftrightarrow \boxed{\vec{F} = I\, \vec{ab} \times \vec{B}}$$



$$\vec{M} = \vec{r} \times \vec{F}$$
$$= -\frac{l_2}{2} \vec{u_y} \times -I l_1 B \vec{u_z}$$
$$= \frac{l_1 l_2}{2} B I \vec{u_x}$$

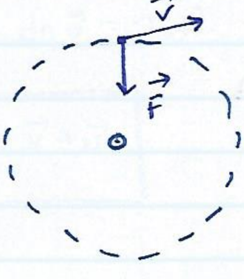
$$M' = \vec{r}' \times \vec{F}'$$
$$= \frac{l_2}{2} \vec{u_y} \times (l_1 B I \vec{u_z})$$

$$\vec{M}_T = \vec{M} + \vec{M}'$$
$$= (l_1 l_2) B I \vec{u_x}$$
$$= A I B \vec{u_x}$$

$\vec{\mu} : |\vec{\mu}| = IA \qquad \vec{\mu} = I\vec{A}$

$\vec{M} = \vec{\mu} \times \vec{B}$

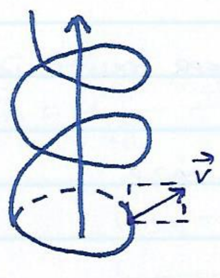
$U = -\vec{\mu}\ \ \vec{B}$



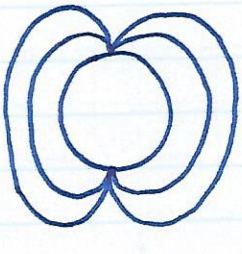
$$\frac{\vec{F}}{m} = \frac{q\vec{v} \times \vec{R}}{m}$$

$$a = \frac{F}{m} = \frac{qvB}{m} = \frac{v^2}{R}$$

$$R = \frac{mv}{qB}$$



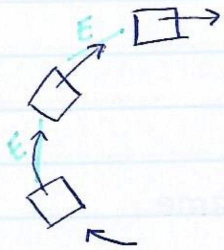
As cargas num campo magnético enrolam em torno das linhas de campo



Cintura Van Allen

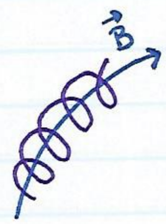
Num acelerador utiliza o campo magnético para confinar a trajectória do electrão.

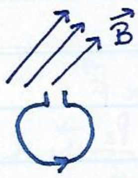
Sincrotrão



Campo Magnético encurva a trajectória.
Campo Eléctrico acelera a particula.

Aula 22 - 20.04.2015





momento magnético

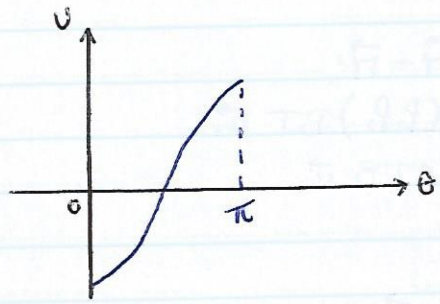
$|\vec{\mu}| = IA$

$\vec{\tau} = \vec{\mu} \times \vec{B}$

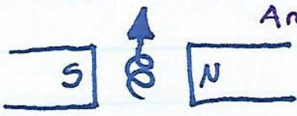
$U = -\mu . \vec{B}$
$= -\mu . B . \cos\theta$

$\vec{\tau} = \vec{p} \times \vec{E}$

$U = -\vec{p} . \vec{E}$



Galvanómetro
Amperímetro



$C \sim KR$

$F = -k\theta \Rightarrow \tau_{mola} = -C\theta$

deformação angular

$C\theta. = \mu B = NIA$

$\theta. = \frac{NA}{C} I$

$\vec{\tau}_{mola} + \vec{\tau}_{\vec{B}} = 0$

$|\vec{\tau}_{mola}| = |\vec{\tau}_B|$

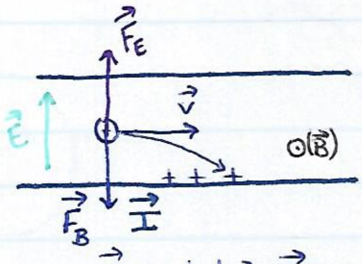
As resistências dos amperímetros têm que ser muito pequenas para não perturbar muito o circuito.

Os amperímetros são sempre colocadas em série.

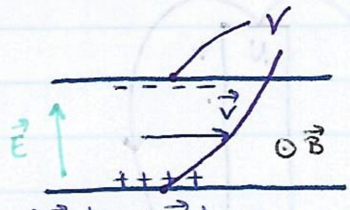
Sensor de Campo Magnético

Sensor de Efeito de Hall $|\vec{F}_E| = |\vec{F}_B|$

$|\Delta V| = Ed$



$\vec{F} = -|q|\vec{v} \times \vec{B}$



$|\vec{F}_E| = |\vec{F}_B| \Rightarrow \cancel{q}E = \cancel{q}v_d B \Leftrightarrow E = v_d B$

Efeito de Hall

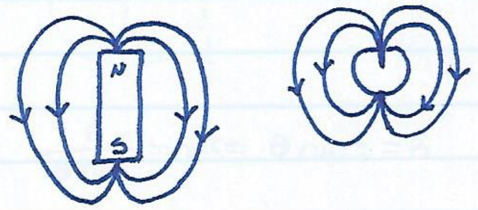
Tensão perpendicular a I e a $\vec{B}$

d.d.p.

$$J = nqv_d \quad v_d = \frac{J}{nq} = \frac{I}{nqA}$$

$$E_H = \frac{J}{nq} B = \underbrace{\frac{1}{nq}}_{R_H = \text{ constante de Hall}} JB$$

$$\frac{\Delta V_H}{d} = \frac{I}{nqA} B \Rightarrow \Delta V_H = \frac{1}{nq} \frac{d}{A} IB$$



$$\boxed{\vec{B} = \frac{1}{4\pi} \mu_0 \frac{I}{r^2} \, d\vec{l} \times \vec{u}_r}$$

Lei de Biot-Savart

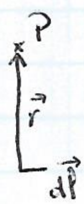
$$\vec{F} = q\vec{v} \times \vec{B} \qquad q\vec{v} = I\vec{l}$$
$$= I\vec{l} \times \vec{B} \qquad dq\,\vec{v} = I\,d\vec{l}$$

Para cargas: $\boxed{\frac{1}{4\pi} \mu_0 \frac{dq}{r^2} \quad \vec{v} \times \vec{u}_r}$

Aula 23 - 22.04.2015

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \vec{u}_r}{r^2} \qquad \vec{B} = \frac{\mu_0}{4\pi} \frac{dq\,\vec{v} \times \vec{u}_r}{r^2}$$

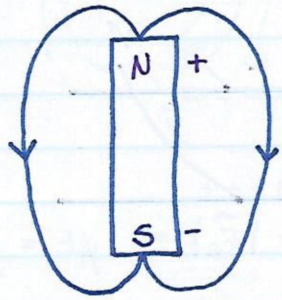
Tesla (T)

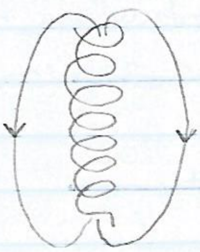


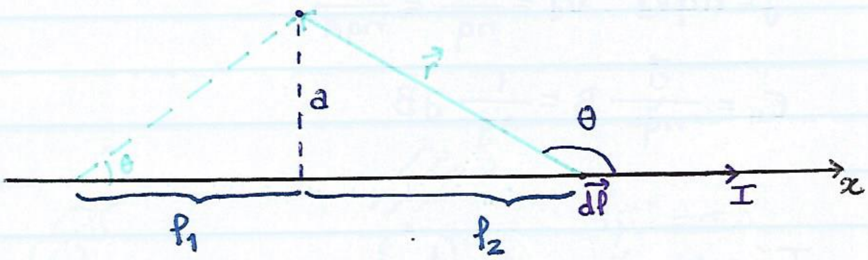
$$\boxed{\mu_0 = 4\pi \times 10^{-7} \ TmA^{-1}}$$

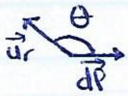
$$\frac{\mu_0}{4\pi} \sim 10^{-7} \qquad \frac{1}{4\pi\varepsilon_0} \sim 9 \times 10^9 \ Nm^2C^{-2}$$

O campo magnético no geral tem intensidades muito inferiores ao campo eléctrico.









$$\vec{dl} \times \vec{u_r} = \vec{dx} \times \vec{u_r} = dx \sin\theta \, \vec{u_z}$$

$$|\vec{dl} \times \vec{u_r}| = |\vec{dl}||\vec{u_r}| \sin\theta$$

$$\vec{B} = \int_{-l_1}^{l_2} \frac{\mu_0}{4\pi} \frac{I \, dx \sin\theta}{r^2} \vec{u_z} \quad (1)$$

$$a = r\sin\theta \Rightarrow r = \frac{a}{\sin\theta}$$

$$\cotan(\theta) = \frac{-x}{a} \Rightarrow x = -a\cotan\theta$$

$$dx = -a\left(-\frac{1}{\sin^2\theta}\right)d\theta$$

$$x = -l_1 \Rightarrow \cotan\theta_1 = \frac{x}{-a} = \frac{l_1}{a}$$

$$\frac{d}{d\theta}\cotan\theta = \frac{d}{d\theta}\frac{\cos\theta}{\sin\theta} = \frac{-\sin^2\theta - \cos^2\theta}{\sin^2\theta} = \frac{-1}{\sin^2\theta}$$

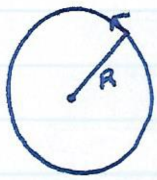
$$(1)\ \vec{B} = \frac{\mu_0}{4\pi}\vec{u_z} I \int_{-l_1}^{l_2} \frac{a}{\sin^2\theta} d\theta \frac{\sin\theta}{\left(\frac{a}{\sin\theta}\right)^2} =$$

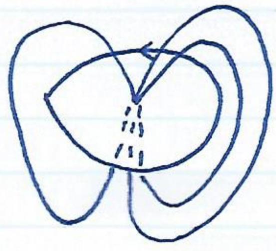
$$= \frac{\mu_0 I}{4\pi}\vec{u_z} \int_{\cotan^{-1}\left(\frac{l_1}{a}\right)}^{\cotan^{-1}\left(\frac{l_2}{a}\right)} \frac{\sin\theta}{a} d\theta = \frac{\mu_0 I}{4\pi a}\left[-\cos\theta\right]_{\theta_1}^{\theta_2} \vec{u_z}$$

$$= \frac{\mu_0 I}{4\pi a}\left[-\cos\theta_2 + \cos\theta_1\right]\vec{u_z}$$

Quando o fio é muito comprido: $\theta_2 \to \pi$ e $\theta_1 \to 0$, logo

$$\vec{B}_\infty = \frac{\mu_0 I}{4\pi d} \vec{u}_z \underbrace{\left(-\cos\pi + \cos 0\right)}_{2} = \frac{\mu_0 I}{2\pi d} \vec{u}_z$$



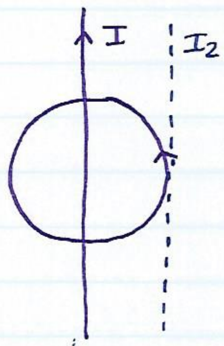


$$\vec{B} = \frac{\mu_0 I}{4\pi R^2} \int d\ell \times 1 \times \underbrace{\sin 90°}_{1} \vec{u}_z$$

$$= \frac{\mu_0 I}{2\pi R^2} \pi R \vec{u}_z = \frac{\mu_0 I}{2R} \vec{u}_z$$

$R = 10$ cm
$I = 1$ A

$$|\vec{B}| = \frac{\mu_0 I}{2R} = \frac{4\pi \times 10^{-7} \times 1}{2 \times 0{,}1} = 2\pi \times 10^{-6}\,T \sim 6 \times 10^{-6}\,T$$



$$|\vec{B}| = \frac{\mu_0}{2\pi} \frac{I}{d}$$

T.P. 8 - 22.04.2015

Série 7 - Resolução

1→ $q = 1{,}60 \times 10^{-19}$ C
$m = 22{,}99$ u.m.a.
$B = 1{,}00$ T
$v = 1{,}00 \times 10^2$ m/s

a) $|\vec{F}| = q\vec{v} \times \vec{B} = 1{,}6 \times 10^{-17}$ N

b) 1 u.m.a $\times N_A = 1$g $\quad m = 22{,}99$ u.m.a $= \frac{22{,}99}{6{,}022 \times 10^{23}}$

$\sim 4 \times 10^{-23}$ g $\sim 4 \times 10^{-26}$ Kg

$F_g = mg \sim 4 \times 10^{-25}$ N → logo pode-se desprezar

* O movimento é circular uniforme

$\vec{F} = q\vec{v} \times \vec{B} \quad |\vec{v}| = 1{,}00 \times 10^2$ m/s

$F = ma = m\frac{v^2}{R} = 1{,}6 \times 10^{-17}$ N *

$$R = \frac{mv^2}{1{,}6 \times 10^{-7}} = \frac{4 \times 10^{-26}\, 10^4}{1{,}6 \times 10^{-17}}$$

$$R = \frac{4 \times 10^{22}}{1{,}6 \times 10^{-17}} \sim 2{,}5 \times 10^{-5}\,m$$

2 → $|\vec{v}| = 1{,}00 \times 10^{7}$ m/s

$a = 2{,}00 \times 10^{13}$ m/s²

$q = 1{,}60 \times 10^{-19}$ C

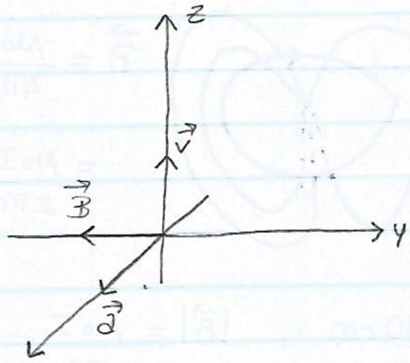
$m = 1{,}67 \times 10^{-27}$ kg

$|\vec{F}| = q|\vec{v}| \times |\vec{B}|$

Como a velocidade é perpendicular ao campo a aceleração só faz variar a direcção → Mov. Circular Uniforme

$$ma = q|\vec{v}| \times |\vec{B}| \Leftrightarrow \frac{ma}{q|\vec{v}|} = |\vec{B}| \Leftrightarrow |\vec{B}| = 2{,}1 \times 10^{-2}\ T$$

Sentido Negativo do Eixo dos Y



4 → Partícula Alfa → Núcleo $^{4}_{2}He$

$^{1}_{1}H$ - prótio $^{2}_{1}H$ → deutério $^{3}_{1}H$ → trítio

$q_p = |e|$ $q_d = |e|$ $q_e = 2|e|$

$m_p = 1u$ $m_d = 2u$ $m_\alpha = 4u$

ΔV

$$\varepsilon_p = q_p \Delta V = |e| \Delta V = \frac{1}{2} m_p v_p^2$$

$$\varepsilon_d = q_d \Delta V = |e| \Delta V = \frac{1}{2} m_d v_d^2$$

$$\varepsilon_\alpha = 2|e| \Delta V = \frac{1}{2} m_\alpha v_\alpha^2$$

$$\frac{\varepsilon_p}{\varepsilon_d} = 1 = \frac{\cancel{1/2} m_p v_p^2}{\cancel{1/2} m_d v_d^2} = \frac{v_p^2}{2 v_d^2} \qquad 2v_d^2 = v_p^2 \Leftrightarrow \boxed{v_d = \frac{1}{\sqrt{2}} v_p}$$

$$2 = \frac{\varepsilon_\alpha}{\varepsilon_p} = \frac{\frac{1}{2} m_\alpha v_\alpha^2}{\frac{1}{2} m_p v_p^2} = \frac{4 v_\alpha^2}{1 v_p^2} \qquad \frac{4 v_\alpha^2}{v_p^2} = 2 \Leftrightarrow \boxed{v_\alpha = \frac{1}{\sqrt{2}} v_p}$$

$\vec{v} \perp \vec{B}$

$$|\vec{F}_{mag}| = q v B = m \frac{v^2}{R} \qquad \frac{1}{R_p} = \frac{|e| B}{m_p v_p} \qquad \frac{1}{R_\alpha} = \frac{2|e| B}{m_\alpha v_\alpha}$$

$$\frac{1}{R} = \frac{q \cancel{v} B}{m v^{\cancel{2}}} \qquad \frac{1}{R_d} = \frac{|e| B}{m_d v_d}$$

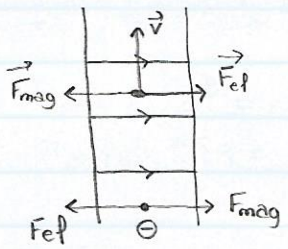
$$\frac{R_p}{R_d} = \frac{\frac{1}{R_d}}{\frac{1}{R_p}} = \frac{\frac{|e|B}{m_d v_d}}{\frac{|e|B}{m_p v_p}} = \frac{m_p v_p}{m_d v_d} = \frac{1}{2}\,\frac{v_p}{\frac{1}{\sqrt{2}} v_R} = \frac{\sqrt{2}}{2}$$

$$\frac{R_p}{R_\alpha} = \frac{\frac{1}{R_\alpha}}{\frac{1}{R_p}} = \frac{\frac{2|e|B}{m_\alpha v_\alpha}}{\frac{|e|B}{m_p v_p}} = \frac{2 m_p v_p}{m_\alpha v_\alpha} = \frac{2 \times v_p}{4 \times \frac{1}{\sqrt{2}} v_p} = \frac{1}{\sqrt{2}}$$

$$\frac{R_d}{R_p} = \sqrt{2} \quad \frac{R_\alpha}{R_p} = \sqrt{2}$$

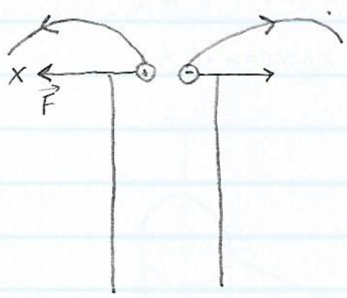
5 → $\vec{E} = 2500\ V/m$

$\vec{B} = 0{,}0350\ T$



$$qE = qvB$$

$$v = \frac{E}{B} = \frac{2500}{0{,}0350} = 7{,}14 \times 10^4\ m$$



$$\vec{F} = q\vec{v} \times \vec{B}$$

$$F = qvB = m\frac{v^2}{R} \qquad R = \frac{mv}{qB} = \frac{2{,}18 \times 10^{-26} \times 7{,}143 \times 10^4}{1{,}6 \times 10^{-19} \times 0{,}0350}$$

$$= 27{,}8\ cm$$

6 → $R = 6{,}00\ cm$

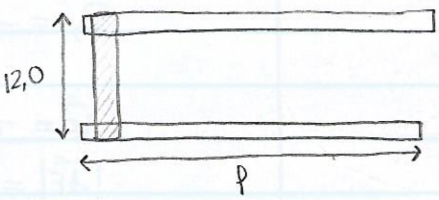
$m = 0{,}720\ Kg$

$d = 12{,}0\ cm$

$\ell = 45{,}0\ cm$

$I = 48{,}0\ A$

$B = 0{,}240\ T$



$$I_{eixo} = \frac{1}{2} m R^2$$

$$|\vec{F}| = q|\vec{v}| \times |\vec{B}|$$

$$|\vec{F}| = IdB = 48{,}0 \times 12{,}0 \times 10^{-2} \times 0{,}240$$

$$= 1{,}38\ N$$

$$\vec{d} = d\,\vec{u}_d$$

$$\vec{B} = -B\,\vec{u}_z \qquad \vec{\ell} \times \vec{B} = \ell\,\vec{u}_x \times (-B\,\vec{u}_z)$$

$$= -\ell B\,(-\vec{u}_y)$$

$$W = \vec{F} \cdot \vec{\Delta y} = 1{,}382 \times 0{,}45$$

$$= 0{,}622\ J$$

$$\Delta E_c = E_{c_f} - E_{c_i} = E_{c_f} = W$$

$$E_c = E_{c_T} + E_{c_R} = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} m v^2 + \frac{1}{2}\left(\frac{1}{2} m R^2\right)\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{4}m(r\omega)^2 = \frac{3}{4}mv^2$$

$v = 1{,}07\ m/s$

7 → $N = 100$ espiras

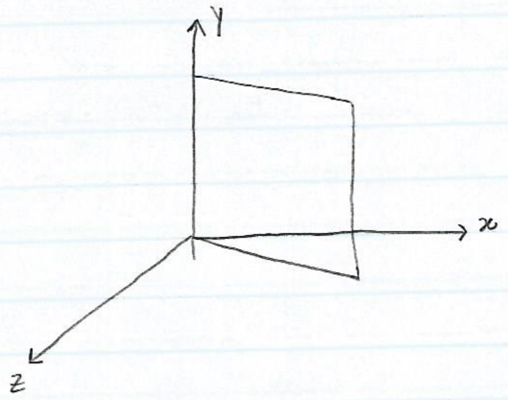
$a = 0{,}400\ m$

$b = 0{,}300\ m$

$\theta = 30^\circ$

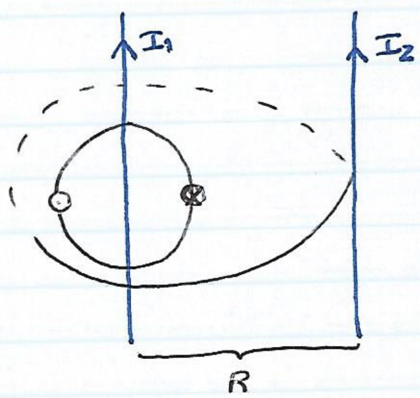
$I = 1{,}20\ A$

Sob uma espira não há força resultante mas há momento magnético.



$\mu = IA \times 100 = 1{,}20 \times a \times b \times 100 =$

Aula 24 - 24.04.2015

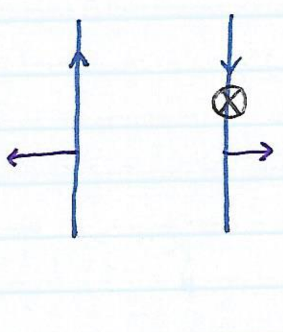


$$\vec{B}_1 = \frac{\mu_0 I}{2\pi R}$$

$$d\vec{F} = I_2 d\vec{\ell} \times \vec{B}_1$$

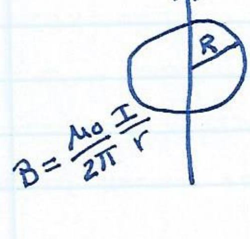
$$|d\vec{F}| = I_2 d\ell B_1$$

$$\vec{F} = \int I_2 d\ell B_1 = I_2 B_1 \ell = \frac{\mu_0 I_1 I_2}{2\pi R}\ell$$



$$\boxed{\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi d_{12}}}$$

Interacção Entre Duas Correntes Paralelas



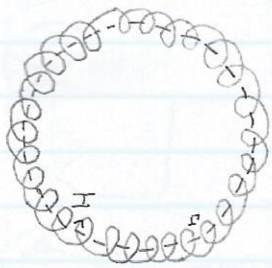
$$\int_{sup\ fechada} \vec{B}\cdot\vec{n}\,dS = 0 \qquad \int_{sup\ fe.} \vec{E}\cdot\vec{n}\,dS = \frac{q}{\varepsilon_0}$$

$$\oint \vec{B}\cdot d\vec{s} = \frac{\mu_0}{2\pi} I \int_{r=R} \frac{1}{r}\,ds = \frac{\mu_0}{2\pi R} I \int ds =$$

$$= \frac{\mu_0}{2\pi R} I \, 2\pi R = \mu_0 I$$

$$\boxed{\oint \vec{B} . dS = \mu I} \rightarrow \text{Lei de Ampere}$$

corrente que atravessa uma superfície limitada por Γ



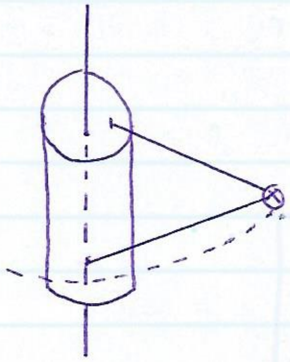
$$\int_{\Gamma} \vec{B} . d\vec{l} = \mu_0 I_{total}$$

$$B_{axial} \int dl = \mu_0 N I$$

$$B_{axial} \, 2\pi R = \mu_0 N I$$

$I_{total} = N I$ ← corrente que percorre o toróide

nº de espiras



$$B_{axial} = \frac{\mu_0}{2\pi R} N I$$

$$\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \vec{u_r}}{r^2}$$

$$\int \vec{B} . d\vec{S} = B \int dS = B 2\pi R$$

$$\Rightarrow 2\pi R B = \mu_0 I$$

$$B = \mu_0 \frac{I}{2\pi R}$$

O campo no exterior do condutor é equivalente a um campo criado por um fio fino no eixo do condutor.

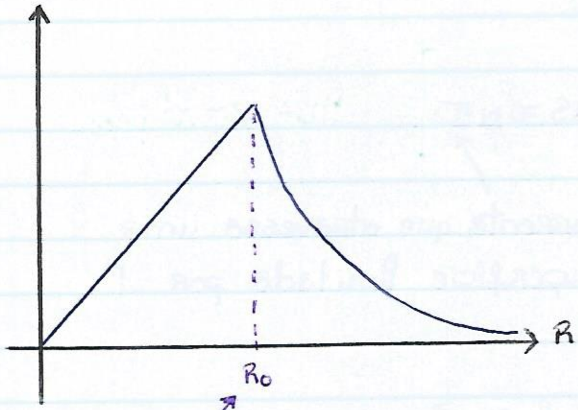
Visto de cima

Para um ponto interior depende da corrente que passa na área da secção.

$$\int \vec{B} . dS = B \, 2\pi R_{int} = \mu_0 I_{total \text{ no interior da trajectória}}$$

$$\Rightarrow B 2\pi R_{int} = \mu_0 J \pi R_{int}^2$$

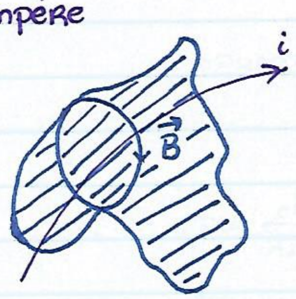
$$B = \frac{\mu_0 J \pi R_{int}^2}{2\pi R_{int}} = \frac{\mu_0 J}{2} R_{int}$$



Raio da secção do condutor

<u>Aula 25</u> - 27.04.2015

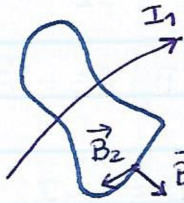
Lei de Ampère

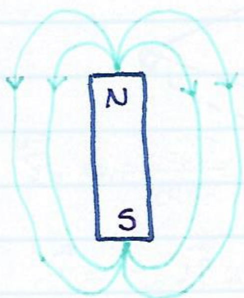


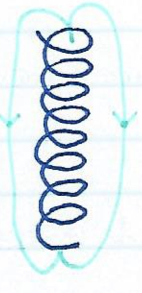
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I = \mu_0 \sum_i I_i$$

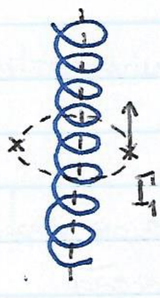
atravessam S

superfície limitada pelo trajecto Γ





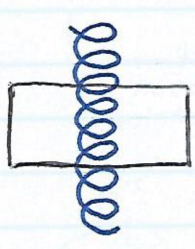


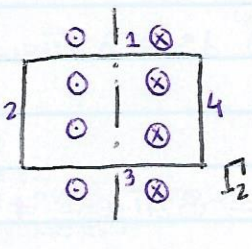


$$\frac{N}{l} = n$$

$$\oint_{\Gamma_1} \vec{B}_{ext} \cdot d\vec{S} = 0$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{u}_r}{r^2}$$





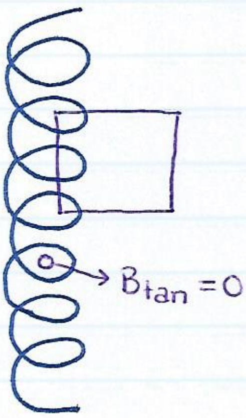
$\Gamma_1 \Rightarrow B_{tan\, \Gamma_1} = 0$

$$\oint_{\Gamma_2} \vec{B} \cdot d\vec{l} = \underbrace{\int_1 \vec{B} \cdot d\vec{l}}_{0} + \int_2 \vec{B} \cdot d\vec{l} + \underbrace{\int_3 \vec{B} \cdot d\vec{l}}_{0} + \int_4 \vec{B} \cdot d\vec{l} = B_2 l_2 - B_4 l_2 = 0$$

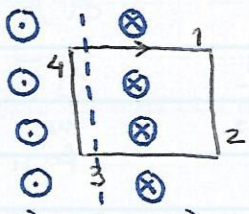
$\Rightarrow B_2 = B_4$

$\oint B = B_2 \ell_2 - B_4 \ell_2 = 0$ e, $B_2 = B_4 = B(\infty) = \phi$

$$\boxed{\begin{array}{c}\text{Solenóide } \infty \\ B_{ext} = 0\end{array}}$$



$\vec{B}$ // eixo solenóide



$$\oint_{\Sigma} \vec{B}.d\vec{\ell} = \underbrace{\int_1 \vec{B}.d\vec{\ell}}_{0} + \int_2 \vec{B}.d\vec{\ell} + \underbrace{\int_3 \vec{B}.d\vec{\ell}}_{0} + \int_4 \vec{B}.d\vec{\ell}$$

$$= B_2 \ell_2 + B_4 \ell_4 = \mu_0 \sum_i I_i \qquad B = \mu_0 n I$$

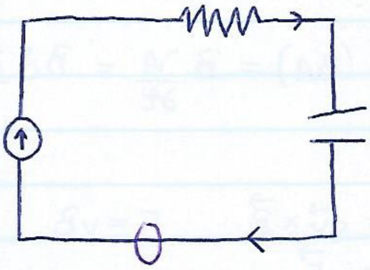
$$B_4 \ell_4 = \mu_0 n \ell_4 I$$

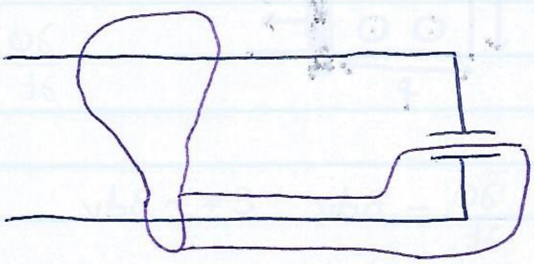
Solenóide ∞: $\vec{B}_{ext} = 0 \quad \vec{B}_{int} = \mu_0 n i \vec{u}_n$ — vector unitário // eixo de simetria
orientação regra da mão direita

$n\ell_2$ = nº de espiras que atravessam

$B = \mu n I$

$\mu_{Fe} = \mu_{r_{Fe}} \mu_0$





$B \, 2\pi r = \mu_0 I \qquad B = \mu_0 \dfrac{I}{2\pi r}$

$$V = \frac{Q}{C} \qquad E = \frac{V}{d} \qquad \phi = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{Cd} = \frac{Q}{\varepsilon_0 \frac{S}{d} d} = \frac{\sigma}{\varepsilon_0} \qquad \frac{d\phi_E}{dt} = \frac{dQ}{dt}\frac{1}{\varepsilon_0} \Rightarrow I = \varepsilon_0 \frac{d\phi_{el}}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

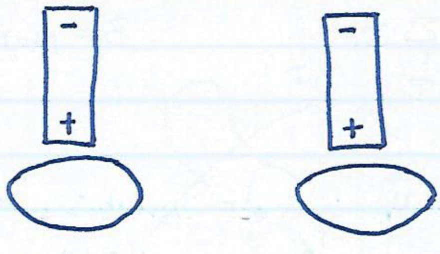
Lei de Ampère-Maxwell

Aula 26 - 29.04.2015

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

$$\oint_{\substack{sup\\fech}} \vec{E} \cdot \vec{n}\, dS = \frac{Q}{\varepsilon_0} \qquad \oint_{\substack{sup\\fech}} \vec{B} \cdot \vec{n}\, dS = 0$$

Indução Electromagnética

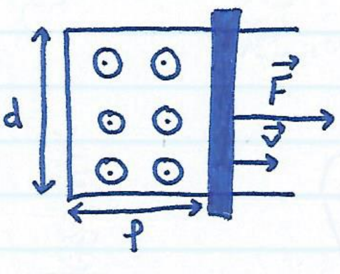


$V_{ind} >$

$$\frac{\partial}{\partial t} \int \vec{B} \cdot \vec{n}\, dS = -\mathcal{E} \qquad \vec{E} = -\nabla V$$

$$\Delta V = \int \vec{E} \cdot d\vec{l}$$

$$\int \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \phi_{mag}$$



$$v \Rightarrow \frac{\partial \phi_{mag}}{\partial t}$$

$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial t}(BA) = B \frac{\partial A}{\partial t} = Bd \underbrace{\frac{\partial l}{\partial t}}_{v}$$

$$\frac{\partial \phi}{\partial t} = Bdv \qquad \mathcal{E} = -Bdv \qquad \vec{F} = q\vec{v} \times \vec{B} = q\vec{E} \qquad E = vB$$

$$\boxed{\frac{\partial \phi}{\partial t} = -\varepsilon}$$ Lei de Lenz

T.P. 9 - 29.04.2015

Série 7

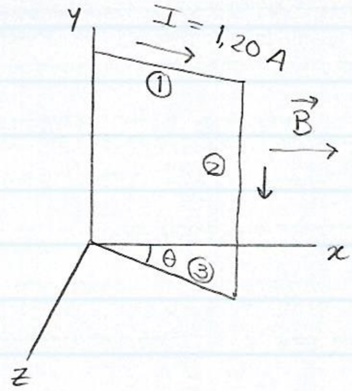
7 → N = 100 espiras

$a = 0{,}400$ m

$b = 0{,}300$ m

$c \ll b$

$\theta = 30°$



a) $B = 0{,}800$ T // Ox

$I = 1{,}20$ A

$F_1 = I \vec{\ell} \times \vec{B}\, \vec{u_x}$

$= I (\ell \cos 30° \, \vec{u_x} + \sin 30° \, \vec{u_z}) \times \vec{B}\, \vec{u_x}$

$= I \ell \sin 30° B \, \vec{u_y}$

$F_2 = I a \vec{u_z} \times B \vec{u_x}$

$= I a B \, \vec{u_z}$

$\vec{\tau} = \vec{r} \times \vec{F_2} + \vec{0}$

$|\vec{\tau}| = b F_2 \sin 60° =$

$= I ab \sin 60° B$

$$\vec{\tau} = \sum_1^{100} \vec{\tau}_1 = 100 \, \vec{\tau}_1$$

$|\vec{\tau}| = 100 \times ab \times \sin 60° B = 9{,}98$ N/m

2ª Maneira

$|\vec{\mu}| = IA = Iab$

$|\vec{\mu}_N| = N I ab$

$\vec{\tau} = \vec{\mu}_N \times \vec{B}$

$|\vec{\tau}| = N I ab \sin 60°$

b) Sentido dos Ponteiros do Relógio

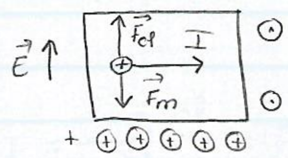
8 → $d_r = 0{,}500$ cm

$I = 8{,}00$ A

$\Delta V = 5{,}10 \times 10^{-12}$ V

$n = 8{,}46 \times 10^{28}$ electroẽs / m³

Efeito de Hall

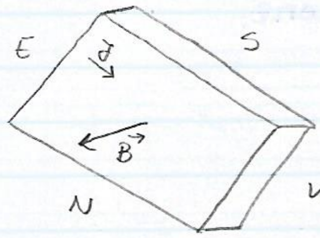


$$\boxed{E_{Hall} = \frac{1}{nq} B J}$$

RH

$qvB = qE \Leftrightarrow v = \frac{E}{B}$

$J = n \cdot q \cdot v = nq \frac{E}{B}$



$$j = \frac{I}{A} = \frac{I}{esp \times h}$$

$$\frac{V_H}{d} = E_H \Leftrightarrow V_H = E_H h$$

$d = h$

$$\frac{V_H}{h} = \frac{1}{nq} B \frac{I}{esp \times h} \Leftrightarrow V_H = \frac{1}{nq} B \frac{I}{esp} \Leftrightarrow B = \frac{nq\, esp}{I} V_H \Leftrightarrow$$

$$\Leftrightarrow B = 4,3 \times 10^{-5}\ T$$

9 → $B = 5,00\ T$ $\qquad m_p = 1,67 \times 10^{-27}\ Kg$

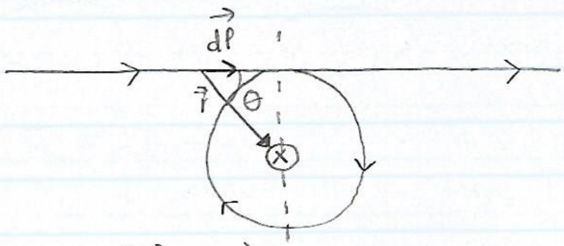
$E_c = 30,0\ MeV = 30,0 \times 10^6\ eV = 30,0 \times 10^6 \times 1,60 \times 10^{-19}\ J = 4,8 \times 10^{-12}\ J$

$$4,8 \times 10^{-12} = \frac{1}{2} m v^2 \Leftrightarrow v^2 = \frac{9,6 \times 10^{-12}}{1,67 \times 10^{-27}} \Leftrightarrow v^2 = 5,75 \times 10^{15} \Leftrightarrow v = 7,58 \times 10^7\ m/s$$

$$F = m\frac{v^2}{R} \Leftrightarrow qvB = m\frac{v^2}{R} \Leftrightarrow qB = \frac{vm}{R} \Leftrightarrow R = \frac{mv}{qB} \Leftrightarrow R = 0,16\ m$$

Série 8

1 →



$$\frac{x}{y} = \cot g\, \theta \Leftrightarrow$$

$$\Leftrightarrow \frac{dx}{d\theta} = \frac{-y}{\sin^2\theta}$$

$$B = \frac{\mu_0}{4\pi} \frac{I}{R^2} (\vec{dl} \wedge \vec{u_r})$$

$$\sin\theta = \frac{y}{r} \Leftrightarrow r = \frac{y}{\operatorname{sen}\theta}$$

$$dB_{fio} = \frac{\mu_0}{4\pi} \frac{I}{r^2} dl \times 1 \times \sin\theta \Leftrightarrow B_{fio} = \int_{-\infty}^{\infty} \frac{\mu_0}{4\pi} \frac{I}{r^2} \operatorname{sen}\theta\, dl \Leftrightarrow$$

$$\Leftrightarrow \frac{B_{fio}}{2} = \int_0^{\infty} \frac{\mu_0}{4\pi} \frac{I}{r^2} \sin\theta\, dl = \int \frac{\mu_0}{4\pi} \frac{I}{r^2} \sin\theta\, dx =$$

$$= -\frac{\mu_0}{4\pi} \times I \int \frac{\sin\theta\ y\, d\theta}{\underbrace{r^2 \sin^2\theta}_{y^2}} = -\frac{\mu_0 I}{4\pi} \int \frac{\sin\theta}{y} d\theta \Leftrightarrow$$

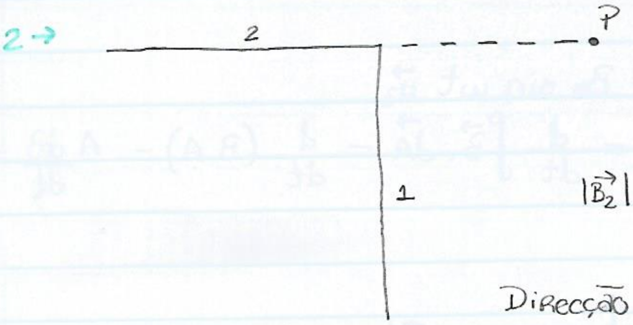
$$\Leftrightarrow B_{fio} = \frac{-\mu_0}{2\pi} \frac{I}{y} \left[-\cos\theta\right]_{\frac{\pi}{2}}^{0} \Leftrightarrow B_{fio} = \frac{\mu_0 I}{2\pi y}$$

$$dB_{esf} = \frac{\mu_0}{4\pi}\frac{I}{r^2}(\vec{dl} \wedge \vec{u_r}) \Leftrightarrow dB_{esf} = \frac{\mu_0}{4\pi}\frac{I}{r^2} \cdot dl \Leftrightarrow$$

$$\Leftrightarrow B_{esf} = \frac{\mu_0}{4\pi}\frac{I}{r^2} 2\pi r = \frac{\mu_0 I}{2r}$$

$$B_{Tot} = B_{fio} + B_{esf} \qquad B = \frac{\mu_0 I}{2R}\left(1 + \frac{1}{\pi}\right)$$

2 →

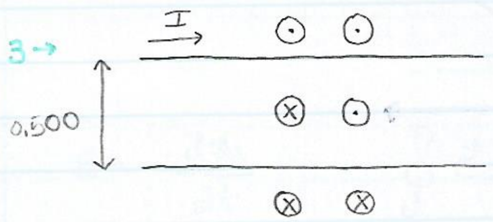


$$|\vec{B_1}| = \frac{\mu_0}{4\pi}\frac{I}{x}$$

$|\vec{B_2}| = 0$ pq $\vec{dl} \times \vec{u_r} = 0$

Direcção perpendicular à folha e sentido para dentro

3 →



$I = 20{,}0\ A$

$F = 320\ \mu N/m$

$$\vec{F} = I_1 l_1 B_2 = I_2 l_2 B_1$$

F_{12} $\qquad$ F_{21}

$$|F_{12}| = \frac{\mu_0}{2\pi}\frac{I_1 I_2 l_1}{D} \qquad \frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{D} =$$

$$|F_{21}| = \frac{\mu_0}{2\pi}\frac{I_1 I_2 l_2}{D} \qquad = 320\ \mu N/m$$

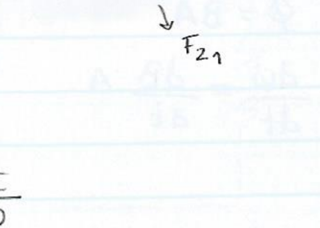
$$B = \frac{\mu_0}{2\pi}\frac{I}{D} \qquad I_2 = \frac{320 \times 10^{-6} \times 0{,}5}{20 \times 2 \times 10^{-7}} = 8 \times 0{,}5 \times 10 = 40\ A$$

$$\vec{B} = \vec{B_1} + \vec{B_2} \qquad |\vec{B}| = |\vec{B_1}| - |\vec{B_2}| = 0$$

$$|\vec{B_1}| - |\vec{B_2}| = 0 \Leftrightarrow |\vec{B_1}| = |\vec{B_2}| \Leftrightarrow \frac{\mu_0}{2\pi}\frac{I_1}{d} = \frac{\mu_0}{2\pi}\frac{I_2}{0{,}5-d}$$

$$\frac{0{,}5-d}{d} = \frac{I_2}{I_1} = 2 \qquad \frac{0{,}5}{d} = 1 = 2 \quad d = \frac{0{,}5}{3} = 0{,}167\ cm$$

Aula 27 - 04.05.2015



$$P = VI = \frac{F_{mag}\,\Delta l}{\Delta t} = F_{mag} V = IlBv$$

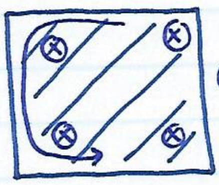
$$\frac{\Delta E}{\Delta t}$$

$$VI = IlBv \Leftrightarrow V = lBv$$

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(B\ell \times \Delta x) = -B\ell \underbrace{\frac{d\Delta x}{dt}}_{v}$$

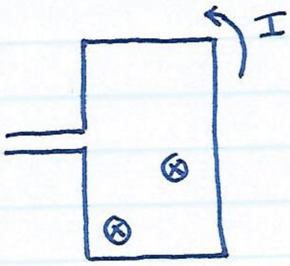
$$\phi = \phi_{\vec{B}ext} + \phi_{\text{criado pela corrente induzida}}$$

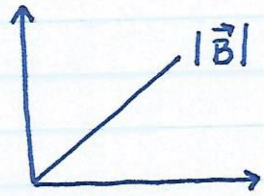
Correntes de Eddy



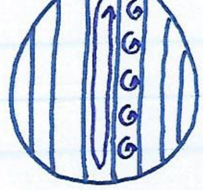
$$\vec{B} = B_0 \sin \omega t \, \vec{u}_z$$

$$\frac{d\phi}{dt} = \frac{d}{dt}\int \vec{B} \cdot d\vec{A} = \frac{d}{dt}(B.A) = A\frac{dB}{dt} = AB_0 \omega \cos$$



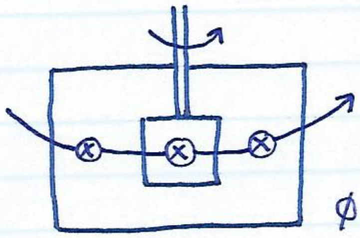


A permeabilidade magnética do ferro aumenta a intensidade do campo magnético.



$$\phi = BA$$

$$\frac{d\phi}{dt} = \frac{dB}{dt} A$$



$$\frac{d\phi}{dt} = B\frac{dA}{dt} = B \alpha \frac{dv}{dt}$$

$$\phi = \vec{B} . dA$$

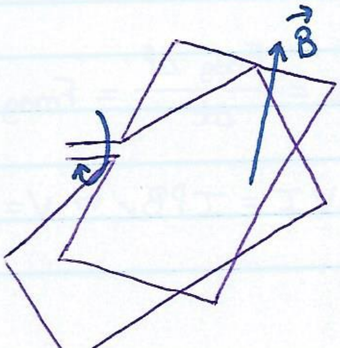
Gerador

motor

campo magnético variável

⇓

mov. mecânico



$$\phi = \vec{B} . \vec{A} = B . A . \cos \theta$$

$$\frac{d\phi}{dt} = BA\left(-\sin\theta \frac{d\theta}{dt}\right)$$

$$V = \underbrace{\overset{320}{310}}\sin(\omega t)$$

$V_{ef} = 220 - 240$ V

$$\frac{-d\phi}{dt} = \mathcal{E} = BA\sin\theta \frac{d\theta}{dt}$$

↓ Corrente Alternada

1ª Eq. de Maxwell: Lei de Gauss: $\oiint \vec{E} \cdot \vec{dA} = \frac{Q}{\varepsilon_0}$

$$\oiint \vec{B} \cdot \vec{dA} = 0$$

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d}{dt}\int \vec{B}\, dS$$

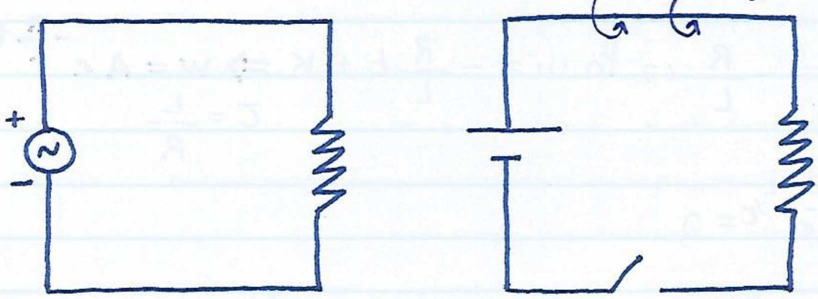
→ $\vec{E}$ não é con em presença $\vec{B}$ variável

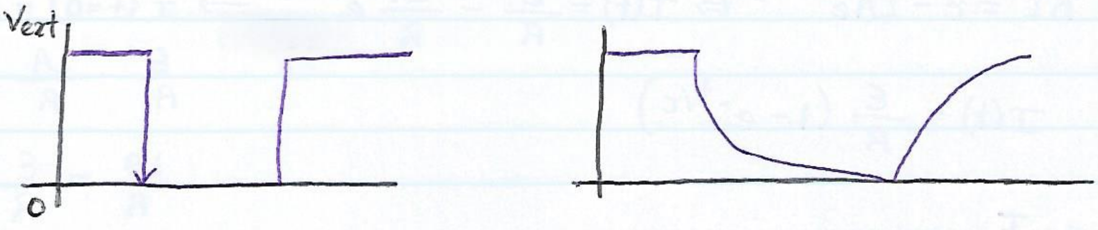
$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \left(I + \varepsilon_0 \int \vec{E} \cdot \vec{dS}\right)$$

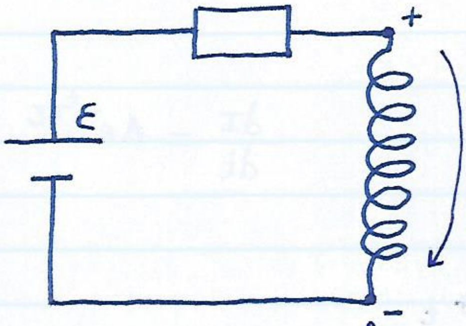
Aula 28 - 06.05.2015

$$\mathcal{E} = -\frac{d\phi_B}{dt} \qquad \oint \vec{E} \cdot \vec{dl} \leftrightarrow \oiint \vec{dA} \cdot \vec{B}$$

$$\mathcal{E} = -\frac{d\phi}{dt}$$







Locais onde aparece a força electromotriz
↓
Traduzida por uma d.d.p.

Sob uma corrente variável

$$\mathcal{E}_{ind} = -\frac{d\phi}{dt} = -N \frac{d\phi}{dt}_{1\text{ espira}}$$

$$\mathcal{E} - RI - \underbrace{\mathcal{E}_{ind}}_{-L\frac{dI}{dt}} = 0$$

$$\mathcal{E} - RI - L\frac{dI}{dt} = 0 \qquad L\frac{dI}{dt} = -\frac{d\phi}{dt}$$

$$\frac{d}{dt}(LI) = -\frac{d}{dt}(N\phi_1)$$

L → autoindutância ou auto-indução

$$\Leftrightarrow LI = N\phi_1$$

$$\phi_1 = \int \vec{B}\cdot d\vec{A} = \mu_0 n I A$$

$$B = \mu_0 n I = \mu_0 \frac{N}{\ell} I$$

$$\boxed{L_{solenóide} = \frac{N\phi_1}{I} = \mu_0 \frac{N^2}{\ell} A}$$

Depende da substância que colocamos no meio

$$\mathcal{E} - RI - L\frac{dI}{dt} = 0$$

$$-R\frac{dI}{dt} - L\frac{d^2I}{dt} = 0 \Leftrightarrow -Rw - L\frac{dw}{dt} = 0 \Leftrightarrow \frac{dw}{dt} = -\frac{R}{L}w \Leftrightarrow$$

$$w = \frac{dI}{dt}$$

$$\Leftrightarrow \frac{1}{w}\frac{dw}{dt} = -\frac{R}{L} \Leftrightarrow \ln w = -\frac{R}{L}t + K \Rightarrow w = Ae^{-\frac{R}{L}t} = Ae^{-t/\tau}$$

$$\tau = \frac{L}{R}$$

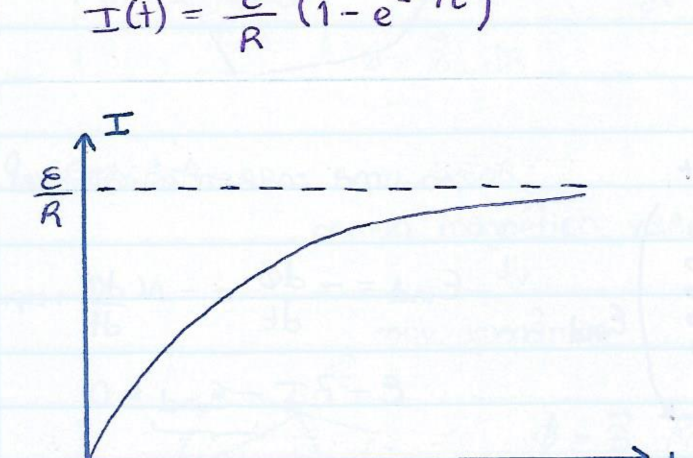
$$\mathcal{E} - RI - LAe^{-t/\tau} = 0$$

$$RI = \mathcal{E} - LAe^{-t/\tau} \Leftrightarrow I(t) = \frac{\mathcal{E}}{R} - \frac{LA}{R}e^{-t/\tau} \rightarrow I(t=0) = 0\,A$$

$$\frac{\mathcal{E}}{R} - \frac{LA}{R} = 0$$

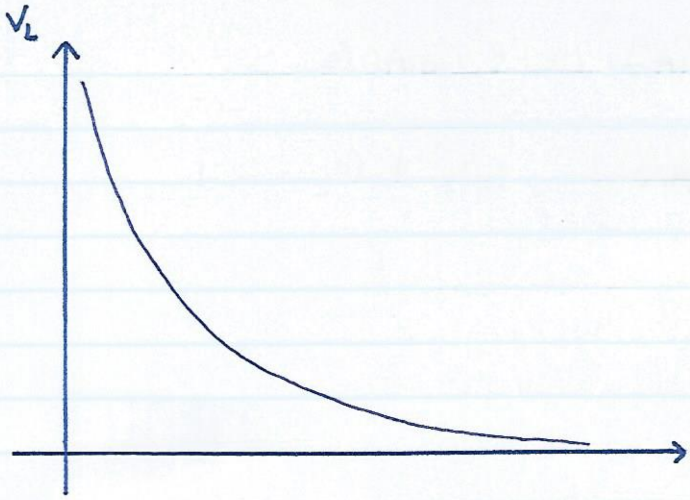
$$\frac{LA}{R} = \frac{\mathcal{E}}{R}$$

$$I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$$



Quando ligamos o interruptor

$$\frac{dI}{dt} = Ae^{-t/\tau}$$



$$\tau = \frac{L}{R} = \frac{10^{-4}}{100} = 10^{-6}\,s = 1\,\mu s$$

$L = 10^{-4}\,H$

$R = 100\,\Omega$

$$L = \frac{|\mathcal{E}_{ind}|}{-\frac{dI}{dt}}$$

$$\left[L\frac{dI}{dt}\right] = [V]$$

$$[L] = \left[V/\frac{dI}{dt}\right] = [V s A^{-1}]$$

henry $\rightarrow$ H

$$\frac{U}{dt} = \frac{1}{2} V_{max} I_{max}$$

$$\frac{U}{dt} = \frac{1}{2} L \frac{I}{\Delta t} I = \frac{1}{2} L \frac{I^2}{dt}$$

T.P. 10 – 06.05.2015

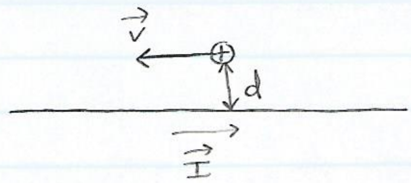
Série 8

4 → $I = 1{,}20\,\mu A$

$v = 2{,}30 \times 10^4\,m/s$

$m = 1{,}673 \times 10^{-27}\,Kg$

$q = 1{,}602 \times 10^{-19}\,C$



$\vec{F}_{mag} + \vec{P} = 0$

$qvB - mg = 0 \Leftrightarrow$

$\Leftrightarrow qvB = mg \Leftrightarrow$

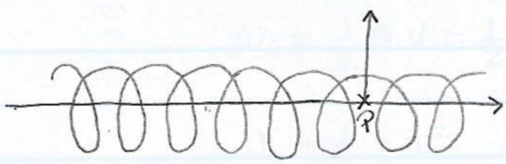
$\Leftrightarrow B = \frac{mg}{qv} \Leftrightarrow$

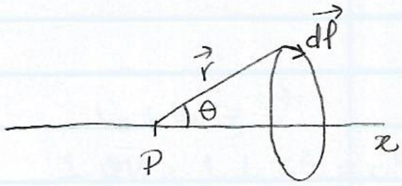
$\Leftrightarrow B = 4{,}45 \times 10^{-12}$

$$B = \frac{\mu_0 I}{2\pi d} \Leftrightarrow 2\pi d B = \mu_0 I \Leftrightarrow d = \frac{\mu_0 I}{2\pi B}$$

$\Leftrightarrow d = 5{,}4 \times 10^{-2}\,m$

6 → a)





$$\vec{dB} = \frac{\mu_0}{4\pi} I \frac{\vec{dl} \times \vec{u_r}}{r^2}$$

$$dB_x = \frac{\mu_0}{4\pi} I \frac{\overbrace{|\vec{dl} \times \vec{u_r}|}^{dl}}{r^2} \sin\theta = \frac{\mu_0 I}{4\pi r^2} \sin\theta\, dl \longrightarrow \text{toda a espira}$$

$$B_x = \int \frac{\mu_0 I}{4\pi r^2} \sin\theta\, dl$$

$\vec{u_r} = -\sin\theta\, \vec{u_y} - \cos\theta\, \vec{u_x}$

$\vec{dl} = dl\, \vec{u_z} \quad |dl||u_r|\,(\sin\theta\, \vec{u_x} - \cos\theta\, \vec{u_y})$

$$B_x = \frac{\mu_0 I}{4\pi r^2} \sin\theta \; 2\pi R = \frac{\mu_0 I R}{2r^2} \sin\theta = \frac{\mu_0 I R^2}{2r^3} = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$$

$\vec{B}_{solenóide} = \sum_{espiras} B_{espira}$

$$= \int \frac{N}{L} B_1 \, dx = \int \frac{N}{L} \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}} dx$$

$x \to \theta$

$$r = \frac{R}{\sin\theta} \quad x = R \cot g\,\theta$$

$$dx = R\, d(\cot g\,\theta)$$

$$= R\, d\left(\frac{\cos\theta}{\sin\theta}\right) =$$

$$= R\left(\frac{-\sin^2\theta - \cos^2\theta}{\sin^2\theta}\right) d\theta$$

$$= -\frac{R}{\sin^2\theta} d\theta$$

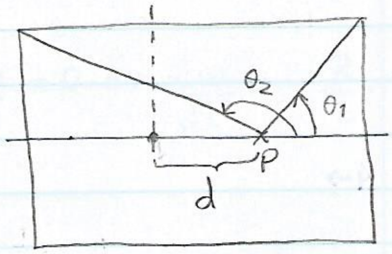
$$\int \frac{N}{L} \frac{\mu_0 I R^2}{2(R/\sin\theta)^3} \left(-\frac{d\theta\, R}{\sin^2\theta}\right)$$

$$= -\int_{\theta_2} \frac{N}{L} \frac{\mu_0 I R^2}{2R^3} R \frac{\sin^3\theta}{\sin^2\theta} d\theta =$$

$$= -\frac{N}{L} \frac{\mu_0 I}{2} \int_{\theta_1} \sin\theta \, d\theta =$$

$$= -\frac{N}{L} \frac{\mu_0 I}{2} \Big[-\cos\theta\Big]_{\theta_1}^{\theta_2}$$

$$B_{solenóide} = \frac{\mu_0 n I}{2} \left[\cos\theta_2 - \cos\theta_1\right]$$



$$B = \frac{\mu_0 n I}{2} (\cos\theta_2 - \cos\theta_1) = -2$$

$$\operatorname{tg}\theta_1 = \frac{\sin\theta_1}{\cos\theta_1} = \frac{R}{\frac{L}{2} - d}$$

$$\operatorname{tg}^2\theta_1 = \frac{\sin^2\theta_1}{\cos^2\theta_1} = \frac{1-\cos^2\theta_1}{\cos^2\theta_1}$$

$$\operatorname{tg}^2\theta_1 \cos^2\theta_1 = 1 - \cos^2\theta_1$$

$$\cos^2\theta_1 (1 + \operatorname{tg}^2\theta_1) = 1$$

$$\cos^2\theta_1 = \frac{1}{1 + \operatorname{tg}^2\theta_1}$$

$$\cos^2\theta_1 = \frac{1}{1 + \frac{R^2}{(\frac{L}{2}-d)^2}} = \frac{(\frac{L}{2}-d)^2}{R^2 + (\frac{L}{2}-d)^2}$$

$$\cos\theta_2 = -\frac{(\frac{L}{2}+d)}{\sqrt{(L/2+d)^2 + R^2}}$$

$$B = -\frac{\mu_0 n I}{2} \left(\frac{L/2 + d}{\sqrt{(\frac{L}{2}+d)^2 + R^2}} + \frac{L/2 - d}{\sqrt{(\frac{L}{2}-d)^2 + R^2}}\right)$$

b)

$$B = -\frac{\mu_0 n I}{2}(1+1)$$

c)

$$B = -\frac{\mu_0 n I}{2} \frac{L}{\sqrt{L^2+R^2}} \qquad B \to -\frac{1}{2}\mu_0 n I$$

8 → $c = 30{,}0\ \text{cm}$

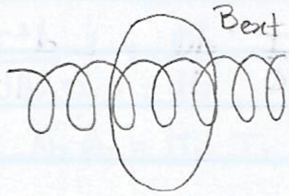
$d = 2{,}50\ \text{cm}$

$N = 300$

$I = 12{,}0\ \text{A}$

a) $\phi = B.A = \left(\dfrac{\mu_0 n I}{\ell}\right)(\pi R^2) = 7{,}4\ \mu\text{Wb}$

b) $\phi = B\,A = \left(\dfrac{\mu_0 n I}{\ell}\right)\left[\pi\left(r_2^2 - r_1^2\right)\right] = 2{,}27\ \mu\text{Wb}$



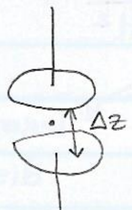
$B_{ext} = 0$

$\phi = \phi_{circ}$ contido no solenóide $+ \phi_{fora}$

$B_{int} = \mu_0 n I$

9 → $r = 10{,}0\ \text{cm}$

$I = 0{,}200\ \text{A}$



$C = \varepsilon_0 \dfrac{A}{d}$

$I = \dfrac{dQ}{dt} = 0{,}2\ \text{A}$

$\dfrac{dE}{dt} = \dfrac{1}{A\varepsilon_0}\dfrac{dQ}{dt} = \dfrac{I}{A\varepsilon_0} = 7{,}19 \times 10^{11}\ \text{V/ms}$

a) $d = 4{,}00\ \text{mm}$

$E = \dfrac{\Delta V}{d}$ $\quad E_{\substack{\text{placas}\\ \text{paralelas}}} = \dfrac{\sigma}{\varepsilon_0}$

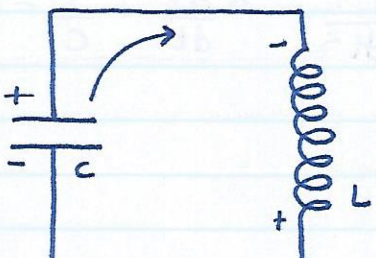
$$E = \frac{\Delta V}{d} = \frac{Q/C}{d} \longrightarrow \frac{Q\,d}{\varepsilon_0 A d}$$

b) $\oint \vec{B}.d\vec{\ell} = \mu_0\left(I + \varepsilon_0 \dfrac{\partial \phi_E}{\partial t}\right)$

$$B \times 2\pi r = \mu_0 \varepsilon_0 \frac{\partial \phi_E}{\partial t} \Leftrightarrow B \times 2\pi r = \mu_0 \varepsilon_0 A \frac{\partial E}{\partial t} \Leftrightarrow$$

$$\Leftrightarrow B \times 2\pi r = \mu_0 \varepsilon_0 \pi r^2 \frac{\partial E}{\partial t} \Leftrightarrow B = \frac{\mu_0 \varepsilon_0 \pi r^2}{2\pi r}\frac{\partial E}{\partial t}$$

Aula 29 - 11.05.2015



$\dfrac{Q}{C} + L\dfrac{dI}{dt} = 0$

$U_C = \dfrac{1}{2} Q V = \dfrac{1}{2}\dfrac{Q^2}{C}$

$U_L = \dfrac{1}{2} L I^2$

$U_C + U_L = c^{te}$

$\dfrac{1}{2} QV + \dfrac{1}{2} L I^2 = c^{te}$

$\omega = \dfrac{1}{\sqrt{LC}}$

$I = 0 \quad \dfrac{1}{2}\dfrac{Q_{max}^2}{C} = c^{te}$

$Q_c = 0 \Leftrightarrow V_c = 0 \Rightarrow \dfrac{1}{2} L I_{max}^2 = \text{constante}$

$$\boxed{\underbrace{\frac{1}{2}\frac{Q_{max}^2}{C}}_{\text{Energia Eléctrica Máxima}} = \underbrace{\frac{1}{2} L I_{max}^2}_{\text{Energia Magnética Máxima}}}$$

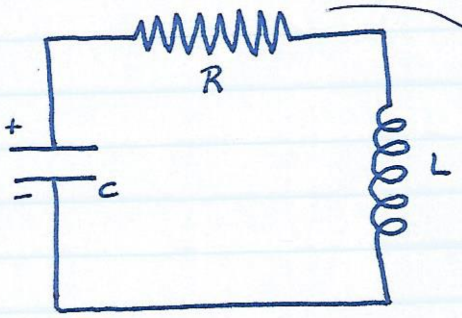
$$\frac{Q_{max}^2}{LC} = I_{max}^2$$

$$\frac{1}{2}\left(2Q\frac{dQ}{dt}\right)\frac{1}{C} + \frac{1}{2}\left(L\,2I\frac{dI}{dt}\right) = 0$$

$$\frac{Q}{C} + L\frac{dI}{dt} = 0 \Leftrightarrow \frac{I}{C}\frac{dQ}{dt} + L\frac{d^2I}{dt^2} = 0 \quad \hookrightarrow \frac{d^2I}{dt^2} = -\frac{1}{LC}I$$

$F = -kx \quad ma = -kx$

$$\frac{d^2x}{dt^2} = -\underbrace{\frac{k}{m}}_{\omega^2}x \qquad \omega^2 = \frac{1}{LC}$$



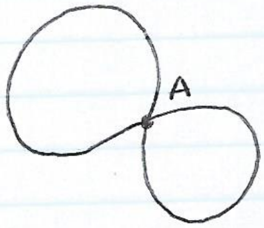
Termo dissipativo porque dissipa energia

$$\frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2 - RI^2 = U$$

$Q = A(t)\cos(\omega t)$

$t = 0 \qquad Q = Q_{max}$

$$\frac{Q}{C} + R\overset{\frac{dQ}{dt}}{I} + L\overset{\frac{d^2Q}{dt^2}}{\frac{dI}{dt}} = 0$$



$$\varepsilon = -\frac{d\phi}{dt}$$

$$\frac{1}{C}\frac{dQ}{dt} - R\frac{dI}{dt} + L\frac{d^2I}{dt^2} = 0 \Leftrightarrow L\frac{d^2I}{dt^2} - R\frac{dI}{dt} + \frac{I}{C} = 0$$

$$L\frac{d^2Q}{dt^2} - R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

$$\updownarrow$$

$$L\frac{d^2I}{dt^2} - R\frac{dI}{dt} + \frac{I}{C} = 0$$

$$Q = Q_0 e^{-\frac{R}{2L}t}\cos(\omega t)$$

$$\omega^2 = \left(\frac{1}{LC} - \frac{R}{2L}\right)$$

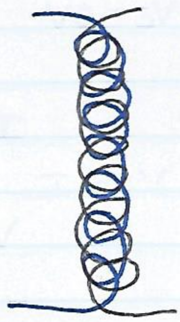
$N\phi = LI$

↳ auto-indutância

$N_2\phi_2 = M I_1$

- Indutância Mútua

$$\mathcal{E}_2 = -\frac{\partial \phi}{\partial t} = -N_2 \frac{\partial \phi_2}{\partial t}$$



$$B_1 = \mu_0 \frac{N_1}{\ell} I_1$$

$$\mathcal{E}_2 = -M \frac{\partial I_1}{\partial t} \qquad |\mathcal{E}_2| = M \frac{\partial I_1}{\partial t}$$

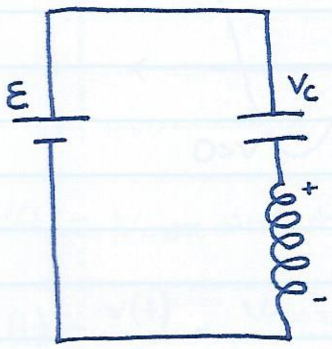
$$\phi_1 = N_1 A B_1 \qquad N_1 \phi_1 = L I_1$$

$$\phi_2 = N_2 A B_1 \qquad N_2 \phi_2 = M_{12} I_1$$

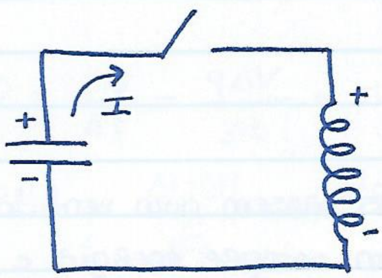
$$\Rightarrow N_2 A \mu_0 \frac{N_1}{\ell} I_1 = M_{12} I_1$$

$$M_{12} = \mu_0 A \frac{N_1 N_2}{\ell}$$

Aula 30 - 13.05.2015



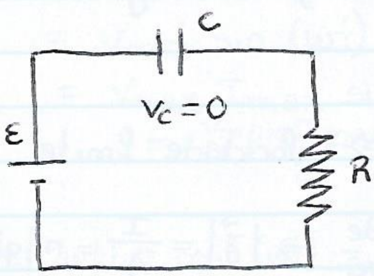
$$\mathcal{E} = V_C + V_L$$
$$= \frac{Q}{C} + L\frac{dI}{dt}$$

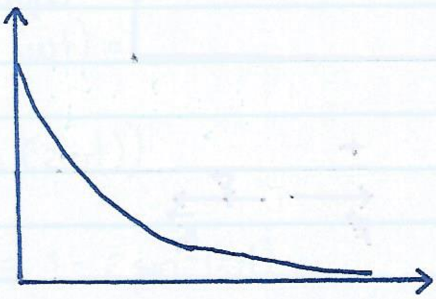


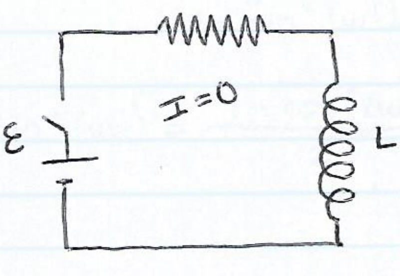
$$V_L = -L\frac{dI}{dt}$$

$$\frac{Q}{C} - \left(-L\frac{dI}{dt}\right) = 0$$

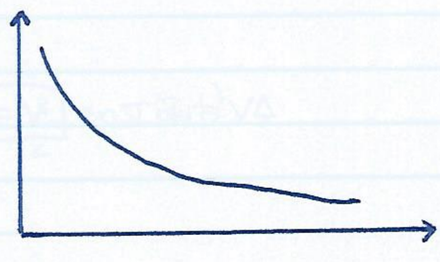
$$\frac{Q}{C} \qquad \frac{dQ}{dt} < 0 \qquad \underbrace{I}_{<0} = \frac{dQ}{dt}$$







$\frac{dI}{dt}$



$$V_L = L\frac{dI}{dt}$$

$t = 0\,s \quad V_c = 0\,V$ (1)

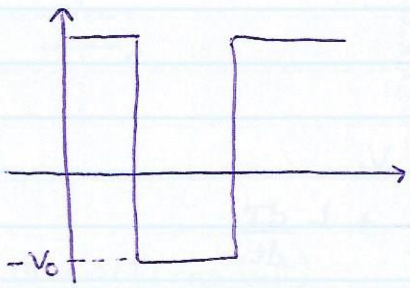
$$I_0 = \frac{\varepsilon}{R} \qquad I_{final} = 0$$

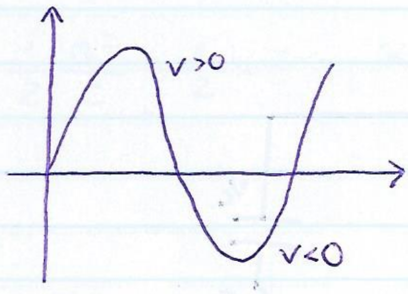
$$V_c = 0 \qquad V_{c\,final} = V + \varepsilon$$

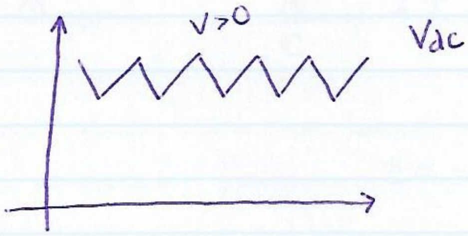
$t = 0\,s \quad I = 0$ (2)

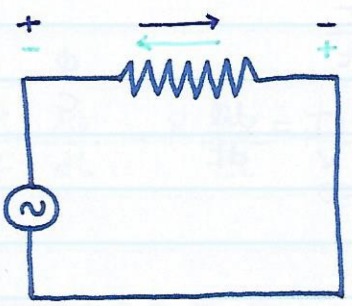
$$\left(\frac{dI}{dt}\right)_0 = \left(\frac{dI}{dt}\right)_{max} \qquad V_L = V_{max} = \varepsilon$$

$$\frac{dI}{dt} \to 0 \quad \text{final} \quad V_c = 0 \quad I = \frac{\varepsilon}{R}$$



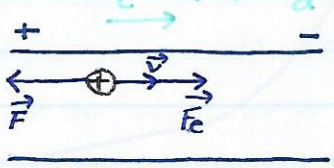






Quer as cargas passem num sentido ou noutro perdem sempre energia e há aquecimento por efeito de Joule

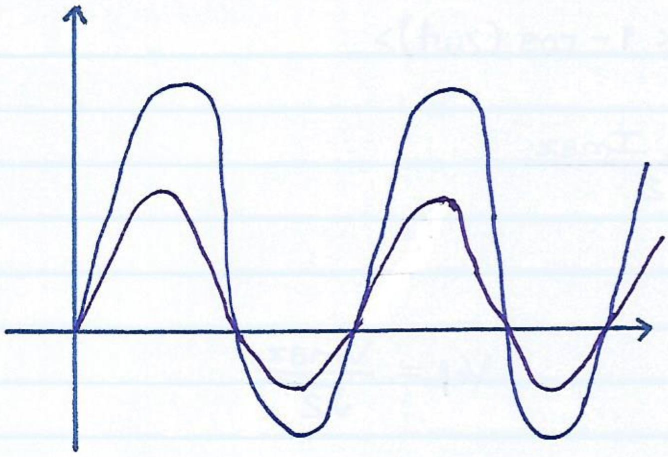
$$\vec{E} = |\vec{E}| = \frac{\Delta V}{d}$$



$V_d \Rightarrow$ Velocidade limite

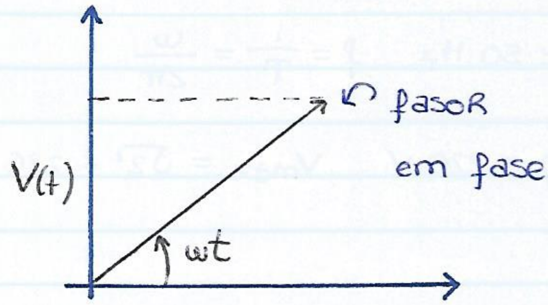
Velocidade de deriva $\rightarrow |\vec{J}| = \frac{I}{A} = n\,|q|\,|\vec{v}|$

$$\Delta V = RI \qquad \boxed{V = RI}$$



I e V estão em fase

$$V = V_{max} \sin(\underbrace{\omega t}_{\text{fase}})$$



$V = RI$

$v = Ri$

$R = \rho \frac{l}{A}$

$$V(t) = V_{max} \sin(\omega t)$$

$$I_{max} = \frac{V_{max}}{R}$$

$$i(t) = \frac{V(t)}{R} = \frac{V_{max}}{R} \sin(\omega t)$$

$$P = \frac{\Delta U}{\Delta t} = \frac{q \Delta V}{\Delta t} = I \Delta V$$

$$P(t) = \frac{\Delta U(t)}{dt} = \frac{dq\, \Delta v(t)}{dt} = I\, \Delta v(t)$$

$$\begin{aligned} p(t) &= v(t)\, i(t) \\ &= V_{max} \sin(\omega t)\, I_{max} \sin(\omega t) = \\ &= V_{max} I_{max} \sin^2(\omega t) = \\ &= \frac{V_{max} I_{max}}{2} (1 - \cos(2\omega t)) \end{aligned}$$

$$\cos(2\omega t) = \underset{1 - \sin^2(\omega t)}{\cos^2(\omega t)} - \sin^2(\omega t) = 1 - 2\sin^2(\omega t)$$

$$\sin^2(\omega t) = \frac{1 - \cos(2\omega t)}{2} = \frac{1}{2} - \frac{1}{2} \cos(\omega t)$$

$$\langle P \rangle = \frac{V_{max} I_{max}}{2} \langle 1 - \cos(2\omega t) \rangle$$

$$= \frac{V_{max} I_{max}}{2}$$

(50 - 60) Hz

(220 - 270) V

$$P = V^* I^* = R I^{*2}$$

$$R I^{*2} = R \frac{I_{max}^2}{2} \qquad V_{ef} = \frac{V_{max}}{\sqrt{2}}$$

$$I^* = \frac{I_{max}}{\sqrt{2}}$$

I eficaz

$$v = V \sin(\omega t) \qquad f \simeq 50 \text{ Hz} \qquad f = \frac{1}{T} = \frac{\omega}{2\pi}$$

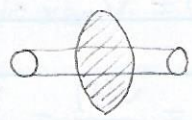
$$\omega \rightarrow T = \frac{2\pi}{\omega}$$

$$V_{ef} = 220 \text{ V} \qquad V_{max} = \sqrt{2} \times 220$$

T.P. 11 - 13.05.2015

1→ $B = \mu_0 n I$

$$\phi_B = \int B . d\vec{A} = \mu_0 n I A \rightarrow \text{solenóide}$$

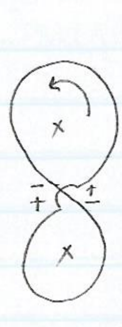


através da secção das espiras exteriores

$$\varepsilon = -N \frac{\partial \phi_B}{\partial t} = -N \mu_0 n \left(\pi r^2_{solenóide}\right) \frac{\partial I}{\partial t} =$$

$$= -14,2 \cos(120 t) \text{ mV}$$

2→



$$2 \text{ T/S} \qquad |\varepsilon_1| = A_1 \frac{\partial B}{\partial t}$$

$$= \pi \times (5 \times 10^{-2})^2 \times 2$$

$$= 15,7 \text{ mV}$$

$$|\varepsilon_2| = \pi \times (9 \times 10^{-2})^2 \times 2$$

$$= 50,9 \text{ mV}$$

$$\varepsilon = \varepsilon_2 - \varepsilon_1$$

$$= 50,9 - 15,7$$

$$= 35,2 \text{ mV}$$

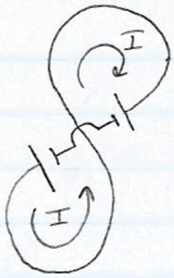
$$l = 2\pi R_1 + 2\pi R_2$$

$$= 0,8796 \text{ m}$$

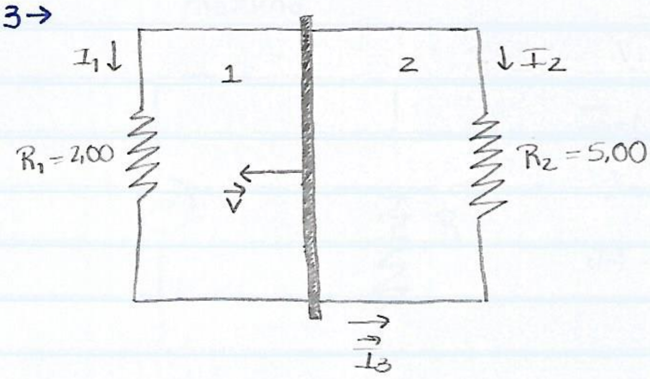
$$R = l \times 3 \ \Omega/m$$

$$= 2,639 \ \Omega$$

$I = 13{,}3 \text{ mA}$



3 →



$\mathcal{E}_1 = -7V$

$\mathcal{E}_2 = 7V$

$$\phi_1 = BA_1 = \frac{\partial \phi_1}{\partial t} = B\frac{\partial A_1}{\partial t} = B\ell\frac{dx}{dt}$$

$$= -B\ell v$$

$$\mathcal{E}_1 = I_1 R_1 \Leftrightarrow \frac{\partial \phi_1}{\partial t} = I_1 R_1 \Leftrightarrow 7{,}00 = 2{,}00 I_1 \Leftrightarrow I_1 = 3{,}50 \text{ A}$$

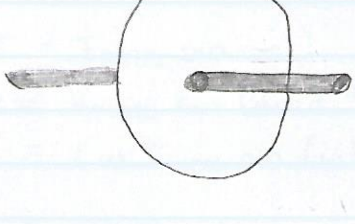
$$\mathcal{E}_2 = I_2 R_2 \Leftrightarrow \frac{\partial \phi_2}{\partial t} = I_2 R_2 \Leftrightarrow 7{,}00 = 5{,}00 I_2 \Leftrightarrow I_2 = 1{,}40 \text{ A}$$

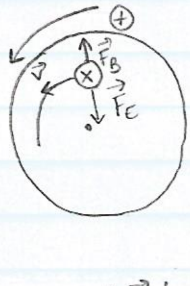
b) $P = R_1 I_1^2 + R_2 I_2^2 = 34{,}3 \text{ W}$

c) $F = I \times \ell \times B = 4{,}29 \text{ N}$ $\quad F = \frac{W}{d} = \frac{Pt}{d} = \frac{P}{v}$

$$P = \frac{W}{t} = \frac{Fd}{t} = Fv$$

4 →





$v = \omega r$

$\vec{F_B} = q\vec{v} \times \vec{B} \Rightarrow |\vec{F_B}| = qvB = q\omega r B$

$|\vec{F_E}| = |\vec{F_B}|$

$qE = q\omega r B \Rightarrow E(r) = \omega r B$

$$\vec{E}(r) = -\omega r B\,\vec{u_r}$$

$$\Delta V = -\int \vec{E} \cdot \vec{dr}$$

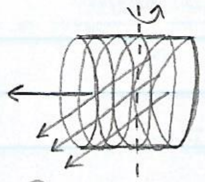
$$= -\int -\omega r B\, dr$$

$$= \omega B \int_A^B r\,dr = \omega B \left[\frac{r^2}{2}\right]_A^B$$

$\Delta V = \omega \frac{BR^2}{2}$ $\quad r_A = 0$, $r_B = R$

$= 24{,}1 \text{ V}$

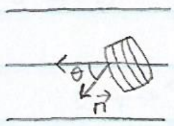
6 →



$60 \text{ Rot/s} = 60 \times 2\pi = 377 \text{ Rad/s}$

$N = 1000$

$A = 0{,}1\, m^2 \qquad B = 0{,}200\, T$



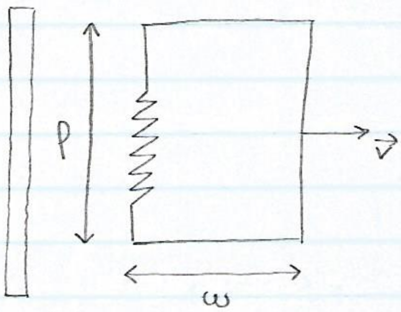
$$\phi_B = \int \vec{B} \cdot \vec{n}\, dA = B \times A \times \cos\theta$$

$$\mathcal{E}_{ind} = -\frac{d\phi_B}{dt} = -B \times A \times \frac{d}{dt}\cos\theta$$

$$= -BA\frac{d}{dt}\cos(\omega t) = \omega BA \sin(\omega t)$$

$$\mathcal{E}_N(t) = N\omega BA \sin(\omega t)$$
$$= (7{,}54 \times \sin(\omega t))\ KV$$

8 →



$1 \to \vec{B} \to$ Podemos usar Lei de Ampère

$2 \to \phi_B$

$$\oint \vec{B} \cdot d\ell = \mu_0 I$$

$$2\pi r B = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$|\vec{B}| = \frac{\mu_0 I}{2\pi r} \qquad \phi_B = \frac{\mu_0 I}{2\pi x}(\ell\, dx) \Rightarrow \phi_B = \frac{\mu_0 I \ell}{2\pi}\int_r^{r+\omega} \frac{1}{x}\, dx$$

$$= \frac{\mu_0 I \ell}{2\pi}\left[\ln x\right]_r^{r+\omega} = \frac{\mu_0 I \ell}{2\pi}\left(\ln(\omega + r) - \ln(r)\right)$$

$$= \frac{\mu_0 I \ell}{2\pi}\ln\left(\frac{\omega + r}{r}\right) = \frac{\mu_0 I \ell}{2\pi}\ln\left(1 + \frac{\omega}{r}\right)$$

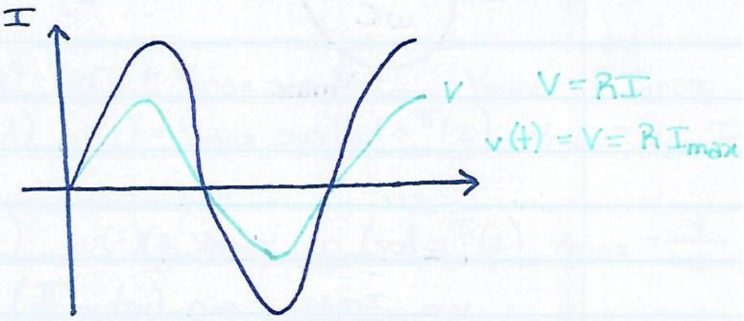
$$\mathcal{E} = -\frac{d\phi}{dt} = \frac{-\mu_0 I \ell}{2\pi}\ \frac{-\frac{\omega}{r^2}}{1 + \frac{\omega}{r}}\ \frac{dv}{dt} = \frac{\mu_0 I \ell \omega}{2\pi(r + \omega) r}\, v$$

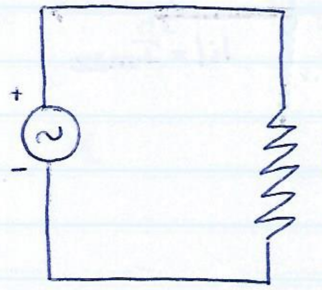
$$I = \frac{\mathcal{E}}{R} = \frac{\mu_0 I \ell v}{2\pi R r}\ \frac{\omega}{r + \omega}$$

Aula 31 - 15.05.2015

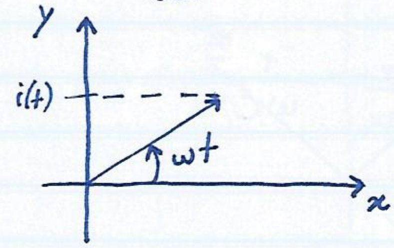
$\omega = \frac{2\pi}{T}$ frequência angular

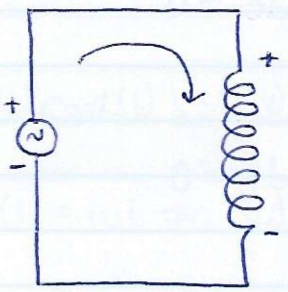
$I = \underbrace{I_{max}}_{\text{amplitude máxima}} \sin \underbrace{(\omega t)}_{\text{fase}}$





$I_{ef} = \frac{I_{max}}{\sqrt{2}} = \sqrt{\langle i^2 \rangle}$





$v = -L \frac{di}{dt}$

$-v(t) = -L \frac{d}{dt} (I_{max} \sin(\omega t))$

$= L I_{max} \omega \cos(\omega t)$

$= L\omega I_{max} \sin\left(\frac{\pi}{2} - \omega t\right)$

$= -L\omega I_{max} \sin\left(\omega t - \frac{\pi}{2}\right)$

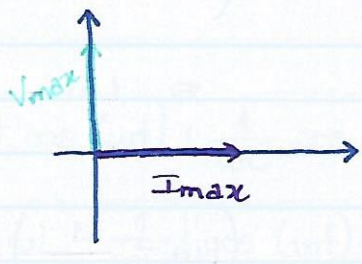
$= L\omega I_{max} \sin\left(\frac{\pi}{2} + \omega t\right)$

Para o indutor:

$I = I_{max} \sin(\omega t)$

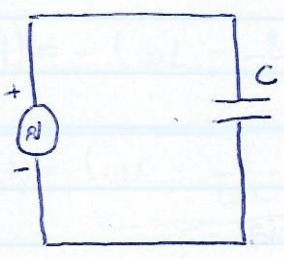
$v = V_{max} \sin(\omega t + \pi/2)$

$= L\omega I_{max} \sin(\omega t + \pi/2)$



$\frac{V_{max}}{I_{max}} = L\omega$

$X_L = \omega L = \frac{V_{max_L}}{I_{max_L}}$



$v_C = \frac{1}{C} \int I_{max} \sin(\omega t)$

$= \frac{1}{C} \left(\frac{-I_{max} \cos(\omega t)}{\omega} \right)$

$= -\frac{I_{max}}{\omega C} \cos(\omega t) = -\frac{I_{max}}{\omega C} \sin\left(\frac{\pi}{2} - \omega\right.$

$$= \frac{I_{max}}{\omega C} \sin\left(\omega t - \frac{\pi}{2}\right)$$

V_{max}

$$V_c = \frac{Q}{C} = \frac{1}{C}\int i\,dt$$

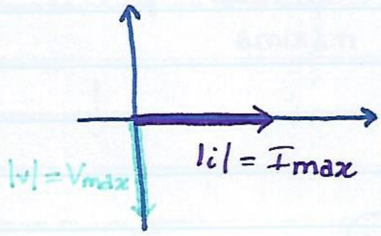
$$i = \frac{dQ}{dt} \Rightarrow Q = \int i\,dt$$

Para o condensador:

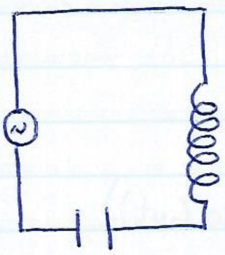
$$i = I_{max} \sin(\omega t)$$

$$v = \frac{I_{max}}{\omega C} \sin\left(\omega t - \frac{\pi}{2}\right)$$

V_{max}



$$\frac{V_{max}}{I_{max}} = \frac{1}{\omega C}$$



$$V_g - L\frac{d}{dt} - \frac{1}{C}\int i\,.dt = 0$$

carregando o condensador e retirando o gerador

$$L\frac{d}{dt} + \frac{1}{C}\int i\,.dt = 0$$

$$i = I_{max} \sin(\omega t)$$

$$L I_{max} \omega \cos(\omega t) + \frac{1}{\omega C}\left(-I_{max}\cos(\omega t)\right) = 0 \Leftrightarrow$$

$$\Leftrightarrow \left(L I_{max} \omega - \frac{I_{max}}{\omega C}\right)\cos(\omega t) = 0 \Leftrightarrow$$

$$\Leftrightarrow L\omega - \frac{1}{\omega C} = 0 \Rightarrow \omega^2 = \frac{1}{LC} \Leftrightarrow \omega = \frac{1}{\sqrt{LC}}$$

$$\underbrace{\left(L\omega - \frac{1}{\omega C}\right)}_{X} I_{max}\cos(\omega t) = V_{max}\sin(\omega t + \phi)$$

e utilizámos $\phi = 0$ para $i = I_{max}\sin(\omega t)$

$$X > 0 \Rightarrow \sin\left(\frac{\pi}{2} + \omega t\right)$$

$$X < 0 \Rightarrow \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$X = X_L - X_C$$

Reactância = Reactância indutiva − Reactância capacitiva

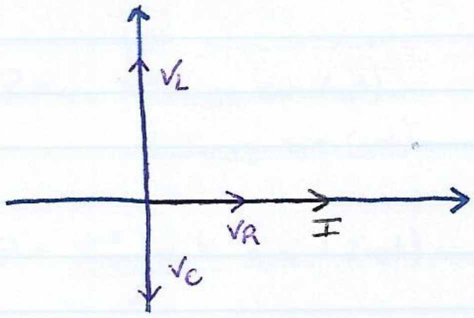
Aula 32 - 18.05.2015

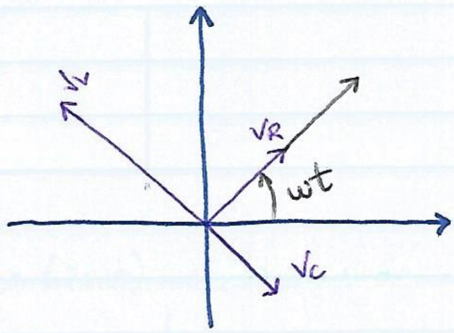
R) $v(t) = R\, i(t)$ $\quad i(t) = I_{max} \sin(\omega t)$ $\quad v(t) = V_{max} \sin(\omega t)$ $\quad V_{max} = R I_{max}$

L) $v(t) = L \frac{di}{dt}$ $\quad i(t) = I_{max} \sin(\omega t)$ $\quad v(t) = V_{max} \sin(\omega t + \pi/2)$ $\quad V_{max} = \omega L I_{max}$

c) $v(t) = \frac{1}{C} \int i\, dt$ $\quad i(t) = I_{max} \sin(\omega t)$ $\quad v(t) = V_{max} \sin(\omega t - \pi/2)$ $\quad V_{max} = \frac{1}{\omega C} I_{max}$

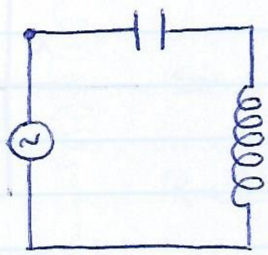
$i = \frac{dQ}{dt} = C \frac{dV}{dt}$ $\qquad V = \frac{Q}{C}$

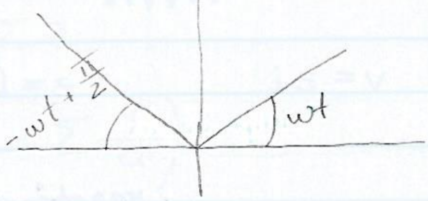




$v(t) = V_C(t) + V_L(t)$

$v(t) = \omega L \sin(\omega t + \pi/2) + \frac{1}{\omega C} \sin(\omega t - \pi/2)$





$v(t) = \omega L(-\cos(\omega t)) + \frac{1}{\omega C} \cos(\omega t) = -\omega t \cos(\omega t) + \frac{1}{\omega C} \cos(\omega t)$

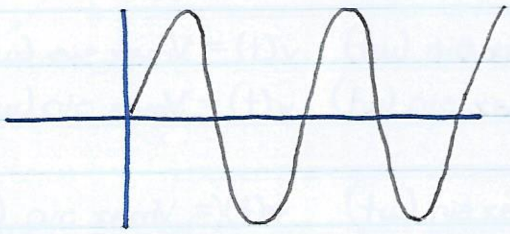
$= -\left(\omega L - \frac{1}{\omega C}\right) \cos(\omega t)$

A Reactância tem as mesmas unidades de Resistência mas implica uma alteração de fase.

$v(t) = -\left(\omega L - \frac{1}{\omega C}\right) \cos(\omega t)$ $\qquad X = X_L - X_C$

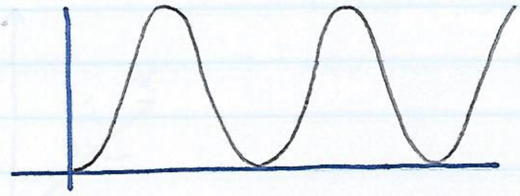
$v(t) = \underbrace{\left(\omega L - \frac{1}{\omega C}\right)}_{X} \sin(\pi/2 + \omega t)$

$I_{max}^2 \times \cos(\omega t)\sin(\omega t) = \frac{1}{2}\sin(2\omega t) \times I_{max}^2$

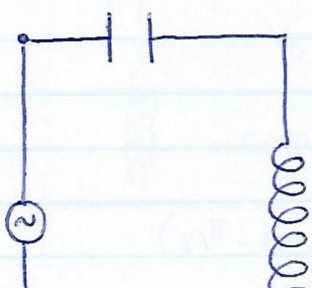


$P = vi$

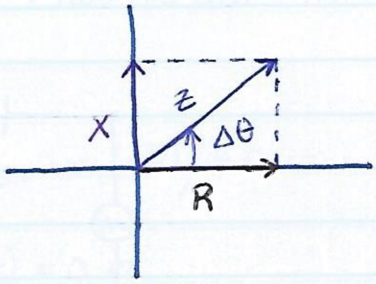
$P = vi = R I_{max}^2 \sin^2(\omega t)$



Circuito RLC

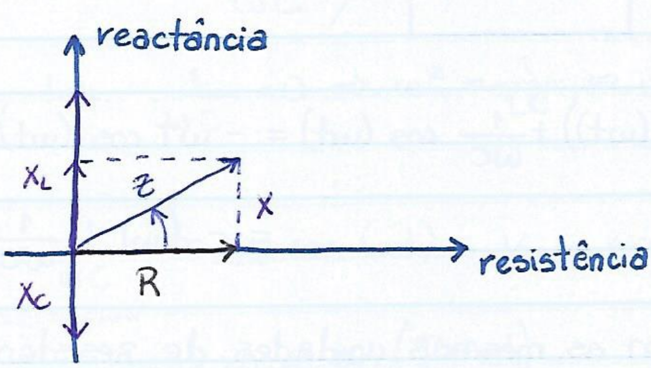


$$v(t) = v_C(t) + v_L(t) + v_R(t)$$
$$= I_{max}\left[\left(\omega_L - \frac{1}{\omega C}\right)\sin(\omega t + \pi/2) + R\sin(\omega t)\right]$$



$v = z\,i$ $\qquad z = (R, X)$

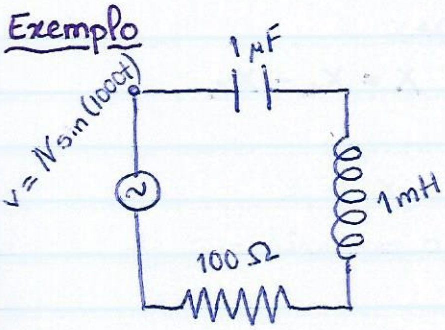
impedância



$$Z = \sqrt{R^2 + X^2}$$
$$\Delta\theta = \mathrm{arctg}\,\frac{X}{R}$$

$$X_L = \omega L$$
$$X_C = \frac{1}{\omega C}$$

Exemplo



$$X_L = \omega L = 1000 \times 10^{-3} = 1\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{1000 \times 10^{-6}} = \frac{1}{10^{-3}} = 1000\,\Omega$$

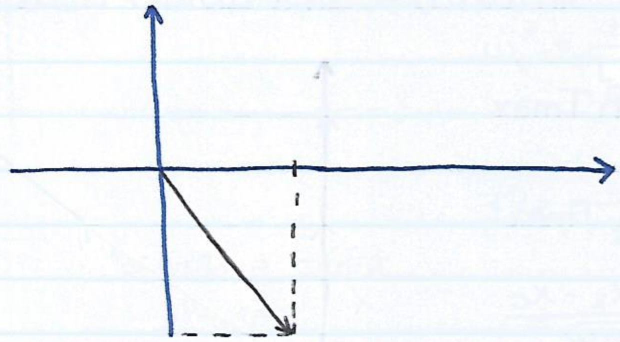
$R = 100\ \Omega$

impedância $Z = (R, X)$

$Z = (100, -999)$

$|z| = \sqrt{100^2 + 999^2}$

$\frac{V_{max}}{I_{max}} = |z|$

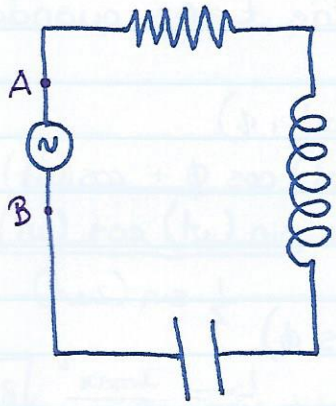


$P = vi = I_{max} \sin(\omega t)$

$z\, I_{max} \sin(\omega t)$

$$P = \frac{I_{max}^2 X}{2} \sin(2\omega t) + \frac{I_{max}^2}{2} R \cos(2\omega t)$$

Aula 33 - 20.05.2015



$Z = (R, X) = (R, X_L - X_C)$

Série $Z_{eq} = \sum Z_i$

Paralelo $\left(\frac{1}{Z_{eq}} = \sum \frac{1}{Z_i}\right)$

$i(t) = I_{max} \sin(\omega t)$

$V_A - V_B = V_R + V_L + V_C = R$

$= R\, I_{max} \sin(\omega t) + V_L + V_C$

$v_L = L \frac{di}{dt} = L\omega I_{max} \cos(\omega t)$

$= R\, I_{max} \sin(\omega t) + \omega L I_{max} \cos(\omega t) - \frac{1}{\omega C} I_{max} \times \cos(\omega t$

$= R\, I_{max} \sin(\omega t) + \left(\omega L - \frac{1}{\omega C}\right) I_{max} \cos(\omega t)$

$$\boxed{V_A - V_B = R\, I_{max} \sin \omega t + X\, I_{max} \cos(\omega t)}$$

$X = X_L - X_C = \omega L - \frac{1}{\omega C}$

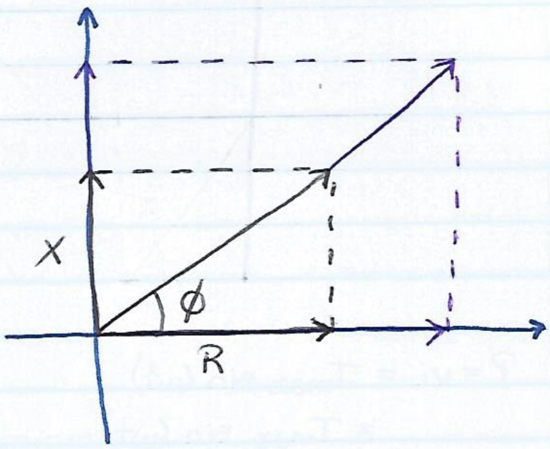
$$V_A - V_B = V_{max} \sin(\omega t + \phi) = V_{max} \sin(\omega t) \cos\phi + V_{max} \cos(\omega t) \sin\phi$$

porque $\sin(a+b) = \sin a \cos b + \cos a \sin b$

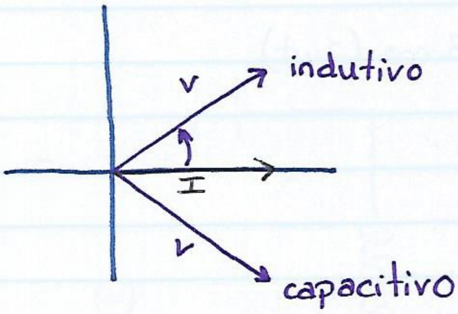
Então, $V_{max} \cos\phi = R I_{max}$

$$V_{max} \sin\phi = X I_{max}$$

$$tg\ \phi = \frac{X}{R} = \frac{X_L - X_C}{R}$$



$$V = Zi$$



$$V = V_0 \sin(\omega t)$$
$$i(t) = I_{max} \sin(\omega t - \phi)$$
$$\Downarrow$$

Porque a tensão vai adiantada e define $t = 0\,s$ quando $V = 0$.

$$P = vi = I_{max} \sin(\omega t) \times V_{max} \sin(\omega t + \phi)$$
$$= I_{max} V_{max} \sin(\omega t) \left[\sin(\omega t) \cos\phi + \cos(\omega t) \sin\phi \right]$$
$$= I_{max} V_{max} \left[\sin^2(\omega t) \cos\phi + \underbrace{\sin(\omega t) \cos(\omega t)}_{\frac{1}{2}\sin(2\omega t)} \sin\phi \right]$$

$$\Rightarrow \langle P \rangle = \frac{I_{max} V_{max}}{2} (\cos\phi)$$

$$\hookrightarrow = \frac{I_{max}^2}{2} \sqrt{R^2 + X^2} \frac{R}{\sqrt{R^2 + X^2}}$$

$$= R I_{ef}^2$$

$$\langle P \rangle = I_{ef} V_{ef} \cos\phi$$

↑ rms → root mean square

$$\frac{V_{max}}{I_{max}} = |Z| = \sqrt{R^2 + X^2}$$

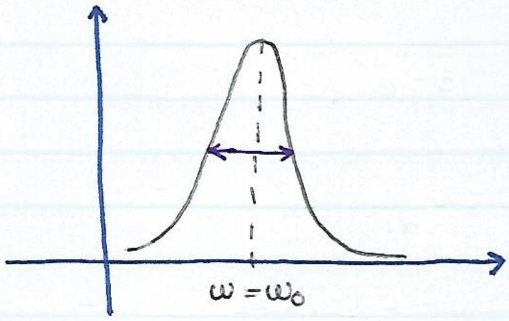
$$\langle P \rangle = \frac{V_{max}^2}{2|Z|} \frac{R}{|Z|}$$

$$\cos\phi = \frac{R}{|Z|}$$

$$= \frac{R V_{max}^2}{2(R^2 + X^2)} = \frac{R V_{max}^2}{2\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)}$$

$$I_{max} = \frac{V_{max}}{|Z|}$$

$$= \frac{R V_{max}^2}{2\left(R^2 + \left(\frac{\omega^2 LC - 1}{\omega C}\right)^2\right)} = \frac{c^2 \omega^2 R}{2\left(\omega^2 c^2 R^2 + L^2 c^2 \left(\omega^2 - \frac{1}{LC}\right)^2\right)}$$



$$\langle P \rangle = \frac{\omega^2 R \; V_{max}^2}{2\left(\omega^2 R^2 + L^2(\omega^2 - \omega_0^2)^2\right.}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\langle P \rangle = \frac{\omega^2 R}{2\left(\omega^2 R^2 + L^2(\omega^2 - \omega_0^2)^2\right.} V_{max}^2$$

$$= \frac{1}{2R} V_{max}^2$$

$Q = \frac{\omega_0}{\Delta\omega}$ ↝ largura a média altura

$$\langle P \rangle = \frac{\langle P_{max} \rangle}{2} \Rightarrow \frac{\omega^2 R}{2\left(\omega^2 R^2 - L^2(\omega^2 - \omega_0^2)^2\right.} = \frac{1}{2} \quad \times \frac{1}{2}$$

$$\frac{\omega^2 R}{\omega^2 R^2 - L^2(\omega^2 - \omega_0^2)^2} = \frac{1}{2R}$$

$$2R^2\omega^2 = \omega^2 R^2 - L^2(\omega^2 - \omega_0^2)^2 = 0$$

$$R^2\omega^2 + L^2(\omega^4 - 2\omega^2\omega_0^2 + \omega_0^4) = 0$$

$$\omega^2 = \frac{L^2}{R^2}$$

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{\frac{1}{\sqrt{LC}}}{\frac{L^2}{R^2}} = \frac{R^2}{L^2(\sqrt{LC})}$$

T.P. 12 – 20.05.2015

Série 10

1→ $\mathcal{E} = 24{,}0 \text{ mV}$

$N = 500$

$I = 4{,}00 \text{ A}$

$\frac{dI}{dt} = 10{,}0 \text{ A/s}$

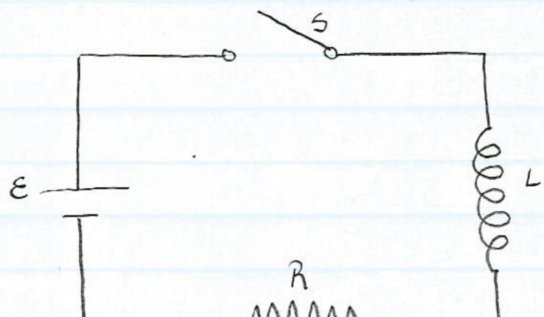
$$\mathcal{E} = -L\frac{di}{dt}$$

$$L = -\frac{\mathcal{E}}{\frac{di}{dt}} = \left|-2{,}4\times10^{-3}\right| = 2{,}4\times10^{-3} \text{ H}$$

$$N\phi = LI \Leftrightarrow \phi = \frac{LI}{N} \Leftrightarrow \phi = \frac{2{,}4\times10^{-3}\times 4{,}00}{500} \Leftrightarrow$$

$$\Leftrightarrow \phi = 1{,}92\times10^{-5} \text{ Wb}$$

2→



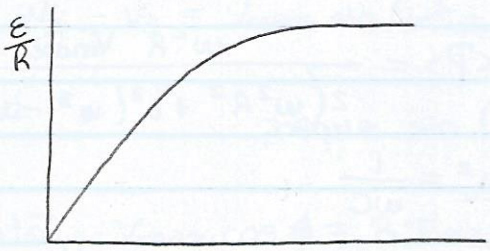
$\mathcal{E} = 6{,}00 \text{ V}$

$L = 8{,}00 \text{ mH}$

$R = 4{,}00\ \Omega$

$\rho_u = \frac{U_{el}}{V} = \frac{1}{2}\varepsilon_0 E^2$

a)



$$\mathcal{E} - \mathcal{E}_{ind} - RI = 0$$

$$\mathcal{E} - L\frac{dI}{dt} - RI = 0 \Leftrightarrow$$

$$\Leftrightarrow 0 - L\frac{d^2I}{dt^2} - R\frac{dI}{dt} = 0 \Leftrightarrow$$

$$\Leftrightarrow 0 - L\frac{dw}{dt} - Rw = 0 \Leftrightarrow$$

$$\Leftrightarrow -L\frac{dw}{dt} = Rw \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{w}\frac{dw}{dt} = -\frac{R}{L} \Leftrightarrow *$$

$$* \ \ln w = -\frac{R}{L}t + K$$

$$\Leftrightarrow w = e^{-\frac{R}{L}t \cdot c} \Leftrightarrow$$

$$\Leftrightarrow \frac{dI}{dt} = Ke^{-\frac{R}{L}t} \Leftrightarrow$$

$$\Leftrightarrow i(t) = Ke^{-\frac{R}{L}t}$$

$$\tau = \frac{L}{R} = 2{,}0 \times 10^{-3}$$

b) $$\frac{dI}{dt} = Ke^{-\frac{t}{\tau}} \Leftrightarrow dI = Ke^{-\frac{t}{\tau}}dt \Leftrightarrow I = \frac{Ke^{-\frac{t}{\tau}}}{|\frac{1}{\tau}|} + c$$

$$I(t=0) = 0 \Leftrightarrow -\frac{K}{|\frac{1}{\tau}|} + c = 0 \Leftrightarrow$$

$$\Leftrightarrow c = \frac{K}{(\frac{1}{\tau})} = K\tau$$

$$\Leftrightarrow I = -\tau Ke^{-\frac{t}{\tau}} + K\tau \Leftrightarrow$$

$$\Leftrightarrow I = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$$

$t = 250\ \mu s = 250 . 10^{-6} s$

$\tau = 2\ ms$

$$I(250\mu s) = \frac{\mathcal{E}}{R}\left(1 - e^{\frac{-250.10^{-6}}{2.10^{-3}}}\right) = \frac{\mathcal{E}}{R}\left(1 - e^{-125.10^{-3}}\right)$$

$$= \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{1}{8}}\right) = 1{,}5\left(1 - e^{-\frac{1}{8}}\right) \simeq 0{,}176\ A$$

c) $I = 1{,}5\ A$

d) $$e^{-\frac{t}{\tau}} = 0{,}2 \Leftrightarrow \frac{-t}{\tau} = \ln(0{,}2) \Leftrightarrow t = \tau \ln(0{,}2) \simeq 3{,}22\ ms$$

$$1 - e^{-\frac{t}{\tau}} = 0{,}8$$

3 → $B = 4{,}50\ T$

$d = 6{,}20\ cm$

$l = 26{,}0\ cm$

$$\rho_u = \frac{U_{mag}}{V} = \frac{1}{2}\frac{B^2}{\mu_0}$$

$$\rho_u = \frac{1}{2}\frac{4{,}50^2}{4\pi \times 10^{-7}} = 8{,}1 \times 10^6\ J/m^3$$

$B = 0$

b) $\rho_u = \frac{U_{mag}}{V} = \frac{U_{mag}}{\pi\left(\frac{d}{2}\right)^2 \times \ell} \Leftrightarrow$

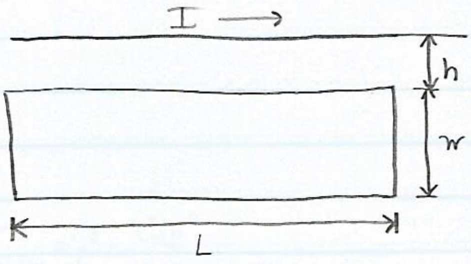
$U_{mag} = \rho_u \times \pi\left(\frac{d}{2}\right)^2 \times \ell \Leftrightarrow U_{mag} = 8{,}06 \times 10^6 \times 7{,}85 \times 10^{-4} \Leftrightarrow$

$\Leftrightarrow U_{mag} = 6{,}33 \times 10^3$ J

4 → $h = 0{,}400$ mm

$w = 1{,}30$ mm

$L = 2{,}70$ mm



Circulação

$\int \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$\phi = \oint \vec{B} \cdot d\vec{A} = \iint B\, dA$

$\mathcal{E}_2 = M_{12} \frac{dI_1}{dt}$

$\mathcal{E}_1 = M_{21} \frac{dI_2}{dt}$

$B\, 2\pi r = \mu_0 I$

$\Leftrightarrow B = \mu_0 \frac{I}{2\pi r}$

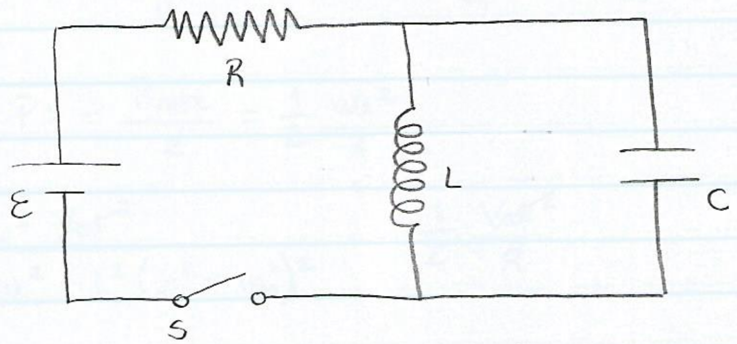
$\phi_{\text{através da espira}} = \int_{-(h+w)}^{-h} \int_0^L \frac{\mu_0 I}{2\pi(-y)}\, dx\, dy$

$= L \frac{\mu_0 I}{2\pi} \int \frac{dy}{y}$

$\phi = \mu_0 \frac{L I}{2\pi} \left[\ln y\right]_{-h-w}^{-h} = \mu_0 \frac{LI}{2\pi} \ln \frac{h}{h+w} = -\mu_0 \frac{LI}{2\pi} \ln \frac{h+w}{h}$

$\phi = M I \Leftrightarrow M = \frac{\phi}{I} \Leftrightarrow M = \frac{\mu_0 L}{2\pi} \ln \frac{w+h}{h} \Leftrightarrow M = 781$ pH

7 →



$\mathcal{E} = 50{,}0$ V

$R = 250\ \Omega$

$C = 0{,}500\ \mu F$

$V_{max} = 150$ V

$U_{cond} = \frac{1}{2} C V^2$

$U_{total} = U_{cond_{max}} = U_{indutor_{max}}$

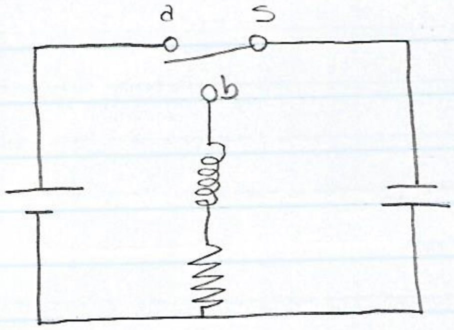
$U_{cond} \hookrightarrow máx \Rightarrow U_{ind} = 0$

$\cancel{\frac{1}{2}} C V_{max}^2 = \cancel{\frac{1}{2}} L I_{max}^2$

$I_{indutor} = \frac{\mathcal{E}}{R}$

$L = \frac{C V_{max}^2}{(\mathcal{E}/R)^2} = 0{,}281$ H

8 →



$R = 7{,}60\ \Omega$

$L = 2{,}20\ mH$

$C = 1{,}80\ \mu F$

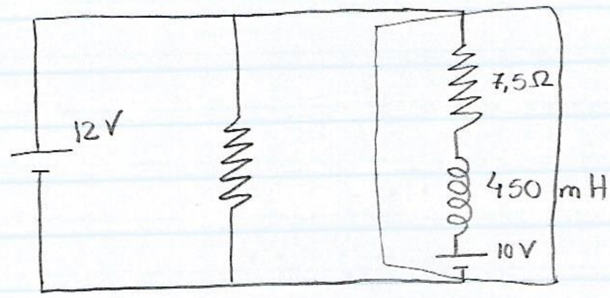
$$\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^{2}}$$

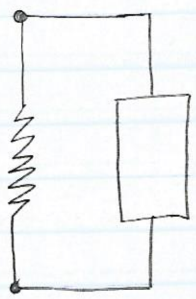
$\omega = 15{,}8\ \text{Krad/s}$

$$f = \frac{\omega}{2\pi} = 2{,}51\ \text{KHz}$$

$$\frac{R^2}{4L^2} = \frac{1}{LC} \Rightarrow R = 69{,}9\ \Omega$$

10 →





$$I_{\text{passar no momento em que abro o motor}} = \frac{12-10}{7{,}5} = \frac{2}{7{,}5}\ A$$

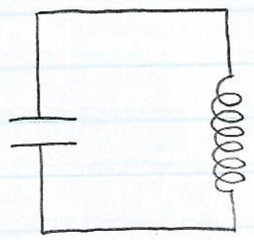
$$RI \leq 80 \Leftrightarrow R \leq \frac{80}{\frac{2}{7{,}5}} \Leftrightarrow R \leq 300\ \Omega$$

5 → $L = 20{,}0\ mH$

$C = 0{,}500\ \mu F$

$I = 0{,}100\ A$

$I_{máx} = 0{,}100\ A$



$$\frac{Q^2}{2C} + \frac{1}{2}Li^2 = U \xrightarrow{\frac{d}{dt}} \frac{2Q}{2C}\frac{dQ}{dt} + \frac{1}{2}L\,2i\,\frac{di}{dt} = 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{Q}{C}\cancel{I} + L\cancel{i}\frac{di}{dt} = 0 \Leftrightarrow \frac{Q}{C} + L\frac{di}{dt} = 0 \Leftrightarrow$$

$$\Leftrightarrow Q(t) = Q_{max} \times \cos\left(\frac{1}{\sqrt{LC}} \times t + \phi\right)$$

$$i(t) = -\frac{Q_{max}}{\sqrt{LC}} \sin\left(\frac{t}{\sqrt{LC}} + \phi\right)$$

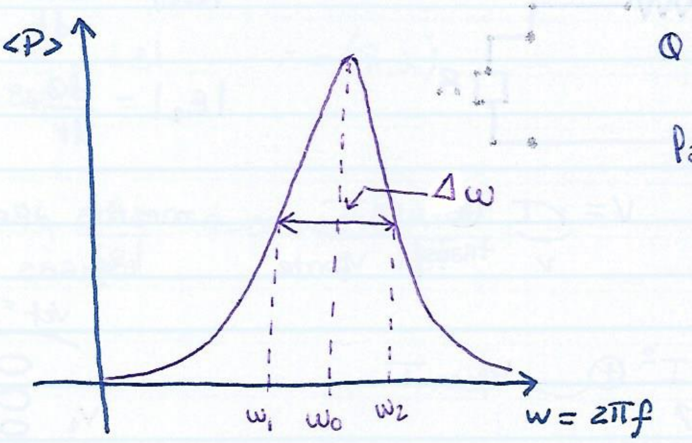
$$I_{max} = \frac{Q_{max}}{\sqrt{LC}} \Leftrightarrow \frac{I_{max} \times \sqrt{LC}}{C} = \Delta V_{max} \Leftrightarrow$$

$$\Leftrightarrow \Delta V_{max} = \frac{0{,}1 \times \sqrt{10 \times 10^{-9}}}{0{,}5 \times 10^{-6}} = \frac{10^{-5}}{0{,}5 \times 10^{-6}} = 20V$$

<u>Aula 34</u> - 22.05.2015

$$\langle P \rangle = \frac{R\omega^2}{R\omega^2 + L^2(\omega^2 - \omega_0^2)^2} V_{ef}^2$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



$$Q = \frac{\omega_0}{\Delta\omega}$$

Largura à meia altura

$$|X_L| = |X_C|$$

$$Z = (R, \underbrace{|X_L| - |X_C|}_{0})$$

$$\frac{V_{ef}}{ef} = \frac{V_{max}}{I_{max}} = |Z|$$

$$\langle P \rangle = \frac{P_{max}}{2} = \frac{1}{2} \frac{V_f^2}{R}$$

$$\frac{R\omega^2 \cancel{V_{ef}^2}}{R^2\omega^2 + L^2(\omega^2 - \omega_0^2)^2} = \frac{1}{2} \frac{\cancel{V_{ef}^2}}{R}$$

$$\frac{R^2 \omega^2}{R^2\omega^2 + L^2(\omega^2 - \omega_0^2)^2} = \frac{1}{2} \Leftrightarrow 2R^2\omega^2 = R^2\omega^2 + L^2(\omega^2 - \omega_0^2)^2$$

$$\Leftrightarrow R^2\omega^2 = L^2(\omega^2 - \omega_0^2)^2 \Leftrightarrow$$

$$\Leftrightarrow L(\omega^2 - \omega_0^2) = + R\omega$$

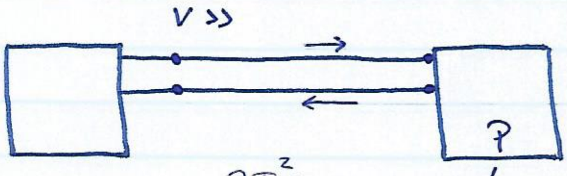
$$\omega^2 - \omega_0^2 = \pm \frac{R}{L}\omega$$

$$(\omega_i - \omega_0)(\omega + \omega_0) = \pm \frac{R}{L}\omega$$

$$\omega - \omega_0 = \pm \frac{R}{L} \frac{\omega}{\underbrace{\omega + \omega_0}_{2\omega}} = \frac{1}{1 + \frac{\omega_0}{\omega}} \simeq \frac{1}{2}$$

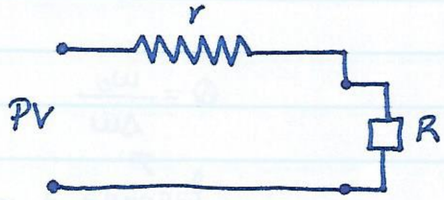
$$\omega - \omega_0 = \pm \frac{1}{2}\frac{R}{L}$$

$$\omega_1 - \omega_0 = \frac{1}{2}\frac{R}{L} \quad \wedge \quad \omega_2 - \omega_0 = -\frac{1}{2}\frac{R}{L} \Rightarrow \omega_1 - \omega_2 = \frac{R}{L} = \Delta\omega$$



$$\frac{P = RI_{ef}^2}{P = VI}$$

$P \swarrow V_{ef} I_{ef}$, $V_{ef} \swarrow$ 220V

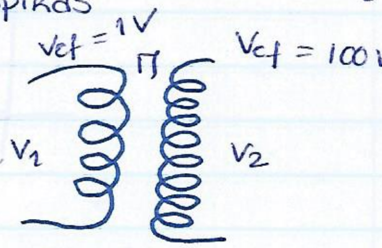


$$|\varepsilon_1| = \frac{d\phi_{1^\circ \text{ enrolamento}}}{dt} = N_1 \frac{d\phi_{1 \text{ espira}}}{dt}$$

$$|\varepsilon_2| = \frac{d\phi_{2^\circ \text{ enrolamento}}}{dt} = N_2 \frac{d\phi_{2 \text{ espira}}}{dt}$$

$$V = \underbrace{rI}_{V} \circledast_{\text{transf}} \underbrace{RI}_{V_{forn}}$$

mesma área das espiras $= \dfrac{\varepsilon_1}{\varepsilon_2} = \dfrac{N_1}{N_2}$



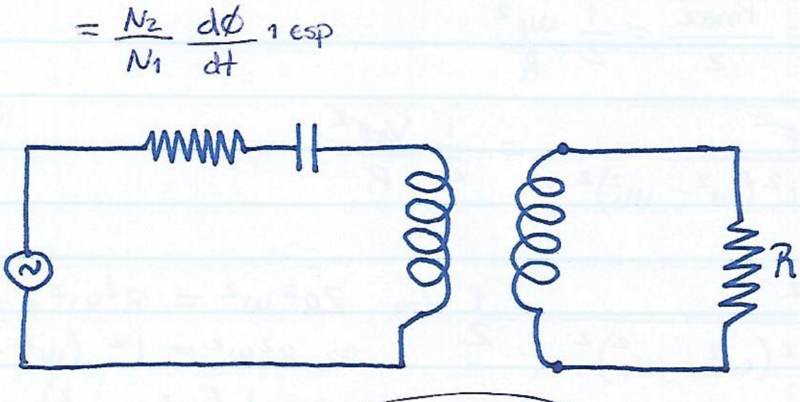
$$VI = rI^2 \oplus V_{forn} I$$

VI: Valor de Tensão Elevada; rI^2: pequeno

$$\frac{V_2}{V_1} = \frac{300}{3} = 100$$

$$V_2 = \varepsilon_2 = M \frac{di_1}{dt}$$

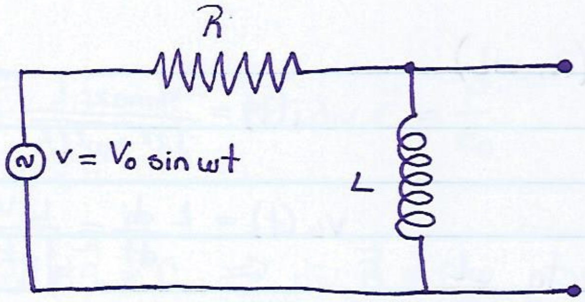
$$= \frac{N_2}{N_1} \frac{d\phi_{1 \text{ esp}}}{dt}$$



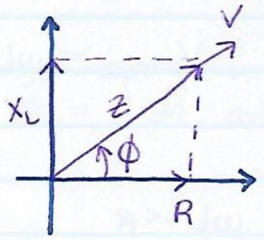
$$\begin{cases} v = Ri + v_c + \left(L \dfrac{di_p}{dt} - M \dfrac{di_s}{dt}\right) \\ M \dfrac{di_p}{dt} = Ri_s \Rightarrow i_s = \dfrac{M}{R}\dfrac{di_p}{dt} \end{cases}$$

$$\to L\frac{di_p}{dt} - M\frac{d}{dt}\left(\frac{M}{R}\frac{di_p}{dt}\right)$$

$$L\frac{di_p}{dt} - \frac{M^2}{R}\frac{d^2 i_p}{dt^2}$$



$$v = v_R + v_L$$
$$= R_i + X_L i$$
$$= R I_{max} \sin(\omega t) + X_L I_{max} \sin(\omega t + \frac{\pi}{2})$$



$v = V_{max} \sin(\omega t)$

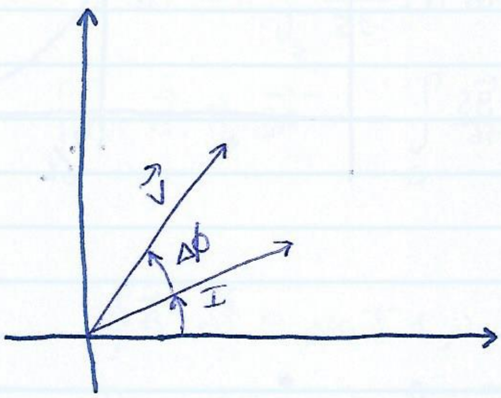
$i = I_{max} \sin(\omega t - \phi)$

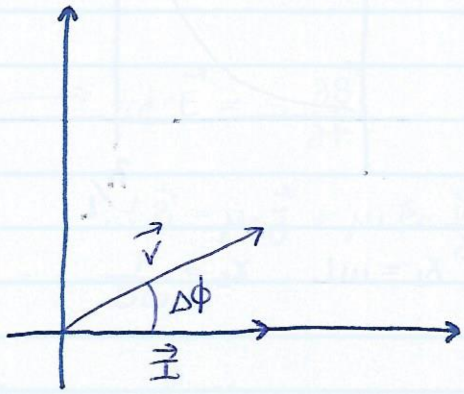
$$I_{max} = \frac{V_{max}}{|z|} = \frac{V_{max}}{\sqrt{R^2 + L^2 w}}$$

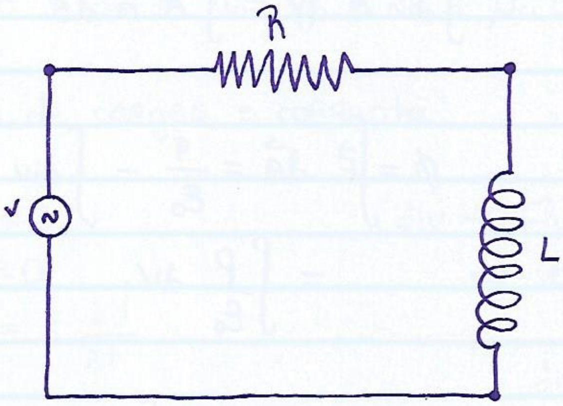
Aula 35 - 25.05.2015

$$\frac{V_{max}}{I_{max}} = \frac{V_{ef}}{I_{ef}} = |z| \qquad z = (R, X)$$

$$\Delta\phi = \mathrm{arctg} \frac{|X|}{|R|} \qquad \Delta\phi = \mathrm{arc\,cos} \frac{|R|}{|z|}$$







$v = V_{max} \sin(\omega t)$

$i = I_{max} \sin(\omega t + \phi)$

$$\frac{V_{max}}{I_{max}} = |z|$$

$$\phi_v - \phi_I = \mathrm{arctg} \frac{|X|}{|R|}$$

$$|Z| = \sqrt{R^2 + \omega^2 L^2} \qquad Z = (R, \omega L)$$

$$I_{max} = \frac{V_{max}}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\phi_v - \phi_i = \operatorname{arctg} \frac{X}{R} = \operatorname{arctg} \frac{\omega L}{R}$$

$$i(t) = \frac{V_{max}}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t - \operatorname{arctg} \frac{\omega L}{R}\right)$$

$$V_L(t) = L \frac{di}{dt} = \frac{L\, V_{max}\, \omega}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \operatorname{arctg} \frac{\omega L}{R}\right)$$

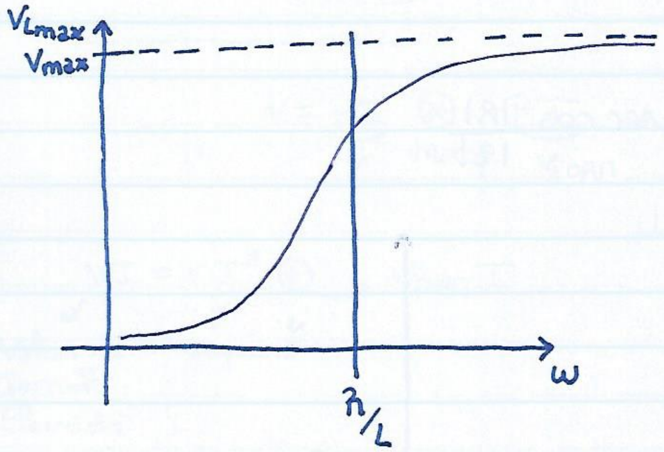
$$V_{L\,max} = \omega L \frac{V_{max}}{\sqrt{R^2 + \omega^2 L^2}}$$

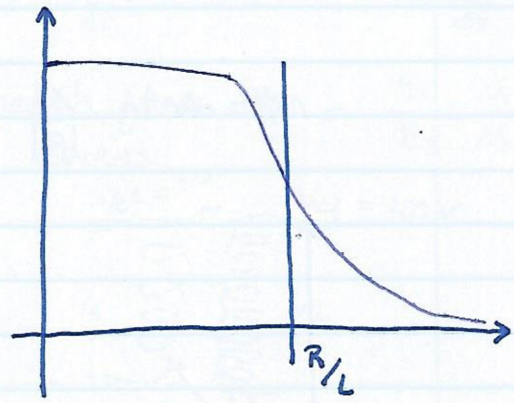
$\omega \gg \quad \omega L \gg R$

$$V_{Lmax} = \frac{\omega L}{\sqrt{\omega^2 L^2}} V_{max}$$

$\omega \ll \quad \omega L \ll R$

$$V_{Lmax} \simeq \frac{\omega L}{R} V_{max} \ll V_{max}$$

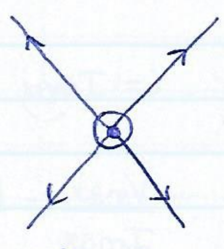




$X_L = \omega L \qquad X_C = \dfrac{1}{\omega C}$

$$\operatorname{div} \vec{A} = \nabla . \vec{A}$$
$$\operatorname{rot} \vec{A} = \nabla \times \vec{A}$$

$$\int \operatorname{div} \vec{A}\, dV = \int \vec{A} . \vec{n}\, dS$$

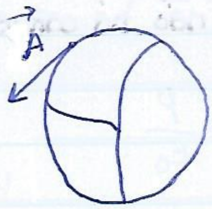


$$\phi = \int \vec{E} . d\vec{A} = \frac{q}{\varepsilon_0} = \int \operatorname{div} \vec{\ } . dV = \frac{q}{\varepsilon_0}$$
$$= \int \frac{\rho}{\varepsilon_0} dV$$

$$\int \vec{E} . d\vec{A} = \frac{q}{\varepsilon_0}$$
$$\operatorname{div} \vec{E} = \frac{q}{\varepsilon_0 V} = \frac{\rho}{\varepsilon_0}$$

$$\int \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0} \Rightarrow \operatorname{div} \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\int \vec{B} \cdot d\vec{A} = 0 \Rightarrow \operatorname{div} \vec{B} = 0$$



$$\oint \vec{v} \cdot d\vec{l} = \int d\vec{A} \cdot \operatorname{rot} \vec{v}$$

$$\int \vec{B} \cdot d\vec{l} = \int d\vec{A} \cdot \operatorname{rot} \vec{B}$$

Circulação do vector
= fluxo do rotacional na área
limitada pelo caminho

$$-\frac{\partial}{\partial t} \phi = \varepsilon = \int \vec{E} \cdot d\vec{l}$$

$$\int \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{n} \, dA$$

$$\int_S \operatorname{rot} \vec{E} \cdot \vec{n} \, d\vec{A} = -\int_S \frac{\partial \vec{B}}{\partial t} \vec{n} \, dA \longrightarrow \operatorname{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\operatorname{rot} \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{\partial \phi_E}{\partial t}$$

$$\int \operatorname{rot} \vec{B} \cdot \vec{n} \, d\vec{A} = \mu_0 \int \vec{J} \cdot \vec{n} \, dA + \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot \vec{n} \, dA$$

Na ausência de cargas e corrente

$$\operatorname{div} \vec{E} = 0$$
$$\operatorname{div} \vec{B} = 0$$
$$\operatorname{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\operatorname{rot} \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

div → Fluxo Elementar por unidade de volume

$$\operatorname{div} |\vec{C}| = \lim_{V \to 0} \frac{\int \vec{C} \cdot \vec{n} \, dA}{V}$$

$$\operatorname{rot} \vec{C} = \lim_{A \to 0} \frac{\oint \vec{C} \cdot d\vec{l}}{A}$$

Aula 36 - 27.05.2015

A forma diferencial é útil para qualquer ponto do espaço, enquanto que a forma integral precisa de um volume definido do espaço.

se não há cargas nem correntes

$$\begin{cases} \operatorname{div} \vec{E} = \dfrac{\rho}{\varepsilon_0} \\ \operatorname{div} \vec{B} = 0 \\ \operatorname{Rot} \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \operatorname{Rot} \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} \end{cases} \qquad \begin{cases} \operatorname{div} \vec{E} = 0 \\ \operatorname{div} \vec{B} = 0 \\ \operatorname{Rot} \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \operatorname{Rot} \vec{B} = \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} \end{cases}$$

$$\operatorname{Rot} \vec{E} = \nabla \times \vec{E} = \begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = -\frac{\partial E_y}{\partial z} \vec{u}_x + \frac{\partial E_y}{\partial x} \vec{u}_z$$

$$\frac{\partial E_y}{\partial x} \vec{u}_z - \frac{\partial E_y}{\partial z} \vec{u}_x = -\frac{\partial \vec{B}}{\partial t}$$

$\vec{u}_y \,//\, \vec{E}$ $\quad \vec{E} = E_y \vec{u}_y$

$\vec{u}_z \,//\, \vec{B}$ $\quad \vec{B} = B_z \vec{u}_z$

$$\frac{\partial E_y}{\partial x} \vec{u}_z - \underbrace{\frac{\partial E_y}{\partial z}}_{=0} \vec{u}_x = -\frac{\partial \vec{B}_z}{\partial t}$$

1 $\Rightarrow \dfrac{\partial E_y}{\partial x} \vec{u}_z = -\dfrac{\partial B_z}{\partial t}$

$$\nabla \times \vec{B} = \begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & 0 & B_z \end{vmatrix} = \underbrace{\frac{\partial B_z}{\partial y}}_{=0} \vec{u}_x - \frac{\partial B_z}{\partial x} \vec{u}_y = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t} \vec{u}_y$$

2 $\Rightarrow -\dfrac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \dfrac{\partial E_y}{\partial t}$

$$\begin{cases} \dfrac{\partial E_y}{\partial x} = -\dfrac{\partial B_z}{\partial t} \\[2ex] -\dfrac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \dfrac{\partial E_y}{\partial t} \end{cases} \Leftrightarrow \begin{cases} \dfrac{\partial^2 E_y}{\partial x^2} = -\dfrac{\partial}{\partial x}\dfrac{\partial B_z}{\partial t} \\[2ex] -\dfrac{\partial \partial B_z}{\partial t \partial x} = \mu_0 \varepsilon_0 \dfrac{\partial^2 E_y}{\partial t^2} \end{cases}$$

$$\boxed{\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}}$$

$$(A\sin(kx-\omega t))' = -A\omega\cos(\ )$$
$$(A\sin(kx-\omega t))'' = -A\omega^2\sin(.)$$

$$\frac{\partial f(x,t)}{\partial x^2} = -k^2 f \qquad f = -\frac{1}{k^2}\frac{\partial^2 f}{\partial t^2} = -\frac{1}{\omega^2}\frac{\partial^2 f}{\partial t^2}$$

$$\frac{\partial^2}{\partial t^2} f(x,t) = -\omega^2 f \qquad \frac{\partial^2 f}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2 f}{\partial t^2}.$$

$$\omega = \frac{2\pi}{T} \quad k = \frac{2\pi}{\lambda} \qquad \frac{k^2}{\omega^2} = \frac{T^2}{\lambda^2} = \frac{1}{(\lambda/T)^2}$$

$$\frac{\partial^2 f}{\partial x} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

$$\begin{cases} \dfrac{\partial \partial E_y}{\partial t} = -\dfrac{\partial^2 B_z}{\partial t^2} \\[2ex] -\dfrac{\partial B_z}{\partial x^2} = \mu_0 \varepsilon_0 \dfrac{\partial}{\partial x}\dfrac{\partial E_y}{\partial t} \Rightarrow \dfrac{\partial B_z}{\partial x^2} = \mu_0 \varepsilon_0 \dfrac{\partial^2 B_z}{\partial t^2} \end{cases}$$

$$\frac{1}{v^2} = \varepsilon_0 \mu_0 \Rightarrow v^2 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times \frac{1}{4\pi \times 9 \times 10^9}}} = \sqrt{\frac{9\times 10^9}{10^7}} =$$

$$= \sqrt{9\times 10^{16}} = 3\times 10^8 \text{ m/s} \qquad \mu_0 = 4\pi \times 10^{-7}$$
$$\varepsilon_0 = \frac{1}{4\pi \times 9 \times 10^9}$$

$$E_y = E_{max} \sin(Kx - \omega t)$$
$$B_z = B_{max} \sin(Kx - \omega t)$$

$$\frac{\partial E_y}{\partial x} = K E_{max} \cos(Kx - \omega t)$$

$$\frac{\partial B_z}{\partial t} = \omega B_{max} \cos(Kx - \omega t)$$

$$\begin{cases} \dfrac{\partial E_y}{\partial x} = -\dfrac{\partial B_z}{\partial t} \\ -\dfrac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \dfrac{\partial E_y}{\partial t} \end{cases}$$

$\Rightarrow$ existem ondas electromagnéticas $c \sim 3 \times 10^8$ m/s

$$K E_{max} \cos(Kx - \omega t) = -\omega B_{max} \cos(Kx - \omega t)$$

$$K E_{max} = \omega B_{max}$$
$$E_{max} = \frac{\omega}{k} B_{max}$$

$$\boxed{E_{max} = c B_{max}}$$

$$\frac{E_{max}}{B_{max}} \quad \frac{1}{2}\frac{B_{max}^2}{\mu_0} \times \frac{E_{max}}{B_{max}}$$

$$\frac{U_E}{V} = \frac{1}{2}\varepsilon_0 E^2 = u_E \qquad u_B(t) = \frac{1}{2}\frac{B(t)^2}{\mu_0} = \frac{1}{4}\frac{B_{max}^2}{\mu_0}$$

$$\frac{U_B}{V} = u_B = \frac{1}{2}\frac{B^2}{\mu_0} \qquad u_E(t) = \frac{1}{2}\varepsilon_0 E^2(t)$$

$$\langle u_E \rangle = \frac{1}{4}\varepsilon_0 E^2$$

$$B(t)^2 = B_{max}^2 \sin^2(Kx - \omega t) \qquad E_{max}^2 = \frac{1}{\varepsilon_0 \mu_0} B_{max}^2$$
$$\langle B^2 \rangle = B_{max}^2 \frac{1}{2}$$

$$B_{max}^2 = \varepsilon_0 \mu_0 E_{max}^2$$

$$\langle U \rangle = \langle u_E \rangle + \langle u_B \rangle$$
$$= \frac{1}{4}\varepsilon_0 E_{max}^2 + \frac{1}{4}\frac{B_{max}^2}{\mu_0}$$

$$u = \frac{1}{2}\varepsilon_0 E_{max}^2 = \frac{1}{2}\frac{B_{max}^2}{\mu_0}$$

$$U/A_t = J_u = \frac{1}{2}\varepsilon_0 E_{max}^2 c = \frac{1}{2}\varepsilon_0 E_{max} c B_{max} c$$

$$= \frac{1}{2}\varepsilon_0 c^2 E_{max} B_{max}$$

$$\frac{U}{At} = \frac{U}{Vt} d = \frac{U}{V} v = u \underset{c}{\underset{\shortparallel}{v}}$$

$$< J_u > = \frac{U}{At} = uc = \frac{1}{2}\frac{E_{max}B_{max}}{\mu_0}$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$\frac{\partial B}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

$$\boxed{\vec{J}_u(t) = \vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}}$$

Vector de Pointing

T.P. 13 - 27.05.2015

Série 11

1 → $V_{max} = 15,0\ V$

$R_{eq} = 8,20 - 10,40 = 18,60\ \Omega$

$$I_{max} = \frac{V_{max}}{R_{eq}} = \frac{15,0}{18,60} = 0,806\ A$$

$$<P> = I_{ef}^2 R = \left(\frac{0,806}{\sqrt{2}}\right)^2 10,4 = 3,38\ W$$

2 → $\Delta V_{max} = 80,0\ V$

$\omega = 65,0\pi$

$L = 70,0\ mH$

a) $X_L = L\omega = 70,0\times10^{-3} \times 65,0\pi = 14,3\ \Omega$

b) $t = 15,5\ ms = 15,5\times10^{-3}\ s$

$$\Delta V = V_{max}\sin(\omega t) = 80,0\sin(\omega t)$$

$$\Delta V_L = L\frac{di}{dt} \Leftrightarrow \frac{di}{dt} = \frac{\Delta V_L}{L} \Leftrightarrow \frac{di}{dt} = \frac{80,0\sin(\omega t)}{70,0\times10^{-3}} \Leftrightarrow$$

$$\Leftrightarrow I = \frac{80,0}{70,0\times10^{-3}}\left(-\frac{\cos(65,0\pi t)}{65,0\pi}\right) + \underset{0}{\underset{\shortparallel}{K}} \Leftrightarrow$$

$$\Leftrightarrow I = -5,597\cos(65,0\pi t)$$

$t = 15,5\times10^{-3} \Rightarrow i = 5,59\ A$

$$I_{max} = \frac{V_{max}}{|z|} = \frac{V_{max}}{\omega L} = 5,597$$

$$i = 5,597 \sin(\omega t - \pi/2)$$

$$t = 15,5 \times 10^{-3} \qquad i = 5,59 \text{ A}$$

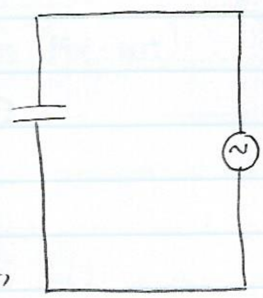
$$v = V_{max} \sin(\omega t)$$

$$i = I_{max} \sin(\omega t - \pi/2)$$

3→ $C = 2,20\ \mu F = 2,20 \times 10^{-6}\ F$

a) $\Delta V_{ef} = 120\ V$

$f = 60,0\ Hz$



$\omega = 2\pi f = 120\pi$

$$X_c = \frac{1}{\omega C}$$

$$\Delta V_{ef} = \frac{\Delta V_{max}}{\sqrt{2}} \Leftrightarrow \Delta V_{max} = \sqrt{2}\, \Delta V_{ef} \Leftrightarrow$$

$\Leftrightarrow \Delta V_{max} = 169,7\ V$

$$I_{max} = \frac{\Delta V_{max}}{X_c} \Leftrightarrow I_{max} = \frac{169,7}{\frac{1}{120\pi\ 2,20 \times 10^{-6}}} \Leftrightarrow$$

$\Leftrightarrow I_{max} = 0,141\ A$

b) $\Delta V_{ef} = 240\ V$

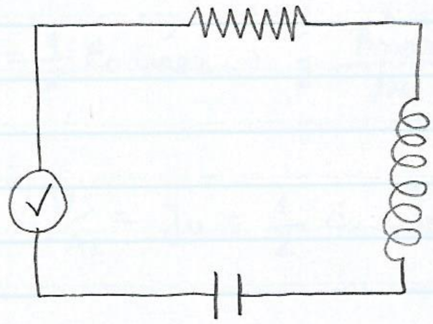
$f = 50,0\ Hz$

$\omega = 100\pi$

$\Delta V_{max} = 339,4\ V$

$I_{max} = 0,235\ A$

4→



$R = 150\ \Omega$

$L = 250\ mH$

$C = 2,00\ \mu F$

$\Delta V_{max} = 210\ V$

$f = 50,0\ Hz$

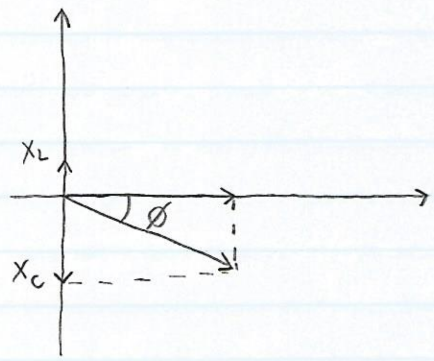
a) $X_L = \omega L = 2\pi f L = 2\pi \times 50,0 \times 250 \times 10^{-3} = 78,5\ \Omega$

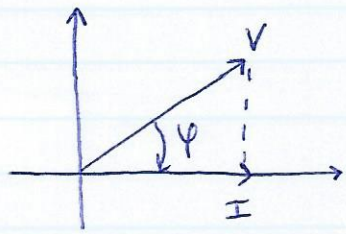
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \left(2\pi \times 50,0 \times 2,00 \times 10^{-6}\right)^{-1} = 1,59\ k\Omega$$

b) $|z| = \sqrt{R^2 + (X_L - X_C)^2} = 1,52\ k\Omega$

c) $I_{max} = \dfrac{V_{max}}{|z|} = \dfrac{210}{1,52 \times 10^3} = 138\ mA$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}(-10,1) = -84,3°$$





$v = Ri$

$P = vi$

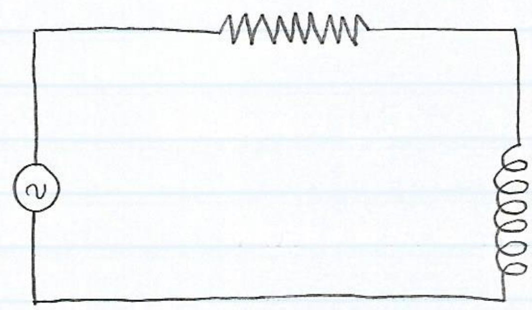
$$\langle P_R \rangle = \frac{V_{max} I_{max}}{2} = \frac{V_{max} \cos\varphi\, I_{max}}{2}$$

$= P_{max}$ que o circuito gastava

$P = vi = vi \cos\phi$

$$\langle P \rangle = \frac{V_{max} I_{max}}{2} \underbrace{\cos\phi}_{\text{factor de potência}}$$

5→



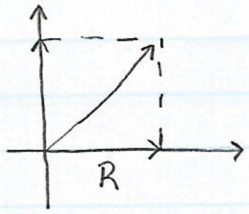
$\Delta V_{ef} = 120\ V$

$f = 60,0\ Hz$

$L = 25,0\ mH$

$R = 20,0\ \Omega$

$$-\Delta V + Ri = -L\frac{di}{dt} \Leftrightarrow -V_{max}\sin(\omega t) + Ri + L\frac{di}{dt} = 0 \quad -V_{max}\sin(\omega t) + R i_{max}\, \mathrm{sen}(\omega t + \alpha) + L i_{max}\, \omega \cos(\omega t + \alpha)$$



$$I_{ef} = \frac{V_{ef}}{|z|}$$

$$|z| = \sqrt{R^2 + (X_L)^2} = 22{,}11\,\Omega$$

$$i_{ef} = \frac{\Delta V_{ef}}{z} = \frac{120}{22{,}11} = 5{,}43\ A$$

$$\cos\phi = \frac{R}{|z|} = 0{,}905$$

$$P_{diss} = P_{máx}\cos\phi$$

$$tg\,\phi = \frac{X_L}{R}$$

$$P_{max} = \frac{1}{2} V_{max} I_{max} = V_{ef} I_{ef}$$

$$P_{diss} = I_{ef}^2 R = 5{,}892 \times 10^2\ W$$

b) $\cos\phi = 1 \Rightarrow \phi = 0$

$\Rightarrow X = 0$

$\Rightarrow |X_L| - |X_c| = 0 \Leftrightarrow |X_L| = |X_c|$

$$\omega L = \frac{1}{\omega C}$$

$P_{max}\cos\phi = P'_{max}$

$V_{ef} I_{ef}\cos\phi = V'_{ef} I'_{ef}$

$$\frac{V_{ef}^2}{|z|}\cos\phi = \frac{V_{ef}'^2}{R} \Rightarrow \left(\frac{V'_{ef}}{V_{ef}}\right)^2 = \frac{R}{|z|}\cos\phi = \cos^2\phi \Rightarrow \frac{V'_{ef}}{V_{ef}} = \cos\phi$$

7 → $\frac{N_P}{N_S} = \frac{13}{1} = 13$ $\frac{V_{efP}}{V_{efS}} = \frac{N_P}{N_S} = 13$

$$V_{efS} = \frac{120\,V}{13} = 9{,}23\ V$$

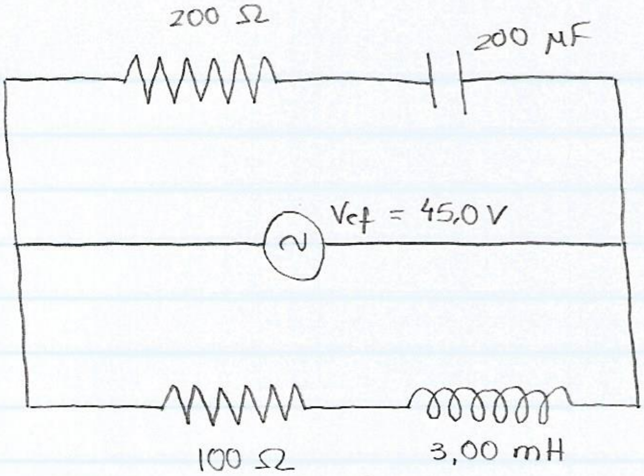
transf. ideal $P_P = P_S$ $I_{efP} = 0{,}350\ A$

$$P_P = V_{ef} I_{ef} = 120\,V \times 350\ V$$

$P_{sec} = V_{efS} I_{efS} = 9{,}23\ I_{efS}$ $I_{efS} = \frac{120 \times 0{,}350}{9{,}23} = 4{,}55\ A$

$$P_{transf} = V_{ef} I_{ef} = 9{,}23 \times 4{,}55 = 42{,}0\ W$$

9 →



$|X_c| = \dfrac{1}{\omega C}$ ↘ $\dfrac{1}{\omega C} \ll 200\ \Omega$ ↗ ω_{baixo} $\quad \dfrac{1}{\omega C} \gg 200\ \Omega$

$\begin{cases} \dfrac{1}{\omega} \ll 200\,C \\ \omega \gg \dfrac{100}{L} \end{cases}$ $\quad \begin{cases} \omega \gg \dfrac{1}{200C} \\ \omega \gg \dfrac{100}{L} \end{cases}$

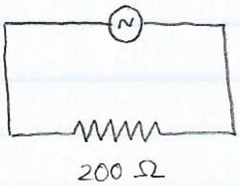
$X_L = \omega L$ ↗ $\omega L \gg 100\ \Omega$ ↘ ω_{baixo} $\quad \omega L \ll 100\ \Omega$

$$\begin{cases} \omega \gg \dfrac{1}{4\times10^{4}\times10^{-6}}\ \text{rad/s} = \dfrac{1}{4\times10^{-2}} = 25\ \text{Rad/s} \\ \omega \gg \dfrac{100}{3\times10^{-3}} = \dfrac{10^{5}}{3} = 3{,}33\times10^{4}\ \text{rad/s} \end{cases}$$

$\omega \ll 25$

$\omega \ll 3{,}33\times10^{4}$

$f \gg \dfrac{\omega}{2\pi} \simeq \dfrac{1}{2}\times10^{4}$



$I_{ef} = \dfrac{45}{200} = 225$ mA

$I_{ef} = 450$ mA