Semiempirical Methods

Semiempirical Methods are simplified versions of Hartree-Fock theory using empirical (= derived from experimental data) corrections in order to improve performance. These methods are usually referred to through acronyms encoding some of the underlying theoretical assumptions. The most frequently used methods (MNDO, AM1, PM3) are all based on the Neglect of Differential Diatomic Overlap (NDDO) integral approximation, while older methods use simpler integral schemes such as CNDO and INDO. All three approaches belong to the class of Zero Differential Overlap (ZDO) methods, in which all two-electron integrals involving two-center charge distributions are neglected. A number of additional approximations are made to speed up calculations (see below) and a number of parameterized corrections are made in order to correct for the approximate quantum mechanical model. How the parameterization is performed characterizes the particular semiempirical method. For MNDO, AM1, and PM3 the parameterization is performed such that the calculated energies are expressed as heats of formations instead of total energies (see earlier remarks on this difference).

acronym	full name	underlying approximation	Parameters	Fitted- Parameters
CNDO	Complete Neglect of Differential Overlap	CNDO	-	-
INDO	Intermediate Neglect of Differential Overlap	INDO	-	-
MINDO/3	Modified Intermediate Neglect of Differential Overlap, version 3	INDO	10	2
MNDO	Modified Neglect of Differential Overlap	NDDO	10	5
AM1	Austin Model 1	NDDO	13	8
PM3	Parametric Model number 3	NDDO	13	13

New versions of the NDDO methods have recently been developed that include d-Orbitals for second-row and higher elements (MNDO/d and PM3(tm)). Also, a slightly extended and reparameterized version of PM3 termed PM5 has recently been made available in the program package MOPAC 2000. In the following we will concentrate on the three methods available in *Gaussian* (MNDO, AM1, PM3).

The first strategy used to reduce computational effort is to consider only valence electrons in the quantum mechanical treatment. This is immediately obvious when comparing Hartree-Fock with, for example, AM1 energy calculations on a molecule such as methanol:

```
#P AM1 scf=tight
AM1 energy of CH30H (Cs)
0 1
C1
H2
   1
       r2
03
      r3
          2 a3
H4
       r4
           1 a4
                     180.0
H5
   1 r5 2 a5 3 d5
                     -d5
r2=1.11900473
r3=1.41043172
r4=0.9641002
r5=1.11868093
a3=105.12806298
a4=107.16494018
a5=110.03331541
d5=119.51560095
```

Even though methanol CH_3OH is composed of overall 6 nuclei and 18 electrons, only 14 of the electrons are treated explicitly. The electrons located in the 1s orbitals of oxygen and carbon are not considered explicitly, but combine with the nuclei to provide an effective core potential for the valence electrons:

```
AM1
                                                        HF/STO-3G
Standard basis: VSTO-3G (5D, 7F)
                                                        Standard basis: STO-3G (5D, 7F)
12 basis functions
                         36 primitive gaussians
                                                         14 basis functions
                                                                                  42 primitive gaussians
                          7 beta electrons
                                                                                  9 beta electrons
 7 alpha electrons
                                                          9 alpha electrons
 nuclear repulsion energy
                                25.1935849033 Hartrees.
                                                         nuclear repulsion energy
                                                                                         40.2200678489 Hartrees
```

The basis sets used in semiempirical calculations are specially optimized minimal basis sets composed of Slater-type orbitals. As only valence electrons are considered and the core electrons are treated together with the nuclei as one effective core potential, it is clear that the simple point charge model used in Hartree-Fock theory to calculate the nuclear repulsion energies is inappropriate for semiempirical calculations. In order to compensate for some of the approximations made in calculating the attractive core-electron energies, the core-core potential used in NDDO methods goes beyond the use of a simple point charge model with reduced nuclear charges such as:

$$E_{AB} = Z'_A Z'_B e^2 / R_{AB}$$

with R_{AB} being the internuclear separation and Z'_A being the effective core charge including the nuclear charge and all core electrons. A general expression for calculation of the core-core repulsion energies between nuclei A and B at distance R_{AB} in NDDO methods is:

$$E_{AB} = Z'_A Z'_B < s_A s_A, s_B s_B > [1 + F(A) + F(B)]$$

The core repulsion energy is here a function of both the electron-electron repulsion integral $\langle s_A s_A, s_B s_B \rangle$ as well as atom-type dependent functions F(A) and F(B) which in turn depend on the internuclear separation R_{AB}. Functions F(A) and F(B) have a relatively simple form in MNDO:

```
F_A = \exp(-\alpha_A R_{AB})
```

including only one additional parameter. In order to improve some of the deficiencies of MNDO (especially those concerning hydrogen bonding), a slightly more complex function was chosen for AM1:

```
F_{A} = \exp(-\alpha_{A}R_{AB}) + \Sigma K_{Ai} \exp[L_{Ai}(R_{AB} - M_{Ai})^{2}]
```

The sum over additional exponentials inloudes either three or four terms and introduces three new parameters K_{Ai} , L_{Ai} , and M_{Ai} for each element constituting the main difference (aside from the actual fitting procedure) between AM1 and MNDO. Which set of parameters is used in a given semiempirical calculation is specified in the *Gaussian* output file as:

```
References:
H: (AM1): M.J.S. DEWAR ET AL, J. AM. CHEM. SOC. 107 3902-3909 (1985)
C: (AM1): M.J.S. DEWAR ET AL, J. AM. CHEM. SOC. 107 3902-3909 (1985)
O: (AM1): M.J.S. DEWAR ET AL, J. AM. CHEM. SOC. 107 3902-3909 (1985)
```

For some of the elements, the core repulsion functions F(A) for a given element depend on the interaction partner B. Taking the element boron as an example, there are four different sets of parameters describing the core of boron implying that F_{boron} is different in E_{BB} , E_{BH} , E_{BC} , and E_{BX} !! Please note that this functionality is not implemented in all programs, *Gaussian* being one of the problematic cases. It is therefore not possible to perform AM1 calculations on boron-containing compounds in *Gaussian*. Any attempt to do so anyway will lead to a calculation with mixed AM1 and MNDO parameters identified in the output file as follows:

```
References:
H: (AM1): M.J.S. DEWAR ET AL, J. AM. CHEM. SOC. 107 3902-3909 (1985)
B: (MNDO): M.J.S. DEWAR, M.L. MCKEE, J. AM. CHEM. SOC., 99, 5231, (1977)
Warning. AM1 has been requested, along with some elements
for which only MNDO parameters are available. Such mixtures
of methods are very risky and have not been fully tested.
```

Indeed, these types of calculations are extremely unreliable and not worth doing under any circumstance.

The performance of NDDO methods for a large number of molecular systems has been studied repeatedly and the mean signed and unsigned errors for the heat of formation (kJ/mol) in a set of 194 typical organic systems containing the elements C, H, N, and O has been collected in the following table:

method	MAD unsigned	MAD signed
MNDO	47.7	+20.1
AM1	30.1	+10.9
PM3	18.4	+0.9

The performance is much worse for all three methods in cases involving second-row elements such as S or P, the description of hypervalent compounds being particularly problematic.

Another critical area of application concerns calculations of nitrogen-containing compounds. While the inversion barriers for trivalent nitrogen are usually too low with AM1, they are predicted to be too high with PM3. As a consequence, some non-planar nitrogen containing compounds are predicted to be flat by AM1, while some flat compounds are predicted to be pyramidal by PM3. This is particularly deplorable for peptide structures as a realistic description of conformational space is impossible without a proper description of the amide bond. The performance of the NDDO models might be appreciated better using the <u>dipeptide system</u> used before in Hartree-Fock geometry optmization. The planarity of the central amide bond can be characterized through the C(O)-N-H-C dihedral angle, being 180° for a fully planar conformation:

method	C(O)-N-H-C dihedral	
MNDO	157	
AM1	161	
PM3	143	
PM3MM	157	

Pyramidalization of the amide nitrogen is indeed largest with PM3 and smallest with AM1. In order to rescue the situation the PM3 method is also available in a PM3MM version, adding a simple molecular mechanics correction term to the amide linkage. It can be seen quite nicely in the above example, that the amide bond becomes more planar upon inclusion of this correction term. Whether or not this correction term is in use can be seen in the output file as:

```
NNHCO= 2.
References:
H: (PM3): J. J. P. STEWART, J. COMP. CHEM. 10, 209 (1989).
C: (PM3): J. J. P. STEWART, J. COMP. CHEM. 10, 209 (1989).
N: (PM3): J. J. P. STEWART, J. COMP. CHEM. 10, 209 (1989).
0: (PM3): J. J. P. STEWART, J. COMP. CHEM. 10, 209 (1989).
```

A reliable prediction of peptide conformations is, however, not possible even with these corrections.

One important point to consider when comparing the performance of semiempirical calculations with other theoretical methods such as Hartree-Fock or DFT is that the former have been parameterized to include all thermochemical corrections to yield heat of formations at 300K, while the latter have not. Also, through parameterization with reference to experimental data, semiempirical methods *might* be expected to recover some part of electron correlation effects. This is, of course, only true for the ground state systems included in the parameterization procedure and not necessarily true for transition states or electronically excited states.

last changes: 12.12.2005, HZ
questions & comments to: zipse@cup.uni-muenchen.de