Physics Homework 3

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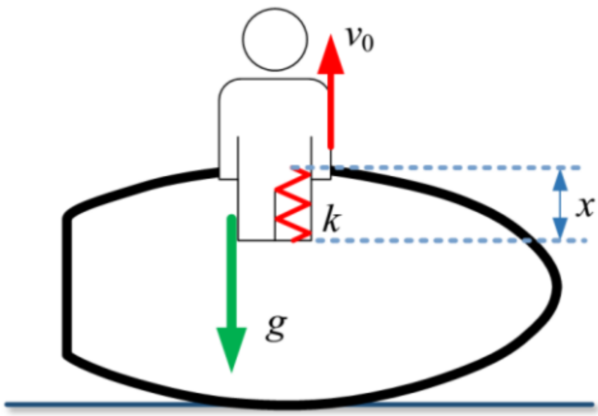
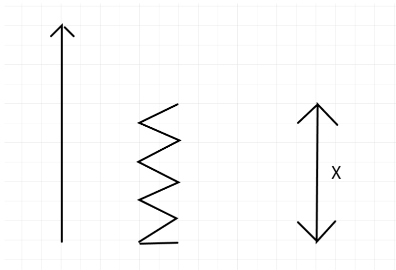
Problem 1 (25 pts)

One particular glitch in the Mafia 3 video game resulted in the player's character getting launched into the air after being intermittently stuck inside a motor boat. A hyper-realistic representation of this scene is shown in the Figure below. The known quantities are: mass of the character m = 80 kg, maximum launch height H = 70 m, initial penetration of the character's feet into the boat's hull x = 0.5 m. Answer the following:

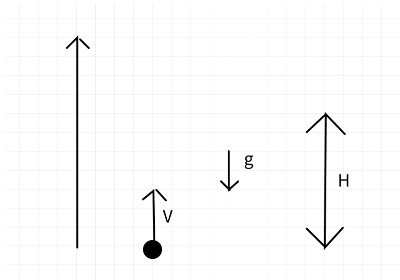
1. Assuming that the boat's hull can be modeled as an ideal spring with the stiffness coefficient k, what should be the value of k (in N/m, round to nearest thousand) so that the character is launched into the air by a given height H. Neglect air drag.
2. What is the maximum character's speed V (in km/h, round to 1 decimal place)?



Solution:

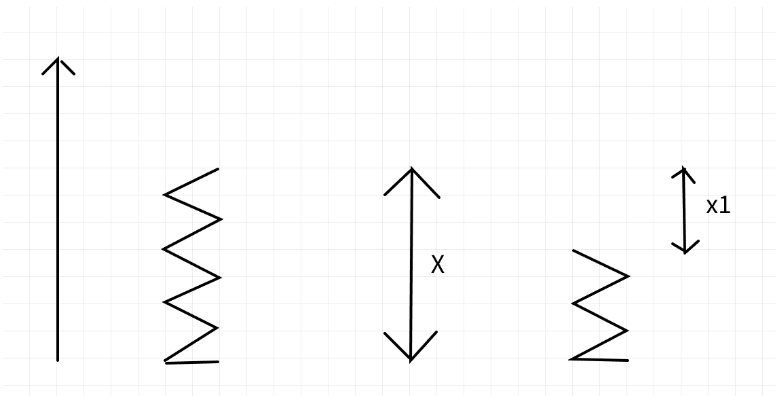
1. 

(Spring compressed then unclenches)

2. 

(Character goes up with velocity)

For the second part of the task

1. 

Given:
$m = 80\ kg$
$H = 70\ m$
$x = 0,5\ m$
g = 9,8 m/s^2

Find:
k –?
v_{max} –?

1. Let us divide this problem it two cases:
   1.spring is pushing on x
   2.character goes up

First potential energy of ideal spring goes to kinetic energy of character and in the same moment kinetic energy of character goes to his potential energy:

$E_{p\ of\ spring} + E_{k\ of\ ch} + E_{p\ of\ ch} = E_{full}$

In the first moment:

$E_{k\ of\ ch} = E_{p\ of\ ch} = 0$

So $E_{p\ of\ spring} = E_{full}$

In the last moment: $E_{p\ of\ spring} = 0$;

$E_{k\ of\ ch} + E_{p\ of\ ch} = E_{full}$

Then:

$E_{k\ of\ ch} = E_{full} - E_{p\ of\ ch}$

$E_{k\ of\ ch} = E_{p\ of\ spring} - E_{p\ of\ ch}$

Second all kinetic energy that we obtain goes to potential energy

$E_{k\ of\ ch} + E_{p1\ of\ ch} = E_{full}$

In the first moment:

$E_{p1\ of\ ch} = 0$

So $E_{k\ of\ ch} = E_{full}$

In the last moment: $E_{k\ of\ ch} = 0$

$E_{k\ of\ ch} = E_{p1\ of\ ch}$

$E_{p\ of\ spring} - E_{p\ of\ ch} = E_{p1\ of\ ch}$;

Where:

$E_{p\ of\ spring} = \frac{kx^2}{2}$

$E_{p\ of\ ch} = mgx$

$E_{p1\ of\ ch} = mgH$

$\frac{kx^2}{2} = mgH + mgx$

$\frac{kx^2}{2} = mg(H + x)$

$k = \frac{2mg(H+x)}{x^2}$

Count:

k = 442176 $\frac{N}{m}$≈442000$\frac{N}{m}$

Check:

$\frac{kg*m*m}{s^2*m^2} = N * \frac{m}{m^2} = \frac{N}{m}$

Answer: k = 442000 $\frac{N}{m}$

Second task:

$F_{spring} = k * \Delta x$. In the start it will be maximum:

Write second Newton law:

$F_{spring} + F_{gravity} = ma$

So our velocity will increase while

$k * x_1 - mg > 0$

If $k * x_1 - mg < 0$ velocity will decrease

Then velocity will be max in $k * x_1 - mg = 0$

Our velocity will grow during F_{spring}+$F_{gravity}$ not equal to zero, then it will grows down as Δx from the start will go down and F_{spring} will go down find new x_1 when it reaches(x_1):

$k * x_1 - mg = 0$

$x_1 = \frac{mg}{k}$:

Write the law of conservation of energy:

Potential energy of a spring goes to Kinetic and Potential energy of character and some energy of spring remain:

$E_{p\ of\ spring} + E_{p\ of\ spring\ remaining} + E_{k\ of\ ch} + E_{p\ of\ ch} = E_{full}$

In point :

$\frac{k(x)^2}{2} - \frac{k(x_1)^2}{2} - \frac{mv_{max}^2}{2} - mg(x - x_1)$ =0

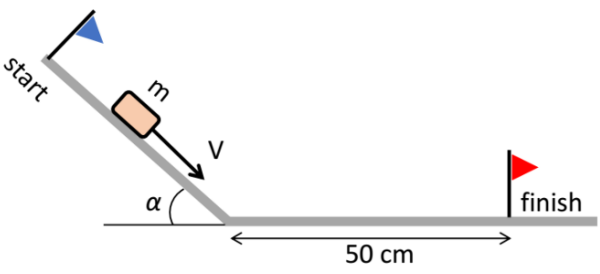
v_{max}=$\sqrt{\frac{kx^2 - kx_1^2 - 2mg(x - x_1)}{m}}$

$v = 37.04\ \frac{m}{s} = 133,3\ \frac{km}{h}$

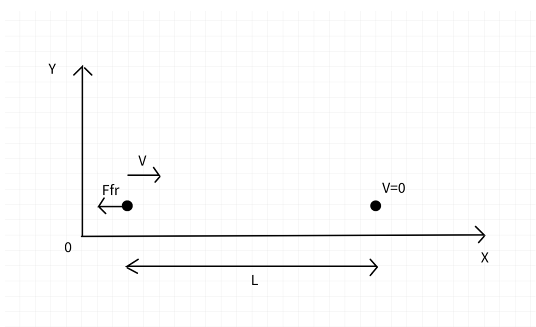
Answer: $v = 133,3\ \frac{km}{h}$

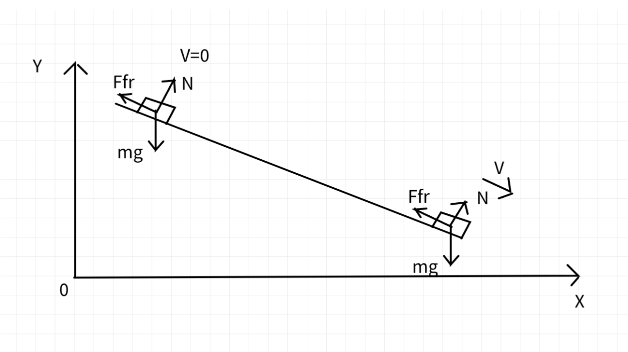
Problem 2 (25 pts)

An object of mass m = 50 g slides with the zero initial velocity down an inclined plane set at an angle α = 30° to the horizontal. It successfully slides down, then covers the distance of 50 cm on the horizontal plane, and stops. Find the work (in J, round to 2 decimal places) performed by the friction forces over the whole distance, assuming the friction coefficient k = 0.15 for both inclined and horizontal planes



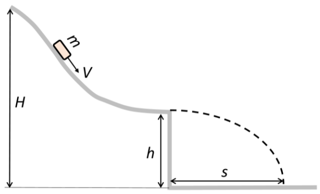
Solution:





Given: $m = 50\ g$ $\alpha = 30°$ $L = 50\ cm$ $k = 0,15$ g = 9,8 m/s^2	After moment it slides down we can write the law of conservation of energy: $E_k + A_{fr\ 2nd} + E_p = E_{full}$ In start point: $A_{fr} = E_p = 0$ $E_k = E_{full}$ In last point: $E_k = 0$; $E_p = 0$ So $A_{fr\ 2nd} = E_{full}$ $A_{fr\ 2nd} = E_k$ $E_k = mV^2/2$ $A_{fr\ 2nd}$ = -kN*L ($F_{fr\ 2nd} * L$) As it goes in horizontal plane projection of N is mg than $A_{fr\ 2nd}$ = -k*mg*L $E_p = 0$ $\frac{mV^2}{2} = k * mg * L$ $V=\sqrt{\frac{2*k*mg*L}{m}} = \sqrt{2 * k * g * L}$ We have found speed on the end of the slope. Now we want to find length of the slope to find work of the friction force on the slope Write second Newton law for object on the slope: $F_{mg} + F_N = ma$ Make projection $mgsin\alpha - k * N = ma$ $mgsin\alpha - k * mgcos\alpha = ma$ $gsin\alpha - k * gcos\alpha = a$ $L_{slope} = \frac{V^2 - V_0^{\ 2}}{2a}$; $V_0 = 0$ $L_{slope} = \frac{V^2}{2*(gsin\alpha - k*gcos\alpha)}$ $A_{fr\ 1st} = F_N * L_{slope}$ $A_{fr\ 1st} = - k * mgcos\alpha * \frac{V^2}{2*(gsin\alpha - k*gcos\alpha)}$ $A_{total} = A_{fr\ 1st} + A_{fr\ 2nd}$ $A_{total} = -k * mgcos\alpha * \frac{V^2}{2*(gsin\alpha - k*gcos\alpha)} - k * mg * L$ $A_{total} = -k * mgcos\alpha * \frac{2*k*gL}{2*(gsin\alpha - k*gcos\alpha)} - k * mg * L$ $A_{total} = -k * mgcos\alpha * \frac{k*L}{(sin\alpha - k*cos\alpha)} - k * mg * L$ m =50g=0,05kg $L = 50\ cm$ = 0,5m Count: A_{total} = -0,05 J **Answer:** A_{total} = -0,05 J
Find: A_{fr} –?	

Problem 3 (25 pts)
A small mass slides down with zero initial velocity from the top of a smooth hill of height H. The foot of the hill has a portion of horizontal surface before the vertical cliff of height h as shown on Figure.
1. What must be the height of the horizontal portion (h/H = ?) to ensure the maximum distance S covered by the flying mass?
2. What the maximum distance s measured in H equals to
Solution:



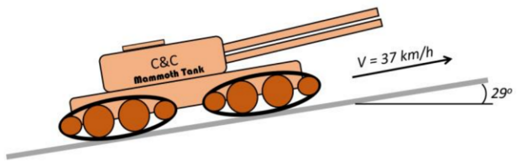
<table>
<tr>
<td>Given:
H
1. S should be max
2. S should be max</td>
<td rowspan="2">The law of conservation of energy:
$E_{k\ in\ start} + E_{p\ in\ start} + E_{k\ before\ fall} + E_{p\ before\ fall} = E_{full}$
If we consider it in two cases (like in previous task we will obtain)
$E_{k\ in\ start} + E_{p\ in\ start} = E_{k\ before\ fall} + E_{p\ before\ fall}$
$E_{k\ in\ start} = 0;$
$E_{p\ in\ start} = E_{k\ before\ fall} + E_{p\ before\ fall}$
$E_{p\ in\ start} = mgH$
$E_{k\ before\ fall} = \frac{mV^2}{2}$

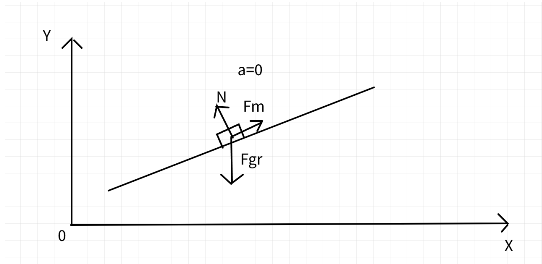
$E_{p\ before\ fall} = mgh$
$mgH = \frac{mV^2}{2} + mgh$
$V = \sqrt{2 * g(H - h)}$

Now consider falling with start speed V:
We have formule for the horizontal flight range, S should be maximum:
$S = V * \sqrt{\frac{2h}{g}}$
$S = \sqrt{2 * g(H - h)} * \sqrt{\frac{2h}{g}}$
$S = \sqrt{2 * (H - h) * 2h}$
$S = \sqrt{4 * Hh - 4 * h^2}$
$4 * Hh - 4 * h^2$ is a parabola than the maximum value it will have is its vertex of parabola.
We know that S should be max: vertex h $= \frac{-4*H}{-8} = \frac{H}{2}$
Answer: h $= \frac{H}{2}$
$S_{max} = \sqrt{2 * g(H - \frac{H}{2})} * \sqrt{\frac{2H}{2g}}$
$S_{max} = \sqrt{gH} * \sqrt{\frac{H}{g}}$
$S_{max} = H$
Answer: $S_{max} = H$</td>
</tr>
<tr>
<td>Find:
h/H—?
$S_{in\ H}$ —?</td>
</tr>
</table>

Problem 4 (25 pts)

Problem 4 (25 pts) The “Mammoth” tank’s electric motor has the maximum power of 5 MW. The tank can move (with no sliding) up a hill, which makes the angle of 29° with the horizontal, at maximum speed of 37 km/h. What is the mass(in tons, round to 2 decimal places) of the tank





Solution:

Given: W = 5MW $\alpha = 29°$ V = 37 km/h g = 9,8 m/s^2	We have formula for power of motor: $W = F_{motor} * V$ $F_{motor} = \frac{W}{V}$ Second newton law: we know that tank moves with constant speed V so a = 0 $F_{motor} + F_{mg} = 0$ make projection: $F_{motor} - F_{mg} = 0$ $F_{mg} = mgsin\alpha$ $\frac{W}{V} - mgsin\alpha = 0$ $m = \frac{W}{V*gsin\alpha}$ W = 5MW = $5*10^6$ W V = 37 km/h = 37/3,6 m/s Count: m = 102393 kg = 102,39 t **Answer:** 102,39 t
Find: m−?	