

## Codeforces Round #751 (Div. 1)

### A. Array Elimination

time limit per test: 2 seconds  
 memory limit per test: 512 megabytes  
 input: standard input  
 output: standard output

You are given array  $a_1, a_2, \dots, a_n$ , consisting of non-negative integers.

Let's define operation of "elimination" with integer parameter  $x$  ( $1 \leq x \leq n$ ) as follows:

- Choose  $x$  distinct array indices  $1 \leq i_1 < i_2 < \dots < i_x \leq n$ .
- Calculate  $b = a_{i_1} \& a_{i_2} \& \dots \& a_{i_x}$ , where  $\&$  denotes the [bitwise AND operation](#) (notes section contains formal definition).
- Subtract  $b$  from each of  $a_{i_1}, a_{i_2}, \dots, a_{i_x}$ ; all other elements remain untouched.

Find all possible values of  $x$ , such that it's possible to make all elements of array equal to 0 using a finite number of elimination operations with parameter  $x$ . It can be proven that exists at least one possible  $x$  for any array  $a$ .

Note that you **firstly choose**  $x$  **and only after that perform elimination operations with value**  $x$  **you've chosen initially**.

#### Input

Each test contains multiple test cases. The first line contains the number of test cases  $t$  ( $1 \leq t \leq 10^4$ ). Description of the test cases follows.

The first line of each test case contains one integer  $n$  ( $1 \leq n \leq 200\,000$ ) — the length of array  $a$ .

The second line of each test case contains  $n$  integers  $a_1, a_2, \dots, a_n$  ( $0 \leq a_i < 2^{30}$ ) — array  $a$  itself.

It's guaranteed that the sum of  $n$  over all test cases doesn't exceed 200 000.

#### Output

For each test case, print all values  $x$ , such that it's possible to make all elements of  $a$  equal to 0 in a finite number of elimination operations with the given parameter  $x$ .

**Print them in increasing order.**

#### Example

input
5 4 4 4 4 4 4 13 7 25 19 6 3 5 3 1 7 1 1 1 5 0 0 0 0 0
output
1 2 4 1 2 1 1 1 2 3 4 5

#### Note

In the first test case:

- If  $x = 1$ , we can make four elimination operations with sets of indices  $\{1\}, \{2\}, \{3\}, \{4\}$ . Since  $\&$  of one element is equal to the element itself, then for each operation  $a_i = a_i - a_i = 0$ .
- If  $x = 2$ , we can make two elimination operations with, for example, sets of indices  $\{1, 3\}$  and  $\{2, 4\}$ :  $a_1 = a_1 \& a_3 = a_2 \& a_4 = 4 \& 4 = 4$ . For both operations  $a_1 - 4 = 0$  and  $a_3 - 4 = 0$ , and after the second operation  $a_2 - 4 = 0$  and  $a_4 - 4 = 0$ .
- If  $x = 3$ , it's impossible to make all  $a_i$  equal to 0. After performing the first operation, we'll get three elements equal to 0 and one equal to 4. After that, all elimination operations won't change anything, since at least one chosen element will always be equal to 0.
- If  $x = 4$ , we can make one operation with set  $\{1, 2, 3, 4\}$ , because  $a_1 \& a_2 \& a_3 \& a_4 = 4$ .

In the second test case, if  $x = 2$  then we can make the following elimination operations:

- Operation with indices  $\{1, 3\}$ :  $= a_1 \& a_3 = 13 \& 25 = 9$ .  $a_1 - a_3 = 13 - 9 = 4$  and  $a_3 - a_1 = 25 - 9 = 16$ . Array will become equal to  $[4, 7, 16, 19]$ .
- Operation with indices  $\{3, 4\}$ :  $= a_3 \& a_4 = 16 \& 19 = 16$ .  $a_3 - a_4 = 16 - 16 = 0$  and  $a_4 - a_3 = 19 - 16 = 3$ . Array will become equal to  $[4, 7, 0, 3]$ .
- Operation with indices  $\{2, 4\}$ :  $= a_2 \& a_4 = 7 \& 3 = 3$ .  $a_2 - a_4 = 7 - 3 = 4$  and  $a_4 - a_2 = 3 - 3 = 0$ . Array will become equal to  $[4, 4, 0, 0]$ .
- Operation with indices  $\{1, 2\}$ :  $= a_1 \& a_2 = 4 \& 4 = 4$ .  $a_1 - a_2 = 4 - 4 = 0$  and  $a_2 - a_1 = 4 - 4 = 0$ . Array will become equal to  $[0, 0, 0, 0]$ .

Formal definition of bitwise AND:

Let's define bitwise AND ( $\&$ ) as follows. Suppose we have two non-negative integers  $x$  and  $y$ , let's look at their binary representations (possibly, with leading zeroes):  $\dots 2^1 0$  and  $\dots 2^1 0$ . Here,  $a_i$  is the  $i$ -th bit of number  $x$ , and  $b_i$  is the  $i$ -th bit of number  $y$ . Let  $z = x \& y$  is a result of operation  $\&$  on number  $x$  and  $y$ . Then binary representation of  $z$  will be  $\dots 2^1 0$ , where:

$$z_i = \begin{cases} 1, & \text{if } a_i = 1 \text{ and } b_i = 1 \\ 0, & \text{if } a_i = 0 \text{ or } b_i = 0 \end{cases}$$

B. Frog Traveler

time limit per test: 2 seconds  
memory limit per test: 512 megabytes  
input: standard input  
output: standard output

Frog Gorf is traveling through Swamp kingdom. Unfortunately, after a poor jump, he fell into a well of  $d$  meters depth. Now Gorf is on the bottom of the well and has a long way up.

The surface of the well's walls vary in quality: somewhere they are slippery, but somewhere have convenient ledges. In other words, if Gorf is on  $x$  meters below ground level, then in one jump he can go up on any integer distance from 0 to  $h_x$  meters inclusive. (Note that Gorf can't jump down, only up).

Unfortunately, Gorf has to take a break after each jump (including jump on 0 meters). And after jumping up to position  $x$  meters below ground level, he'll slip exactly  $s_x$  meters down while resting.

Calculate the minimum number of jumps Gorf needs to reach ground level.

**Input**  
The first line contains a single integer  $d$  ( $1 \leq d \leq 300\,000$ ) — the depth of the well.

The second line contains  $n$  integers  $h_1, h_2, \dots, h_n$  ( $0 \leq h_i \leq d$ ), where  $h_i$  is the maximum height Gorf can jump from  $i$  meters below ground level.

The third line contains  $n$  integers  $s_1, s_2, \dots, s_n$  ( $0 \leq s_i \leq d - i$ ), where  $s_i$  is the distance Gorf will slip down if he takes a break on  $i$  meters below ground level.

**Output**  
If Gorf can't reach ground level, print  $-1$ . Otherwise, firstly print integer  $k$  — the minimum possible number of jumps.

Then print the sequence  $x_1, x_2, \dots, x_k$  where  $x_i$  is the depth Gorf'll reach after the  $i$ -th jump, but before he'll slip down during the break. Ground level is equal to 0.

If there are multiple answers, print any of them.

Examples

<b>input</b>
3 0 2 2 1 1 0
<b>output</b>
2 1 0

<b>input</b>
2 1 1 1 0
<b>output</b>
-1

<b>input</b>
10 0 1 2 3 5 5 6 7 8 5

9 8 7 1 5 4 3 2 0 0
output
3 9 4 0

**Note**

In the first example, Gorf is on the bottom of the well and jump to the height 1 meter below ground level. After that he slip down by meter and stays on height 2 meters below ground level. Now, from here, he can reach ground level in one jump.

In the second example, Gorf can jump to one meter below ground level, but will slip down back to the bottom of the well. That's why he can't reach ground level.

In the third example, Gorf can reach ground level only from the height 5 meters below the ground level. And Gorf can reach this height using a series of jumps  $10 \Rightarrow 9 \quad 9 \Rightarrow 4 \quad 5$  where  $\Rightarrow$  is the jump and  $\quad$  is slipping during breaks.

C. Optimal Insertion

time limit per test: 3 seconds  
memory limit per test: 512 megabytes  
input: standard input  
output: standard output

You are given two arrays of integers  $a_1, a_2, \dots, a_n$  and  $b_1, b_2, \dots, b_m$ .

You need to insert all elements of  $b$  into  $a$  in an arbitrary way. As a result you will get an array  $c_1, c_2, \dots, c_{n+m}$  of size  $n+m$ .

Note that you are not allowed to change the order of elements in  $a$ , while you can insert elements of  $b$  at arbitrary positions. They can be inserted at the beginning, between any elements of  $a$ , or at the end. Moreover, elements of  $b$  can appear in the resulting array in any order.

What is the minimum possible number of inversions in the resulting array  $c$ ? Recall that an inversion is a pair of indices  $(i, j)$  such that  $i < j$  and  $c_i > c_j$ .

**Input**

Each test contains multiple test cases. The first line contains the number of test cases  $t$  ( $1 \leq t \leq 10^4$ ). Description of the test cases follows.

The first line of each test case contains two integers  $n$  and  $m$  ( $1 \leq n, m \leq 10^6$ ).

The second line of each test case contains  $n$  integers  $a_1, a_2, \dots, a_n$  ( $1 \leq a_i \leq 10^9$ ).

The third line of each test case contains  $m$  integers  $b_1, b_2, \dots, b_m$  ( $1 \leq b_i \leq 10^9$ ).

It is guaranteed that the sum of  $n$  for all tests cases in one input doesn't exceed  $10^6$ . The sum of  $m$  for all tests cases doesn't exceed  $10^6$  as well.

**Output**

For each test case, print one integer — the minimum possible number of inversions in the resulting array  $c$ .

Example
input
3 3 4 1 2 3 4 3 2 1 3 3 3 2 1 1 2 3 5 4 1 3 5 3 1 4 3 6 1
output
0 4 6

**Note**

Below is given the solution to get the optimal answer for each of the example test cases (elements of  $a$  are underscored).

- In the first test case,  $c = [1, 1, \underline{2}, 2, \underline{3}, 3, 4]$ .
- In the second test case,  $c = [1, 2, \underline{3}, \underline{2}, \underline{1}, 3]$ .
- In the third test case,  $c = [1, 1, 3, \underline{3}, \underline{5}, \underline{3}, \underline{1}, 4, 6]$ .

D. Difficult Mountain

time limit per test: 2 seconds

A group of  $n$  alpinists has just reached the foot of the mountain. The initial difficulty of climbing this mountain can be described as an integer  $d$ .

Each alpinist can be described by two integers  $s_i$  and  $t_i$ , where  $s_i$  is his skill of climbing mountains and  $t_i$  is his neatness.

An alpinist of skill level  $s_i$  is able to climb a mountain of difficulty  $d$  only if  $s_i \leq d$ . As an alpinist climbs a mountain, they affect the path and thus may change mountain difficulty. Specifically, if an alpinist of neatness  $t_i$  climbs a mountain of difficulty  $d$  the difficulty of this mountain becomes  $\max(d - t_i, 0)$ .

Alpinists will climb the mountain one by one. And before the start, they wonder, what is the maximum number of alpinists who will be able to climb the mountain if they choose the right order. As you are the only person in the group who does programming, you are to answer the question.

Note that after the order is chosen, each alpinist who can climb the mountain, must climb the mountain at that time.

Input

The first line contains two integers  $n$  and  $d$  ( $1 \leq n \leq 500\,000$ ;  $0 \leq d \leq 10^9$ ) — the number of alpinists and the initial difficulty of the mountain.

Each of the next  $n$  lines contains two integers  $s_i$  and  $t_i$  ( $0 \leq s_i, t_i \leq 10^9$ ) that define the skill of climbing and the neatness of the  $i$ -th alpinist.

Output

Print one integer equal to the maximum number of alpinists who can climb the mountain if they choose the right order to do so.

Examples

input
3 2 2 6 3 5 5 7
output
2

input
3 3 2 4 6 4 4 6
output
2

input
5 0 1 5 4 8 2 7 7 6 3 2
output
3

Note

In the first example, alpinists 2 and 3 can climb the mountain if they go in this order. There is no other way to achieve the answer of 2.

In the second example, alpinist 1 is not able to climb because of the initial difficulty of the mountain, while alpinists 2 and 3 can go up in any order.

In the third example, the mountain can be climbed by alpinists 5, 3 and 4 in this particular order. There is no other way to achieve optimal answer.

E. Phys Ed Online

gym is open for  $n$  days and has a ticket system. At the  $i$ -th day, the cost of one ticket is equal to  $c_i$ . You are free to buy more than one ticket per day.

You can activate a ticket purchased at day  $i$  either at day  $i$  or any day later. Each activated ticket is valid only for  $k$  days. In other words, if you activate ticket at day  $i$ , it will be valid only at days  $i, i + 1, \dots, i + k - 1$ .

You know that the  $s$ -th student wants to visit the gym at each day from  $l_s$  to  $r_s$  inclusive. Each student will use the following strategy of visiting the gym at any day  $i$  ( $l_s \leq i \leq r_s$ ):

1. person comes to a desk selling tickets placed near the entrance and buy several tickets with cost  $c_i$  apiece (possibly, zero tickets);
2. if the person has at least one activated and still valid ticket, they just go in. Otherwise, they activate one of tickets purchased today or earlier and go in.

Note that each student will visit gym only starting  $l_s$ , so each student has to buy at least one ticket at day  $l_s$ .

Help students to calculate the minimum amount of money they have to spend in order to go to the gym.

**Input**  
The first line contains three integers  $n, s, k$  ( $1 \leq n \leq 300\,000; 1 \leq s \leq 10^5$ ) — the number of days, the number of students and the number of days each ticket is still valid.

The second line contains  $n$  integers  $c_1, c_2, \dots, c_n$  ( $1 \leq c_i \leq 10^9$ ) — the cost of one ticket at the corresponding day.

Each of the next  $s$  lines contains two integers  $l_s, r_s$  and  $k$  ( $1 \leq l_s \leq r_s \leq n$ ) — the segment of days the corresponding student want to visit the gym.

**Output**  
For each student, print the minimum possible amount of money they have to spend in order to go to the gym at desired days.

Example				
<table><tr><th>input</th></tr><tr><td>7 5 2 2 15 6 3 7 5 6 1 2 3 7 5 5 7 7 3 5</td></tr><tr><th>output</th></tr><tr><td>2 12 7 6 9</td></tr></table>	input	7 5 2 2 15 6 3 7 5 6 1 2 3 7 5 5 7 7 3 5	output	2 12 7 6 9
input				
7 5 2 2 15 6 3 7 5 6 1 2 3 7 5 5 7 7 3 5				
output				
2 12 7 6 9				

**Note**  
Let's see how each student have to spend their money:

- The first student should buy one ticket at day 1.
- The second student should buy one ticket at day 3 and two tickets at day 4. Note that student can keep purchased tickets for the next days.
- The third student should buy one ticket at day 5.
- The fourth student should buy one ticket at day 7.
- The fifth student should buy one ticket at day 3 and one at day 4.

## F. Two Sorts

time limit per test: 4 seconds  
memory limit per test: 512 megabytes  
input: standard input  
output: standard output

Integers from 1 to  $n$  (inclusive) were sorted lexicographically (considering integers as strings). As a result, array  $a_1, a_2, \dots, a_n$  was obtained.

Calculate value of  $(a_{-1}((a_1 - a_2) \bmod 998244353)) \bmod 10^9 + 7$ .

$a \bmod b$  here means the remainder after division  $a$  by  $b$ . This remainder is always non-negative and doesn't exceed  $b - 1$ . For example,  $5 \bmod 3 = 2, (-1) \bmod 6 = 5$ .

**Input**  
The first line contains the single integer  $n$  ( $1 \leq n \leq 10^{12}$ ).

**Output**  
Print one integer — the required sum.

Examples

input
3
output
0

input
12
output
994733045

input
21
output
978932159

input
1000000000000
output
289817887

Note

A string `s` is lexicographically smaller than a string `t` if and only if one of the following holds:

- `s` is a prefix of `t`, but `s`  $\neq$  `t`;
- in the first position where `s` and `t` differ, the string `s` has a letter that appears earlier in the alphabet than the corresponding letter in `t`.

For example, `42` is lexicographically smaller than `6`, because they differ in the first digit, and `4` < `6`; `42` < `420`, because `42` is a prefix of `420`.

Let's denote `998244353` as `mod`.

In the first example, array `a` is equal to `[1, 2, 3]`.

- `(1 - 1) mod = 0 mod = 0`
- `(2 - 2) mod = 0 mod = 0`
- `(3 - 3) mod = 0 mod = 0`

As a result, `(0 + 0 + 0) mod 109 + 7 = 0`

In the second example, array `a` is equal to `[1, 10, 11, 12, 2, 3, 4, 5, 6, 7, 8, 9]`.

- `(1 - 1) mod = 0 mod = 0`
- `(2 - 10) mod = (-8) mod = 998244345`
- `(3 - 11) mod = (-8) mod = 998244345`
- `(4 - 12) mod = (-8) mod = 998244345`
- `(5 - 2) mod = 3 mod = 3`
- `(6 - 3) mod = 3 mod = 3`
- `(7 - 4) mod = 3 mod = 3`
- `(8 - 5) mod = 3 mod = 3`
- `(9 - 6) mod = 3 mod = 3`
- `(10 - 7) mod = 3 mod = 3`
- `(11 - 8) mod = 3 mod = 3`
- `(12 - 9) mod = 3 mod = 3`

As a result, `(0 + 998244345 + 998244345 + 998244345 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3) mod 109 + 7 = 2994733059 mod 109 + 7 = 994733045`