CODEFORCES β
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Mathforces: Tech Scouts Online Test 2018 (just fun and practice, unofficial, unrated)

A. Three Friends (1 point)

Three friends D, J and Y met at a party, some of them always tell the truth, others — always lie. One of them said "`D and J are both liars`", another said "`D and Y are both liars`". How many liars are there among the friends?

Test 1 (1 point)

Specify the answer

B. Triangle Area (1 point)

A triangle ABC of area 16 is given. Let M, N, P be points on the sides AB, BC and CA respectively such that $AM : MB = BN : NC = CP : PA = 1 : 3$. Find the area of the triangle MNP.

Test 1 (1 point)

Specify the answer

C. 7-digit Number (1 point)

John wants to compose a 7-digit number by using each of the digits $1, 2, 3, 4, 5, 6, 7$ exactly once, so that the digits 1 and 2 are not neighbouring. In how many ways can he compose such a number?

Test 1 (1 point)

Specify the answer

D. Multiple of 29 (1 point)

What is the smallest multiple of 29 whose last two digits are 29 and whose sum of digits is 29?

Test 1 (1 point)

Specify the answer

E. Least Possible Difference (1 point)

Let x, y, z and w be real numbers such that $|x - y| = 3$, $|y - z| = 7$ and $|z - w| = 12$. Find the least possible value of $|x - w|$.

Test 1 (1 point)

Specify the answer

F. Regular Polygon (1 point)

A regular n-gon is given in the plane. The length of its side is $\sqrt{2 - \sqrt{2}}$ and the length of its largest diagonal is 2. Find all possible values of n. (Give the answer in the form $a, b, c, \dots$ without spaces.)

Test 1 (1 point)

Give the answer in the form a,b,c,... without spaces.

G. Find Number (1 point)

Find the least natural number that has exactly 20 natural divisors, exactly 4 of which are odd.

Test 1 (1 point)

Specify the answer

H. Circle Radius (2 points)

Let $ABCD$ be a square of side length 6. Let BEC be an equilateral triangle such that E is outside the square. Find the radius of the circle that passes through the points A, E and D.

Test 1 (2 points)

Specify the answer

I. Maximum Value (2 points)

Real numbers a and b satisfy the equality $(a+b-1)^2 = ab+1$. What maximum value may the expression a^2+b^2 take?

Test 1 (2 points)

Specify the answer

J. Eleven Segments (2 points)

One has 11 segments of integer lengths, the smallest length being 1. There are no three segments such that one can build a non-degenerate triangle with them. What is the minimum possible length of the largest of the 11 segments?

Test 1 (2 points)

Specify the answer

K. 2018 Integers (2 points)

2018 integers are written around a circle. Their sum is one. A *good sequence* is a sequence of consecutive (clockwise) numbers on the circle such that their sum is positive. How many good sequences are there?

Test 1 (2 points)

Specify the answer

L. Queens (2 points)

What is the maximum number of queens that can be placed on a 101×101 board so that no three queens are in the same row, column or diagonal (there are 402 diagonals)?

Test 1 (2 points)

Specify the answer

M. Seven Digit Number (2 points)

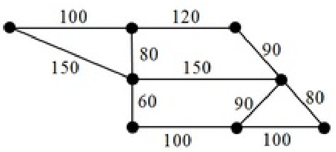
Let N be a 7-digit number that is divisible by each of its digits. If all the digits of N are different, find the sum of the digits of N.

Test 1 (2 points)

Specify the answer

N. All Streets (2 points)

A tourist wants to walk through all the streets in the city and end his journey at the same point where he started. What minimum distance will the tourist have to walk? The map of the city is given in the figure; the numbers near the streets are their lengths.



Test 1 (2 points)

Specify the answer

O. Greatest Prime Divisor (2 points)

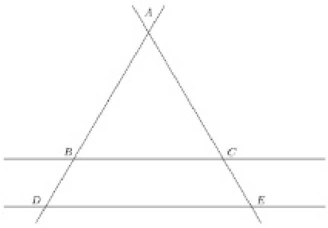
Consider the number $S = 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + 100 \cdot 100!$. What is the greatest prime divisor of the number $S+1$?

Test 1 (2 points)

Specify the answer

P. Equilateral Triangles (2 points)

One draws fifteen lines in the plane. What is the largest number of equilateral triangles (whose sides are on these lines) that one can generate in such a way? For example in the figure below, two equilateral triangles ABC and ADE are generated by four lines.



Test 1 (2 points)

Specify the answer

Q. Surjective Functions (3 points)

Let $M = \{1, 2, 3, 4, 5, 6, 7\}$ be a set with 7 elements. Find the number of surjective functions $f : M \to M$ such that $f(f(a)) \neq a$ and $f(f(f(a))) \neq a$ for all $a \in M$.

Note: A function $f : A \to B$ is called surjective if for each $b \in B$ there exists $a \in A$ such that $f(a) = b$.

Test 1 (3 points)

Specify the answer

R. Divisible by 83 (3 points)

Consider the recurrent sequence given by $x_0 = 0$, $x_1 = 4$ and $x_{n+1} = 4 \cdot x_n - 3 \cdot x_{n-1}$ for all natural n. Find the minimum natural n such that x_n is divisible by 83.

Test 1 (3 points)

Specify the answer

S. Find Distance (3 points)

Let $ABCD$ be a rectangle. A point P is chosen on the segment BC such that $\angle APD = 90^o$. Let M be the projection of B on the line AP, N be the projection of C on the line DP and Q be the midpoint of MN. Given that $BP = 32$ and $PC = 18$, find the distance from Q to the line AD.

Test 1 (3 points)

Specify the answer

T. Compute the Product (3 points)

Let a, b, c, d be real numbers such that $a = \sqrt{7 - \sqrt{6 - a}}$, $b = \sqrt{7 - \sqrt{6 + b}}$, $c = \sqrt{7 + \sqrt{6 - c}}$ and $d = \sqrt{7 + \sqrt{6 + d}}$. Compute the product $abcd$.

Test 1 (3 points)

Specify the answer

U. The Greatest Possible Value (3 points)

Let x and y be positive real numbers such that $x^5 + y^5 = 20xy$. Find the greatest possible value of y^5.

Test 1 (3 points)

Specify the answer