Performance Evaluation of XOR-Based Cooperative Relays Using Finite Buffer and Batch Service Queue Models

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Outline

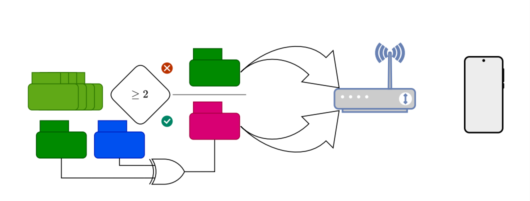
Introduction

Motivation

- Cooperative relays improve network throughput and reliability.
- XOR network coding enables two-packet transmissions in one timeslot.
- Accurate queueing models for XOR relays remain underexplored.

Problem Statement

- Finite buffer at relay (N packets).
- Poisson arrivals (λ), exponential service (μ).
- Two transmission modes:
  - Single packet transmission.
  - XOR batch transmission (2 packets at once).
- Key performance questions:
  - What is the average delay, occupancy, and blocking probability?
  - Can classical queue models approximate it?



Theoretical Queue Models

Finite Buffer Single-Server: $M/M/1/J$

- Finite capacity: N packets (including service).
- Blocking probability:

$$P_{\mathrm{b}} = \frac{\rho^N(1-\rho)}{1-\rho^{N+1}}, \quad \rho = \frac{\lambda}{\mu}$$

- Average number of packets:

$$E[Q] = \frac{\rho}{1-\rho} - \frac{(N+1)\rho^{N+1}}{1-\rho^{N+1}}$$

- Average system time:

$$E[t_{\mathrm{q}}] = \frac{E[Q]}{(1-P_{\mathrm{b}})\lambda}$$

Partial Batch Service Queue: $M/M^K/1$ (with $K = 2$)

- Batch size up to $K = 2$ packets.
- Characteristic equation:

$$\mu r^{K+1} - (\mu + \lambda) r + \lambda = 0$$

- Unique root r_0 with $0 < r_0 < 1$.
- Average number in system:

$$E[Q] = \frac{r_0}{1 - r_0}$$

- Average system time:

$$E[t_{\mathrm{q}}] = \frac{E[Q]}{\lambda}$$

- Limitation: Allows new arrivals to **join ongoing service**.

Modeling Challenges

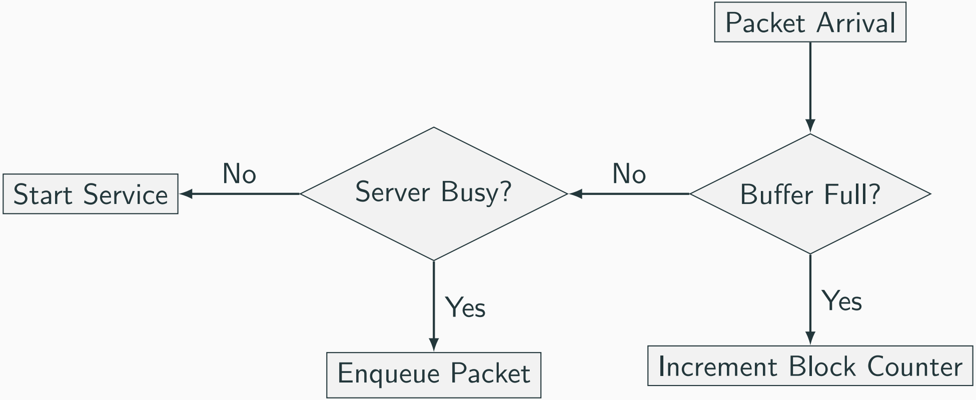
- XOR relay decisions depend on the queue length at transmission start.
- Batch composition fixed at service initiation.
- $M/M^K/1$ assumption of ongoing-batch joinability is **unrealistic for coding**.
- Need for a **state-dependent simulation-based model**.

Proposed Simulation Model

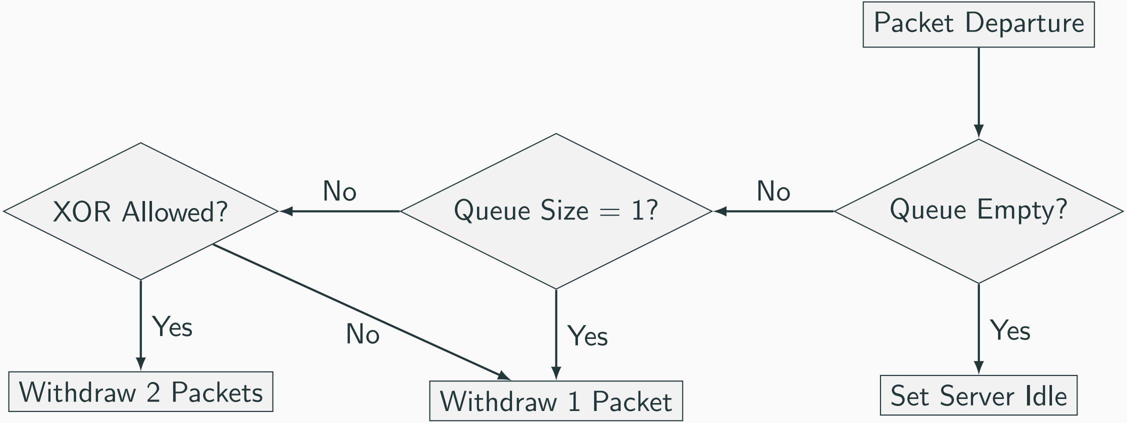
System State Variables

- $Q(t)$: Total number of packets (queue + in service).
- $q(t)$: Queue length (waiting packets only).
- $s(t)$: Server state (number of packets in service).

Arrival Event Handling (Flowchart)



Departure Event Handling (Flowchart)



Simulation Event Handling: Arrival vs Departure

Arrival Event

```
If next event is arrival:
    Update clock: t ← t_arr
    Increment arrivals: A ← A + 1
    If server is busy:
        If queue is not full:
            q ← q + 1
        Else:
            Increment blocked packets: B ← B + 1
    Else:
        Start service immediately
        Schedule next departure
    Schedule next arrival
```

Departure Event

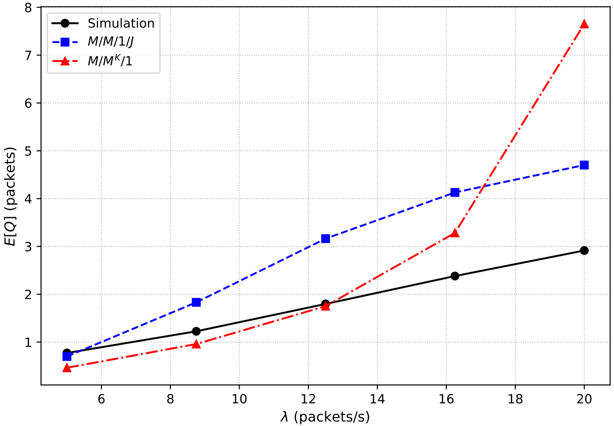
```
If next event is departure:
    Update clock: t ← t_dep
    Increment departures: D ← D + 1
    If q >= 1:
        If q >= 2 and XOR enabled:
            Remove 2 packets: q ← q - 2
        Else:
            Remove 1 packet: q ← q - 1
        Schedule next departure
    Else:
        Set server idle: t_dep ← Inf.
Record occupancy: Q(t) ← q + server state
```

Results

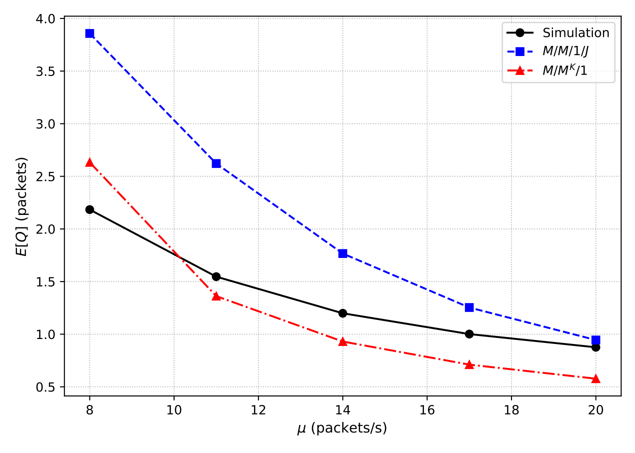
Simulation Parameters

- Each simulation run was executed until a total of 1,000,000 packet departures occurred.
- The random number generator was initialized with a fixed seed.
- Each parameter arrival rate (λ), service rate (μ), and buffer size (N) was varied independently.
- The remaining values were kept constant at their default values:
  - $\lambda = 10$ packets/s
  - $\mu = 12$ packets/s
  - $N = 5$

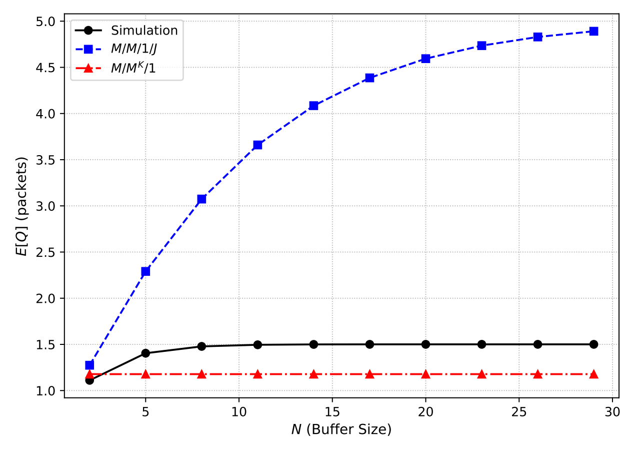
Average Occupancy vs λ



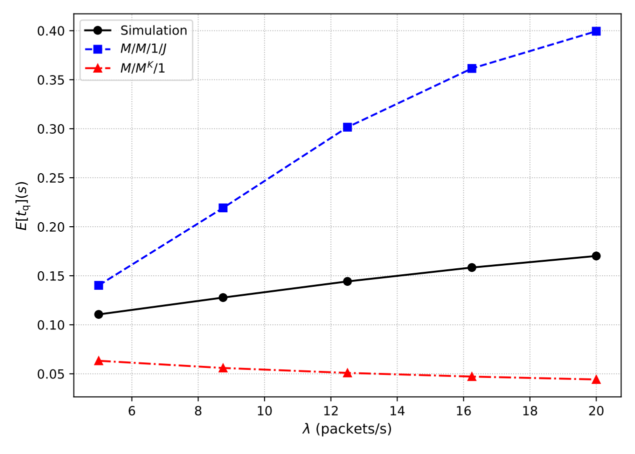
Average Occupancy vs μ



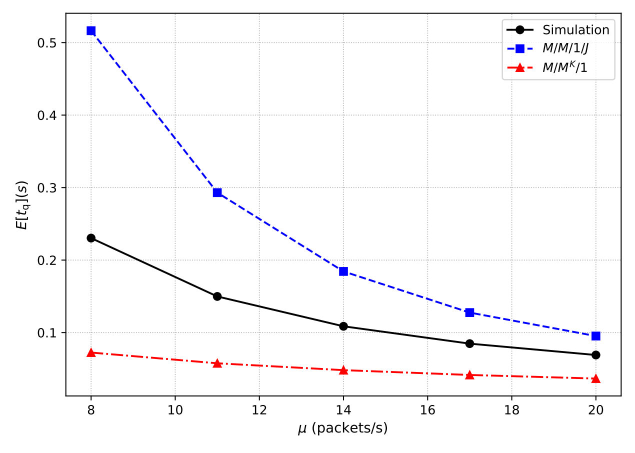
Average Occupancy vs N



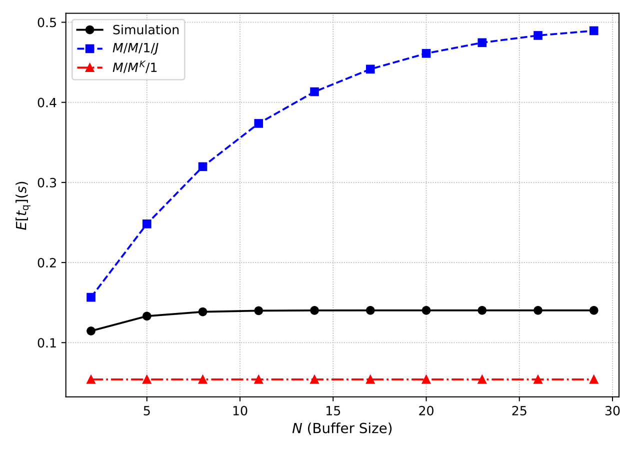
Average System Time vs λ



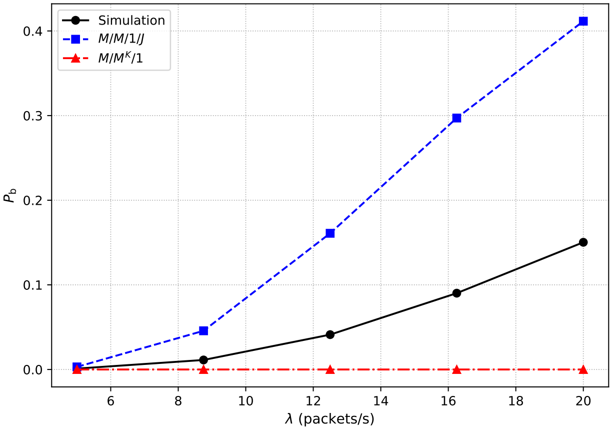
Average System Time vs μ



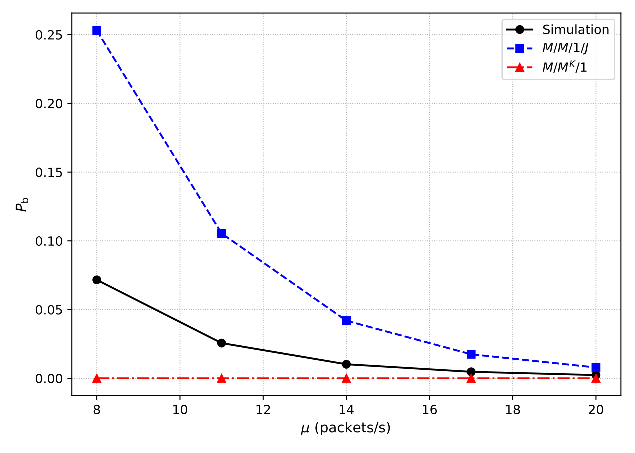
Average System Time vs N



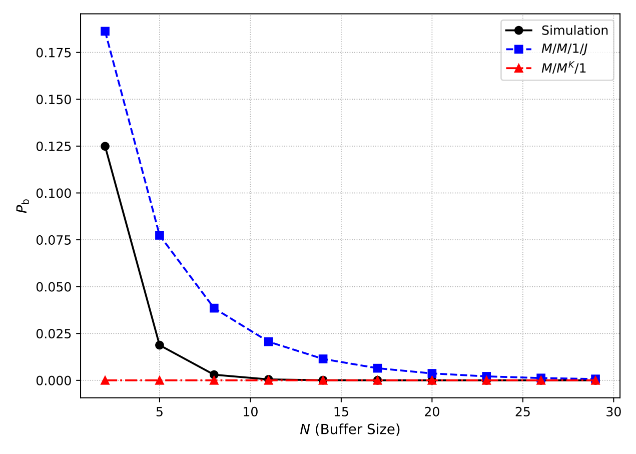
Blocking Probability vs λ



Blocking Probability vs μ



Blocking Probability vs N



Discussion

Observations

- XOR relay performance lies between $M/M/1/J$ and $M/M^K/1$.
- At low traffic:
  - Matches $M/M/1/J$.
- At high traffic:
  - Benefits from batch transmissions.
  - But limited by finite buffer and arrival randomness.

Conclusion

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- XOR queueing offers performance improvements under realistic constraints.
- Neither $M/M/1/J$ nor $M/M^K/1$ fully captures XOR relay dynamics.
- Simulation fills the gap by modeling state-dependent, batch-triggered service.
- Future work: Develop analytical models incorporating queue-length-dependent batch formation.