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Contents

1 Notations

1. **RLS:** Randomised local Search
2. **(1+1) EA:** The Standard (1+1) EA
3. **RSH:** Randomised Search Heuristics referring to all in this context analysed Evolutionary algorithms
4. **bin:** When solving Partition a set of numbers is divided into to distinct subsets and in this paper both subsets are referred to as bins
5. b_F**:** The fuller bin
6. b_F**:** The emptier bin
7. b_{w_i}**:** The bin containig the object w_i
8. **n:** The input length of the problem
9. **x:** A vector $x \in \{0, 1\}^n$ describing a solution
10. **W:** The sum of all values $\sum_{i=1}^{n} w_i$

2 Improving bounds on the Standard RSHs

Lemma 2.1. *If $w_1 \geq W/2$ then the RSH reaches the optimal value in expected time $\Theta(n \log n)$*

Proof. The optimal solution is putting w_1 in one bin and all other elements in the other bin. So the problem is almost identical to OneMax/ZeroMax. A flip of only the first bit can only happen if the emptier bin has a weight of at most $\frac{W-w_1}{2}$. After this flip the weight of the emptier bin is at least $\frac{W-w_1}{2}$ and therefore another single bit flip of w_1 can only happen before a different bit is flipped. After a different bit has been flipped, the RSH wont flip the first bit alone again, because it will never result in an improvement. The first bit could also be flipped with multiple other bits, TODO. So the run can be devided into three phases:

Phase 1: The RSH behaves exactly like OneMax/ZeroMax and flips every bit to the opposite of the first bit (except for the first bit).

Phase 2: The RSH flips only the first bit or bits that do not result in an improvement.

Phase 3: The RSH behaves exactly like ZeroMax/OneMax and flips every bit to the opposite of the first bit (except for the first bit).

The expected lenght of the first phase is $\mathcal{O}(n)$ because the probability of flipping the first bit is at least $\frac{1}{n} \cdot (1 - \frac{1}{n})^{n-1} >= \frac{1}{ne}$ and therefore the expected time for such a step is at most $\mathcal{O}(\frac{1}{ne}^{-1}) = \mathcal{O}(ne) = \mathcal{O}(n)$.
The lenght of the second phase is $\mathcal{O}(n)$ because the solution is either optimal or there is at least one bit that needs to be flipped for an optimal solution. Since the expected length of Phase 1 is $\mathcal{O}(n)$ the solution produced by the RSH won't be optimal in expectation due to the bound of $\Theta(n \log n)$ for OneMax/ZeroMax. This again results in expected time $\mathcal{O}(n)$.
The lenght of the third phase is identical to a run of the RSH on OneMax/ZeroMax where flips of the first bit are ignored as if it was already correctly flipped and therefore the expected time is $\Theta(n \log n)$

So the total expected time is $\mathcal{O}(n) + \mathcal{O}(n) + \Theta(n \log n) = \Theta(n \log n)$ □

Lemma 2.2. *The RSH reaches an approximation ratio of at most 4/3 in expected time $\mathcal{O}(n \log n)$ if $w_1 \leq W/2$*

Proof. Helpful statements

(1) If the weight of the fuller bin is at most $\frac{2}{3}$ the approximation ratio is at most $\frac{2}{3}/opt \leq \frac{2}{3}/\frac{1}{2} \leq \frac{4}{3}$

(2) If $w_1 \geq \frac{1}{3}$ and w_1 is in the emptier bin, then the weight of the fuller bin at most $1 - w_1 \leq 1 - \frac{1}{3} = \frac{2}{3} \rightarrow$ approximation $\leq \frac{4}{3}$ (1)

(3) $b_F - b_E \geq v <-> b_F \geq b_E + v$ and therefore any object of weight at most v can be moved from b_F to b_E if $b_F - b_E \geq v$

(4) $b_F \geq \frac{2}{3} -> b_E \leq \frac{1}{3} -> b_F - b_E \geq \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ -> Every object $\leq \frac{1}{3}$ can be moved from b_F to b_E as long as $b_F \geq \frac{2}{3}$ (3)

(5) In Time $\mathcal{O}(n \log n)$ the weight of the fuller bin can be decreased to $\leq \frac{2}{3}$ if every item besides the biggest in the bin is $\leq \frac{1}{3}$.

Proof of (5):
In time $\mathcal{O}(n \log n)$ the RSH can move every object $\leq \frac{1}{3}$ to the emptier bin as long as $b_F \geq \frac{2}{3}$ (4). So in Time $\mathcal{O}(n \log n)$ the solution can be shifted to w_1 being in the first bin and all other objects in the other bin. The algorithm will only stop moving the elements if the condition $b_F \geq \frac{2}{3}$ is no longer satisfied (4). If every object was moved to the emptier bin, the weight of the fuller bin is at most $\max\{1 - w_1, w_1\} \leq \frac{2}{3}$. So either the RSH moves all objects to the emptier bin or stops moving objects because $b_F < \frac{2}{3}$. Either way the fuller bin is at most $\frac{2}{3}$ and with (1) the result follows.

If $w_1 + w_2 > \frac{2}{3}$ after time $\mathcal{O}(n)$ w_1 and w_2 are separated (Proof by by C.Witt) and will remain separated afterwards. From then on the following holds. If w_1 is in the emptier bin, the result follows by (2). Otherwise the result follows by (5) □

Corollary 2.2.1. *The RSH reaches an approximation ratio of at most 4/3 in expected time* $\mathcal{O}(n \log n)$

Proof. This follows directly from 2.1 and 2.2 □

Erklärung der wissenschaftlichen Redlichkeit

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Passau, den 1. Januar 1970

Lipp Daniel