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Contents

1 Notations

1. **RLS:** Randomised local Search
2. **(1+1) EA:** The Standard (1+1) EA
3. **RSH:** Randomised Search Heuristics referring to all in this context analysed Evolutionary algorithms
4. **bin:** When solving Partition a set of numbers is divided into to distinct subsets and in this paper both subsets are referred to as bins
5. b_F**:** The fuller bin
6. b_F**:** The emptier bin
7. b_{w_i}**:** The bin containig the object w_i
8. **n:** The input length of the problem
9. **x:** A vector $x \in \{0, 1\}^n$ describing a solution
10. **W:** The sum of all values $\sum_{i=1}^{n} w_i$

2 Improving bounds on the Standard RSHs

Lemma 2.1. *If* $w_1 \geq W/2$ *then the RSH reaches the optimal value in expected time* $\Theta(n \log n)$

Proof. The optimal solution is putting w_1 in one bin and all other elements in the other bin. So the problem is almost identical to OneMax/ZeroMax. A flip of the first bit can only happen if the emptier bin has a weight of at most $\frac{W-w_1}{2}$. After this flip the weight of the emptier bin is at least $\frac{W-w_1}{2}$ and therefore another flip of w_1 can only happen before a different bit is flipped. After a different bit has been flipped the RSH wont flip the first bit again because it will never result in an improvement. So the run can be devided into three phases:

Phase 1: The RSH behaves exactly like OneMax/ZeroMax and flips every bit to the opposite of the first bit.

Phase 2: The RSH flips only the first bit or bits that do not result in an improvement.

Phase 3: The RSH behaves exactly like ZeroMax/OneMax and flips every bit to the opposite of the first bit.

The expected lenght of the first phase is $\mathcal{O}(n)$ because the probability of flipping the first bit is at least $\frac{1}{n} \cdot (1 - \frac{1}{n})^{n-1} >= \frac{1}{ne}$ and therefore the expected time for such a step is at most $\mathcal{O}(\frac{1}{ne}^{-1}) = \mathcal{O}(ne) = \mathcal{O}(n)$.
The lenght of the second phase is $\mathcal{O}(n)$ because the solution is either optimal or there is at least one bit that needs to be flipped for an optimal solution. This again results in expected time $\mathcal{O}(n)$. Since the expected length of Phase 1 is $\mathcal{O}(n)$ the solution produced by the RSH won't be optimal in expectation due to the bound of $\Theta(n \log n)$ for OneMax/ZeroMax.
The lenght of the third phase is identical to a run of the RSH on OneMax/ZeroMax where flips of the first bit are ignored as if it was already correctly flipped and therefore the expected time is $\Theta(n \log n)$

So the expected time is $\mathcal{O}(n) + \mathcal{O}(n) + \Theta(n \log n) = \Theta(n \log n)$ □

Lemma 2.2. *The RSH reaches an approximation ratio of at most 4/3 in expected time* $\mathcal{O}(n \log n)$ *if* $w_1 \leq W/2$

Proof. Helpful statements

(1) If the weight of the fuller bin is at most $\frac{2}{3}$ the approximation ratio is at most $\frac{2}{3}/opt \leq \frac{2}{3}/\frac{1}{2} <= \frac{4}{3}$

(2) If $w_1 \geq \frac{1}{3}$ and w_1 is in the emptier bin, then the weight of the fuller bin at most $1 - w_1 \leq 1 - \frac{1}{3} = \frac{2}{3}$ -> approximation $\leq \frac{4}{3}$ 2

(3) $b_F - b_E \geq v <-> b_F \geq b_E + v$ and therefore any object of weight at most v can be moved from b_F to b_E if $b_F - b_E \geq v$

(4) $b_F \geq \frac{2}{3} -> b_E \leq \frac{1}{3} -> b_F - b_E \geq \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ -> Every object $\leq \frac{1}{3}$ can be moved from b_F to b_E as long as $b_F \geq \frac{2}{3}$ 2

(5) In Time $\mathcal{O}(n \log n)$ the weight of the fuller bin can be decreased to <= 2/3 if every item besides the biggest in the bin is $\leq \frac{1}{3}$.

Proof of (5):
In time $\mathcal{O}(n \log n)$ the RSH can move every object $\leq \frac{1}{3}$ to the emptier bin as long as $b_F \geq \frac{2}{3}$ 2. So in Time $\mathcal{O}(n \log n)$ the solution can be shifted to w_1 being in the first bin and all other objects in the other bin. The algorithm will only stop moving the elements if the condition $b_F \geq \frac{2}{3}$ is no longer satisfied 2. If every object was moved to the emptier bin, the weight of the fuller bin is at most $\max\{1 - w_1, w_1\} \leq \frac{2}{3}$. So either the RSH moves all objects to the emptier bin or stops moving objects because $b_F < \frac{2}{3}$. Either way the fuller bin is at most $\frac{2}{3}$ and with 2 the result follows.

If $w_1 + w_2 > \frac{2}{3}$ after time $\mathcal{O}(n)$ w_1 and w_2 are separated (Proof by by C.Witt) and will remain separated afterwards. From then on the following holds. If w_1 is in the emptier bin, the result follows by 2. Otherwise the result follows by 2 □

Corollary 2.2.1. *The RSH reaches an approximation ratio of at most 4/3 in expected time $\mathcal{O}(n \log n)$*

Proof. This follows directly from 2.1 and 2.2 □

Erklärung der wissenschaftlichen Redlichkeit

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Passau, den 1. Januar 1970

Lipp Daniel