Physics Aware Vector Field Synthesis

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Abstract

Vector field(VF) synthesis from sparse constraints allows for the reconstruction of field-gathered vector flow, efficient and minimally lossy storage of flow data, human designed VFs for industrial applications, etc. The traditional method of synthesizing vector fields only interpolates the sparse vector constraints and doesn't consider more physically relevant data, which. Given the importance of VF synthesis, this study explores the use of physical constraints such as Jacobean, curl, and divergence in VF synthesis.

Introduction

Current methods of VF synthesis rely solely on interpolating known vector values. This has been a tried and true method, but we believe we can better improve upon this method.

We hypothesize that including physical constraints such as Jacobean, curl, and divergence may improve accuracy without significantly decreasing performance

Background

Interpolation is a great reconstructing tool, especially when we have data points really close to each other. However, there are situations where we can't capture the continuity of flow no matter how close our points. However, in principle we can better overcome the limitations of discretization by encoding derivative data such as Jacobean.

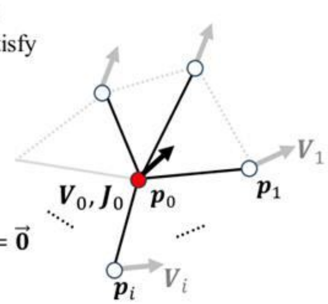
Below we describe the formulation for the two main methods we will be looking at.

Previously, consider a grid point $\boldsymbol{p}_0$, and its direct neighbors $\{\boldsymbol{p}_i\}$. The vector values defined on them satisfy

$$\boldsymbol{V}_0 = \sum_{i=1} w_{i0} \boldsymbol{V}_i \qquad (1)$$

This leads the following linear system

$$\begin{bmatrix} 1 & -w_{10} & 0 & -w_{i0} & \dots & 0 \\ -w_{10} & 1 & \dots & & & \\ & & 1 & & & \\ & & & \dots & & \\ & & & & \dots & \\ 0 & \dots & & & \dots & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_0 \\ \boldsymbol{V}_1 \\ \dots \\ \boldsymbol{V}_i \\ \dots \end{bmatrix} = \vec{0}$$



Given some known vector values, $\{\boldsymbol{V}_k\}$, at selected grid points, the remaining unknown vector values can be solved using the above system.

In the **new framework**, in addition to the above vector constraints, given the Jacobian $\boldsymbol{J}_0$ at $\boldsymbol{p}_0$, we have a

$$\boldsymbol{V}_0 = \boldsymbol{V}_i - \boldsymbol{J}_0(\boldsymbol{p}_i - \boldsymbol{p}_0) \qquad (2)$$

We can (a) estimate vectors at the neighbors of the selected grid points as additional constraints or (b) modify equation (1) using equation (2).

Methods

Each method constructs a linear system from the constraints.
There are 3 types of methods:

- Integrate the Jacobean in the interpolation equations.
- Augment the matrix with the physical constraints.
- Take a first pass that uses the physical constraints to augment the amount of known vectors.

Below we show one result of reconstructing using interpolation and using Jacobean interpolation.

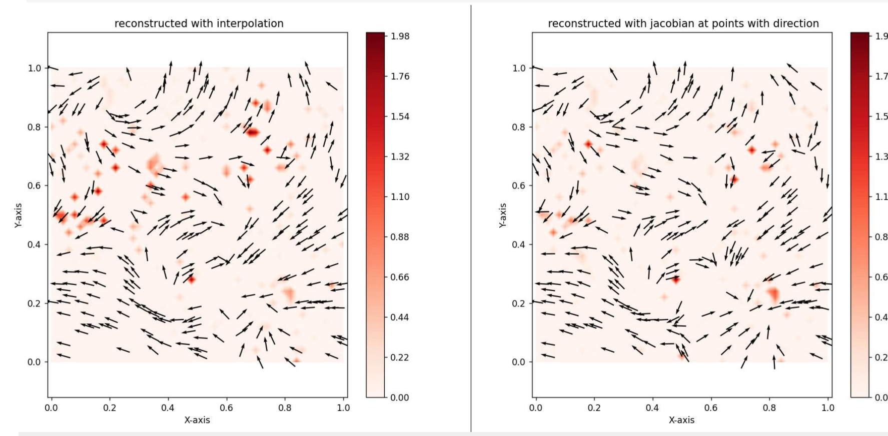


Figure 1. Error over reconstructed VF by interpolation(left) and Jacobean interpolation(right). Redder points indicate higher error.

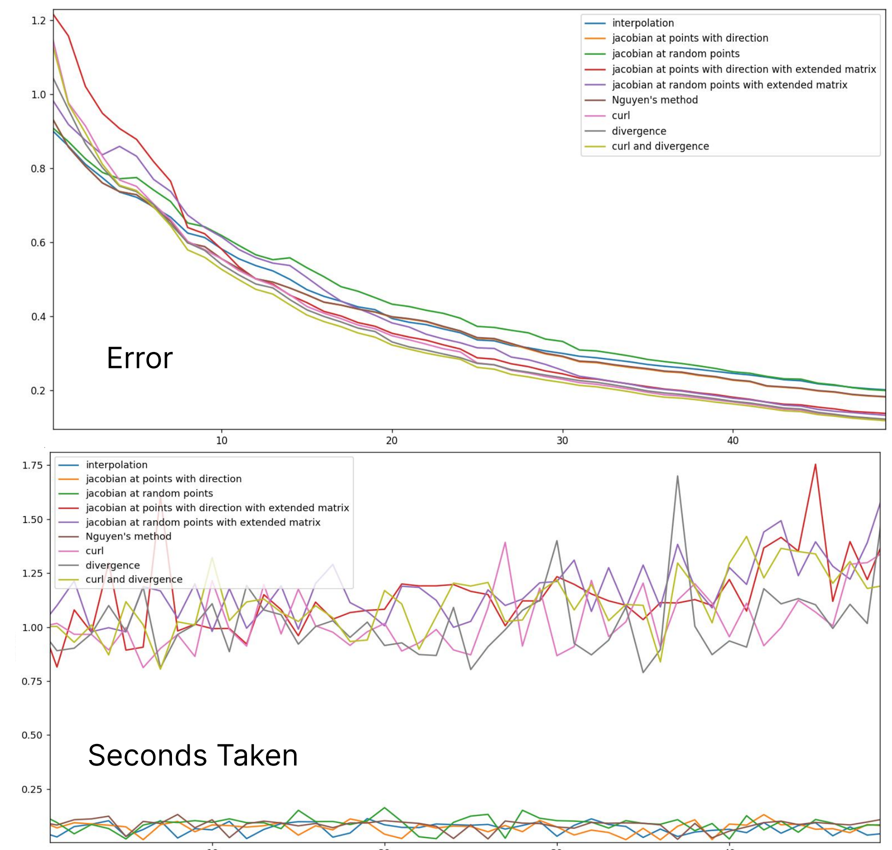


Figure 2. Error(top) and seconds taken(bottom) of each method over runs with (3+x)% sparse data present(averaged over 15 random seeds).

Results

The results from our study support our hypothesis but also show some other unexpected results.

- The accuracy of the first and third type of methods are consistently lower than that of interpolation.
- The second type of methods suffer from over fitting the more information is in the system.
- The second type of methods have significantly worse performance since they rely on least squares.
- Jacobean interpolation performs worse when the Jacobean data is not at points with vector data.

Future Work

- Incorporate other types of constraints such as magnitude.
- Study the over fitting of the second type of method.
- Research other methods of incorporating constraints.
- Investigate the results of our research on 3D fields.

We have hopes these results may lead to improvements in fields where accuracy is critical and that further research may further improve these results.

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