



University of Pisa

Department of computer science

Algorithm design

Ninth hands-on: Game theory 1

Domenico Erriquez

## 1. PROBLEM 1

After Diabolik's capture, Inspector Ginko is forced to free him because the judge, scared of Eva Kant's revenge, has acquitted him with an excuse. Outraged by the incident, the mayor of Clerville decides to introduce a new legislation to make judges personally liable for their mistakes. The new legislation allows the accused to sue the judge and have him punished in case of error. Consulted on the subject, Ginko is perplexed and decides to ask you to provide him with a formal demonstration of the correctness/incorrectness of this law.

### 1.1. SOLUTION

First let's model the problem. We have two players:

- *Judge*, whose actions are  $\{Accuse, Don't\ accuse\}$
- *Accused*, whose actions are  $\{Sue, Don't\ sue\}$

We have also a kind of third player *Society* that has no actions, but his job is to decide if the law is correct or not. We have two states for each player: for the judge the states are *Selfish* where the judge only cares about not being sued and don't get revenge, and *Not selfish* where the judge duty is to respect the law. The accused player's states are: *Guilty, not guilty*. On the payoff matrixes, the first column is the payoff of the judge, the second one for the accused and the third one in brackets is the society.

**Case 1:** The accused is guilty

		Accused	
		Sue	Don't sue
Judge selfish	Accuse	0,0 (1)	1,0 (2)
	Don't accuse	1,1 (0)	<b>2,1</b> (0)

The Nash equilibrium is in the case  $\{Don't\ accuse, Don't\ sue\}$ . The society's utility is low(0), so the law is wrong in this case.

		Accused	
		Sue	Don't sue
Judge not selfish	Accuse	<b>2,0 (1)</b>	<b>3,0 (2)</b>
	Don't accuse	0,1 (0)	1,1 (0)

The Nash equilibriums are  $\{Accuse, Sue\}$  and  $\{Accuse, Don't\ sue\}$ , the society's utility is high in those cases, so the law is correct.

**Case 2:** The accused is not guilty

		Accused	
		Sue	Don't sue
Judge selfish	Accuse	0,1 (1)	1,0 (0)
	Don't accuse	1,2 (0)	<b>2,3 (2)</b>

The Nash equilibrium is in the case  $\{Don't\ accuse, Don't\ sue\}$ . The society's utility is high(2) so the law is correct in this case.

		Accused	
		Sue	Don't sue
Judge not selfish	Accuse	0,1 (1)	1,0 (0)
	Don't accuse	2,2 (0)	<b>3,3 (2)</b>

The Nash equilibrium is in the case  $\{Don't\ accuse, Don't\ sue\}$ . The society's utility is high(2), so the law is correct in this case.

## 2. PROBLEM 2

An investment agency wants to collect a certain amount of money for a project. Aimed at convincing all the members of a group of N people to contribute to the fund, it proposes the following contract: each member can freely decide either to contribute with 100 euros or not to contribute (retaining money on its own wallet). Independently on this choice, after one year, the fund will be rewarded with an interest of 50% and uniformly redistributed among all the N members of the group. Describe the game and find the Nash equilibrium.

### 2.1.SOLUTION PROBLEM 2

First let's define the model of the game. We have  $N$  players  $P_1, P_2 \dots P_n$ , each of the players can take only 2 different actions:  $\{contribute, not\ contribute\}$ . Let be  $k$  the number of players that choose to contribute, then for each player we have the following score  $s_i$ :

$$s_i\{contribute\} = \frac{(k + 1) * 100 * 1,5}{n}$$

$$s_i\{not\ contribute\} = \frac{k * 100 * 1,5}{n} + 100$$

For  $n > 1$ , for every possible value of  $k$ , it is more convenient for the player  $i$ , to don't contribute and since the score function is the same for each player, then for each player is more convenient to don't contribute. In total, there are  $2^n$  possible situations and the Nash equilibrium is reached only when all the players choose to don't contribute. In fact in each of all the other situation, for a player is better to change his strategy because he can have a better score.