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Algorithm design

Tenth hands-on: Game theory 2

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1. PROBLEM 1

Two players drive up to the same intersection at the same time. If both attempt to cross, the result is a fatal traffic accident. The game can be modeled by a payoff matrix where crossing successfully has a payoff of 1, not crossing pays 0, while an accident costs -100.

- Build the payoff matrix
- Find the Nash equilibria
- Find a mixed strategy Nash equilibrium
    - Compute the expected payoff for one player (the game is symmetric)

1.1. SOLUTION

In this problem there are two players: Player1 and Player2. The actions that both players can take are: $\{cross, don't\ cross\}$.
The payoff matrix is the following:

		Player 2	
		Cross	Don't cross
Player 1	Cross	-100, -100	1,0
	Don't cross	0,1	0,0

From the payoff matrix is possible to see that there are two Nash equilibrium: $\{cross, don't\ cross\}, \{don't\ cross, cross\}$.

In order to calculate the mixed strategy, first let's define as p the probability for Player1 to $Cross$ and $1-p$ the probability to $Don't\ cross$. Similarly let's define q as the probability for Player2 to $Cross$ and $1-q$ the probability to $Don't\ cross$. Defined these probabilities we need to solve the following equations:

$$uPlayer1(Cross) = uPlayer1(Don't\ cross)$$
$$uPlayer1(Cross) = q(-100) + (1-q) * 1$$
$$uPlayer1(Don'tcross) = q * 0 + (1-q) * 0$$

Solving the equation, we have that $q = \frac{1}{101}$

For Player 2:

$$uPlayer2(Cross) = uPlayer2(Don't\ cross)$$
$$uPlayer2(Cross) = p(-100) + (1-p) * 1$$
$$uPlayer2(Don'tcross) = p * 0 + (1-p) * 0$$

Solving the equation, we have that $p = \frac{1}{101}$

Therefore, the strategy $Player1(Cross, don't\ cross) = \left(\frac{1}{101}, \frac{100}{101}\right)$ and $Player2(Cross, don't\ cross) = \left(\frac{1}{101}, \frac{100}{101}\right)$ is a mixed strategy Nash equilibrium.

Since the game is symmetric, we have the same expected payoff for the players

		Player 2	
		Cross ($\frac{1}{101}$)	**Don't cross ($\frac{100}{101}$)**
Player 1	**Cross ($\frac{1}{101}$)**	-100, -100	1,0
	Don't cross ($\frac{100}{101}$)	0,1	0,0

$$EP_{Player1} = pq(-100) + (1-p)(q) * 0 + p(1-q) * 1 + (1-p)(1-q) * 0 = 0$$
$$EP_{Player2} = pq(-100) + (1-p)(q) * 0 + p(1-q) * 1 + (1-p)(1-q) * 0 = 0$$

2. PROBLEM 2

Find the mixed strategy and expected payoff for the Back Stravinsky game.

2.1. SOLUTION

In the Back Stravinsky game we have the following payoff table

		Player 2	
		Back	**Stravinsky**
Player 1	**Back**	2, 1	0,0
	Stravinsky	0,0	1,2

In order to calculate the mixed strategy, first let's define as p the probability for Player1 to choose $Back$ and $1-p$ the probability to choose $Stravinsky$. Similarly let's define q as the probability for Player2 to choose $Back$ and $1 - q$ the probability to choose $Stravinsky$. Defined these probabilities we need to solve the following equations:

$$uPlayer1(Back) = uPlayer1(Stravinsky)$$
$$uPlayer1(Back) = q(2) + (1 - q) * 0$$
$$uPlayer1(Stravinsky) = q * 0 + (1 - q) * 1$$

Solving the equation, we have that $q = \frac{1}{3}$

For Player 2:

$$uPlayer2(Back) = uPlayer2(Stravinsky)$$
$$uPlayer2(Back) = p(1) + (1 - p) * 0$$
$$uPlayer2(Stravinsky) = p * 0 + (1 - p) * 2$$

Solving the equation, we have that $p = \frac{2}{3}$

Therefore, the strategy $Player1(Back, Stravinsky) = \left(\frac{2}{3}, \frac{1}{2}\right)$ and $Player2(Back, Stravinsky) = \left(\frac{1}{3}, \frac{2}{3}\right)$ is a mixed strategy Nash equilibrium.

The expected payoff for both players is:

$$EP_{Player1} = pq(2) + p(1 - q) * 0 + (1 - p)(q) * 0 + (1 - p)(1 - q) * 1 = \frac{2}{3}$$
$$EP_{Player1} = pq(1) + p(1 - q) * 0 + (1 - p)(q) * 0 + (1 - p)(1 - q) * 2 = \frac{2}{3}$$

3. PROBLEM 3

The Municipality of your city wants to implement an algorithm for the assignment of children to kindergartens that, on the one hand, takes into account the desiderata of families and, on the other hand, reduces `city traffic caused by taking children to school. Every school has a maximum capacity limit that cannot be exceeded under any circumstances. As a form of welfare the Municipality has established the following two rules:

- in case of a child already attending a certain school, the sibling is granted the same school;
- families with only one parent have priority for schools close to the workplace.

Model the situation as a stable matching problem and describe the payoff functions of the players. Question: what happens to twin siblings?

3.1. SOLUTION

For this exercise, we consider the set C of the children and we model the schools defining a set S containing the seats of all the schools. According to the game's description we can model the preferences for each child C_i as the distance between the child's location, which can be the house H_i or the workplace W_i of the child's parent, and the school.
Families with only one parent have higher priority so we try to minimize the distance between parent's worlplace W_i and the school.

If a child is already in a school, then his sibling will be located in the same school as well, so the distance for the sibling is set to 0. The twins can be seen as unique child of size 2.