Investigating how the radius of a simple pendulum affect its time period.

Physics HL Lab Report 5

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1 Introduction and background knowledge

Note

Use objective, scientific knowledge. Avoid using "we did", "we were", "you can find". Try use scientists in the history.

Simple pendulum is a phenomenon that has been discovered by humans for a long time. Its periodic behaviour made it an ideal tool for timekeeping long before modern physics has been invented. In physics, the simple pendulum is a great instance to illustrate how physics estabilishes models to interprete real-world phenomenons.

The period, which is the time taken for the pendulum to do one complete oscillation, varys according to several factors. The effect of the length of string (or rod, depending on the context) is the main focus of this experiment.

Research question: What is the relationship between the length of the string and the period of simple pendulum?

2 Hypothesis and reasoning

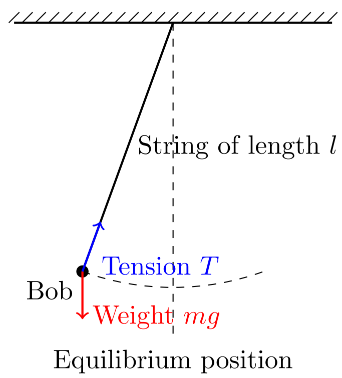

Figure 1: Model of a simple pendulum

As is shown in Figure 1, the bob in a simple pendulum system is subject to two forces: the gravitational force (weight) and the tension from the string. The path of motion is a circle, and therefore the velocity vector must be tangent to the path and perpendicular to the tension all the time.

When the string is at angle θ from equilibrium position, we may decomposite the gravitational force into two forces: one at the opposite direction to tension T, the other perpendicular to that. It can be noticed that the latter part equals to $mg\sin\theta$. Hence the tangential acceleration can be found by

$$a = \frac{mg\sin\theta}{m} = g\sin\theta$$

.

It is also worth noticing that the arclength of the bob from equilibrium can be given by

$$x = l\theta$$

where l is the length of the string. When θ is small we have

$$\theta \sim \sin\theta \sim \tan\theta$$

Therefore it holds that

$$a \approx g\theta = -\frac{g}{l}x$$

(The negative sign comes from different direction of a and x)
Which means that it is valid to say

$$a \propto x$$

showing simple harmonic motion property of the simple pendulum.
According to simple pendulum equations

$$a = -\omega^2 x$$

$$T = \frac{2\pi}{\omega}$$

We can find that

$$T = \frac{2\pi}{\sqrt{\frac{-a}{x}}} = 2\pi\sqrt{\frac{l}{g}}$$

To make the euqation linear, we can square the both side

$$T^2 = \frac{4\pi^2}{g}l$$

This equation shows a linear relationship between T^2 and l, which leads to our hypothesis: T **increases as** l **gets larger.** T^2 **is directly proportional to** l**.**

3 Experiment design

3.1 Variables

> Note
>
> The range should be maximized. Try $L_{max} \approx 100cm$. Or we should explain why the range is not maximized.

- Independent variable: the length of the string l. $(10, 20, 30, 40, 50\text{cm})$
- Dependent variable: the time period T for the pendulum.
- Controlled variables: The size, mass, material of the bob. Initial angle from equilibrium position, etc. They are listed in Table 1.

Note

Mass, volume of the bob, displaced angle/amplitude, acceleration of free fall, released speed, material of the string.

Table 1: Controlled Variables

Variable	**Value**	**Reason to control**	**Method to control**
The diameter of the bob d	2.2 cm	Make air resistance consistent in different trials	Use the same bob
The mass of the bob m	about 5 g	Avoid the effect of mass change	Use the same bob
Initial angle θ_i	about 20°	Make sure the pendulum is equally like SHM	Use a protractor to control the angle.

3.2 Materials

* 1× Iron stand (taller than 55 cm).
* 1× Electric force gauge (that can display force - time graph on the laptop connected to it).
* 1× Pendulum (contains a string longer than 50 cm and a bob attached to it).
* 1× Meter rule.
* 1× Protractor.

3.3 Setup diagram

As is shown in Figure 2, the force gauge is fixed on the top of the iron stand, and the pendulum is attached to the force gauge.

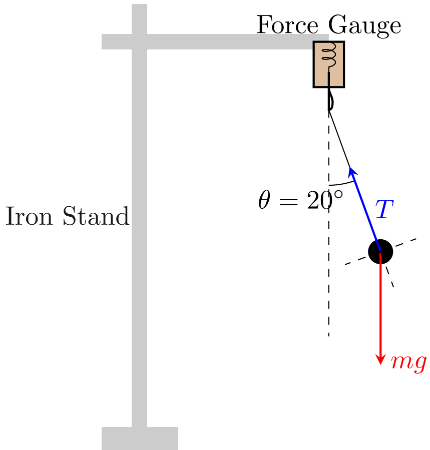

Figure 2: Setup diagram

3.4 Procedure

Note

Make sure each measured value is clearly stated. (length below the hook → the distance from the hooking point to the bob's center of mass) (also where is the center of mass, how to get it?) Make sure the measuring value is clearly stated.

1. $l = 10cm$.
2. Use a vernier caliper to measure the diameter and derive the radius of r the bob.
3. Make sure the length of the wire from the top of the bob to the hook is $l - r$.

Also, how to measure the angle displaced from vertical? (e.g. Use a protractor and a hanging line)

1. Attach the string to the hook of the force gauge. Use the meter rule to ensure that the length below the hook is $l = 10\,\text{cm}$.
2. Lift the bob of the pendulum. Make sure the string remains straight. Use the protractor to make sure its angle to vertical is approximately $20°$.
3. Freely release the bob from stationary. Record the diagram shown on the laptop.
4. Find a suitable range. Take down the beginning time, ending time and number of crests in the range.
5. Repeat steps 2-4 four times to get four additional trials.
6. Repeat steps 1-5 with $l = 20, 30, 40, 50$cm.

Note

Risk assessment. At least show one of the safety consideration, ethic consideration and environmental consideration.

4 Results

4.1 Raw data

As is mentioned in Procedure part, the string length, number of time span, start time and end time are recorded in the experiment. Five groups of experiment were done with five trials each group.

Table 2: Raw Data (full table in appendix)

Note

Still need uncertainty. and explanation on why 2 period on force = 1 period on motion. Also consistent decimal places is needed.

Group No.	String length l(cm)	Trial No.	Start time t_0	End time t_1	Number of periods n
1	10	1	15.1	28.9	40
1	10	2	6.4	19.8	40
1	10	3	9.8	23.3	40
1	10	4	8.6	22.2	40
1	10	5	11.3	24.8	40
2	20	1	27.7	46.8	40
...	...	...	...	...	...
5	50	5	13.9	27.8	19

4.2 Processed data

The processed data can be seen in Table 3. Period for the simple pendulum, its square, and their errors are calculated. The method of calculation is illustrated in Sample Processing part.

$T - L$ graph and $T^2 - L$ graph are plotted, as is shown in Figure 3. Linear regression is also done on $T^2 - L$ graph.

4.3 Sample processing

Take processing of group 1 data as an example, the pendulum time period can be given by $\frac{t_1 - t_0}{\frac{1}{2}n}$ (since in one complete oscillation, the force gauge reaches the crest twice)

Table 3: Processed data

Group	T(s)	ΔT(s)	$\%\Delta T$(s)	T^2(s^2)	ΔT^2(s^2)
1	0.678	$\pm$ 0.2	$\pm$ 1.45%	0.46	$\pm$ 0.013
2	0.953	$\pm$ 0.25	$\pm$ 1.31%	0.91	$\pm$ 0.024
3	1.141	$\pm$ 0.15	$\pm$ 0.88%	1.30	$\pm$ 0.023
4	1.304	$\pm$ 0.05	$\pm$ 0.38%	1.70	$\pm$ 0.013
5	1.446	$\pm$ 0.2	$\pm$ 1.48%	2.09	$\pm$ 0.062

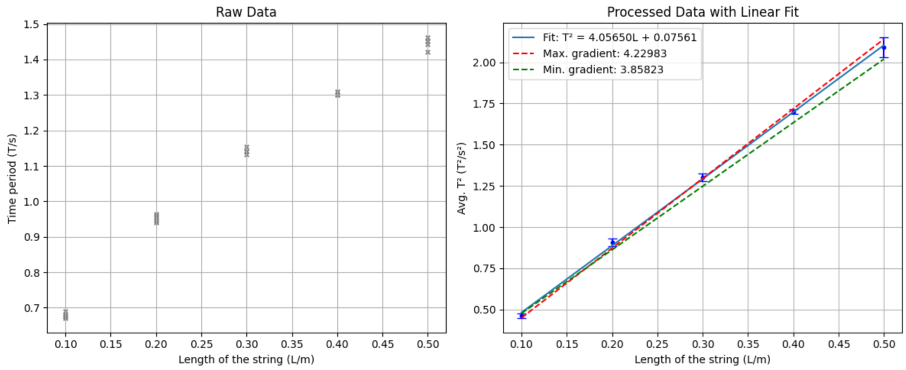

Figure 3: $T-L$ graph and T^2-L graph

$$T_a = \frac{t_{1a} - t_{0a}}{\frac{1}{2}n} = \frac{28.90\text{s} - 15.10\text{s}}{20} = 0.69\text{s}$$

$$T_b = \frac{t_{1b} - t_{0b}}{\frac{1}{2}n} = \frac{19.80\text{s} - 6.40\text{s}}{20} = 0.67\text{s}$$

$$\cdots$$

$$T = \frac{t_a + t_b + t_c + t_d + t_e}{5} \approx 0.68\text{s}$$

And the error is calculated according to error propangation,

$$\pm 0.5\Delta T = \pm \frac{\max\{T\} - \min\{T\}}{2} = \pm 0.01\text{s}$$

$$\%\Delta T = \pm \frac{\Delta T}{T} \times 100\% = \pm 1.47\%$$

$$T^2 = (0.68\text{s})^2 \approx 0.46\text{s}^2$$

$$\Delta T^2 = 2\%\Delta T \times T^2 = \pm 0.013\text{s}^2$$

5 Discussion and conclusion

The diagram shows strong correlation of T^2 and L. The best fit line of $T^2 - L$ has

- a gradent of $4.056\mathrm{s}^2\mathrm{m}^{-1}$.
- a y-intercept of $7.561 \times 10^{-2}\mathrm{s}^2$.

As is mentioned in processed data section, the graph is expected to be a straight line passing through the origin. The diagram show a graph with relatively small error, supporting the initial hypothesis.

The y-intercept is expected to be zero, but 7.561×10^{-2} is found. This is a pretty small amount and it might have resulted from the random error due to limiting accuracy of the device or the sampling frequency has been set too low.

The gradient is expected to be $\frac{4\pi^2}{g}$, where g is the gravitational acceleration near sea level $g \approx 9.81\,\mathrm{ms}^{-2}$. The theoratical value of the gradient should be 4.024, which means there is an 0.7% error. This low error shows high accuracy and precison of this experiment, considering the pendulum is not exactly SHM.

Error bars are relatively small, indicating high precison and low random error.

6 Evaluation

The experiment was conducted successfully, gathering sufficient data and supporting our hypothesis. However, there is still room for improvement.

- **Defect in SHM pendulum model** Our hypothesis is based on $\sin\theta \approx \theta$. However, the θ is not very small in the experiment, indicating the bob is not doing perfect SHM. A better model, which may involve ODE (Ordinary Differential Equation), can provide a better hypothesis.
- **Air resistance and friction** We ignore the air resistance and friction between the string and hook in the experiment. Those force may slow down the pendulum and cause its period to change. It is not practical to completely eliminate those effects, but using a heavier bob and smoother string can reduce its effect on the result.
- **Unfixed apparatus** Some viberation was noticed during the experiment. Additionaly, the string tied to the hook did not remain on the same position either. This may cause the pivot point that is assumed to be fixed to change its position during the experiment, making data inaccurate. Using a steadier iron stand and a smaller hook can fix this issue.
- **Not on the same surface** When releasing the bob, the hand may exert a force that deviate the bob's motion from the horizontal plane it was supposed to be in. This disturbance may make the motion similar to a cone pendulum. To fix that, we may use an iron bob and electromagnet to lift and release the bob.

Note

Other sources of uncertainty

- Streching of the string.
- Friction between string and hook.
- None zero initial speed.
- Inaccurate length measurement.
- Insufficient trials.
- Small range.

Table 4: Full data

Group	l(cm)	Trial	t_0(s)	t_1(s)	n	Δt(s)	T(s)	$\bar{T}$(s)	ΔT(s)	$\%\Delta T$(s)	T^2(s^2)	ΔT^2(s^2)
1	10	1	15.1	28.9	40	13.8	0.69	0.68	0.20	1.45%	0.46	0.01
1	10	2	6.4	19.8	40	13.4	0.67					
1	10	3	9.8	23.3	40	13.5	0.68					
1	10	4	8.6	22.2	40	13.6	0.68					
1	10	5	11.3	24.8	40	13.5	0.68					
2	20	1	27.7	46.8	40	19.1	0.96	0.95	0.25	1.31%	0.91	0.02
2	20	2	4.5	23.3	40	18.8	0.94					
2	20	3	8.6	27.8	40	19.2	0.96					
2	20	4	5	24.3	40	19.3	0.97					
2	20	5	6.6	25.5	40	18.9	0.95					
3	30	1	25.4	42.5	30	17.1	1.14	1.14	0.15	0.88%	1.30	0.02
3	30	2	41.4	58.7	30	17.3	1.15					
3	30	3	8.9	25.9	30	17	1.13					
3	30	4	33	50.1	30	17.1	1.14					
3	30	5	8.9	26	30	17.1	1.14					
4	40	1	13	26	20	13	1.30	1.30	0.05	0.38%	1.70	0.01
4	40	2	15.9	29	20	13.1	1.31					
4	40	3	11.2	24.3	20	13.1	1.31					
4	40	4	6.3	19.3	20	13	1.30					
4	40	5	5.4	18.4	20	13	1.30					
5	50	1	3.8	17.3	19	13.5	1.42	1.45	0.20	1.48%	2.09	0.06
5	50	2	59.4	73.2	19	13.8	1.45					
5	50	3	28.8	42.5	19	13.7	1.44					
5	50	4	18.4	32.2	19	13.8	1.45					