Option 1: driving fuel efficiency

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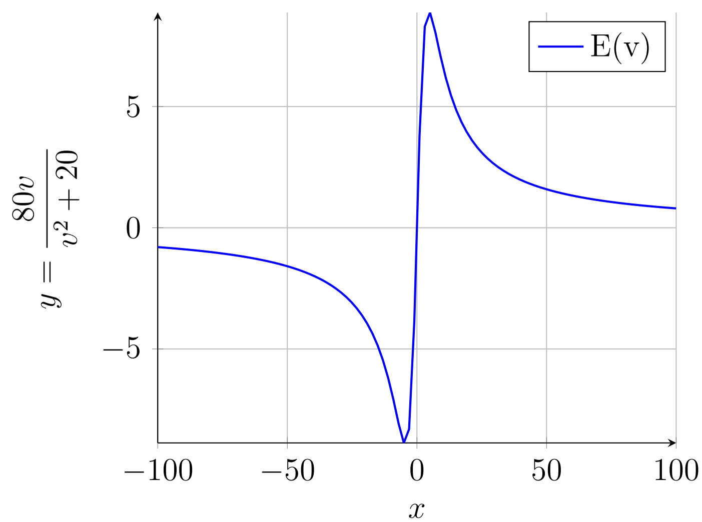
March 27, 2025

$$E(v) = \frac{80v}{v^2 + 20}$$



1 Modeling

As the speed increases, the fuel efficiency first increases and then reduces.

Since

$$
\begin{aligned}
&\lim_{v\to+\infty} E(x)\\
=&\lim_{v\to+\infty} \frac{80v}{v^2+20}\\
=&\lim_{v\to+\infty} \frac{80}{2v}\\
=&0
\end{aligned}
$$

The horizontal asymptote is $y = 0$. It means as the speed increases, the efficiency decreases and gets infinitely close to zero but never reaches zero.

2 Optimization

The optimal driving strategy should be maximizing the efficiency. To find the maximum of a continuous function, we can try find the zeros of its derivative.

The quotient rule can be applied for differentiation:

$$\left(\frac{a}{b}\right)' = \frac{a'b - ab'}{b^2}$$

where $a = 80v$ and $b = v^2 + 20$.

The differentiation of a and b is trivial

$$
\begin{aligned}
a = 80v \quad &\Rightarrow \quad a' = 80,\\
b = v^2 + 20 \quad &\Rightarrow \quad b' = 2v.
\end{aligned}
$$

Therefore

$$
\begin{aligned}
E'(v) &= \frac{a'b - ab'}{b^2} = \frac{(80)(v^2+20) - (80v)(2v)}{(v^2+20)^2}\\
E'(v) &= \boxed{\frac{-80(v^2-20)}{(v^2+20)^2}}.
\end{aligned}
$$

$E'(v)$ is zero only when $v^2 - 20$ is zero. $v = \pm\sqrt{20} = \pm 2\sqrt{5}$. The negative one does not make sense in the real circumstance. Therefore $E(v)$ reaches its maximum when $v = 2\sqrt{5}$.