How does the moment of inertia affect the period of Maxwell's Wheel?

Physics HL Internal Assesment

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1 Introduction and background knowledge

In ancient Greece (around 220 B.C.), Archimedes laid solid foundation for rigid body statics by discovering and formalizing the conditions of equilibrium for levers. People studied Archimedes's work and developed concepts like torque after then, but it was not until Issac Newton (1643-1727) formulated Newton's Laws of Motion that humans are well equipped for rigid body dynamics (Farber, 1961). With the assistance of advanced mathematical tools, Leonhard Eular (1707-1783) successfully formalized the dynamics of rigid bodies and derived the concepts of moment of inertia and principal axes (Marquina, Marquina, Marquina, & Hernández-Gómez, 2016).

Just like inertia (which is determined by mass) in translational motion, the moment of inertia evaluates a body's ability to resist angular acceleration. Generally speaking, the further away the mass is distributed from its axis of rotation, the larger the body's moment of inertia is. Objects with large moment of inertia are good at storing rotational kinetic energy, which makes them suitable for objects like flywheels, while objects with small moment of inertia can rotate faster.

Maxwell's wheel is commonly used as an instrument to illustrate the conservation of energy and the concept of rotational kinetic energy in physics education. It can also be utilized to measure the moment of inertia of a disk-shaped object. This experiment aims at discovering the significance of the moment of inertia in a Maxwell's wheel experiment.

Research question: How does the moment of inertia affect the period of Maxwell's Wheel?

2 Hypothesis and reasoning

As is shown in Figure 1, the Maxwell's wheel mainly consists of three parts: a wheel of radius R, an axle of radius r, and a string. Aside from friction, two forces are acting on the pendulum: the tension from the string and the gravitational force. During the entire process, the gravitational potential energy converts to kinetic energy of the wheel.

The downward movement and the upward movement is basically symmetric, so only the downward reaction needs algebraric analysis.

Since the wheel is not in equilibrium, its hard to analyze the magnitude of tention T. However, the problem can be tackled using conservation of energy.

Due to the negeligibility of friction, we can assume that all the gravitational potential energy lost is converted to the kinetic energy. Or more specificly, the sum of the translational kinetic energy and rotational kinetic energy should equal to the loss in gravitational potential energy. Or formally,

$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \tag{1}$$

where I is the moment of inertia of the entire wheel.

Moreover, using the definition of velocity and angular velocity, we can derive identities 2, 3, and 4.

$$\frac{\mathrm{d}\Delta H}{\mathrm{d}t} = v \tag{2}$$

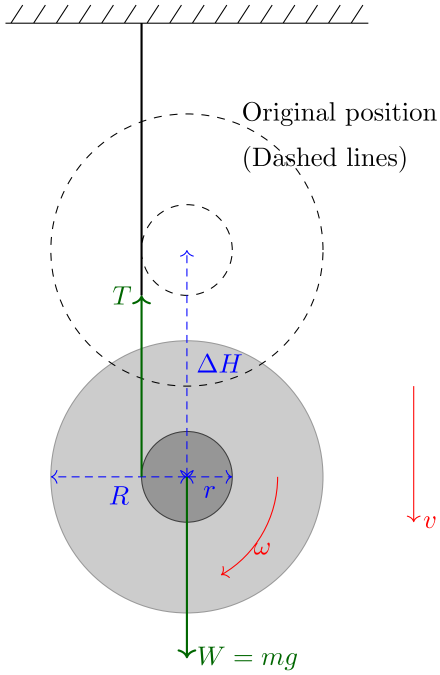

Figure 1: Model of a Maxwell's Wheel

$$\omega r = v \tag{3}$$

$$\Delta H(0) = 0 \tag{4}$$

Therefore we can get Equation 5, in which terms can be moved and combined to derive Equation 6.

$$mg\Delta H = \frac{1}{2}mv^2 + \frac{1}{2}\frac{I}{r^2}v^2 \tag{5}$$

$$\frac{2mg\Delta H}{m + \frac{I}{r^2}} = (\frac{\mathrm{d}\Delta H}{\mathrm{d}t})^2 \tag{6}$$

Moving $\mathrm{d}\Delta H/\mathrm{d}t$ to the left size and make its index 1, a simple ODE displaced as Equation 7 is obtained.

$$\frac{\mathrm{d}\Delta H}{\mathrm{d}t} = \sqrt{\frac{2mg}{m + \frac{I}{r^2}}}\Delta H^{0.5} \tag{7}$$

With the help of formula 4, the ODE is solved, and the solution is Equation 8.

$$\Delta H(t) = \frac{mg}{2m + \frac{2I}{r^2}}t^2 \tag{8}$$

The downward movement terminates when $\Delta H(t) = l$, which means

$$t_{down} = \sqrt{\frac{(2m + \frac{2I}{r^2})l}{mg}} \tag{9}$$

The entire period of the Maxwell's Wheel is

$$T = 2t_{down} = 2\sqrt{\frac{(2m + \frac{2I}{r^2})l}{mg}} \tag{10}$$

which can be re-written as

$$T^2 = 8l\frac{(m + \frac{I}{r^2})}{mg} \tag{11}$$

Or

$$T^2 = \frac{8l}{mgr^2}I + \frac{8l}{g} \tag{12}$$

From this formula, the hypothesis can be derived: T **increases as** I **increases.** T^2 **and** I **has a linear relationship.**

3 Experiment design

3.1 Variables

- Independent variable: Moment of inertia (manipulated by changing the distance from the magnets to the pivot: $r' = 2.5, 3.5, 4.5, 5.5, 6.5, 7.5, 8.5$cm).
- Dependent variable: "Period" of the Maxwell's Wheel (Time between two lowest positions).
- Controlled variables: The material, mass and size of the plate and magnets. The length of the string. The mass, radius and length of the axle, etc. They are listed in table 1.

Independent variable turned out to be $0.668, 0.776, 0.919, 1.09, 1.31, 1.56$ and $1.85 \times 10^{-3}\text{kgm}^2$ (as is shown in Section 4). The independent variable is hard to manipulate, as the moment of inertia results from the arithmatical addition of two parts: the acrylic disk and the magnets. The former part is independent of r', making it impossibe to reach very small r'. The latter part is proportional to r'^2, resulting in difficulty to place the magnets on the acrylic plane if even distribution between I is needed.

3.2 Materials

* 2 Retort stands (height ≈ 50 cm) with clamps
* 2 Cotton strings (length ≈ 70 cm)
* 1 Acrylic disc with axle (radius ≈ 10 cm, mass ≈ 110 g)
* 16 Round magnets (radius ≈ 3.0 cm, mass ≈ 23 g)

Table 1: Controlled variables

Variable	Value	Reason to control	Method to control
Total mass of the wheel m	$\approx 400\,\mathrm{g}$	To make sure the gradient is not affected.	Use the same set of magnets, plate and axle.
Material of the wheel	Acrylic + NdFeB magnet	To make sure the mass distribution is even and consistent.	Use the same set of magnets and plate.
Height of displacement	$\approx 20\,\mathrm{cm}$	Unify the initial kinetic energy and length of path.	Determine and mark a release position on the string.
Temperature and humidity	Around 25°C, 40% RH	Avoid the change in air resistance	Conduct the experiment in a short period of time.

* 1 Electric force gauge (50 N), with compatible data cable
* 1 Tape rule (range $\geq 50\,\mathrm{cm}$, resolution $\leq 0.1\,\mathrm{cm}$)
* 1 Electric balance (range $\geq 400\,\mathrm{g}$, resolution $\leq 0.1\,\mathrm{g}$)
* 1 Vernier caliper (range $\geq 5\,\mathrm{cm}$, resolution $\leq 0.01\,\mathrm{cm}$)
* 1 Hot glue gun

3.3 Setup diagram

The apparatus is set up by hanging the Maxwell's Wheel between two iron stands using cotton string, as is shown in Figure 2.

3.4 Procedure

1. Measure the total mass of the magnets, mass of the acrylic plate, radius of the acrylic plate, radius of the axle and maximum vertical displacement after setting up the apparatus as is shown in Figure 2
2. Find a pair of diameters of the acrylic plane that is perpendicular to each other. This can be done by drawing the perpendicular bisector of an arbitrary pair of perpendicular chords.
3. Calibrate both axis with the assistance of a ruler. Make sure the center of the circle is scaled zero.
4. Attach four pairs of magnets to the plane.
5. Adjust the position of the magnets to make the distance for the magnets' centers of mass are $r = 8.5\,\mathrm{cm}$. This can be done by placing the magnet (with radius $r' = 3\,\mathrm{cm}$) between $r = 10\,\mathrm{cm}$ mark and $r = 7\,\mathrm{cm}$ mark, while touching the both marks.
6. Fix the magnets with glue guns.

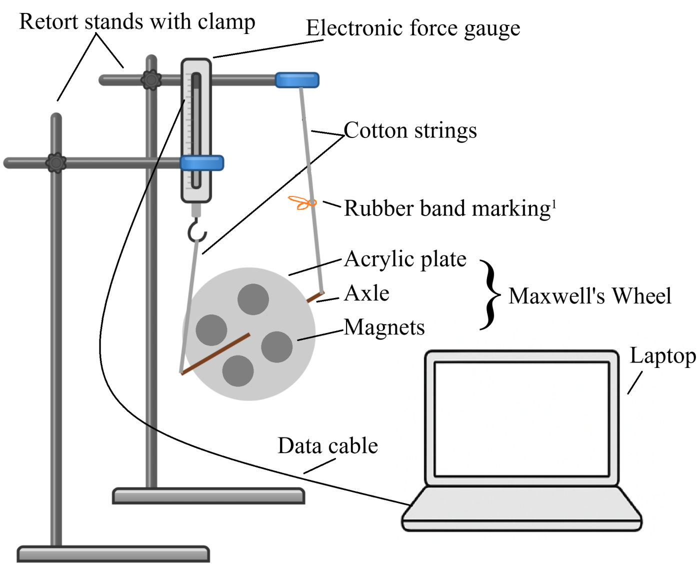

Figure 2: Setup diagram [2]

1 The rough location of rolling up is marked with a rubber band and the precise location is marked with a pen.
2 The graph is made with chemix and mspaint.

7. Swirl up the axle until it reaches the designated point, start the forcemeter and release the wheel.
8. Record the time for the first and second time where the forcemeter reading reaches local maxima [1]
9. Repeat Steps 7 and 8 for 4 additional trials.
10. Repeat Steps 5 to 9 with $r = 7.5, 6.5, 5.5, 4.5, 3.5$cm.

3.5 Risk assessment

The experiment is relatively safe with little potential hazard.

It is worth keeping in mind that one should excercise caution when using the magnets, since the strong attraction of the magnets can cause unintended damage to the operator (e.g. their skin may be pinched when the magnets snaps together).

The experiment does not involve ethical issues as no human or animal subjects are involved.

The experiment has little environmental issues as little material is disposed and none of the materials are toxic.

[1] This is done with the assistance of a program. If the first local maximum is not prominant, the second and the third local maxima is used instead.

4 Results

4.1 Raw data

Raw data is collected as follows:

- Total mass of the magnets M_m : 193.70 ± 0.01g
- Total mass of the acrylic plate M_a: 193.17 ± 0.01g
- Radius of the acrylic plate R_a: 10.0 ± 0.05cm
- Radius of the axle r: 4.0 ± 0.1mm
- Maximum vertical displacement H: 20 ± 0.5cm
- Other data is shown in Table 2.

Table 2: Raw Data

Experiments 1-4					Experiments 5-7				
No.	r(cm) ± 0.1 cm	Trial	t_1(s) ± 0.01 s	t_2(s) ± 0.01 s	No.	r(cm) ± 0.1 cm	Trial	t_1(s) ± 0.01 s	t_2(s) ± 0.01 s
1	2.5	1	2.28	6.68	5	6.5	1	3.58	9.48
		2	2.25	6.63			2	9.28	15.23
		3	2.48	7.03			3	3.45	9.48
		4	6.73	11.28			4	3.53	9.43
		5	2.33	6.78			5	9.28	15.23
2	3.5	1	2.73	7.23	6	7.5	1	10.43	17.68
		2	7.03	11.53			2	3.93	10.68
		3	2.78	7.63			3	4.08	11.03
		4	2.58	7.38			4	4.23	11.13
		5	2.83	7.33			5	4.03	10.53
3	4.5	1	2.88	8.13	7	8.5	1	4.08	11.78
		2	3.18	8.63			2	11.68	18.33
		3	3.08	8.53			3	11.48	18.38
		4	2.88	7.88			4	12.03	20.58
		5	3.03	8.28			5	11.78	18.63
4	5.5	1	3.28	8.93			6	4.13	11.63
		2	3.23	8.73					
		3	3.23	8.83					
		4	3.18	8.73					
		5	3.48	9.23					

4.2 Processed data

4.3 Sample processing

Take the data processing of experiment 1 as an example.

Table 3: Processed data

Experiment	$I(\times 10^{-3}\text{kgm}^2)$	$T^2(\text{s}^2)$
1	0.668 ± 0.0201	19.9 ± 0.759
2	0.776 ± 0.0237	21.4 ± 1.62
3	0.919 ± 0.0528	27.9 ± 2.38
4	1.09 ± 0.0309	31.5 ± 1.40
5	1.31 ± 0.0345	35.4 ± 0.773
6	1.56 ± 0.0381	47.2 ± 5.15
7	1.85 ± 0.0417	50.7 ± 7.48

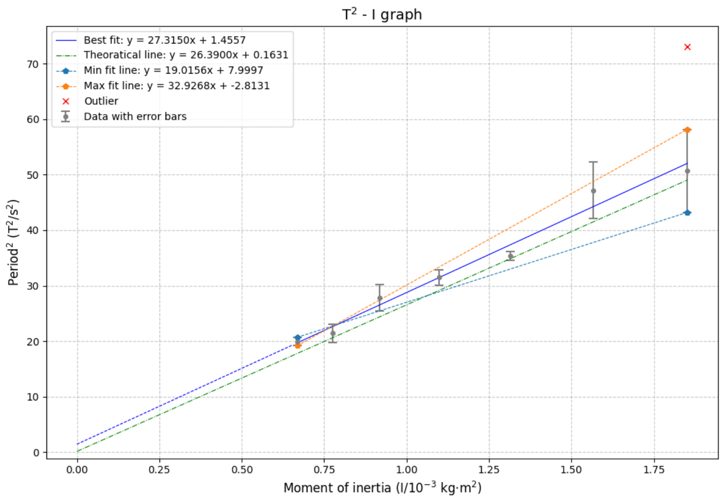

Figure 3: $T^2 - I$ graph

The R, distance from the magnets' center of mass to the pivot, is measured to be 2.5 ± 0.1cm. Therefore the moment of inertia contributed by the magnets is calculated as

$$\begin{aligned} I_m &= MR^2 \\ &= 179.39 \times 10^-3\text{kg}(\times 2.5 \times 10^-2)^2\text{m}^2 \\ &= 1.1 \times 10^{-3}\text{kgm}^2 \end{aligned} \tag{13}$$

$$\begin{aligned} \Delta I_m &= \%\Delta I_m \times I_m \\ &= (\frac{\Delta M}{M} + 2\frac{\Delta R}{R}) \times I_m \\ &= \pm 9 \times 10^{-6}\text{kgm}^2 \end{aligned} \tag{14}$$

The force gauge data is recorded and plotted against time in Figure 4. It can be noticed that the reading of the force gauge is noisy but shows clear peaks. With the assistance of a program, prominant peaks are marked in the figure using red dots.

The x-index of the first two peaks are taken as t_1 and t_2 of this trial. The "period" of the experiment T_a is calculated as in Equation 15, whose uncertainty

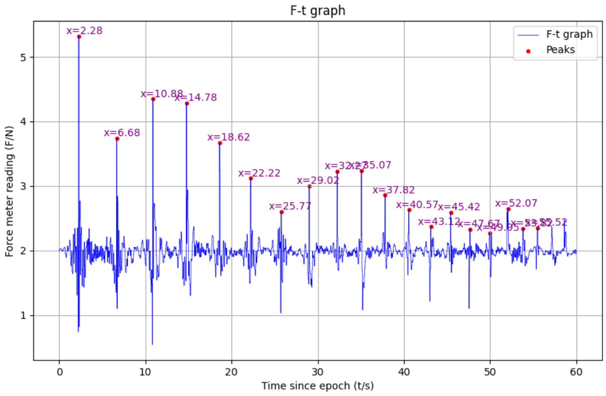

Figure 4: $F - t$ graph

is calculated as in Equation 16.

$$
\begin{aligned}
T_a &= t_1 - t_0 \\
&= 6.68\,\mathrm{s} - 2.28\,\mathrm{s} \\
&= 4.40\,\mathrm{s}
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
\Delta T_a &= \Delta t_1 + \Delta t_0 \\
&= (\pm 0.01\,\mathrm{s}) + (\pm 0.01\,\mathrm{s}) \\
&= \pm 0.02\,\mathrm{s}
\end{aligned}
\tag{16}
$$

Applying this two five trials, we can get the average of the first experiment and the corresponding uncertainties, as is shown in Equations .

$$
\begin{aligned}
\bar{T} &= \frac{T_a + T_b + T_c + T_d + T_e}{5} \\
&= 4.466\,\mathrm{s}
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
\Delta \bar{T} &= \frac{\max\{T_i\} - \min\{T_i\}}{2} \\
&= \frac{4.55\,\mathrm{s} - 4.38\,\mathrm{s}}{2} \\
&= 0.085\,\mathrm{s}
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
\bar{T}^2 &= (4.466\,\mathrm{s})^2 \\
&= 19.94\,\mathrm{s}^2
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
\Delta \bar{T}^2 &= \frac{\Delta \bar{T}}{\bar{T}} \times 2 \times \bar{T}^2 \\
&= 1.9032\% \times 2 \times 19.94\,\mathrm{s}^2 \\
&= 0.7592\,\mathrm{s}^2
\end{aligned}
\tag{20}
$$

5 Discussion and conclusion

The diagram shows strong correlation between T^2 and I. The best fit line of $T^2 - I$ has

- a gradient of $(27.3150 \pm 6.9556) \times 10^3\ \mathrm{s^2kg^{-1}m^{-2}}$.
- a y-intercept of $1.4557\mathrm{s}^2$.

As is mentioned in Section 4.2, the graph is expected to be a straight line very close to the origin. The diagram shows a graph with relatively small error, supporting the initial hypothesis.

The y-intercept is expected to be $\frac{8l}{mgr^2} \approx 0.16\mathrm{s}^2$, but $1.4557\mathrm{s}^2$ is found. This is almost 9 times the theoretical value. However, considering the large value of T in data, $1.4557\mathrm{s}^2$ is still less than 10% of the minimum dependent variable. This means it can still be considered an error, whose cause will be discussed in Section 6.

The gradient is expected to be $\frac{8l}{g} = 26.3900 \times 10^3\ \mathrm{s^2kg^{-1}m^{-2}}$, where l is the length for lifting (controlled to be close to 20 cm)and g is the gravitational acceleration near sea level. There is a small 3.51% error between the actual gradient and predicted gradient, and the predicted gradient lies within the uncertainty range of the gradient. The low error shows high accuracy and precision of this experiment.

Error bars are small when I is small, but becomes larger as I gets larger. That might result from the significant horizontal perturbation get greater as I increases. Horizontal movements make the wheel start to swing like a pendulum and makes the motion more chaotic and less trackable.

The best fit line fails to pass through the fifth error bar, which is also coincidentially a very small error bar. That might result form coincidence, since there is only five trials in each group.

6 Evaluation

The experiment was conducted successfully, gathering sufficient data and supporting the initial hypothesis. However, there are some uncertainties in the experiment that can be improved.

- **Defect in modelling** It is assumed in the hypothesis that the wheel do not swing horizontally and the string is always vertical, but small horizontal movement is inevitable during the experiment process.
- **Screw-threaded axle** For an ideal Maxwell's Wheel, the radius of the small axle should be constant. However, a screw-threaded axle is utilized to ensure better installation of the wheel, making the string swirl around the thread with inconsistant radius. This can be improved by using an axle that has screw thread only in the middle point.
- **String attaching** When attaching the wire to the axle, some adhesive (hot glue) is used to ensure connection stability. This also contributed to inconsistant radius. Opt for an axle with a hole for attachment to avoid the use of hot-melt adhesive.

- **Horizontal movement** The hypothesis is based on negeligibile horizontal movement, but in the real experiment horizontal movement is inevitable because the wheel is controlled and released by hand. Using a plumb line to adjust the string before releasing can minimize the horizontal deflection.

- **Imprecise marking on the string** The string is not vertical to the ground, which means the marked distance is not precisely the vertical displacement. Additionaly, axle's size can also lead to inconsistant vertical displacement. This might have lead to unexpected large y-intercept mentioned in Section 5.

References

Farber, E. (1961, 5). A History of Mechanics. *Science*, *133*(3463), 1472. Retrieved from `https://doi.org/10.1126/science.133.3463.1472.a` doi: 10.1126/science.133.3463.1472.a

Marquina, J., Marquina, M., Marquina, V., & Hernández-Gómez, J. (2016). Leonhard euler and the mechanics of rigid bodies. *European Journal of Physics*, *38*(1), 015001. doi: 10.1088/0143-0807/38/1/015001