Problem 5) Isnad Perez

Derive the order of the error with respect to the sin and cosine approximations.

Part A $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

Approximation
$\sin(\frac{1}{N}) \approx \frac{1}{N}$

$\text{error } \sin(\frac{1}{N}) = O(?)$

$\text{error}_{\sin} = \sin(\frac{1}{N}) - \frac{1}{N}$ (Using series expansion, only using first three terms $x = \frac{1}{N}$)

$\sin(\frac{1}{N}) = \frac{1}{N} - \frac{(\frac{1}{N})^3}{3!} + \frac{(\frac{1}{N})^5}{5!} - \dots$

* Substitue

$\text{error}_{\sin} = - \frac{(\frac{1}{N})^3}{3!} + \frac{(\frac{1}{N})^5}{5!} - \dots - \frac{1}{N}$

$\text{error}_{\sin} = - \frac{(\frac{1}{N})^3}{3!} + \frac{(\frac{1}{N})^5}{5!} - \dots$

Part A
Leading term is $-\frac{(\frac{1}{N})^3}{3!}$, so error sin behaves like $O(\frac{1}{N^3})$

Part B $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

Approximation
$\cos(\frac{1}{N}) \approx 1 - \frac{1}{2N^2}$

$\text{error}_{\cos}(\frac{1}{N}) = O(?)$

$\text{error}_{\cos} = \cos(\frac{1}{N}) - (1 - \frac{1}{2N^2})$ (Using series expansion only using first three terms: $x = \frac{1}{N}$)

$\cos(\frac{1}{N}) = 1 - \frac{(\frac{1}{N})^2}{2!} + \frac{(\frac{1}{N})^4}{4!} - \dots$

* Substituting

$\text{error}_{\cos} = \frac{(\frac{1}{N})^4}{4!} - \frac{(\frac{1}{N})^6}{6!} + \dots - (1 - \frac{1}{2N^2})$

$\text{error}_{\cos} = \frac{(\frac{1}{N})^4}{4!} - \frac{(\frac{1}{N})^6}{6!} + \dots$

Part B
Leading term is $\frac{(\frac{1}{N})^4}{4!}$, so error cos behaves like $O(\frac{1}{N^4})$