Simulating Focal Plane Array Observations with MeqTrees

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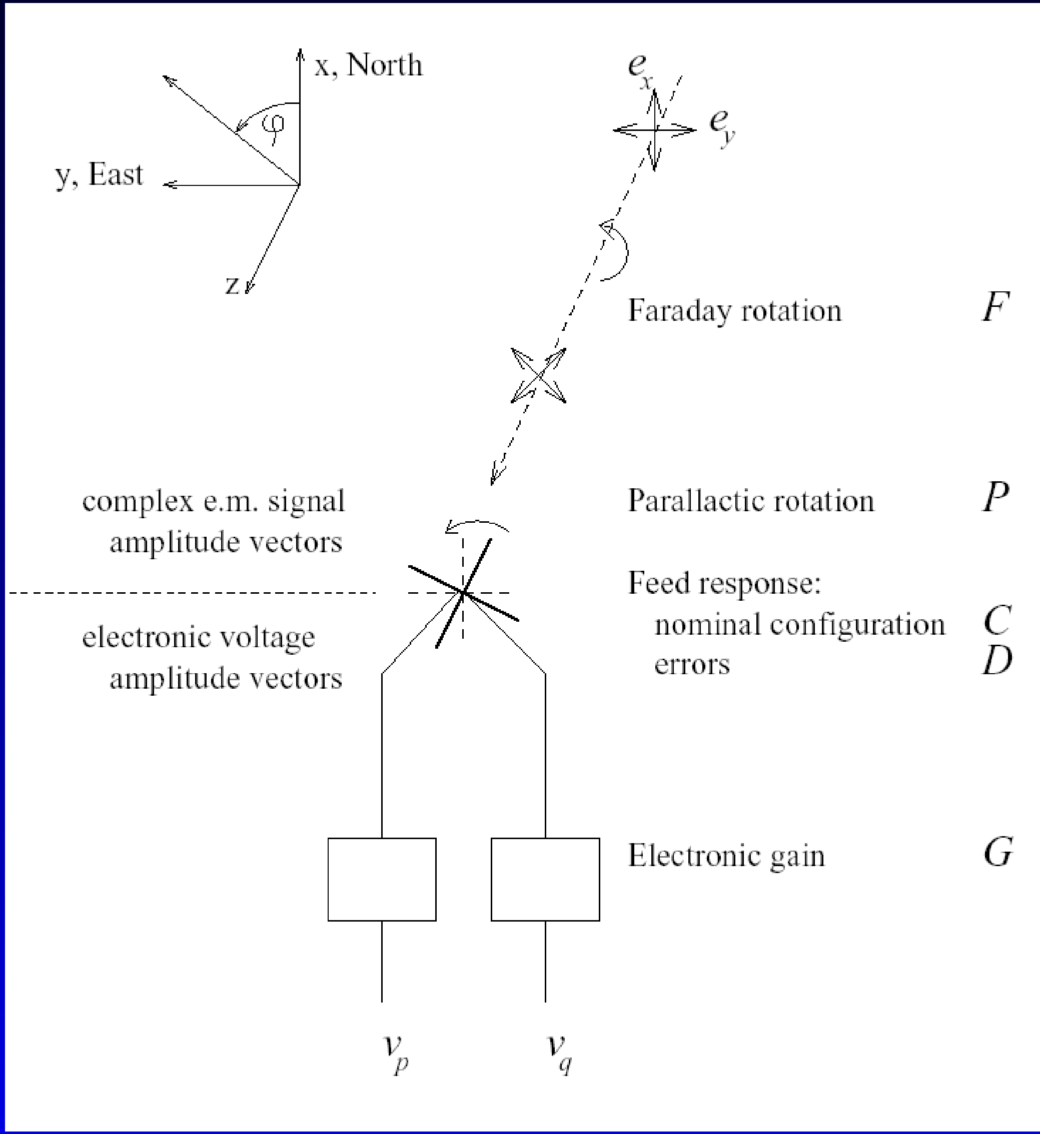
Topics

- Overview of Measurement Equation
- Overview of MeqTrees
- Example of MeqTrees Configuration
- Correction for E-Jones effects
- Simulation Setup
- Examples of MeqTrees Simulations
  - Phase-Conjugate Weighting
  - Optimization for Gaussian beam shape
  - AzEl observation tracking a fixed offset position
- What’s Next?

Measurement Equation - HBS



Jones Matrices

- The real heart of the Measurement Equation (M.E.) is composed of of two 2×2 *station*-based response matrices, called ‘Jones matrices’.
- The 2×2 Jones matrix J_i for *station* i can be decomposed into a product of several 2×2 Jones matrices, each of which models a specific *station*-based instrumental effect in the signal path (see Hamaker, Bregman, Sault papers and aips++ notes from Noordam and Cornwell).

$$\mathsf{J}_\mathsf{i} \;=\; \mathsf{G}_\mathsf{i}\, [\mathsf{H}_\mathsf{i}]\, \mathsf{E}_\mathsf{i}\, \mathsf{P}_\mathsf{i}\, \mathsf{K}_\mathsf{i}\, \mathsf{T}_\mathsf{i}\, \mathsf{F}_\mathsf{i}$$

- The visibility for an interferometer composed of *station* i and *station* j with linearly polarized receptors is given by the following equation, where $\vec{V}_{\mathsf{ij}}$ is the visibility, $\vec{I}$ is the incoming electromagnetic coherency matrix, and J_j^* is the complex conjugate of J_j.

$$\vec{V}_{\mathsf{ij}} \;=\; \mathsf{J}_\mathsf{i}\, \vec{I}\, \mathsf{J}_\mathsf{j}^*$$

$$\vec{I} \;=\; 0.5 \begin{pmatrix} I+Q & U-iV \\ U+iV & I-Q \end{pmatrix}$$

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Jones Matrix Definitions

$\mathsf{F}_{\mathsf{i}}(\vec{\rho}, \vec{r}_{\mathsf{i}})$	ionospheric Faraday rotation
$\mathsf{T}_{\mathsf{i}}(\vec{\rho}, \vec{r}_{\mathsf{i}})$	atmospheric complex gain
$\mathsf{K}_{\mathsf{i}}(\vec{\rho}.\vec{r}_{\mathsf{i}})$	factored Fourier Transform kernel
P_{i}	projected *receptor* orientation(s) w.r.t. the sky
$\mathsf{E}_{\mathsf{i}}(\vec{\rho}, \vec{r}_{\mathsf{i}})$	voltage primary beam
$[\mathsf{H}_{\mathsf{i}}]$	hybrid (conversion to circular polarization coord)
G_{i}	electronic complex gain (*station* contributions)

- E-Jones definition

$$\mathsf{E}_{\mathsf{i}}^{+}(\vec{\rho}, \vec{r}_{\mathsf{i}}) = \mathsf{E}_{\mathsf{i}}^{\odot}(\vec{\rho}, \vec{r}_{\mathsf{i}}) = \mathsf{E}_{\mathsf{i}}(\vec{\rho}, \vec{r}_{\mathsf{i}}) = \begin{pmatrix} \mathsf{e}_{\mathsf{iaa}} & \mathsf{e}_{\mathsf{iba}} \\ \mathsf{e}_{\mathsf{iab}} & \mathsf{e}_{\mathsf{ibb}} \end{pmatrix}$$

- On axis diagonal terms describe position dependant primary beam attenuation
- Non-zero off-diagonal terms $\mathsf{e}_{\mathsf{iba}}$ and $\mathsf{e}_{\mathsf{iab}}$ describe ‘leakage’ between *receptor*s

MeqTrees Summary

- M.E. predicts data measured with a particular instrument.
  - Model the instrument and observed data
  - Use for both system calibration and extraction of data parameters
  - Work mostly with Fourier (Visibility) data
- Procedure
  - Implement model in software using tree structure
  - Use a priori guesses to set model parameters
  - Compare observed data with predicted values
  - Solver/Condeq nodes adjust model parameters for best fit
  - Can solve for many discrepant parameters at same time
    - Hubble constant not yet done
- Multi-threaded processing available
- In on-going development
- NOT an antenna / FPA design tool or a synthesis imaging tool

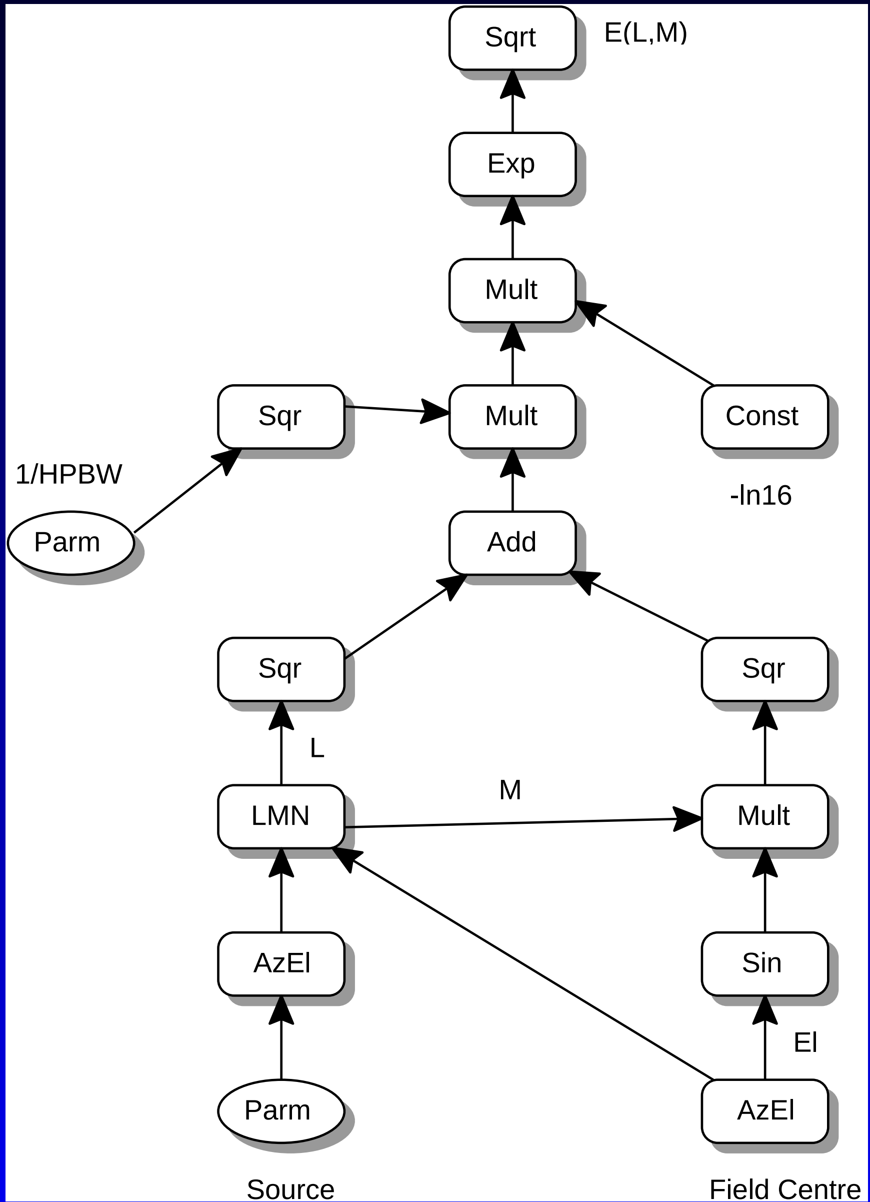
Example E-Jones Calculation

- The voltage beam pattern, E, of a Large Aperture Reflector (LAR) measured at the position of a source whose direction coordinates L and M are defined with respect to the field centre in an AzEl reference frame can be given as:

$$\mathrm{E(L,M)} = \sqrt{\exp(-\ln 16 \times (\frac{1}{\mathrm{HPBW}})^2(\mathrm{L}^2 + (\mathrm{M}\sin(\mathrm{El}))^2))}$$

- $\mathrm{HPBW} =$ half power beam width at zenith
- $\mathrm{El} =$ elevation of field or tracking centre

The LAR Beam as a MeqTree



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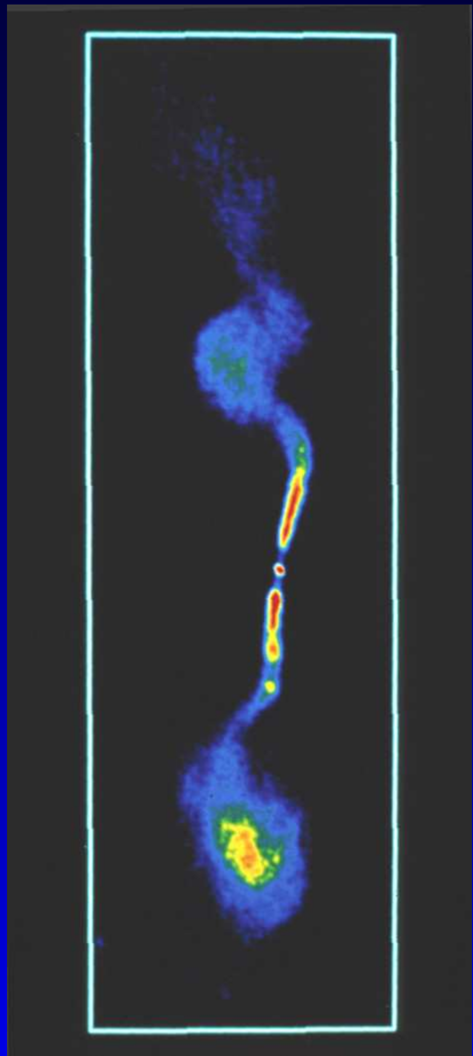
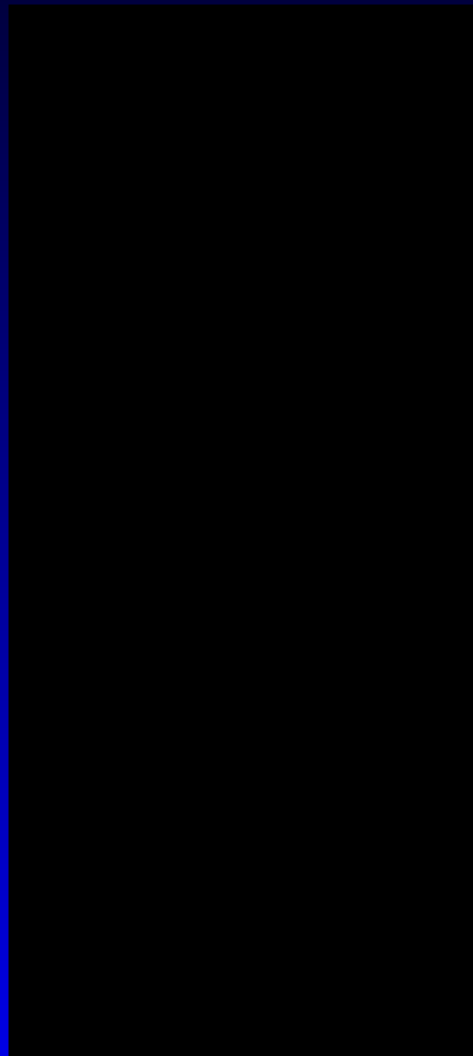
Reduction Goals

- Left - most reduction packages; Right - MeqTrees

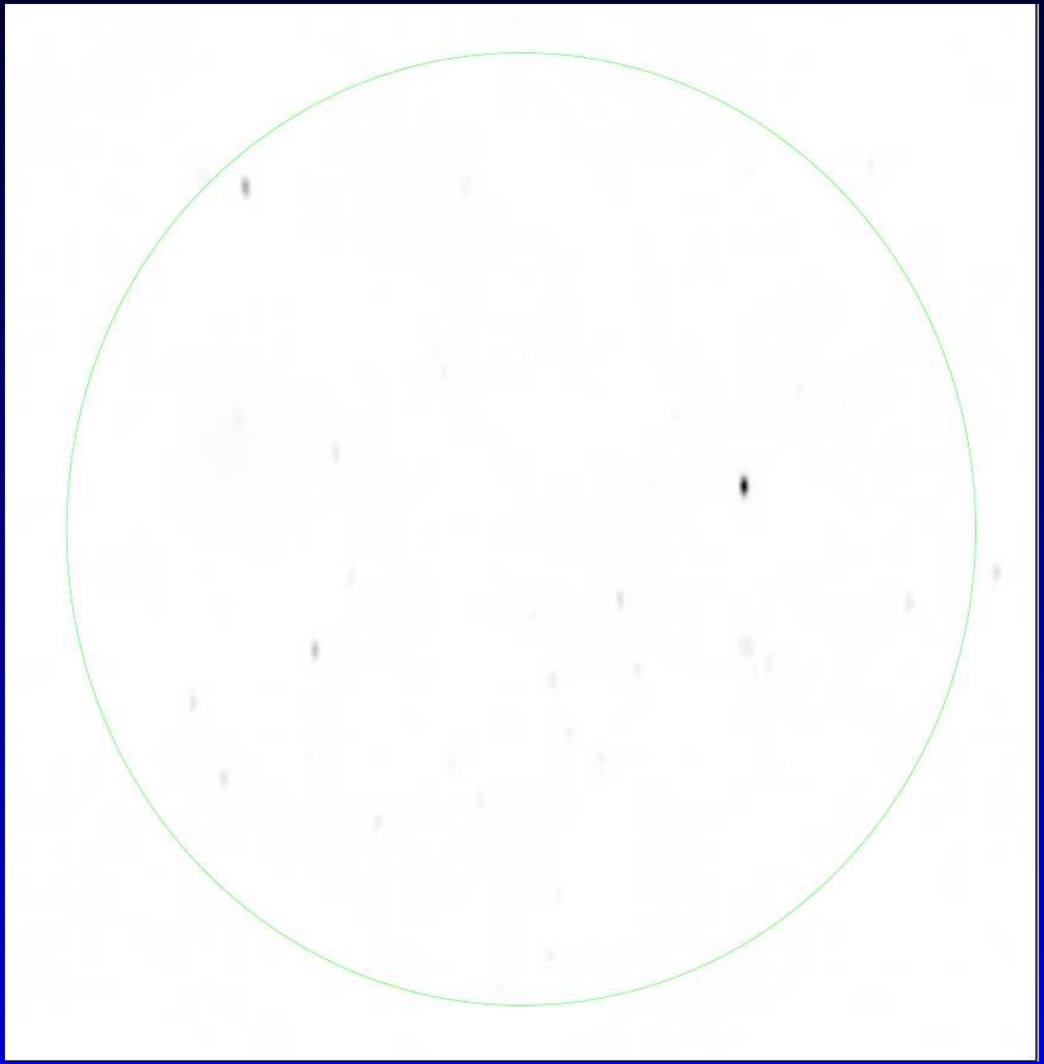




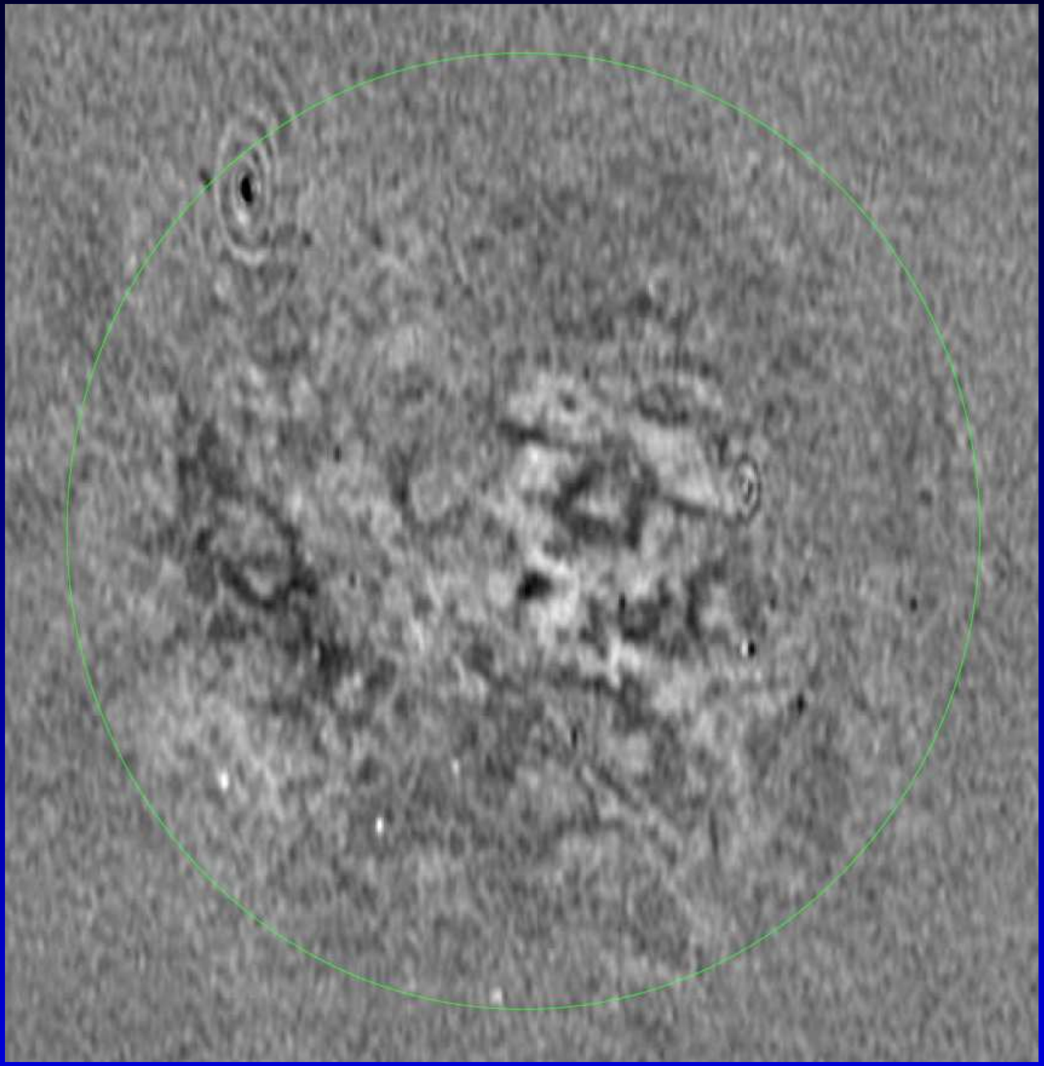
Know Thy E-Jones

- No longer acceptable to model primary beams as simple Gaussians
- South Africa SKA Calibration and Imaging Workshop 2006
  - At least 4 or 5 presentations concerned with detailed measurements of telescope primary beams
  - Example - work of R. Reid et al. at DRAO on polarization leakage
    - Each telescope of DRAO SST has different E-Jones voltage pattern
    - Detailed measurements made of the pattern for each dish
    - Accurate correction for instrumental polarization now possible

DRAO Stokes I



Stokes U No Correction



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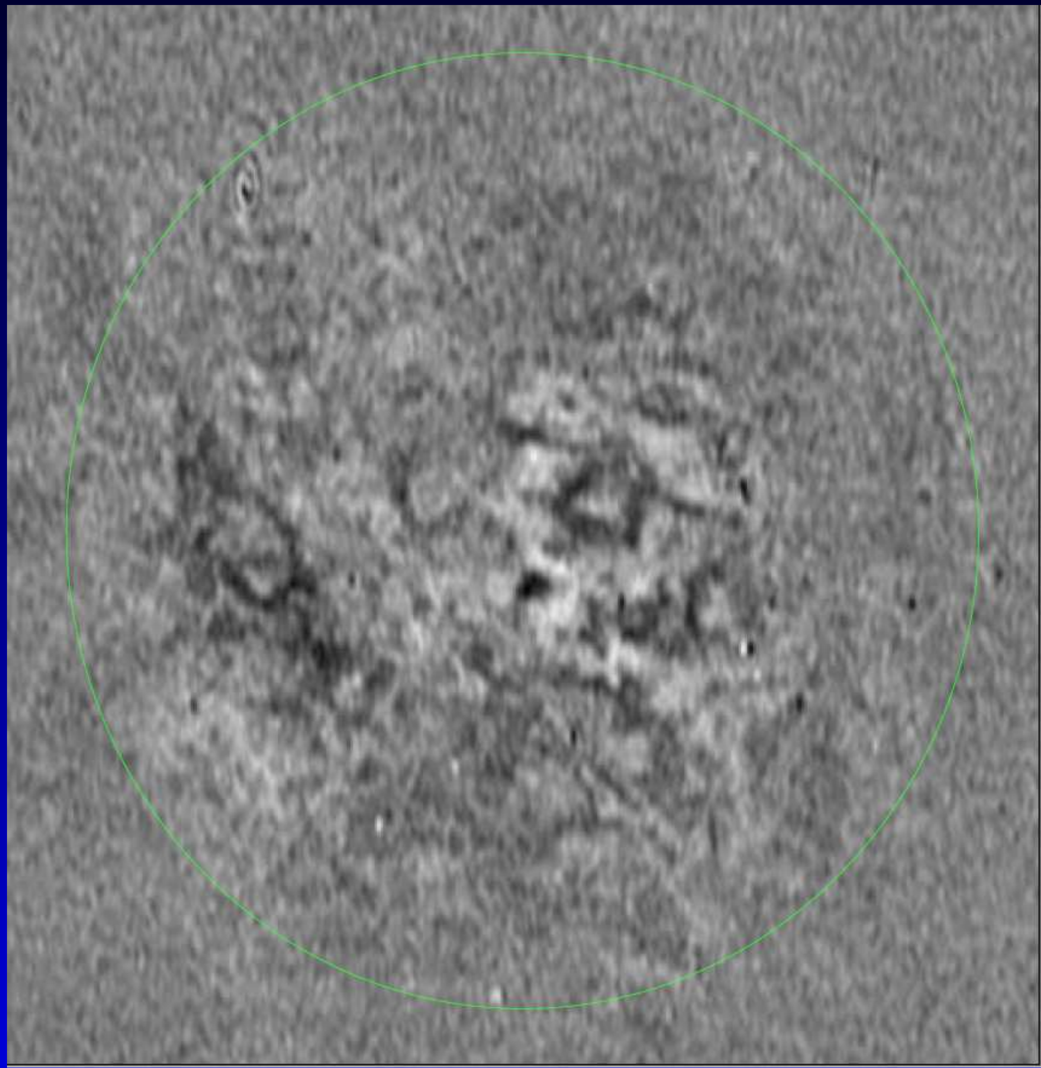
Stokes U Corrected

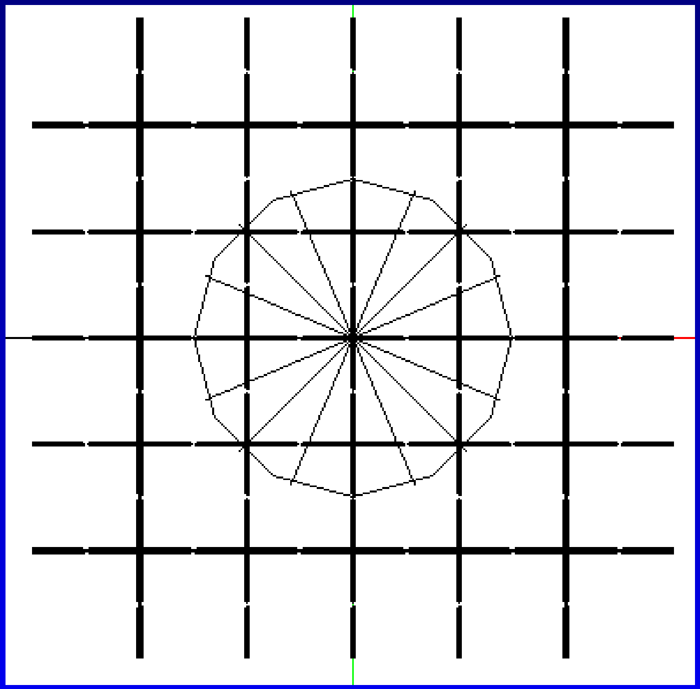


Know Thy FPA E-Jones

- Detailed knowledge of individual FPA voltage patterns allows accurate ‘first order’ prediction of phased array beam shapes
  - Resampling and interpolation tools allow extrapolation from coarse ‘grid’ measurements of actual FPA elements to finer grid for prediction of actual values associated with radio sources in the field
- Assuming MIRANdA / SKA dishes and receiver elements are stamped out of uniform molds, detailed measurements of FPA voltage patterns on ‘representative’ dishes should allow us to model entire array.
- GRASP calculations are the equivalent of the above activity for purposes of the simulations presented here.

Simulated FPA

- 30 dipole elements in each of X and Y directions
- Frequency = 1500 MHz; Spacing = lambda / 2
- Dish diameter = 10m; Focal length = 4.5m
- No coupling between elements; No feed struts in simulation
- Not meant as a ‘realistic’ final FPA design, but a good testbed for various aspects of software development and data processing



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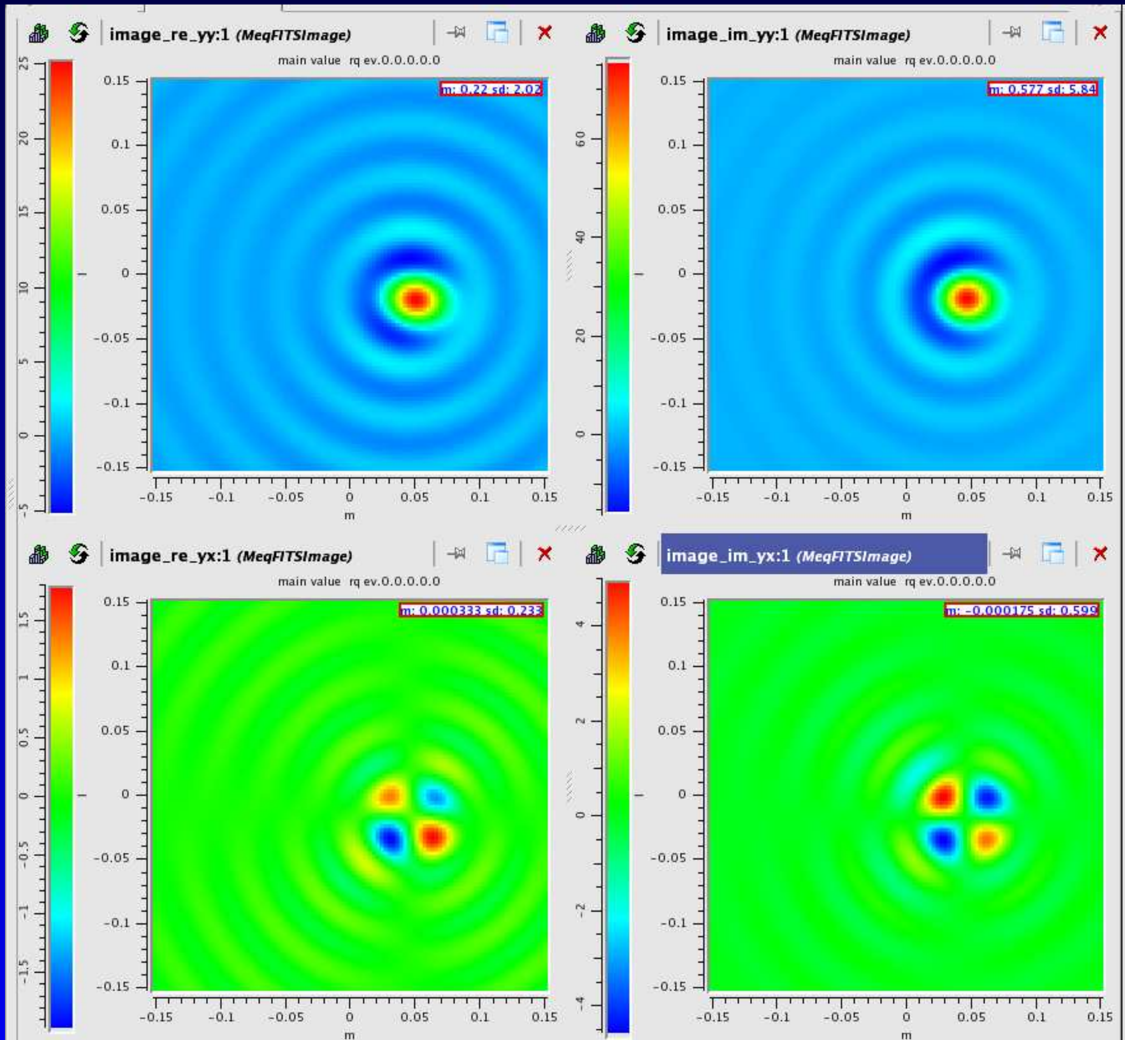
Simulation Procedure

- Do GRASP calculations of voltage radiation patterns for each of the X and Y dipoles used in this simulation
  - We get both co-polarization and cross-polarization leakage terms
- Convert GRASP 'grd' files to FITS images
- MeqTrees reads in radiation patterns from the FITS images
- Phase up X and Y radiation patterns, depending on optimization criteria, for requested observing position. In most of the simulations shown here we observe on a 5 x 5 grid centred on L=M=0, in steps of 82 arcmin (HPBW).
- Form E-Jones Matrix (fully complex) from weighted combinations
- Simulate observations of the 'visible' sky via our equation:

$$\vec{V}_{\mathsf{ij}} = \mathsf{E}_{\mathsf{i}}\, \vec{I}\, \mathsf{E}_{\mathsf{j}}^{*}$$

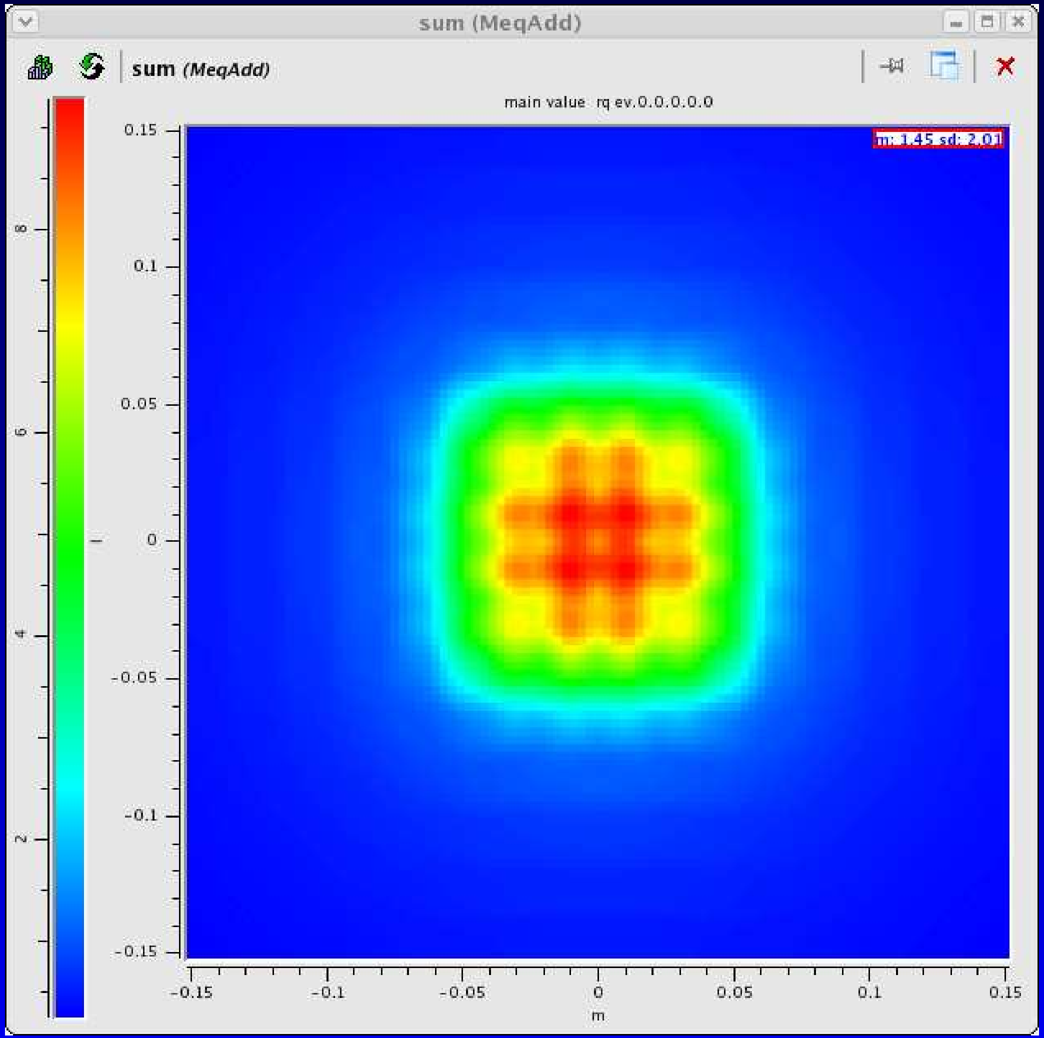
Typical GRASP Dipole Pattern

- In reality, we must measure these patterns in order to do accurate predicts, and thus compare with observations



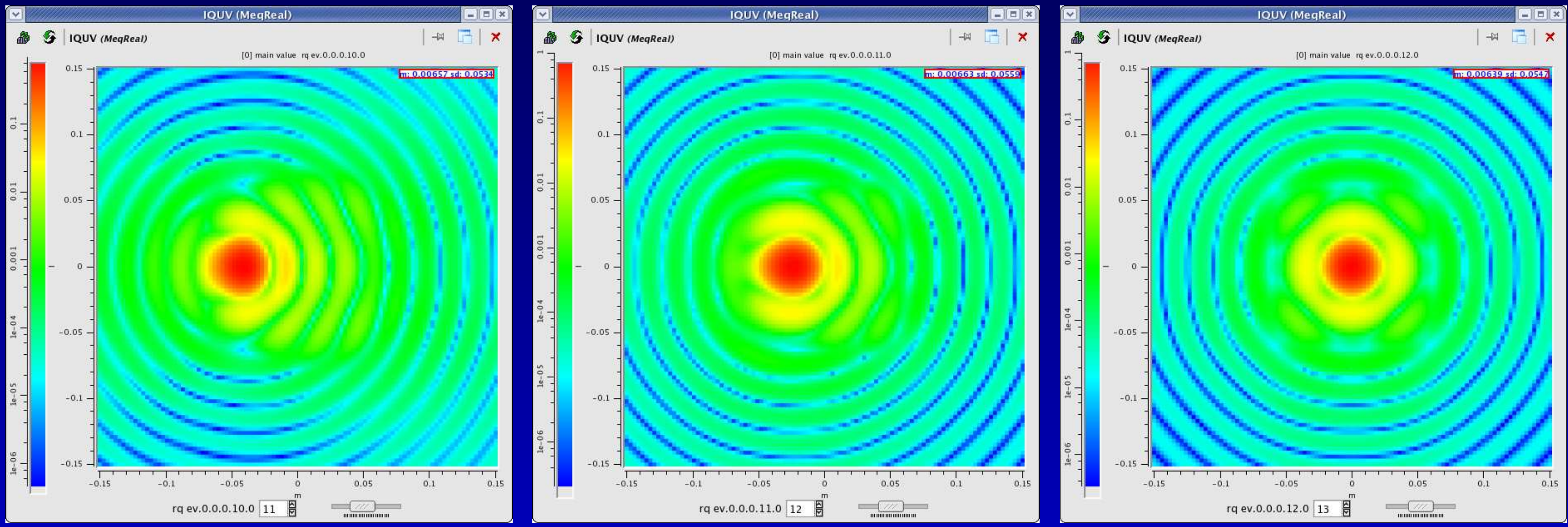
Sky Coverage

- Basically we can attempt to do beam-forming over the range -0.05 to 0.05 radians in L and M.



Phase Conjugate Weighting - I

- Phase conjugate weighting maximizes gain in observed direction, but does nothing particular for beam shape
- demo shows I beams for central row as we move from left edge toward centre of array in steps of 82 arcmin (HPBW)



Phase Conjugate Weighting - I

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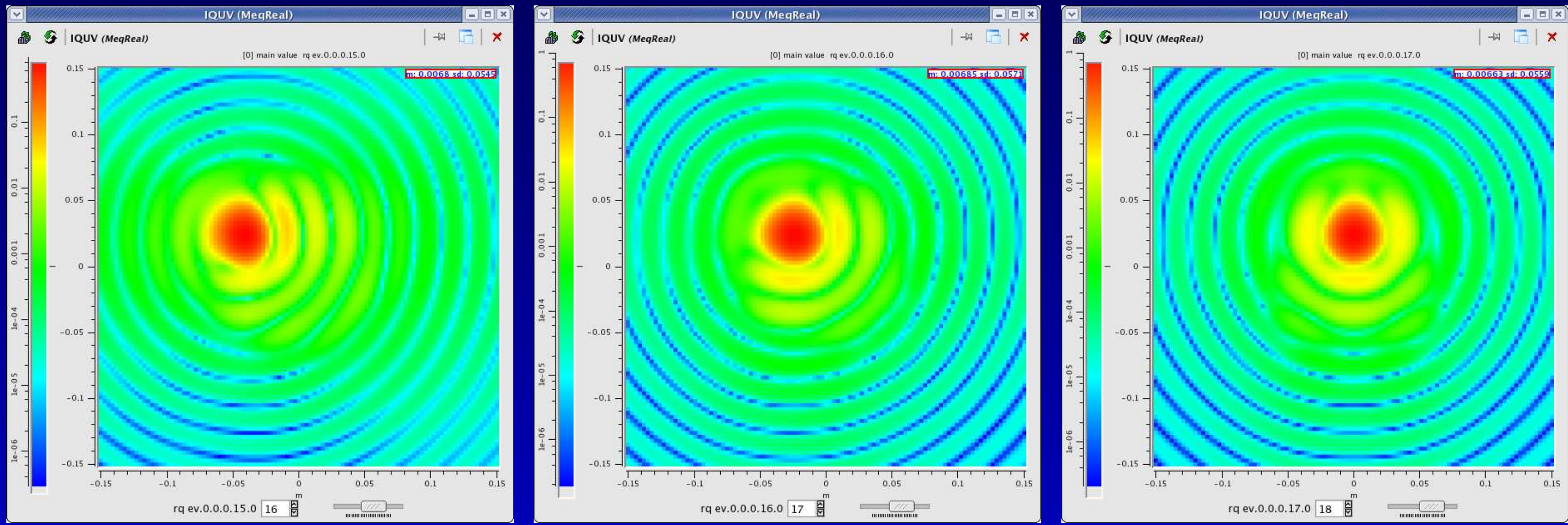

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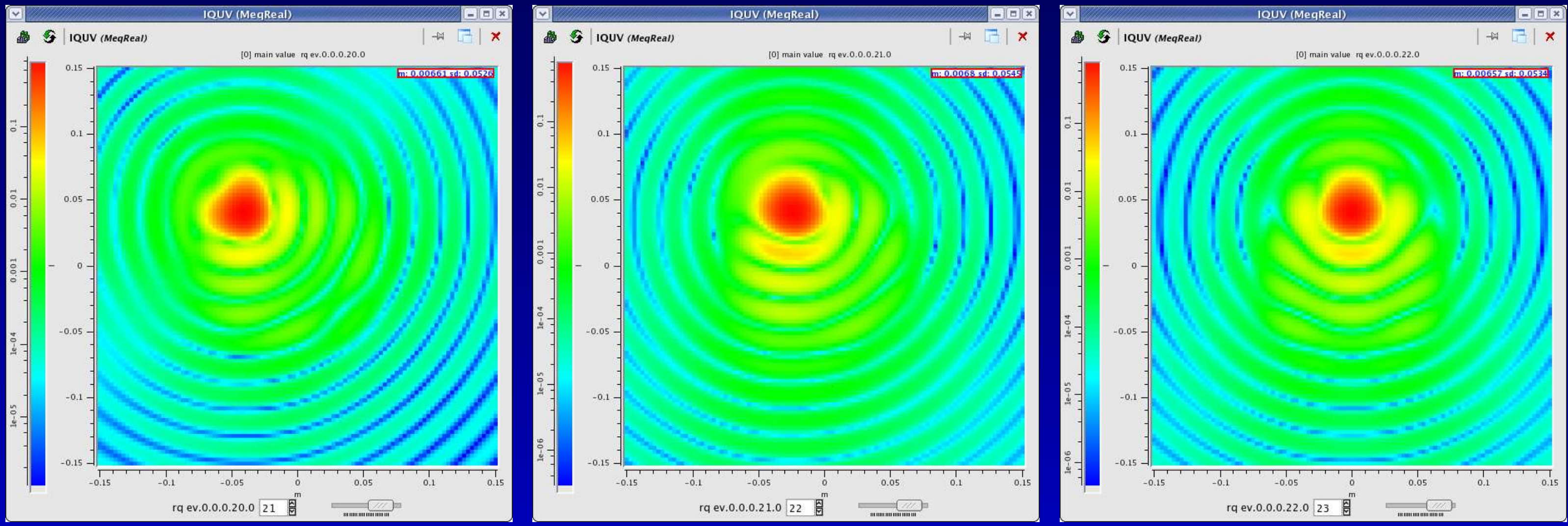
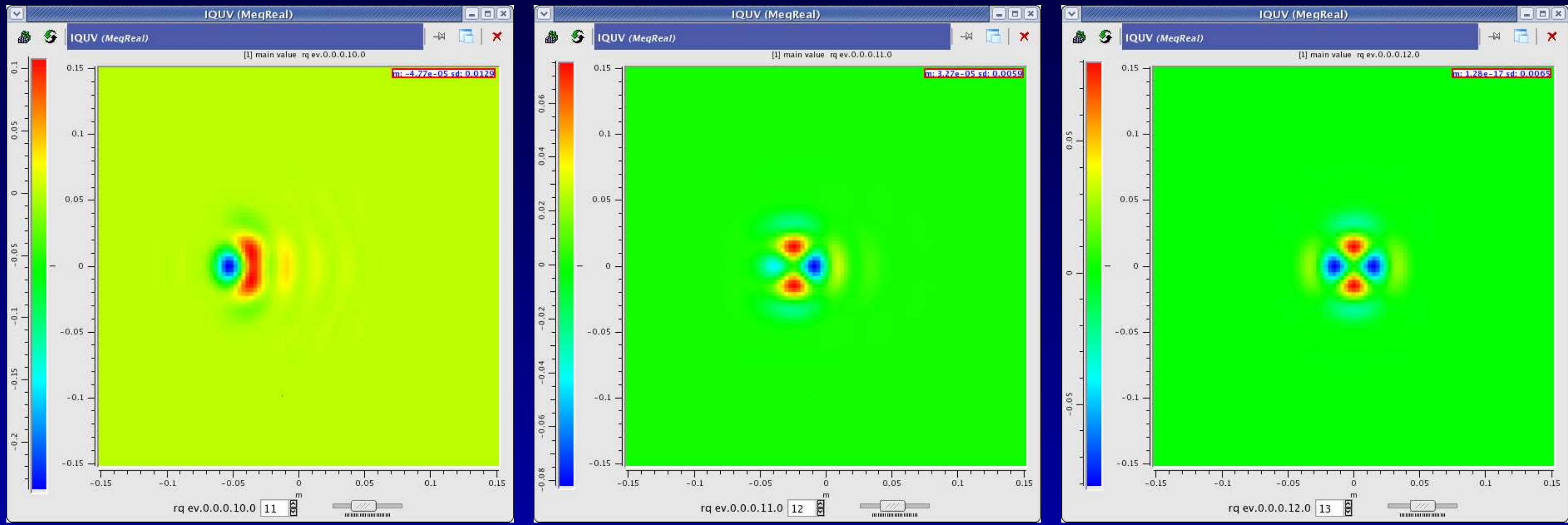
Phase Conjugate Weighting - I

- Phase conjugate weighting maximizes gain in observed direction, but does nothing particular for beam shape
- demo shows I beams as we move along top edge of array in steps of 82 arcmin (HPBW)



Phase Conjugate Weighting - Q

- demo shows Q response for central row as we move from left edge toward centre of array in steps of 82 arcmin (HPBW)



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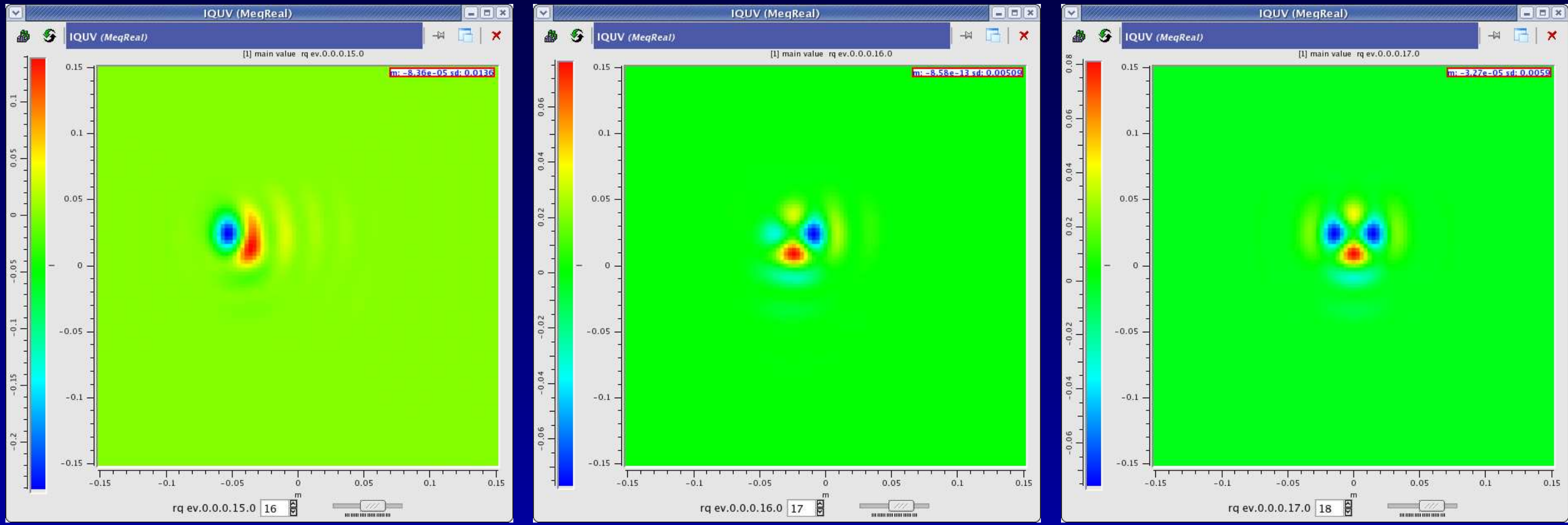
Phase Conjugate Weighting - Q

- demo shows Q response for middle row as we move from left edge toward centre of array in steps of 82 arcmin (HPBW)



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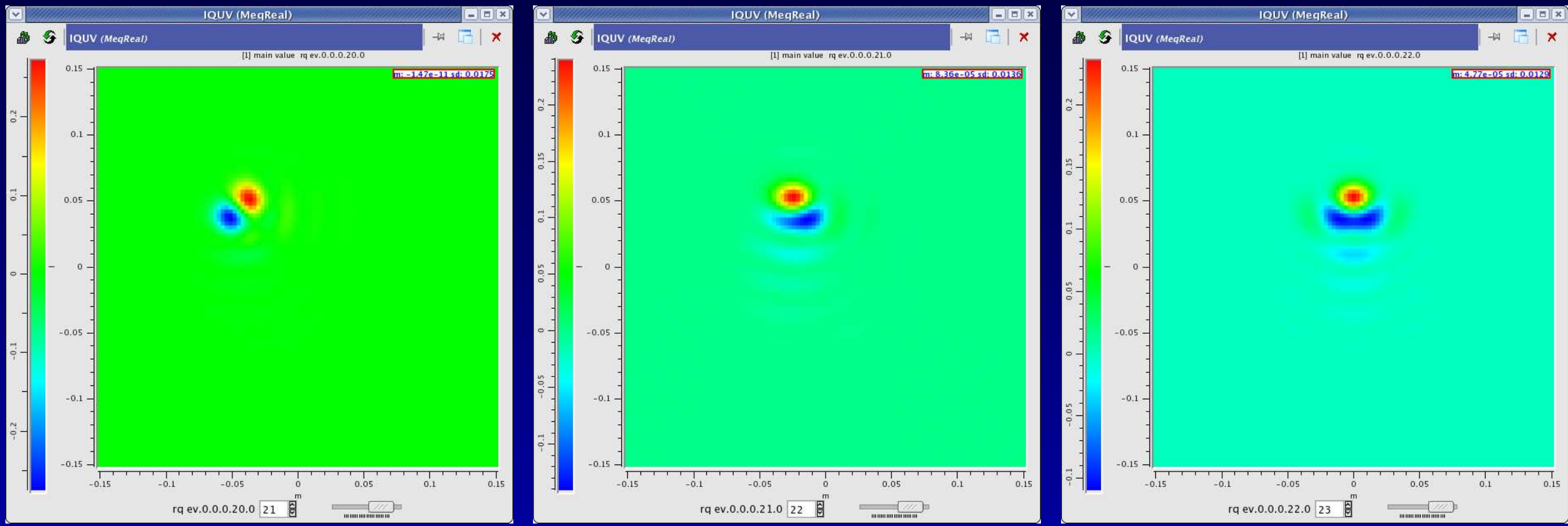
Phase Conjugate Weighting - Q

- demo shows Q response as we move along top edge of array in steps of 82 arcmin (HPBW)



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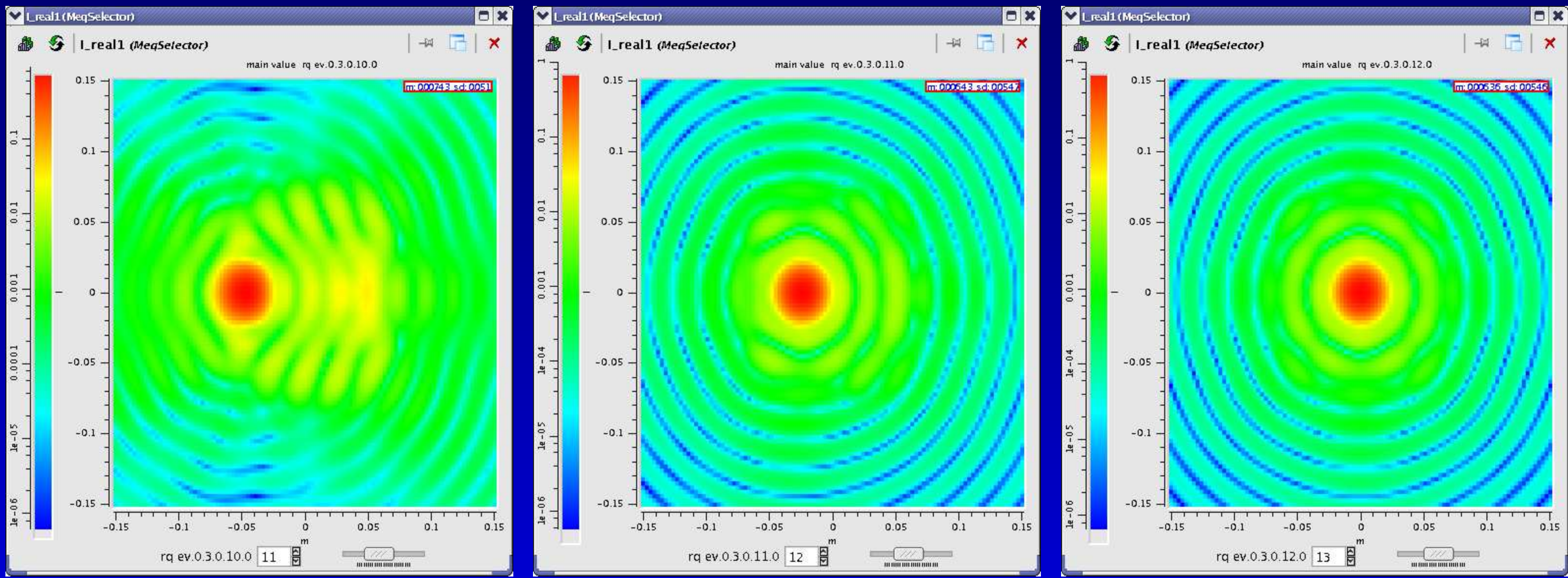
Optimized Gaussian Beam - I

- Obtain values for phase-conjugate weighting in a particular direction
- Provide these values as initial guess for weights to MeqTrees solver
- Solver adjusts weights until phased beam has optimal gaussian shape
- demo shows I beams for central row as we move from left edge toward centre of array in steps of 82 arcmin (HPBW)



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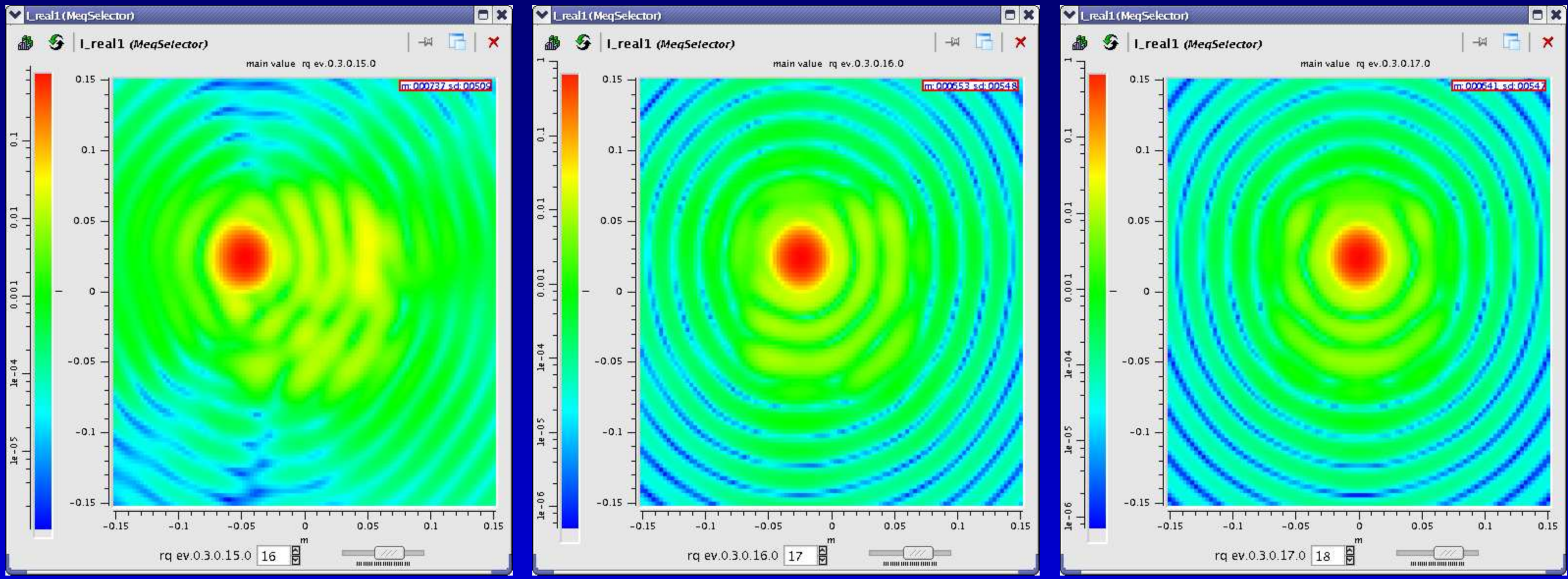

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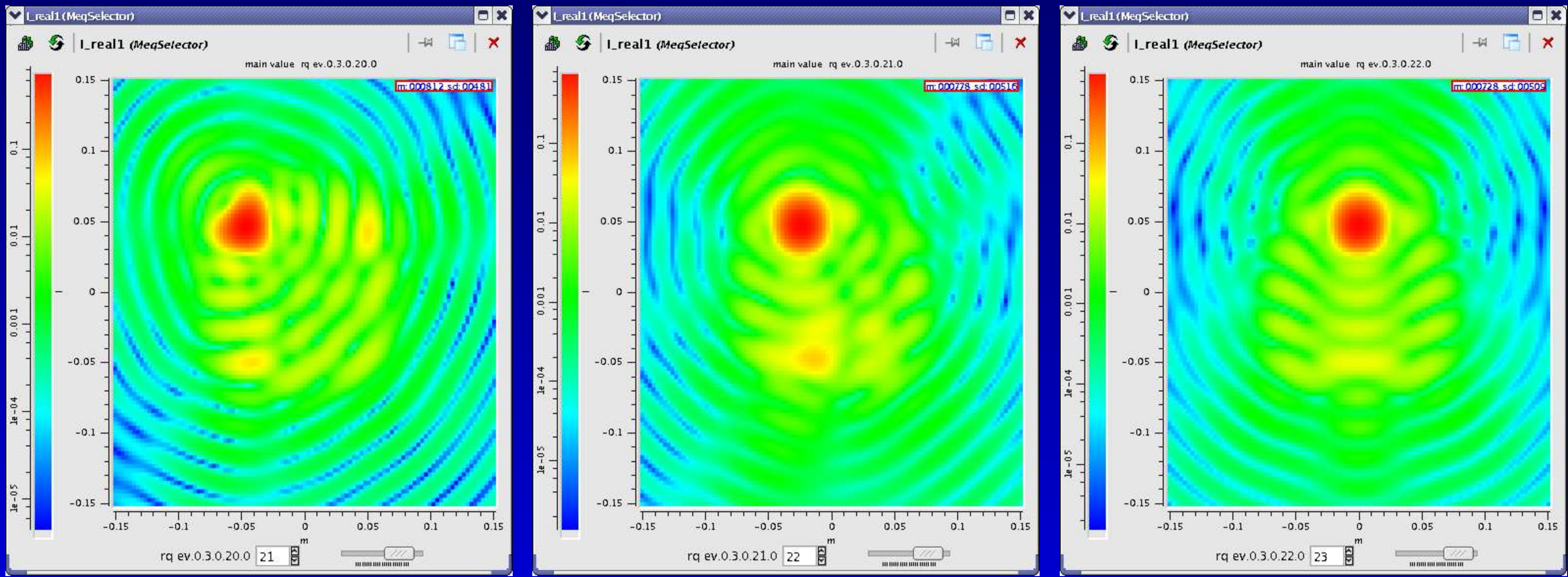
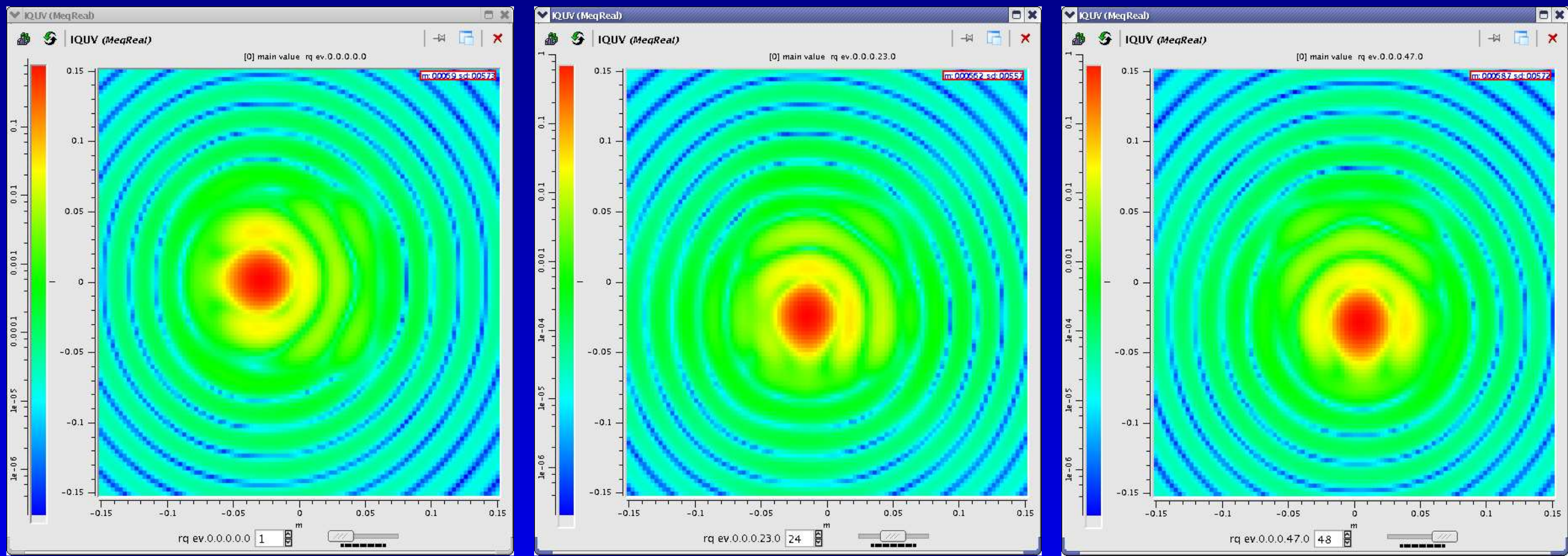


AzEl Telescope Simulation - I

- Calculate Parallactic Angle as a function of time for AzEl-mounted telescope stationed at VLA site which tracks position RA = 0 hr, Dec = 0 deg
- Phase up FPA at a position whose offset with respect to the tracking centre is -0.02 radians in both L and M when the Parallactic Angle is zero (transit)
- Adjust FPA phase conjugate weights to keep beam centred on this position.
  - 8 hour observation; calculate FPA beam every 10 minutes
- Total Intensity beam shown for start, middle and end of observation



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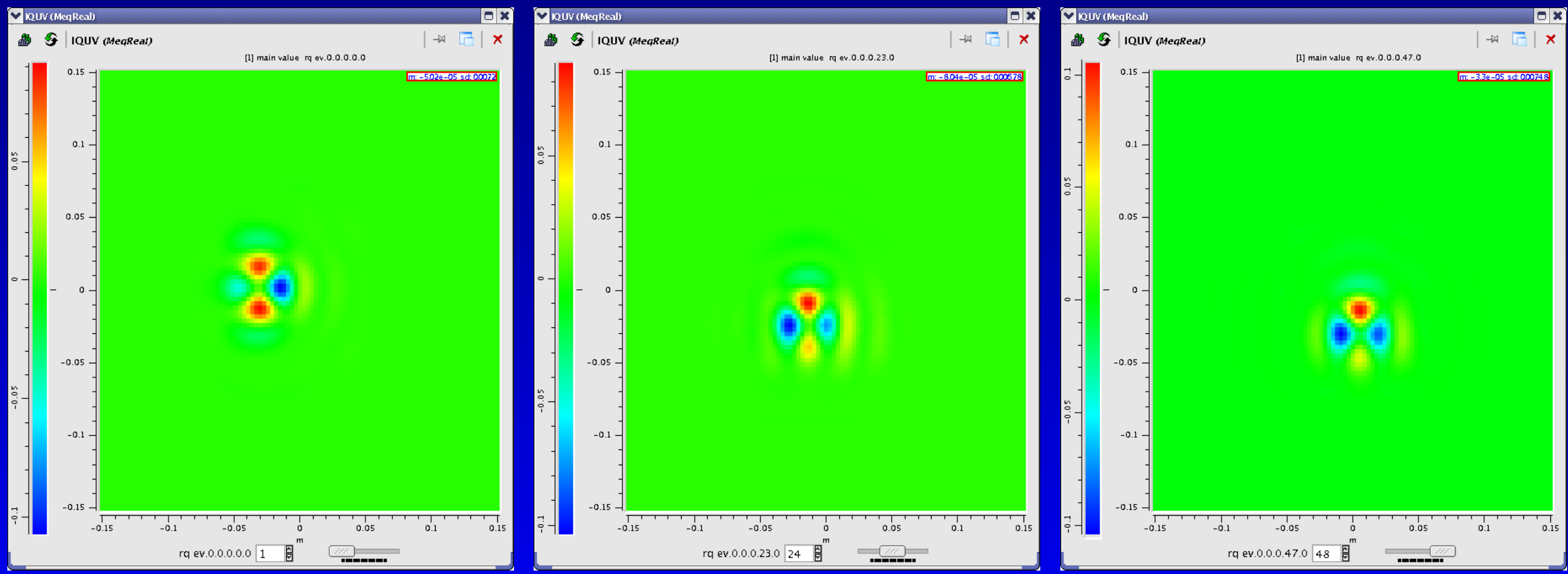
AzEl Telescope Simulation - Q

- Calculate Parallactic Angle as a function of time for AzEl-mounted telescope stationed at VLA site which tracks position RA = 0 hr, Dec = 0 deg
- Phase up FPA at a position whose offset with respect to the tracking centre is -0.02 radians in both L and M when the Parallactic Angle is zero (transit)
- Adjust FPA phase conjugate weights to keep beam centred on this position.
  - 8 hour observation; calculate FPA beam every 10 minutes
- Q response shown for start, middle and end of observation



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Modcal - Remove Anything

- Algorithm developed at DRAO to get rid of unwanted sources when you don’t have a good understanding of your E-Jones.
- Baseline-based rather than antenna-based so not really part of the Jones Matrix formalism.
- Can be useful as a method of last resort.
- Only about 20 lines of python code with MeqTrees.

Modcal - Example

- Right image shows source in sidelobe which does not clean properly; left image shows source vaporised by modcal algorithm.

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Conclusion: Know Thy E-Jones

- Heuristics
- Learning

What’s Next?

- Need Better Optimization than Gaussian Beam
  - Spheroids
  - Kaiser-Bessel
- Generate GRASP models of antennas more suitable for FPA such as Vivaldis and simulate observations with them.
- Look at effects of system gain variations on formed beams.

‘Solving for the Hubble constant (say as a polc in time) should be possible too, but you need a machine big enough to model the universe on....’

- Oleg M Smirnov, Russian/Dutch computer scientist

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Questions?

- Email: tony.willis@nrc.ca

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