VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

School of Advanced Sciences
Department of Mathematics
Continuous Assessment Test –I
Fall Semester 2023-24

Programme Name & Branch: M.C.A | **Exam Duration:** 90 mins
Slot: D1+TD1 **Semester:** I | **Maximum Marks:** 50
Course Code: PMAT501L | **Course Title:** Probability and Statistics
Faculty Name: M. Gowsalya, M.Nalliah, G. Mokeshrayalu | **Class Number:** 6405

General instructions: Answer all the questions (5X10=50 Marks)

1. (a).The probability that a regularly scheduled flight departs on time is $P(D) = 0.83$; the probability that it arrives on time is $P(A) = 0.82$; and the probability that it departs and arrives on time is $P(D \cap A) = 0.78$. Find the probability that a plane (i) arrives on time given that it departed on time, and (ii) departed on time given that it has arrived on time. **(5M)**

   (b).Suppose that we have a fuse box containing 20 fuses, of which 5 are defective. If 2 fuses are selected at random and removed from the box in succession without replacing the first, what is the probability that both fuses are defective? **(5M)**

2. A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact, plans 1, 2, and 3 are used for 30%, 20% and 50% of the products respectively. The "defect rate" is different for the three procedures as follows: $P(D/P_1) = 0.01$, $P(D/P_2) = 0.03$, $P(D/P_3) = 0.02$, where $P(D/P_j)$ is the probability of a defective product, given plan j. If a random product was observed and found to be defective, which plan was most likely used and thus responsible? **(10M)**

3. A discrete Random variable X has the following probability distribution.

x:	0	1	2	3	4	5	6	7	8
$p(x)$:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

Find the value of a, $P(X < 3)$, mean, variance and cumulative distribution function of X. **(10M)**

4. The Joint density for the random variables (X, Y), where X is the unit temperature change and Y is the proportion of spectrum shift that a certain atomic particle produces, is

$$f(x,y) = \begin{cases} 10xy^2, & 0 < x < y < 1 \\ 0, & elsewhere \end{cases}$$

   (a) Find the marginal densities $g(x), h(y)$ and the conditional density $f(y/x)$.
   (b) Find the probability that the spectrum shifts more than half of the total observations, given that the temperature is increased to 0.25 unit. **(10M)**

5. Let X and Y be the random variables with joint probability distributions: **(10M)**

x / y	0	1	2
0	$\frac{3}{28}$	$\frac{9}{28}$	$\frac{3}{28}$
1	$\frac{3}{14}$	$\frac{3}{14}$	0
2	$\frac{1}{28}$	0	0

Find the expected values E(X), E(Y), E(XY), Marginal distributions and Covariance of X and Y.