

Linear inequality representation of convex domains

Computational Intelligence, Lecture 7

by Sergei Savin

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- Convex polytopes
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CONVEX POLYTOPES

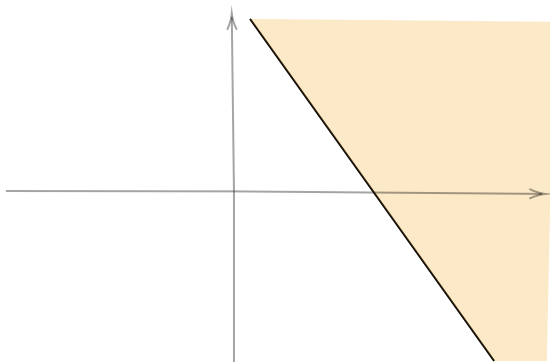
Before defining what a convex polytope is, let us look at examples:



HALF-SPACES

Definition

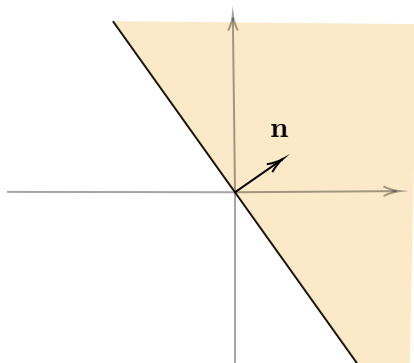
We can define half-space as a set of all points \mathbf{x} , such that $\mathbf{a}^\top \mathbf{x} \leq b$. It has a very clear geometric interpretation. In the following image, the filled space is **not** in the half space.



HALF-SPACES

Construction. Simple case

Consider half-space that passes through the origin, and defined by its normal vector \mathbf{n} :

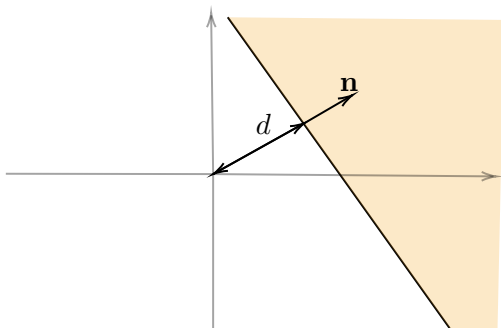


It is easy to see that this half-space can be defined as "all

HALF-SPACES

Construction. General case

In the general case there is some distance between the boundary of the half-space and the origin, let's say d .

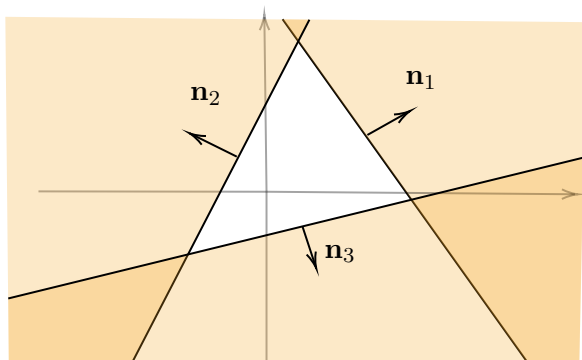


The same way we see, that the half space can be defined as "all vectors \mathbf{x} , such that $\mathbf{n} \cdot \mathbf{x} \leq d$ ". This is the same as making

HALF-SPACES

Combination

We can define a region of space as an *intersection* of half-spaces
 $\mathbf{a}_i^\top \mathbf{x} \leq b_i$:



$$\begin{bmatrix} \mathbf{a}_1^\top \end{bmatrix} \quad \begin{bmatrix} b_1 \end{bmatrix}$$

HALF-SPACES

Formal description via inequalities

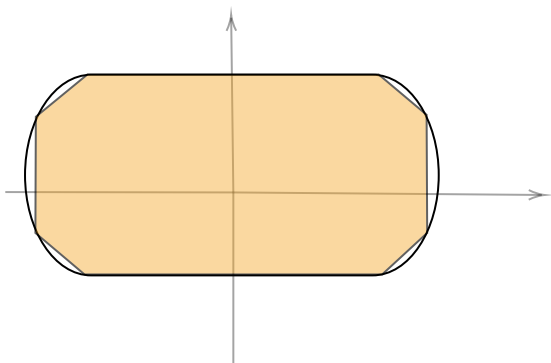
The last result allows us to write any convex polytope as a matrix inequality:

$$\mathbf{Ax} \leq \mathbf{b} \tag{1}$$

And conversely, any matrix inequality (1) represents either an empty set or a convex polytope.

LINEAR APPROXIMATION OF CONVEX REGIONS

Some convex regions can be easily approximated using polytopes.



Which allows to represent constraints on \mathbf{x} to belong in such a region as a matrix inequality

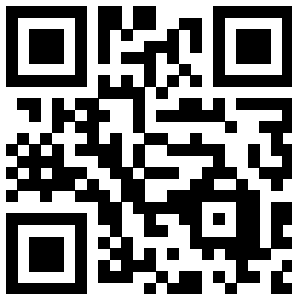
Represent in matrix inequality form the following figures:

- Equilateral triangle
- A square
- Parallelepiped
- Trapezoid

Lecture slides are available via Moodle.

You can help improve these slides at:

github.com/SergeiSa/Computational-Intelligence-Slides-Spring-2021



Check Moodle for additional links, videos, textbook suggestions.