

CENG213 Homework 2

Deadline: 23/01/2021 10 a.m.

Question 1 (30 points)

Determine whether the string $w = aabbb$ is in the language generated by the grammar:

$$\begin{aligned} S &\rightarrow AB \\ A &\rightarrow BB|a \\ B &\rightarrow AB|b \end{aligned}$$

Implement the algorithm for the membership check (Please refer to the pseudocode on the page 155 of the textbook.) in the Python programming language. Update the algorithm to output the derivation steps as well.

Question 2 (40 points)

$$\begin{aligned} L_1 &= \{0^n 1^n 2^i \mid n \geq 1, i \geq 1\} \\ L_2 &= \{0^i 1^n 2^n \mid n \geq 1, i \geq 1\} \end{aligned}$$

1. Write a CFG for L_1 .
2. Write a CFG for L_2 .
3. Design a Turing machine for their intersection language.

Question 3 (20 points)

$$L = \{a^n w w^R b^n \mid w \in \{a, b\}^*, n \geq 1\}$$

- Write a context-free grammar to generate L.
- Show a natural PDA that accepts L.

Question 4 (30 points)

Design a Turing machine to compute the following function for an x positive integer represented in unary.

$$f(x) = \begin{cases} \frac{x}{2}, & \text{if } x \text{ is even} \\ \frac{x+1}{2}, & \text{if } x \text{ is odd} \end{cases}$$