CENG213 Homework 1

Deadline: 05/12/2020 10 a.m.

Question 1 (15 points)

For each part, give a relation that satisfies the condition.

- Reflexive and symmetric but not transitive
- Reflexive and transitive but not symmetric
- Symmetric and transitive but not reflexive

Question 2 (20 points)

Please implement the transitive closure algorithms that we discussed at the lecture (1.6 Closures and Algorithms from the textbook) in Python.

Please use the following test case to test your algorithms (This is just a sample, your algorithms must be responsive to other input relations as well).

Input relation:

$R = \{(a, b), (a, c), (b, d), (d, e)\}$ on $A = \{a, b, c, d, e\}$.

The reflexive transitive closure R^*:

$R = \{(a, a), (b, b), (c, c), (d, d), (e, e), (a, b), (a, c), (b, d), (d, e), (a, d), (b, e), (a, e)\}$

Question 3 (20 points)

Problem 2.4.12 from the texbook (Lewis H., and Papadimitriou, C. 1998):

2.4.12. Let $D = \{0, 1\}$ and let $T = D \times D \times D$. A correct *multiplication* of two numbers in binary notation can also be represented as a string in T^*. For example, the multiplication $10 \times 5 = 50$, or

$$\begin{array}{r} 0\ 0\ 1\ 0\ 1\ 0 \\ \times\ 0\ 0\ 0\ 1\ 0\ 1 \\ \hline 1\ 1\ 0\ 0\ 1\ 0 \end{array}$$

would be pictured as the following string of six symbols.

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

Show that the set of all strings in T^* that represent correct multiplications is *not* a regular language. (*Hint:* Consider the multiplication $(2^n + 1) \times (2^n + 1)$.)

Question 4 (15 points)

Find a regular expression that generates the set of all strings of triplets defining correct binary addition.

Question 5 (15 points)

Please construct a DFA for the following language:

$L = \{\text{a}^k y | y \in \{a, b\}^* :$ y contains at least k a's, for $k \geq 1\}$

Show that L is a regular language.

Question 6 (15 points)

Please construct a DFA for the following language:

$L = \{\text{w} \in \{0, 1\}^* : w$ is any string except 11 and $111\}$.