Seminar on non-archimedean geometry

Graduate Student Seminar in Arithmetic Geometry
Universität Regensburg

Winter term 2014/15

The goal of this seminar is to understand the basic ideas, some results and examples of the different approaches to non-archimedean geometry. For the most part, we will follow closely the lecture notes of Brian Conrad [1] at the Arizona Winter School 2007 – in particular for the theories of Tate, Raynaud and Berkovich. Here, we want to take advantage of the numerous exercises and examples that can be found in the text.
Helpful are also Kiran Kedlaya's notes on rigid analytic geometry [3].

If time allows, we will try to discuss in more or less depth Huber spaces.

Here are some suggestions what the talks can contain.

1 Access to non-archimedean geometry based on ideas of Tate.

1.1 Affinoid algebras

Cover Section 1 of [1]. Discuss Tate algebras, the idea behind them and how they generalize to affinoid algebras. Define Banach structures and why they are important for us.

1.2 The local theory

Cover Subsection 2.1 and 2.2 of [1]. Some basic algebraic and topological properties of affinoid algebras.The set of maximal ideal. A Hausdorff "canonical" topology. Some subtle issues to be discussed in the exercises. Weierstraß , Laurent, rational domains. Admissible opens.

1.3 Globalisation

Cover Subsections 2.3 and 2.4 of [1]. Explain the idea of Grothendieck topologies. Look at [3] Tate's acyclicity theorem. Affinoid spaces. Topologisation. The problem of stalks. The global definition.

1.4 Coherent sheaves

Cover Subsection 3.1 of [1]. Kiehl's theorems. Definition of coherent sheaves, why it makes sense. Some notions from algebraic geometry.

1.5 Cohomology

Cover Subsection 3.2 of [1]. Discuss cohomology especially in the proper and flat cases.

2 Raynaud's theory

Cover Subsection 3.3 of [1]. Formal scheme models for rigid spaces. When they are useful.

3 Berkovich theory

3.1 Topological basics and Banach algebras

Cover Subsections 4.1 to 4.3 of [1]. Differences to Tate's theory and in which sense this approach is more general. The spectrum of Banach algebras.

3.2 Local theory

Cover Subsections 4.4 and 4.5. Affinoid subdomains. Relative boundary.

3.3 Globalisation

Cover Subsection 5.1 and 5.2 until the top of page 51. Topology on affinoid. Structure sheaves. The subtlety of openness and closedness. Examples for the gluing process (maybe in the discussion part).

3.4 Applications

Rest of Subsection 5.2 and Subsection 5.3 of [1]. Topological properties of analytic spaces. Proper and ́etale morphisms.

4 Huber's approach

4.1 Adic spaces I

Section 1 of [2]. Local theory, pre sheaf. ....

4.2 Adic spaces II

Section 2 of [2]. Spaces. Some structures on them. .....

References

[1] Brian Conrad. Several approaches to non-archimedean geometry. In *p-adic Geometry*, volume 78 of *AMS University Lecture Series*. Amer. Math. Soc., Providence, RI 41, 2008. Lectures from the 2007 Arizona Winter School.

[2] Roland Huber. A generalization of formal schemes and rigid analytic varieties. *Mathematische Zeitschrift*, 217(4):513–552, 1994.

[3] Kiran S. Kedlaya. Topics in Algebraic Geometry: rigid analytic geometry. 2004.