2-1 Gain, Attenuation, and Decibels

Most electronic circuits in communication are used to process signals, i.e., to manipulate signals to produce a desired result. All signal processing circuits involve either gain or attenuation.

Gain

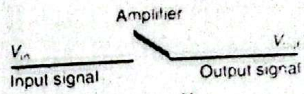
Gain

Gain means amplification. If a signal is applied to a circuit such as the amplifier shown in Fig. 2-1 and the output of the circuit has a greater amplitude than the input signal, the circuit has gain. Gain is simply the ratio of the output to the input. For input (V_{in}) and output (V_{out}) voltages, voltage gain A_V is expressed as follows:

$$A_V = \frac{\text{output}}{\text{input}} = \frac{V_{out}}{V_{in}}$$

1

Figure 2-1 An amplifier has gain.



$A = \text{gain} = \frac{V_{out}}{V_{in}}$

The number obtained by dividing the output by the input shows how much larger the output is than the input. For example, if the input is 150 μV and the output is 75 mV, the gain is $A_V = (75 \times 10^{-3})/(150 \times 10^{-6}) = 500$.

The formula can be rearranged to obtain the input or the output, given the other two variables: $V_{out} = V_{in} \times A_V$ and $V_{in} = V_{out}/A_V$.

If the output is 0.6 V and the gain is 240, the input is $V_{in} = 0.6/240 = 2.5 \times 10^{-3} = 2.5$ mV.

Example 2-1

What is the voltage gain of an amplifier that produces an output of 750 mV for a 30-μV input?

$$A_V = \frac{V_{out}}{V_{in}} = \frac{750 \times 10^{-3}}{30 \times 10^{-6}} = 25{,}000$$

Since most amplifiers are also power amplifiers, the same procedure can be used to calculate power gain A_P:

$$A_P = \frac{P_{out}}{P_{in}}$$

where P_{in} is the power input and P_{out} is the power output.

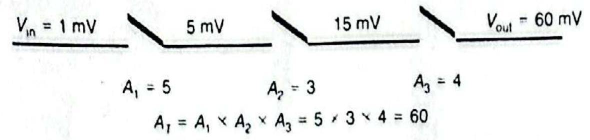
Example 2-2

The power output of an amplifier is 6 watts (W). The power gain is 80. What is the input power?

$$A_P = \frac{P_{out}}{P_{in}} \quad \text{therefore} \quad P_{in} = \frac{P_{out}}{A_P}$$

$$P_{in} = \frac{6}{80} = 0.075 \text{ W} = 75 \text{ mW}$$

Figure 2-2 Total gain of cascaded circuits is the product of individual stage gains.



When two or more stages of amplification or other forms of signal processing are cascaded, the overall gain of the combination is the product of the individual circuit gains. Fig. 2-2 shows three amplifiers connected one after the other so that the output of one is the input to the next. The voltage gains of the individual circuits are marked. To find the total gain of this circuit, simply multiply the individual circuit gains: $A_T = A_1 \times A_2 \times A_3 = 5 \times 3 \times 4 = 60$.

If an input signal of 1 mV is applied to the first amplifier, the output of the third amplifier will be 60 mV. The outputs of the individual amplifiers depend upon their individual gains. The output voltage from each amplifier is shown in Fig. 2-2.

Example 2-3

Three cascaded amplifiers have power gains of 5, 2, and 17. The input power is 40 mW. What is the output power?

$$A_P = A_1 \times A_2 \times A_3 = 5 \times 2 \times 17 = 170$$

$$A_P = \frac{P_{out}}{P_{in}} \quad \text{therefore} \quad P_{out} = A_P P_{in}$$

$$P_{out} = 170(40 \times 10^{-3}) = 6.8 \text{ W}$$

Example 2-4

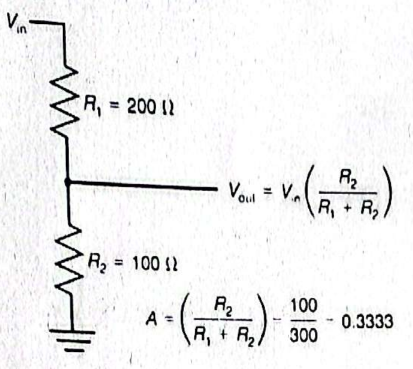
A two-stage amplifier has an input power of 25 μW and an output power of 1.5 mW. One stage has a gain of 3. What is the gain of the second stage?

$$A_P = \frac{P_{out}}{P_{in}} = \frac{1.5 \times 10^{-3}}{25 \times 10^{-6}} = 60$$

$$A_P = A_1 \times A_2$$

If $A_1 = 3$, then $60 = 3 \times A_2$ and $A_2 = 60/3 = 20$.

Figure 2-3 A voltage divider introduces attenuation.



Attenuation

Attenuation

Attenuation refers to a loss introduced by a circuit or component. Many electronic circuits, sometimes called stages, reduce the amplitude of a signal rather than increase it. If the output signal is lower in amplitude than the input, the circuit has loss, or attenuation. Like gain, attenuation is simply the ratio of the output to the input. The letter A is used to represent attenuation as well as gain:

$$\text{Attenuation } A = \frac{\text{output}}{\text{input}} = \frac{V_{out}}{V_{in}}$$

Circuits that introduce attenuation have a gain that is less than 1. In other words, the output is some fraction of the input.

An example of a simple circuit with attenuation is a voltage divider such as that shown in Fig. 2-3. The output voltage is the input voltage multiplied by a ratio based on the resistor values. With the resistor values shown, the gain or attenuation factor of the circuit is $A = R_2/(R_1 + R_2) = 100/(200 + 100) = 100/300 = 0.3333$. If a signal of 10 V is applied to the attenuator, the output is $V_{out} = V_{in}A = 10(0.3333) = 3.333$ V.

When several circuits with attenuation are cascaded, the total attenuation is, again, the product of the individual attenuations. The circuit in Fig. 2-4 is an example. The attenuation factors for each circuit are shown. The overall attenuation is

$$A_T = A_1 \times A_2 \times A_3$$

With the values shown in Fig. 2-4, the overall attenuation is

$$A_T = 0.2 \times 0.9 \times 0.06 = 0.0108$$

Given an input of 3 V, the output voltage is

$$V_{out} = A_T V_{in} = 0.0108(3) = 0.0324 = 32.4 \text{ mV}$$

Figure 2-4 Total attenuation is the product of individual attenuations of each cascaded circuit.

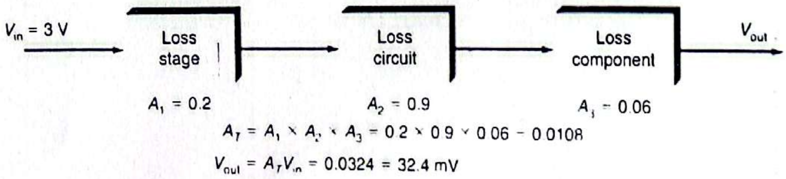
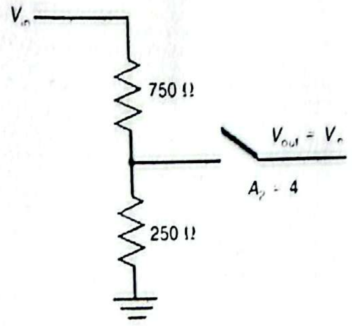


Figure 2-5 Gain exactly offsets the attenuation.



$$A_1 = \frac{250}{750 + 250} \qquad A_T = A_1 A_2 = 0.25(4) = 1$$

$$A_1 = \frac{250}{1000} = 0.25$$

It is common in communication systems and equipment to cascade circuits and components that have gain and attenuation. For example, loss introduced by a circuit can be compensated for by adding a stage of amplification that offsets it. An example of this is shown in Fig. 2-5. Here the voltage divider introduces a 4-to-1 voltage loss, or an attenuation of 0.25. To offset this, it is followed with an amplifier whose gain is 4. The overall gain or attenuation of the circuit is simply the product of the attenuation and gain factors. In this case, the overall gain is $A_T = A_1A_2 = 0.25(4) = 1$.

Another example is shown in Fig. 2-6, which shows two attenuation circuits and two amplifier circuits. The individual gain and attenuation factors are given. The overall circuit gain is $A_T = A_1 A_2 A_3 A_4 = (0.1)(10)(0.3)(15) = 4.5$.

For an input voltage of 1.5 V, the output voltage at each circuit is shown in Fig. 2-6.

In this example, the overall circuit has a net gain. But in some instances, the overall circuit or system may have a net loss. In any case, the overall gain or loss is obtained by multiplying the individual gain and attenuation factors.

Example 2-5

A voltage divider such as that shown in Fig. 2-5 has values of $R_1 = 10\ \text{k}\Omega$ and $R_2 = 470\ \Omega$.

a. What is the attenuation?

$$A_1 = \frac{R_2}{R_1 + R_2} = \frac{470}{10{,}470} \qquad A_1 = 0.045$$

b. What amplifier gain would you need to offset the loss for an overall gain of 1?

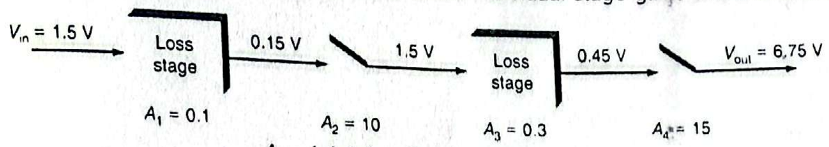
$$A_T = A_1A_2$$

where A_1 is the attenuation and A_2 is the amplifier gain.

$$1 = 0.045A_2 \qquad A_2 = \frac{1}{0.045} = 22.3$$

Note: To find the gain that will offset the loss for unity gain, just take the reciprocal of attenuation: $A_2 = 1/A_1$.

Figure 2-6 The total gain is the product of the individual stage gains and attenuations.



$$A_T = A_1A_2A_3A_4 = (0.1)(10)(0.3)(15) = 4.5$$

Example 2-6

An amplifier has a gain of 45,000, which is too much for the application. With an input voltage of 20 μV, what attenuation factor is needed to keep the output voltage from exceeding 100 mV? Let A_1 = amplifier gain = 45,000; A_2 = attenuation factor; A_T = total gain.

$$A_T = \frac{V_{out}}{V_{in}} = \frac{100 \times 10^{-3}}{20 \times 10^{-6}} = 5000$$

$$A_T = A_1A_2 \quad \text{therefore} \quad A_2 = \frac{A_T}{A_1} = \frac{5000}{45{,}000} = 0.1111$$

Decibels

Decibel (dB)

The gain or loss of a circuit is usually expressed in *decibels (dB)*, a unit of measurement that was originally created as a way of expressing the hearing response of the human ear to various sound levels. A decibel is one-tenth of a bel.

When gain and attenuation are both converted to decibels, the overall gain or attenuation of an electronic circuit can be computed by simply adding the individual gains or attenuations, expressed in decibels.

It is common for electronic circuits and systems to have extremely high gains or attenuations, often in excess of 1 million. Converting these factors to decibels and using logarithms result in smaller gain and attenuation figures, which are easier to use.

Decibel Calculations. The formulas for computing the decibel gain or loss of a circuit are

$$\text{dB} = 20 \log \frac{V_{out}}{V_{in}} \tag{1}$$

$$\text{dB} = 20 \log \frac{I_{out}}{I_{in}} \tag{2}$$

$$\text{dB} = 10 \log \frac{P_{out}}{P_{in}} \tag{3}$$

Formula (1) is used for expressing the voltage gain or attenuation of a circuit; formula (2), for current gain or attenuation. The ratio of the output voltage or current to the input voltage or current is determined as usual. The base-10 or common log of the input/output ratio is then obtained and multiplied by 20. The resulting number is the gain or attenuation in decibels.

Formula (3) is used to compute power gain or attenuation. The ratio of the power output to the power input is computed, and then its logarithm is multiplied by 10.

Example 2-7

a. An amplifier has an input of 3 mV and an output of 5 V. What is the gain in decibels?

$$\text{dB} = 20 \log \frac{5}{0.003} = 20 \log 1666.67 = 20(3.22) = 64.4$$

b. A filter has a power input of 50 mW and an output of 2 mW. What is the gain or attenuation?

$$\text{dB} = 10 \log \frac{2}{50} = 10 \log 0.04 = 10(-1.398) = -13.98$$

Note that when the circuit has gain, the decibel figure is positive. If the gain is less than 1, which means that there is an attenuation, the decibel figure is negative.

Now, to calculate the overall gain or attenuation of a circuit or system, you simply add the decibel gain and attenuation factors of each circuit. An example is shown in Fig. 2-7, where there are two gain stages and an attenuation block. The overall gain of this circuit is

$$A_T = A_1 + A_2 + A_3 = 15 - 20 + 35 = 30 \text{ dB}$$

Decibels are widely used in the expression of gain and attenuation in communication circuits. The table on the next page shows some common gain and attenuation factors and their corresponding decibel figures.

Ratios less than 1 give negative decibel values, indicating attenuation. Note that a 2:1 ratio represents a 3-dB power gain or a 6-dB voltage gain.

tilog

Antilogs. To calculate the input or output voltage or power, given the decibel gain or attenuation and the output or input, the *antilog* is used. The antilog is the number obtained when the base is raised to the logarithm, which is the exponent:

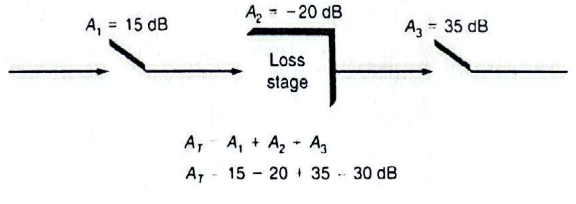
$$\text{dB} = 10 \log \frac{P_{out}}{P_{in}} \quad \text{and} \quad \frac{\text{dB}}{10} = \log \frac{P_{out}}{P_{in}}$$

and

$$\frac{P_{out}}{P_{in}} = \text{antilog} \frac{\text{dB}}{10} = \log^{-1} \frac{\text{dB}}{10}$$

The antilog is simply the base 10 raised to the dB/10 power.

Figure 2-7 Total gain or attenuation is the algebraic sum of the individual stage gains in decibels.



dB GAIN OR ATTENUATION

Ratio (Power or Voltage)	Power	Voltage
0.000001	−60	−120
0.00001	−50	−100
0.0001	−40	−80
0.001	−30	−60
0.01	−20	−40
0.1	−10	−20
0.5	−3	−6
1	0	0
2	3	6
10	10	20
100	20	40
1000	30	60
10,000	40	80
100,000	50	100

Remember that the logarithm y of a number N is the power to which the base 10 must be raised to get the number.

$$N = 10^y \quad \text{and} \quad y = \log N$$

Since

$$\text{dB} = 10 \log \frac{P_{out}}{P_{in}}$$

$$\frac{\text{dB}}{10} = \log \frac{P_{out}}{P_{in}}$$

Therefore

$$\frac{P_{out}}{P_{in}} = 10^{\text{dB}/10} = \log^{-1} \frac{\text{dB}}{10}$$

The antilog is readily calculated on a scientific calculator. To find the antilog for a common or base-10 logarithm, you normally press the (Inv) or (2nd) function key on the calculator and then the (log) key. Sometimes the log key is marked with 10^x, which is the antilog. The antilog with base e is found in a similar way, by using the (Inv) or (2nd) function on the (ln) key. It is sometimes marked e^x, which is the same as the antilog.

Example 2-8

A power amplifier with a 40-dB gain has an output power of 100 W. What is the input power?

$$\text{dB} = 10 \log \frac{P_{out}}{P_{in}} \qquad \text{antilog} = \log^{-1}$$

$$\frac{\text{dB}}{10} = \log \frac{P_{out}}{P_{in}}$$

$$\frac{40}{10} = \log \frac{P_{out}}{P_{in}}$$

$$4 = \log \frac{P_{out}}{P_{in}}$$

$$\text{antilog } 4 = \text{antilog}\left(\log \frac{P_{out}}{P_{in}}\right)$$

$$\log^{-1} 4 = \frac{P_{out}}{P_{in}}$$

$$\frac{P_{out}}{P_{in}} = 10^4 = 10{,}000$$

$$P_{in} = \frac{P_{out}}{10{,}000} = \frac{100}{10{,}000} = 0.01 \text{ W} = 10 \text{ mW}$$

Example 2-9

An amplifier has a gain of 60 dB. If the input voltage is 50 μV, what is the output voltage?

Since

$$\text{dB} = 20 \log \frac{V_{out}}{V_{in}}$$

$$\frac{\text{dB}}{20} = \log \frac{V_{out}}{V_{in}}$$

Therefore

$$\frac{V_{out}}{V_{in}} = \log^{-1} \frac{\text{dB}}{20} = 10^{\text{dB}/20}$$

$$\frac{V_{out}}{V_{in}} = 10^{60/20} = 10^3$$

$$\frac{V_{out}}{V_{in}} = 10^3 = 1000$$

$$V_{out} = 1000V_{in} = 1000(50 \times 10^{-6}) = 0.05 \text{ V} = 50 \text{ mV}$$

dBm. When the gain or attenuation of a circuit is expressed in decibels, implicit is a comparison between two values, the output and the input. When the ratio is computed, the units of voltage or power are canceled, making the ratio a dimensionless, or relative, figure. When you see a decibel value, you really do not know the actual voltage or power values. In some cases, this is not a problem; in others, it is useful or necessary to know the actual values involved. When an absolute value is needed, you can use a *reference value* to compare any other value.

Reference value

An often used reference level in communication is 1 mW. When a decibel value is computed by comparing a power value to 1 mW, the result is a value called the *dBm*. It is computed with the standard power decibel formula with 1 mW as the denominator of the ratio:

dBm

$$\text{dBm} = 10 \log \frac{P_{out}(\text{W})}{0.001(\text{W})}$$

Here P_{out} is the output power, or some power value you want to compare to 1 mW, and 0.001 is 1 mW expressed in watts.

The output of a 1-W amplifier expressed in dBm is, e.g.,

$$\text{dBm} = 10 \log \frac{1}{0.001} = 10 \log 1000 = 10(3) = 30 \text{ dBm}$$

Sometimes the output of a circuit or device is given in dBm. For example, if a microphone has an output of -50 dBm, the actual output power can be computed as follows:

$$-50 \text{ dBm} = 10 \log \frac{P_{out}}{0.001}$$

$$\frac{-50 \text{ dBm}}{10} = \log \frac{P_{out}}{0.001}$$

Therefore

$$\frac{P_{out}}{0.001} = 10^{-50 \text{ dBm}/10} = 10^{-5} = 0.00001$$

$$P_{out} = 0.001 \times 0.00001 = 10^{-3} \times 10^{-5} = 10^{-8} \text{ W} = 10 \times 10^{-9} = 10 \text{ nW}$$

GOOD TO KNOW

From the standpoint of sound measurement, 0 dB is the least perceptible sound (hearing threshold), and 120 dB equals the pain threshold of sound. This list shows intensity levels for common sounds. (Tippens, *Physics*, 6th ed., Glencoe/McGraw-Hill, 2001, p. 497)

Sound	*Intensity level, dB*
Hearing threshold	0
Rustling leaves	10
Whisper	20
Quiet radio	40
Normal conversation	65
Busy street corner	80
Subway car	100
Pain threshold	120
Jet engine	140–160

Example 2-10

A power amplifier has an input of 90 mV across 10 kΩ. The output is 7.8 V across an 8-Ω speaker. What is the power gain, in decibels? You must compute the input and output power levels first.

$$P = \frac{V^2}{R}$$

$$P_{in} = \frac{(90 \times 10^{-3})^2}{10^4} = 8.1 \times 10^{-7} \text{ W}$$

$$P_{out} = \frac{(7.8)^2}{8} = 7.605 \text{ W}$$

$$A_P = \frac{P_{out}}{P_{in}} = \frac{7.605}{8.1 \times 10^{-7}} = 9.39 \times 10^6$$

$$A_P(\text{dB}) = 10 \log A_P = 10 \log 9.39 \times 10^6 = 69.7 \text{ dB}$$

dBc. This is a decibel gain attenuation figure where the reference is the carrier. The carrier is the base communication signal, a sine wave that is modulated. Often the amplitude's sidebands, spurious or interfering signals, are referenced to the carrier. For example, if the spurious signal is 1 mW compared to the 10-W carrier, the dBc is

$$\text{dBc} = 10 \log \frac{P_{\text{signal}}}{P_{\text{carrier}}}$$

$$\text{dBc} = 10 \log \frac{0.001}{10} = 10(-4) = -40$$

Example 2-11

An amplifier has a power gain of 28 dB. The input power is 36 mW. What is the output power?

$$\frac{P_{out}}{P_{in}} = 10^{\text{dB}/10} = 10^{2.8} = 630.96$$

$$P_{out} = 630.96P_{in} = 630.96(36 \times 10^{-3}) = 22.71 \text{ W}$$

Example 2-12

A circuit consists of two amplifiers with gains of 6.8 and 14.3 dB and two filters with attenuations of −16.4 and −2.9 dB. If the output voltage is 800 mV, what is the input voltage?

$$A_T = A_1 + A_2 + A_3 + A_4 = 6.8 + 14.3 - 16.4 - 2.9 = 1.8 \text{ dB}$$

$$A_T = \frac{V_{out}}{V_{in}} = 10^{\text{dB}/20} = 10^{1.8/20} = 10^{0.09}$$

$$\frac{V_{out}}{V_{in}} = 10^{0.09} = 1.23$$

$$V_{in} = \frac{V_{out}}{1.23} = \frac{800}{1.23} = 650.4 \text{ mV}$$

Example 2-13

Express $P_{out} = 12.3$ dBm in watts.

$$\frac{P_{out}}{0.001} = 10^{\text{dBm}/10} = 10^{12.3/10} = 10^{1.23} = 17$$

$$P_{out} = 0.001 \times 17 = 17 \text{ mW}$$