System of Linear Equation

1 System of Linear

1.1 Definition

In mathematics, a **system of linear equation** consist of collections of linear equations involving the same set of variable. For example,

two Linear Equation consist of two system,

$$\begin{aligned} 2x + 4y &= 7 \\ 8x + 5y &= 9 \end{aligned}$$

three Linear Equation consist of three system.

$$\begin{aligned} 3x + 2y - z &= 1 \\ 2x + 2y - 4z &= -2 \\ -x + 2y - z &= 0 \end{aligned}$$

The **solution** to which **Linear Equation** valid is an assignment of number to the variables such that all the variables withing the equation is satisfied.

General form of writing **linear equation** in variables $x_1 + x_2 + \cdots + x_n$ is in the equation form.

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b,$$

Where $a_1, a_2, \ldots, a_n$ and b are constants. The constants which a_i is called ***coefficient*** to which x_i is; and b are called the **constant term** of the equation.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \qquad\qquad\qquad\qquad\qquad\quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases} \tag{1.1}$$

Solution to which **linear equation 1.1** are number sets of $(s_1, s_2, \ldots, s_n)$ to which when substitute to $x_1, x_2, \ldots, x_n$ are true within the b_m.

Theorem 1.1 *Any linear equation has one of the following conditions*

(0) No solution.

(1) Unique solutions.

(2) Infinitely many solutions

1.2 Ways to Solve Linear Equation

1.2.1 Substitution

Let say,

$$\begin{aligned} 3x - 2y &= 7 \\ 4x + y &= 11 \end{aligned}$$

Turn one of the equation into $y = x$ or $x = y$.

$$\begin{aligned} 4x + y &= 11 \\ y &= 11 - 4x \\ y &= -4x + 11 \end{aligned}$$

Substitute $y = -4x + 11$ to first equation.

$$\begin{aligned} 3x - 2y =& \quad 7 \\ 3x - 2(-4x + 11) =& \quad 7 \\ 3x + 8x - 22 =& \quad 7 \\ 11x - 22 =& \quad 7 \\ 11x =& \quad 29 \\ x =& \frac{29}{11} \end{aligned}$$

Now substitute $x = -\frac{29}{11}$ to $y = -4x + 11$.

$$
\begin{aligned}
y &= -4x + 11 \\
y &= -4\left(\frac{29}{11}\right) + 11 \\
y &= \frac{-116}{11} + 11 \\
y &= \frac{-116}{11} + \frac{121}{11} \\
y &= \frac{-116 + 121}{11} \\
y &= \frac{5}{11}
\end{aligned}
$$

So the **solution** to the equation.

$$\boxed{x = \tfrac{29}{11},\ y = \tfrac{5}{11}}$$

To prove if this solution satisfied the equation is to substitute x and y to both equation or one of the equation.
So to proof it, choose one of the equation $3x - 2y = 7$.

$$
\begin{aligned}
3x - 2y &= 7 \\
3\left(\frac{29}{11}\right) - 2\left(\frac{5}{11}\right) &= 7 \\
\frac{87}{11} - \frac{10}{11} &= 7 \\
\frac{87 - 10}{11} &= 7 \\
\frac{77}{11} &= 7 \\
7 &= 7
\end{aligned}
$$

Second equation $4x + y = 11$.

$$\begin{aligned} 4x - 2y &= 11 \\ 4\left(\frac{29}{11}\right) - \left(\frac{5}{11}\right) &= 11 \\ \frac{116}{11} + \frac{5}{11} &= 11 \\ \frac{116 + 5}{11} &= 11 \\ \frac{121}{11} &= 11 \\ 11 &= 11 \end{aligned}$$

Both equation are true and the solution satisfied the variable.