Leiden University

An AMUSE Based Research Project

THE MILKY WAY AND ITS EXOTIC MILLISECOND PULSAR POPULATION: A STATISTICAL ANALYSIS

Is there the possibility that millisecond pulsars living in the Milky Way migrated from the Large Magellanic Cloud?

Author:
Shiyang Zhang *(s2746425)*
Arend Moerman *(s1749323)*
Erwan Hochart *(s2009296)*

Supervisor:
Prof.dr. Simon Portegies Zwart
Martijn Wilhelm

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Universiteit Leiden
Leiden Observatory

Abstract

From testing the theory of general relativity to finding the first extra-solar planetary system, millisecond pulsars are a class of rapidly rotating neutron stars with a wide range of applications in physics. This paper investigates whether millisecond pulsars found in the Milky Way may have originated from another galactic system, in this case, the Large Magellanic Cloud, or LMC for short and uses AMUSE to achieve this. AMUSE is a programming framework that allows for the bridging of multiple systems as well as resolving issues with a multi-scale nature. The millisecond pulsars are generated using formation rates from literature. They are generated inside a globular cluster and follow natal kick velocity distributions. The globular cluster is modeled by an existing globular cluster in the LMC, namely NGC 1783. The total evolution of the simulation is 1Gyr, with it finishing at the present day. Although results show that there is a possibility for millisecond pulsars migration, only 0.80% of the ejected millisecond pulsars are found to be close-encounters with the Milky Way. This result suggests that the capturing of exotic millisecond pulsars is not a likely scenario and one with a minimal effect on the population distribution of Milky Way millisecond pulsars.

1 Introduction

From being used to detect the first extra-solar planetary system (Wolszczan and Frail, 1992) to being the centre of discussion in Nobel Prize-winning papers testing the theory of general relativity (Taylor and Weisberg, 1982), millisecond pulsars have become a popular research topic in the field of physics since their discovery in 1967 (Hewish et al., 1968).

Pulsars are rapidly rotating neutron stars formed after a supernova explosion and may be thought of as the cosmic clock. The periodic signal observed giving them this 'clock-like' trait emerges after charged particles accelerate along with the rotating pulsars magnetic field lines which gives rise to electromagnetic radiation. Since they rotate quickly about their axis, once this beam of radiation crosses our line of sight we observe a signal which is incredibly periodic and stable. To date, there are upwards of 2800 detected pulsars (Manchester et al., 2005)[1] and a lot of unanswered questions, making them an exciting field of research for astronomers.

One such mystery is the significant fraction of millisecond pulsars within the Milky Way having no companions. Following the recycling model (Alpar et al., 1982), millisecond pulsars originate from a binary system since this allows them to achieve their rapid rotational velocities. Although it is proposed that isolated millisecond pulsars may have gotten rid of their companions via ablation (as is the case for PSR 1957 +20 (Fruchter et al., 1988)) the timescales of such a process make this explanation inconclusive. This explanation is inconclusive since it would require the evaporation efficiency to be incredibly large with $\eta \approx 0.1$ (Jia and Li, 2016). Another possible explanation for this large fraction of solitary millisecond pulsars may be from the Milky Way capturing rogue millisecond pulsars, pulsars who got ejected from their original galaxy after being in the vicinity of a supernova explosion.

This paper probes this idea and investigates millisecond pulsars that have become unbound from their galactic system after receiving a large enough kick velocity originating from a supernova event. More specifically, we look at the possibility of millisecond pulsars originating from the Large Magellanic Cloud (hereafter LMC). This ejection scenario can either result from a type 1a supernova event where the remnant millisecond pulsar feels a kick velocity or in the case where the millisecond pulsar's companion star undergoes a supernova explosion resulting in the ejection of the millisecond pulsar. We adopt a statistical approach in our analysis utilising the AMUSE[2] (Portegies Zwart and McMillan, 2018) framework to achieve a better understanding on the evolution history of both the Milky Way and millisecond pulsars as well as provide a solid foundation for future research to build from.

[1] The paper was published in 2005, but the catalogue was last updated November 14, 2020

[2] Code available at: https://amusecode.github.io/

The paper is composed of six main sections. Section two gives a brief introduction of millisecond pulsars providing the reader with justification as to why such a topic is interesting to investigate. Section three dissects the mathematical theory implemented within our code, giving for a better understanding of the eventual results. An overall discussion on AMUSE and the code is found in section four in which we also justify some further assumptions made. Section five and six describes the general procedure and discusses the results of the paper respectively. Section seven will summarise the report with a general conclusion as well as suggesting possible research that can be built from our paper.

2 Millisecond Pulsars: What are they?

The millisecond pulsar birthrate in the Milky Way is 2×10^{-5} yr^{-1} (Lorimer, 1999) and form once a massive star exhausts its fuel such that hydrostatic equilibrium is no longer sustained and the star collapses onto itself. If the star is massive enough, it will undergo a supernova explosion in which the remaining object, the neutron star, ends up weighing approximately $1.4M_\odot$ with a significant portion of the original mass now ejected from the system. Since the star becomes only a fraction of its original size (a few kilometres in radius), for it to conserve angular momentum, its spin drastically increases. The periodic signal received from these celestial objects stems from the fact that charged particles accelerate along the magnetic field lines and as the pulsar rotates along its axis if the beam crosses through our line of sight a signal is detected. It is noteworthy to mention, however, that it remains unclear how their magnetic field arises (Cruces et al., 2019) but this is outside of the scope of the paper.

This report focuses on a particular class of pulsars called millisecond pulsars. These pulsars have a period T of $1.4 \leq T$ ms ≤ 30 (Lorimer, 2008), meaning they can spin just shy of a thousand times per second along their axis. The report investigates this class of pulsar specifically as a quicker spin induces a weaker magnetic field, in turn, allowing them to live longer (Phinney and Kulkarni, 1994). A 'normal' pulsar tends to live around 100 Myr (Toscano et al., 1999a), whereas, millisecond pulsars have a lifetime comparable to the age of the Universe (Lorimer, 2008), allowing the opportunity for exotic millisecond pulsars to migrate into the Milky Way.

Nevertheless, a typical supernova explosion will not yield the rapid spin millisecond pulsars exhibit. To explain the emergence of such large rotational velocities, astronomers use the recycling model. The recycling model is built from the premise that millisecond pulsars form in binary systems. In a binary system, the pulsar may accrete mass from its companion star causing its angular momentum (and therefore spin) to increase (Saxton, 2020). Since globular clusters are the densest galactic environment, they are regarded as millisecond pulsar nurseries as there is a larger probability for a neutron star to interact with another celestial object in these regions to obtain such rapid rotational velocities.

As mentioned in section 1, there is a mystery revolving why so many millisecond pulsars are in isolated systems since the recycling model implies they originate in binary systems. Lorimer (2008) states that roughly 20% of all millisecond pulsars are in non-binary systems giving rise to several different possible scenarios to explain this large percentage.

For instance; the system could be disrupted after a gravitational collapse of a white dwarf and its companion star inducing a type 1a supernova, or if the companion star undergoes a supernova explosion ejecting the original millisecond pulsar Burgay et al. (2013). An isolated millisecond pulsar may also form through white dwarf mergers Michel (1987), or via ablation of its companion star or even by the destruction of the companion star through pulsar wind effects (Bhatta, 1991).

Although ablation and the destruction of a companion star through pulsar winds can cause solitary millisecond pulsars, these will not provide the required velocity for the millisecond pulsar to be ejected from the LMC and caught by the Milky Way which is the focal point of our report. This brings forth a crucial assumption made throughout our report. Since we look at the possibility of millisecond pulsar migration; all millisecond pulsars in the simulation get ejected via either a type 1a supernova explosion or by having it's companion star undergo a supernova explosion. These events allow the millisecond pulsar to attain velocities needed to be ejected from the host galaxy and captured by the Milky Way. We apply this assumption by changing the time interval of the millisecond pulsar birth rate (further discussed in section 5).

3 Distribution of our Millisecond Pulsars

3.1 The Velocity Distribution

There have been many papers investigating the velocity distribution of pulsars over the past few decades (Lorimer et al. (1997) ; Hansen and Phinney (1997) ; Toscano et al. (1999b) ; Hobbs et al. (2005)) each providing differences in their results. This paper uses Hansen and Phinney (1997) as a reference point simply due to it calculating the ejection velocities for pulsars after a supernova ejection which is the scenario that allows the pulsars to become unbound from the LMC. Furthermore, this velocity distribution is adopted since it takes into account the fainter, quicker pulsars as well as the slower, brighter ones that some of the other studies overlooked. Using Monte Carlo simulations, they found that pulsars ejected from a supernova tend towards the following velocity distribution:

$$P(v_k) = \sqrt{\frac{2}{\pi}} \frac{\bar{v}_k^2}{\sigma_v^3} e^{-v_k^2/2\sigma_v^2} \tag{1}$$

Where $\bar{v}$ is the mean velocity of the population, σ_v the standard deviation of the distribution and v_k the kick velocity. Upon simulation, they found that the mean velocity is given as $\bar{v}_k = 250 - 300$ km s^{-1} with a standard deviation of $\sigma_v = 190$ km s^{-1}. The mean velocity for this paper was chosen at $\bar{v_k} = 250$ km s^{-1} while keeping the standard deviation at $\sigma_v = 190$ km s^{-1}. The distribution function of equation 1 with the chosen values for this paper is shown in figure 3-1.

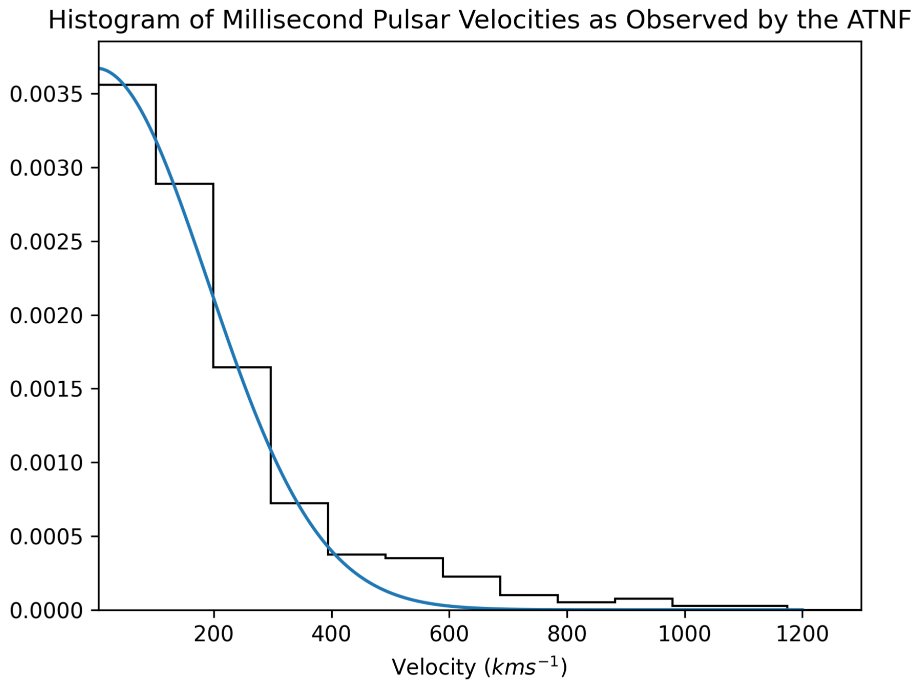

Figure 3-1: Distribution of the function used in the report compared to the histogram of measured pulsar velocities in km s^{-1} using the ATNF catalogue (Manchester et al., 2005) in the background.

As can be seen in the figure above, most millisecond pulsars will have their velocities in the lower end of the spectrum. This trend fits that of the observed data with the histogram shown, further illustrating the strength of the chosen velocity distribution of the report. One should note, however, that our use of the lower constraint on the mean velocity causes the distribution to skew towards smaller velocities. This low-velocity bias gives rise to the notion that the simulation will compute a smaller amount of close-encounter millisecond pulsars compared to possible theoretical values.

The simulation adopts this distribution shown in equation 1 when calculating the velocity in the x, y, z directions of the millisecond pulsar. To further improve the ejection mechanism, the code uses a `numpy` random number generator which multiplies the v_x, v_y, v_z velocity values by -1 or 1 giving the millisecond pulsar a random orientation of ejection.

3.2 The Mass Distribution

Another aspect to keep in mind with the simulated millisecond pulsars is the mass attributed to them. Although this will not play a role in the orbit and evolution of the system as its mass isn't necessary for any of the gravitational codes, it remains defined as it allows calculation on the energy of the system. Calculating the system's energy provides information on the reliability and stability of the simulation.

Although pulsars tend to have a mass of $1.4M_{\odot}$ following the Chandrasekhar limit, millisecond pulsars accrete mass after their supernova explosion meaning they can attain a mass exceeding this. The mass distribution for the sampled population adopted in this report uses a Gaussian distribution, shown in equation 2 while utilising the values found by Antoniadis et al. (2016) as this is one of the more recent works investigating the issue. Since there is a lack of research papers modelling the mass distribution of millisecond pulsars, and following the idea from the Central Limit Theorem a Gaussian distribution was chosen to model the sampled millisecond pulsars mass. The equation is provided below.

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\bar{M_{\odot}})^2}{2\sigma^2}} \tag{2}$$

Antoniadis et al. (2016) found that there is a mean millisecond pulsar mass of $\bar{M_{\odot}} = 1.4$ and a standard deviation of $\sigma = 0.3M_{\odot}$.

4 The Code

The simulation uses several different scripts. This section dissects each of these scripts and describes their role in the simulation.

4.1 Gravitational Potentials

The simulation incorporates four gravitational potentials, one for the SMC, one for the LMC, one for the globular cluster the millisecond pulsar is ejected from and finally, one for the Milky Way. The potentials originate from the code imported from the AMUSE community `galactic_potentials`.

The potential of the LMC and SMC incorporates both a Plummer and Navarro–Frenk–White (NFW) potential (Navarro et al., 1997). Although this is an idealised version of the potential for these two irregular galaxies, numerous papers have approximated their potentials as such with great success (Bes; Yoshizawa and Noguchi (2003); Bekki and Chiba (2005)). The SMC, being the smallest galactic system, will have minimal effect on the trajectory of the pulsar and a general model of its potential suffices for the report. Furthermore, although the pulsar originates from the LMC, given that it has to be ejected from the system to be captured by the Milky Way the pulsar's trajectory will be minimally affected by the LMC potential meaning that it is right to ignore some of the finer details. Lastly, the paper isn't interested in the intricate details of the evolution of the pulsar within these systems, rather, it aims to find which parameters are needed for the pulsar to be bound to the Milky Way and thus such a generalisation of potentials is deemed sufficient for the report.

The Milky Way potential model used was developed by Bovy (2015) and gets imported from the AMUSE catalogue, namely with `MWpotentialBovy2015`. This model provides an idealised version of the potential. Nevertheless, it incorporates a bulge model with a power-law density profile along with a Miyamoto-Nagai potential describing the disk and an NFW potential for the dark matter halo. His paper provides extensive details if one wishes to know further. However, it is noteworthy to realise that both the spiral arms and bar of the Milky Way are neglected in this profile. Nevertheless, their effects are minimal for what the report aims to achieve and so the current model provides an accurate representation of the general galactic potential.

The globular cluster will have the weakest potential within the simulation. However, since the millisecond pulsar originates at this location, it is essential to represent such a system. The globular cluster mimics the cluster NGC 1783 which inhabits the LMC. The potentials used will be that of a Plummer potential, a potential profile first introduced to model globular cluster environments (Plummer, 1911).

One final comment on the potentials is that they are rigid potentials. Rigid potentials mean that the potential of the LMC, SMC, globular cluster and Milky Way remain static throughout the simulation such that dynamical effects such as tidal pulls are ignored throughout the simulation. Although this may seem as a large assumption, as mentioned earlier, the report doesn't look to analyse in intricate detail the trajectory of the millisecond pulsar. Instead, it looks at the possibility of close-encounter millisecond pulsars appearing after being ejected from the LMC. This means that modelling interaction between potentials would only add unnecessary complication to the code and yield it inefficient without much gain in the process.

4.2 The Code's Methodology

The simulation begins by running the two scripts `galpot_init.py` and `particle_init.py`. `galpot_init` sets up and initialises the analytical potentials for the LMC, SMC and a globular cluster as calculated based on the models described in the previous section. The script `coordinates.py` plays an essential role in doing so as it converts coordinates from the International Celestial Reference System (ICRS) system, in which the original data is provided in, into cartesian coordinates while simultaneously converting it into the galactocentric frame with the help of `astropy`. Furthermore, `coordinates.py` contains the coordinates and velocities associated with the galactic systems at the initial time-step - that means the conditions 1Gyr in the past which is the time the simulation starts generating the millisecond pulsar population.

`particle_init.py` initialises the millisecond pulsar to be emitted based on the parameters given in section 3 and for every time-step generates a set of particles that represent and traces the LMC, SMC, globular cluster and millisecond pulsars centre of mass. The object classes governing the potentials and millisecond pulsar are systematically updated as the simulation evolves.

The representation of the galactic centre of mass done in `particle_init.py` allows the simulation to keep track of the location of the potentials and figure 4-1 represents the motion of all three galactic systems through a full simulation in the xy plane. Furthermore, during each time-step the millisecond pulsar generator generates a new star to be emitted from the LMC based on a probability dependent on the calculated millisecond pulsar birth rate in the LMC (discussed in section 5).

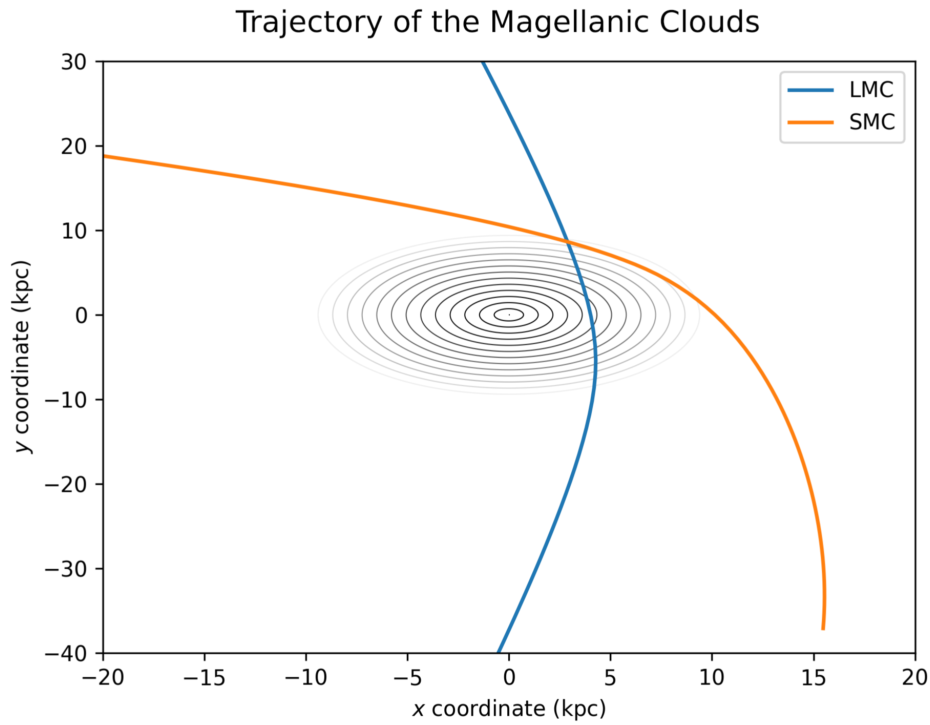

Figure 4-1: The evolution of the Large Magellanic Cloud and Small Magellanic Cloud through a simulation. The contour lines show the Milky Way. It remains still throughout the simulation since with the help of `astropy` the Milky Way has been defined as the reference frame.

The backbone of the program is in `evol.py`. This script is what evolves the particle sets over time. The LMC and SMC centre of mass particles interact with each other through a Hermite integrator and are both bridged with the Milky Way potential with the help of AMUSE's bridge functionality. The globular cluster's centre of the mass particle, on the other hand, uses a `drift_without_gravity` solver which is in turn bridged with the potentials of the LMC, SMC and Milky Way respectively. The orientation of the globular cluster's initial velocity was picked such that it will remain bound to the LMC.

The neutron stars are generated throughout the simulation, potentially adding a new millisecond pulsar each time-step based on a probability dependent on the birthrate of such a celestial object for every time-step within the LMC. The millisecond pulsars may then interact with one another through a Hermite integrator while simultaneously still being bridged with the LMC, SMC, Milky Way and globular cluster potentials.

At the end of the simulation, a check is carried out to filter the data into only showing the millisecond pulsars which are close to the Milky Way. The `evol.py` script outputs three different files: `neut_stars_positions.pkl`, `check.txt` and `gal_lines.pickle`. `neut_stars_positions.pkl` contains a data frame that keeps track of each millisecond pulsars' trajectory in the simulation. The rows represent neutron stars while the columns represent a given time in the simulation. `check.txt` contains the indices in this data frame which represent the neutron stars deemed to be calose encounters. The `gal_lines.pickle` file contains the coordinates of the LMC, SMC and globular cluster's centre of mass particles.

The whole program runs from `interface.py`. This script gives multiple prompts, providing the user with some flexibility in what they want to obtain. First, it asks whether one wants to generate a new set of neutron stars or not, it proceeds by asking if one wants to acquire plots of the trajectories and whether one wants to print in a text file the final positions of the bound millisecond pulsars.

If one wants to generate a new set of millisecond pulsars, the code `evol.py` is called and the simulation is carried out. If one wants to plot, the three output files mentioned earlier are called by another script, `plotters.py`. This script contains all the functions that plot the data. It also contains a function called `pplot`, which is not used in the final simulation but was used to make the GIF files showcasing the time behaviour of our program.

If one wants to calculate distances, the final coordinates of the stars that came close are used to calculate the distance to the galactic centre. The output file is called `distances.txt`. These can then, for example, be compared against known neutron stars to see how the simulated results compare with observations.

The complete program, including results, can be obtained at: https://zenodo.org/record/4166276#.X51s0XVKgrg

The program is also on GitHub at: https://github.com/ErwanH29/Team-B-Millisecond-Pulsars

5 The Simulation

The simulation begins by initialising the LMC, SMC and Milky Way system. The SMC and LMC's position and velocity parameters are provided by SIMBAD (Wenger et al., 2000) and can be referenced in table 1 and table 2. The data imported is has a heliocentric frame of reference, this gets converted into the galactocentric frame with the help of `astropy`. This conversion enables the Milky Way to become the frame of reference. After evolving the system backwards 1Gyr in time, the globular cluster, modelled after the LMC's NGC 1783, is added to the system. Its initial values are also shown in table 2. The code traces the system backwards by reversing the velocity's sign in the gravitational code. The research then initialises the system at this time and takes the values found as the new initial conditions. By doing so, we do not take into account the star formation rate history nor the evolution of the different potentials meaning our initial values may differ to what the actual scenario was 1Gyr back in time.

Object	R.A (ICRS system)	Dec. (ICRS system)	Distance (kpc)
LMC	05h 23m 34.6s	$-69°45'22''$	49.97
SMC	00h 52m 38.0s	$-72°48'01''$	61.7
Globular Cluster	04h 59m 08.6s	$-65°59'15''$	50.1

Table 1: Initial (heliocentric) positional values of the LMC, SMC and NGC 1783 used for the simulation. Data was taken from Wenger et al. (2000), except for the distance of NGC 1783, taken from

Object	v_x (km s^{-1})	v_y (km s^{-1})	v_z (km s^{-1})
LMC	47.0	242	225
SMC	5.35	164	136

Table 2: Initial (heliocentric) velocity values of the LMC and SMC. Data was taken from Wenger et al. (2000). The velocity of the globular cluster is calculated from the circular velocity formula at a given radius due to the minimal amount of measurements regarding the velocity of NGC 1783.

The globular cluster is assumed to be a collisionless system such that the millisecond pulsar doesn't have it's trajectory drastically altered at the beginning of each ejection. The paper simulates a collisionless cluster since this provides a great deal of simplification. Having a collisionless cluster reduces the dependency on arbitrary parameters such as the number of particles within the globular cluster or the proximity they have with one another. Furthermore, for large velocities, neighbouring stars will have a negligible effect on an unbound pulsar.

The simulation proceeds by evolving the system forwards in time for 1Gyr with the help of the bridge code mentioned in section 4.2, allowing for the multi-scale system to be represented at the current time with the emitted millisecond pulsars having their final positions being their present-day positions. By moving forwards in time, once again we neglect the star formation rate of all three galactic systems which may play a role in the overall mass and therefore the potential of the system. Nevertheless, we assume that the LMC emits an isolated millisecond pulsar from a supernova every 1.80 Myr.

This emission rate of a millisecond pulsar every 1.80 Myr is determined based on the average star formation rate (SFR) of both the Milky Way and LMC in the last gigayear and which research has shown to be relatively constant during this period. The values of the SFR for the Milky Way and LMC are $0.68 - 1.45\ M_\odot\ \text{yr}^{-1}$ (Robitaille and Whitney, 2010) and $0.2 M_\odot\ \text{yr}^{-1}$ (Harris and Zaritsky, 2009) respectively. Since millisecond pulsars are born every 5×10^4 years in the Milky Way (Lorimer, 2008), by incorporating the ratio of the SFR between the LMC and the Milky Way the final birth rate of millisecond pulsars in the LMC is obtained. The birthrate ends up being one millisecond pulsar every 0.36 Myr in the LMC, given a Milky Way SFR value of $1.45 M_\odot\ \text{yr}^{-1}$ is chosen. The SFR value of the Milky Way was selected as this would provide an upper limit on the population formed. This millisecond pulsar birth rate provides information on the overall population, the report, however, focuses on those that feel a kick velocity originating from a supernova explosion. If we incorporate the fact that 20% of all millisecond pulsars are in non-binary systems, a final value of a millisecond pulsar birth rate of one every 1.80 Myr is obtained for the LMC, and therefore our simulation.

It is clear from this that many assumptions are made - for instance taking a $1:1$ relation between the birth rate of millisecond pulsars in the LMC and Milky Way as well as the arbitrary value used for the SFR of the Milky Way. However, this was adopted since there are no papers currently that provide a value for a millisecond pulsar birth rate within the LMC. Given this absence of information, the method on which we obtained our final value will have to make due.

In the end, roughly 555 millisecond pulsars get simulated as can be seen in Appendix A, and so given the velocity distribution, one expects that only a minute fraction of these gets captured by the Milky Way. To further simplify data processing, the code filters through the data by only extracting the close-encounter pulsars, defined as millisecond pulsars within ± 30kpc of the galactic centre in all three spatial coordinates (x, y, z).

The chosen constraints on what is labelled a close-encounter has no physical reasoning behind it and is somewhat ambiguously given, allowing for some room to improve for future research. Labelling bound millisecond pulsars as those whose kinetic energy is less than the potential energy of the Milky Way's potential at their given location or by allowing the simulation to integrate over a larger period can provide for some results which are just as enlightening, however this would be for a report looking at the possibility of exotic millisecond pulsars becoming bound to the Milky Way rather than current time close-encounters. Nonetheless, this constraint was chosen as we look to investigate whether millisecond pulsars could have migrated from the LMC, a galactic system 1.1 Gyr old and by doing so it could help explain any current-day observations on the millisecond pulsar population distribution.

6 Results

6.1 Evolution of the Simulation

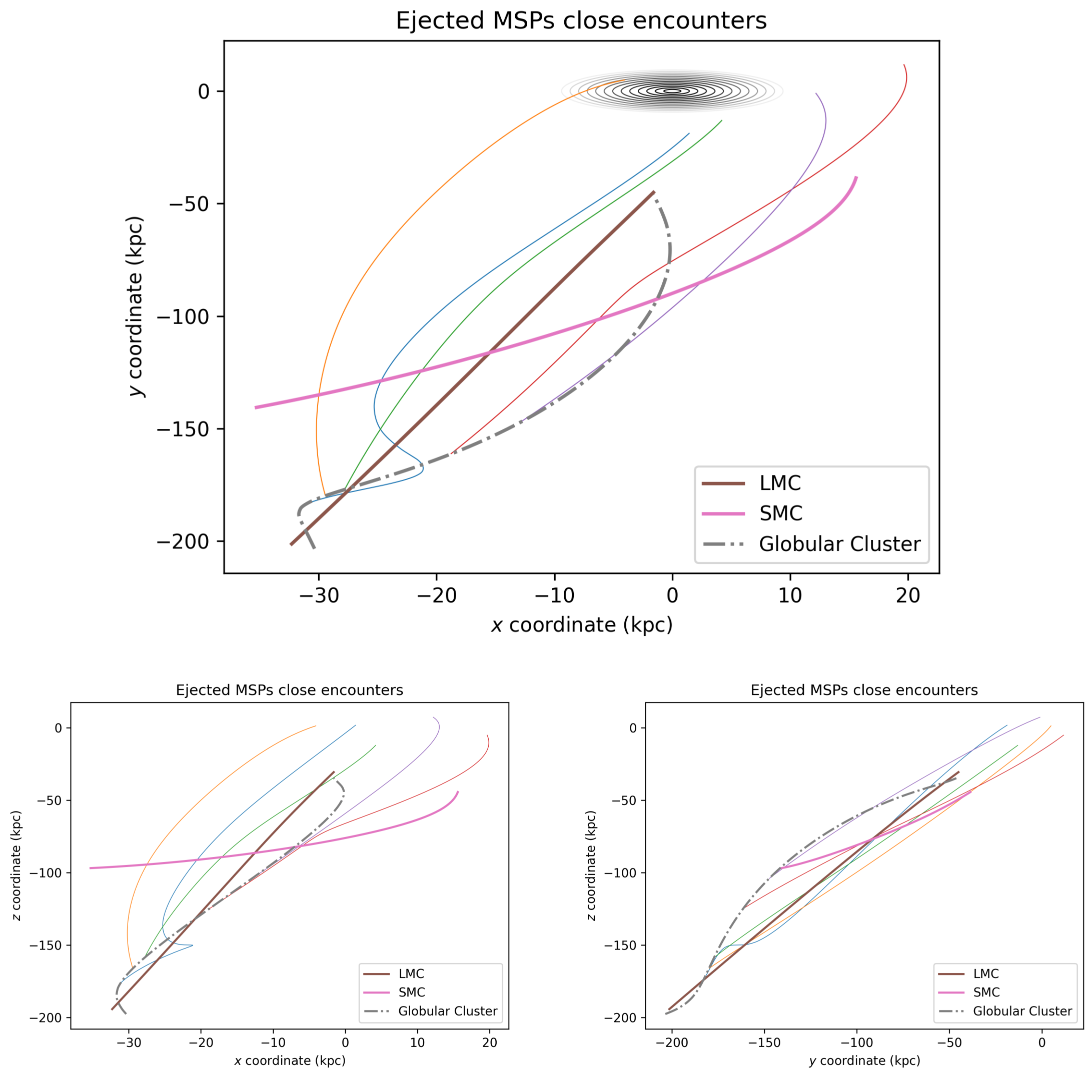

Figure 6-1: Top: Close-encounter millisecond pulsar trajectories in the xy plane. The Milky Way is shown with the contour lines of its potential. Bottom Left: Close-encounter millisecond pulsar trajectories in the xz plane. Bottom right: Close-encounter millisecond pulsar trajectories in the yz plane. The Milky Way is only shown in the xy plane due to the framework only allowing the contours to be plotted in the z-plane.

The three plots above show the trajectories of the bound millisecond pulsars, defined to be pulsars with a final position 30 kpc from the centre of the Milky Way. The specific simulation shown emitted a total of 555 pulsars with a close-encounter chance of 0.90%. Even with the minute close-encounter rate, the simulation illustrates the possibility of millisecond pulsars migrating between different galactic systems.

Figure 6-1 above illustrate the success of utilising the bridging feature provided by AMUSE as the galactic neighbourhood has its evolution all dependent on one another's motion. For instance, the SMC exhibits a curved path tending towards the Milky Way, whereas, the globular cluster in which the millisecond pulsars get ejected from becomes dependent on the evolution of the LMC who in turn depends on the Milky Way and the SMC. By having such a powerful tool to simulate the dynamical evolution of multiple systems interlocked with one another, the user obtains a more reliable result.

For the given simulation, five millisecond pulsars are close-encounters. It is interesting to note that each of these millisecond pulsars has their final trajectories bending towards the Milky Way. Given the law of the conservation of energy and provided the millisecond pulsar doesn't come into the influence of some object with a massive gravitational potential, they will end up bound to the Milky Way given long enough times.

Following this, the plot provides for two enlightening observations. For starters, every simulation exhibited at least one close-encounter millisecond pulsar, meaning that there is a non-negligible chance that there are exotic millisecond-pulsars within our vicinity. The second key takeaway is that none of the close-encounter millisecond pulsars has had their orbits stabilised. If a millisecond pulsar, or any object for that matter, has a stabilised orbit within the Milky Way there is no possibility of tracing them back their original location. Since none of the ejected millisecond pulsars has a stabilised orbit, it is possible to trace back the trajectory of such an object. Future research could build from this by analysing current-day observations of millisecond pulsars and seeing whether they may have originated within the LMC to all but confirm the theoretical reality this paper illustrates.

The report focuses purely on possible close-encounters which could help predict any discrepancy in a position distribution of millisecond pulsars. Nevertheless, future research can build from the idea and investigate the possibility of ejected millisecond pulsars caught within the Milky Way. Such a paper would require a longer integration time as was made clear with the curving of the close-encounter millisecond pulsars in figure 6-1. A comparison of the kinetic energy of the ejected millisecond pulsar with the potential energy of the Milky Way at any given point could allow for preliminary filtering of the millisecond pulsar sample and rendering the final code that tracks their trajectories more efficient and quick. By analysing the final positions of the caught millisecond pulsars it would give for more enlightening results on any potential bias of their final positions and explain for any bumps in future distance distribution models of millisecond pulsars. The integration time for this paper was constraint with the age of the LMC and so such an investigation would necessitate the use of M31, our nearest galactic neighbour. Although its distance is much further away, the much longer lifetime and much larger star formation rate could balance the effects out.

It is also interesting to point out that both the red and the blue solid lines have a significant bump in their trajectory. These bumps may arise via the interaction of newly generated millisecond pulsars with these two objects as they both seem to occur while still following the globular cluster's trajectory. If this is the case, it would give rise to further unaccounted for uncertainty as newly generated pulsars seem to affect each other's orbit. This stems from our assumption that all millisecond pulsars within the LMC are born in one globular cluster with a definite location. In reality, the birth of millisecond pulsars would span all over the LMC, giving for a much smaller chance of interactions between objects. The plots of the other simulations shown in Appendix B do not seem to suggest this is a common occurrence meaning that such an effect wouldn't pollute the final results as much as this particular simulation suggests it too.

6.2 A Quantitative Analysis

6.2.1 Position Distribution

Among the 44 simulations, the mean value of millisecond pulsars formed in each simulation is 550.20 ± 18.25. The close-encounter condition filters most of these pulsars, resulting in the final mean close-encounter rate of 0.00769 ± 0.00342 or roughly 4.270 millisecond pulsars per Gigayear. As mentioned in the previous section, since most of these trajectories are curving back towards the centre of the Milky Way, one can understand that there is a greater amount of millisecond pulsars undetected after filtering that also have their trajectories bound to the Milky Way. This means that if one wishes to investigate bound millisecond pulsars, the value obtained in this paper would only signify a lower limit.

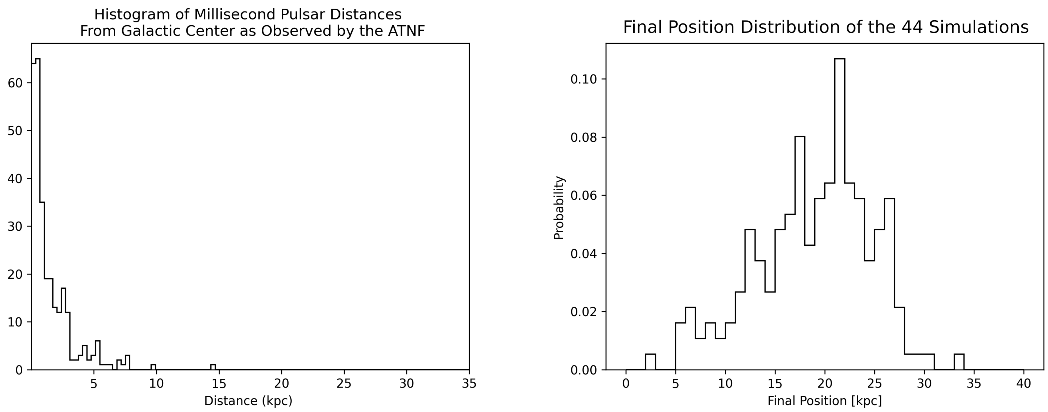

Figure 6-2: Left: Histogram of the distance distribution of the currently observed millisecond pulsar population with respect to the center of the Milky Way. Sample size of 291 using data from Manchester et al. (2005). Right: Histogram of the close-encounter millisecond pulsars' distance distribution encompassing all 44 simulations. Sample size of 187.

The right plot in figure 6-2 shows the discrete probability distribution of the final distances the close-encounter millisecond pulsars have with respect to the centre of the Milky Way. In total, after the 44 simulations ran there were 187 millisecond pulsars classified as close-encounters. The mean and the median value of these close-encounter millisecond pulsars' distance with respect to the centre of the Milky Way are 18.89 kpc and 19.40 kpc respectively. The histogram suggests that a model representing their position distributions would have their distances skewed towards greater distances. It would be interesting to see whether future research on millisecond pulsars observes a bump at these upper extremes.

In order to quantify the symmetry of the probability distribution function (PDF), the statistics introduce Cramér–Rao bound to find the lowest mean squared error among all unbiased methods. This allows us to reach the most unbiased estimator. χ^2 which illustrates the asymmetry of the PDF is given as:

$$\chi^2 = \sum_{i>0} \frac{(n_i - n_{-i})^2}{n_i + n_{-i}} \tag{3}$$

For more information on why this is so Zhang et al. (2017) provides a more thorough explanation. At the end, the least biased estimator has a value of 18.94 kpc with a χ^2 reaching a minimum of 6.38.

This paper only shows a theoretical prediction of any bias in the distance distribution of millisecond pulsars which could also explain why so many millisecond pulsars are in non-binary systems. If one compares the observed data with the one obtained after simulating, we see a massive discrepancy in the final position distribution of millisecond pulsars. One expects this, however, since observations primarily looked at the galactic plane since millisecond pulsars are faint objects, the ones that are far from Earth will be too faint to detect. This lack of signal from millisecond pulsars causes less research analysing the outskirts of the Milky Way to be conducted, resulting in a smaller amount of detections. What is interesting, however, is to notice that there are millisecond pulsars at a distance larger than 10 kpc. For these objects, future research could look at tracing back their velocities to all but confirm the possibility of exotic millisecond pulsars, or any celestial object for that matter, living within the Milky Way.

It is also interesting that the observed data show a slight bump at a distance ≈ 5 kpc. A distance of 5 kpc also corresponds to the one in which our close-encounter millisecond pulsars start to appear. Of course, this is more likely a coincidence with millisecond pulsars drifting to the outskirts of the galactic planes as their orbits stabilise, but, it could also be interesting to analyse these objects. Nevertheless one has to realise that the histogram on the left showing real-life observations equates to one 'simulation', whereas, the one on the right 44 simulations.

6.2.2 Energy Stability

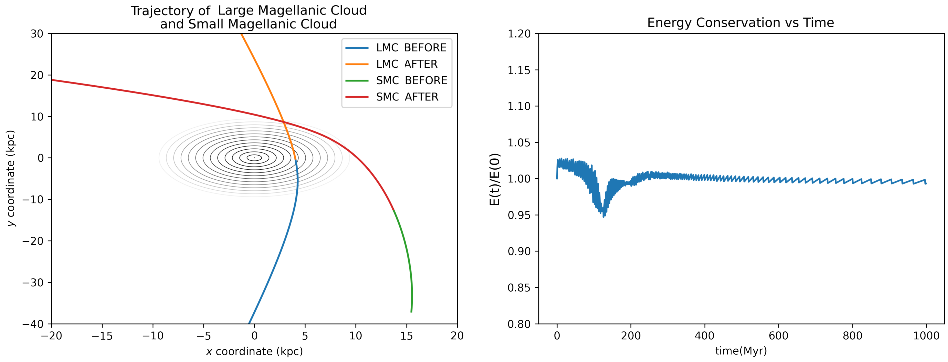

Figure 6-3: Left: The evolution of the LMC and SMC through a simulation. Before and after represent the trajectories before and after the energy ratio reaches the minimum value. Right: A time evolution of the ratio between energy and initial energy of the system.

Figure 6-3 shows the ratio of the energy $E(t)$ at a given time t with respect to the initial energy E_0 of the simulation. One observes from this figure that the energy of the system varies drastically over the evolution of the system, plateauing as the time approaches 1 Gigayears. This large periodic variation, signified by the thicker lines, suggests that the system is unstable at the beginning of the simulation and this instability stems from the many assumptions taken when modelling the galactic system.

One of the assumptions in the report was that the potentials were rigid and will not interact with one another. Having rigid potentials means that when systems are approaching each other, the tidal forces felt will remain unaccounted for giving rise to a disparity with the real-life situation and this links to the trough observed at 126 Myr. This trough corresponds to the moment of closest approach between the LMC and the Milky Way, and in turn, corresponds to the largest amount of energy neglected since interactions get disregarded throughout the simulation. The fact that the energy ratio plateau's after this closest approach further signifies the uncertainties brought from assuming rigid potentials. Once the two galactic systems start to distance from one another, the tidal forces felt on either system decreases. One sees this with the total system energy approach its initial value. However, the ratio doesn't converge back to $E/E_0 = 1$, rather it converges to 0.9984 meaning there is a 0.16% loss of energy. This value is very small and so gives strength to the assumption taken to treat potentials as rigid as the benefits (i.e computing time, code efficiency...) in doing so outweighs the loss of 0.16% of energy loss.

7 Conclusion

The report looks to see whether the Milky Way may have caught millisecond pulsars ejected from other galactic systems, more precisely ones originating in the Large

Magellanic Cloud. Information on this could give insights on possible position distribution fluctuations observed as well as possibly providing a reason why such a large fraction of observed millisecond pulsars roam the Milky Way in isolated systems.

After utilising the bridging technique and gravitational codes with AMUSE, 44 simulations were conducted ejecting a total of 24209 total millisecond pulsars, 186 of which classified as close-encounters. This is a small capture rate amounting to 0.00769 or 0.769% throughout all simulations. This value is interpreted as 4.270 close encounter millisecond pulsars from the LMC per Gigayear. As small as this value is, it brings forth an interesting topic in which papers can build upon.

Papers can investigate the same effect using a larger and older galactic system such as M31 rather than LMC and see the population capture rate. Another possibility would be for researchers to trace back millisecond pulsars far from the galactic centre given that the plots found in section 6 and Appendix B show most millisecond pulsars curve back towards the centre of the Milky Way. This could confirm evidence to such a mechanism happening rather than the theoretical approach provided by the paper.

Furthermore, there is a wide range of room for improvement in the given simulation. Incorporating tidal effects and interactions of the galactic system can give for a more realistic situation however can be deemed unnecessary, yet it would be interesting to model ejected millisecond pulsars from more than just one globular cluster to see the effect possible millisecond pulsar interactions had with one another. Another possibility is to look at the bound millisecond pulsars rather than the close-encounters, although this would look at a different research question altogether. Nevertheless, as mentioned in section 1, the code can be played with following the GitHub link https://github.com/ErwanH29/Team-B-Millisecond-Pulsars.

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Appendix A: Graph showing all the millisecond pulsar trajectories

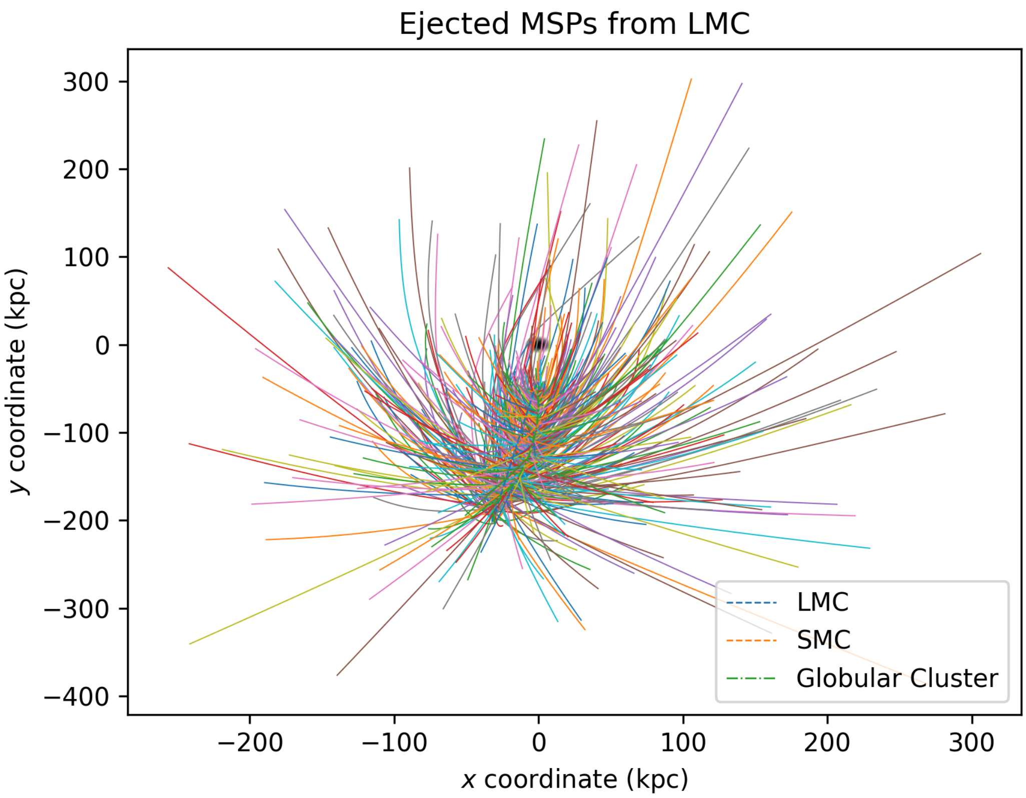

Figure A.1: Graph showing all the emitted millisecond pulsars during one run of a simulation.

The plot above shows the trajectories of all the emitted millisecond pulsars of a given simulation. When compared to the other plots showing the xy plane it is interesting to note that although a large portion of the simulated pulsars are ejected from the LMC, they do not get close to the Milky Way (as was reflected in the results). This could be due to a combination of having too high a velocity to be close to the Milky Way as well as the evolution of the system being too small since over time, the pulsars can curve back inwards towards the Milky Way as it feels it's potential.

Nevertheless it is interesting to see that a large portion of the population of isolated millisecond pulsars could be roaming in deep space unaffected by galactic systems and motivates the idea of investigating emitted millisecond pulsars from M31.

Appendix B: More plots showing the simulations evolution through time.

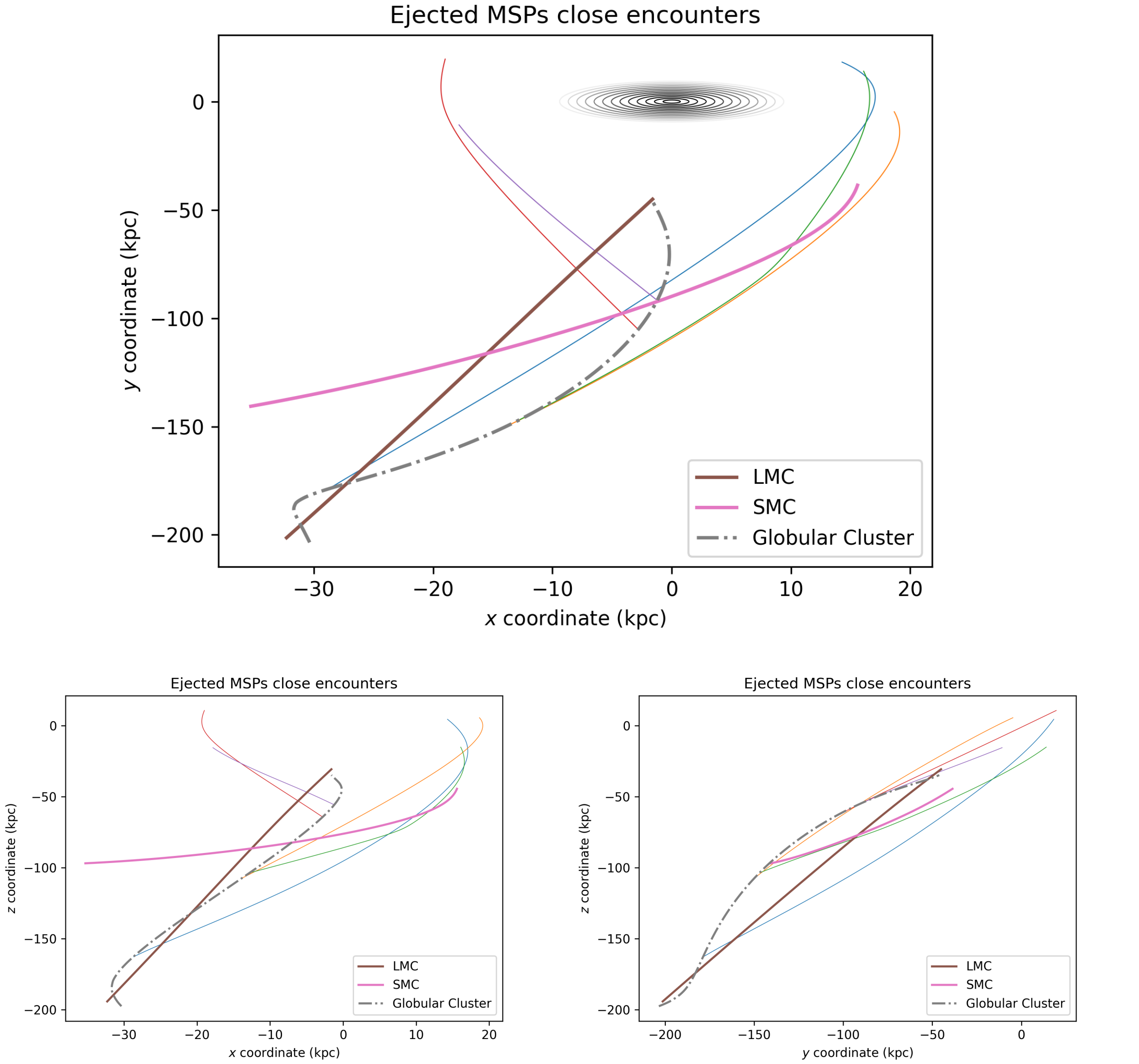

Figure B.1: Top: Close-encounter millisecond pulsar trajectories in the xy plane. The Milky Way is shown with the contour lines of its potential. Bottom Left: Close-encounter millisecond pulsar trajectories in the xz plane. Bottom right: Close-encounter millisecond pulsar trajectories in the yz plane.

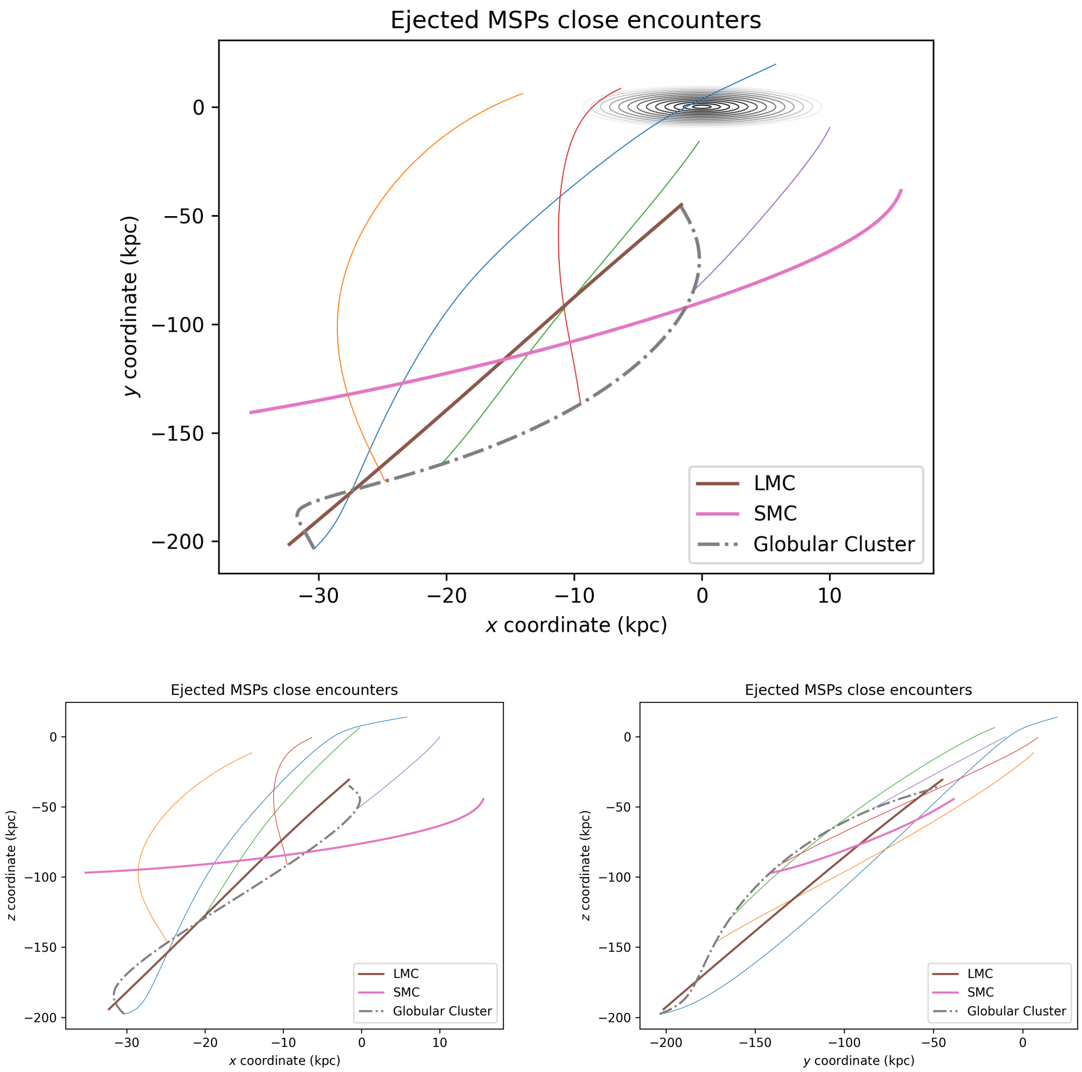

Figure B.2: Top: Close-encounter millisecond pulsar trajectories in the xy plane. The Milky Way is shown with the contour lines of its potential. Bottom Left: Close-encounter millisecond pulsar trajectories in the xz plane. Bottom right: Close-encounter millisecond pulsar trajectories in the yz plane.