

A METHOD FOR SCENARIO RISK QUANTIFICATION FOR AUTOMATED DRIVING SYSTEMS

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ABSTRACT

Recent innovations, such as automated driving and smart mobility, have elevated the safety-criticality of automotive systems due to the impact of these technologies on the traffic behavior and safety. New safety validation and assessment methodologies are required to provide the level of assurance that matches the societal impact of these systems. The objective of this paper is to introduce a novel method for assessment and quantification of the risk of a driving scenario considering the operational design domain. For our proposed method, we assume that a scenario consists of activities (performed by different actors) and environmental conditions that leads to a potentially hazardous consequence. The risk of a driving scenario is the product of the probability of the exposure of a scenario and the severity of the hazardous consequence of that scenario. We introduce a systematic method for calculating the probability of exposure, where we assume a causal relation between the activities that constitute a scenario. By making educated assumptions on the dependencies among the different activities and environmental conditions, we simplify the calculation of the probability of the exposure. For estimating the severity, we employ Monte Carlo simulations. We illustrate the use of our proposed method by applying it to an example of a collision avoidance system in a cut-in scenario. We use naturalistic driving data acquired from field studies on the Dutch highways to determine the risk. The presented example illustrates the potential of our proposed risk estimation method. Using our proposed method, we can compare the safety criticality of various scenarios in a quantitative manner, which can be used as a safety metric for evaluating automated driving systems. This can lead to stronger justification for design decisions and test coverage for developing automated vehicle functionalities.

INTRODUCTION

New developments in the automotive industry towards higher levels of automation are introducing new safety concerns for vehicles. Test procedures and performance measures need to be adapted for evaluation of vehicles with an Automated Driving (AD) system. The safety and reliability of the AD vehicles must be validated in principle for all possible traffic situations that an AD system may encounter on the road, before these systems can be taken into production.

Scenario-based safety validation for automated driving is one of the proposed approaches that is broadly supported by the automotive community. This is reflected in the ISO/PAS 21448:2019 standard on the safety of the intended functionality (SOTIF) [1]. Related projects in Germany (Pegasus [2]), The Netherlands (StreetWise [3]), and Singapore [4] strongly support this approach. Risk assessment is an essential component of the safety validation as it indicates the acceptance criteria of the AD system.

The ISO 26262:2018 [5] captures the state of the art in automotive functional safety. It defines the safety lifecycle and the related safety activities such as Hazard Analysis and Risk Assessment. Other methodologies, such as STPA [6], give guidelines on safety engineering based on systems theory. From the mentioned sources, the only one that offers a framework for measuring risk is ISO 26262. It defines risk as:

Definition 1 (Risk [5]) *The combination of the probability of occurrence of harm and the severity of that harm.*

ISO 26262:2018 gives guidelines to assess risk based on vehicle level hazardous events. A hazardous event is the combination of a vehicle level hazard with operational situation or scenario. It requires analyzing each hazardous event risk individually based on three parameters of Severity, Probability of exposure, and Comparability. The combination of these parameters contributes to constructing the Automotive Safety Integrity Level (ASIL). In this framework, each parameter is quantified in three or four levels that construct the ASIL ranking A, B, C, D, and QM, where ASIL A represents the least critical level and in ascending order, ASIL D the most critical level. Quality Management (QM) means that the identified hazard is not critical enough for the safety processes, and the quality management system of the manufacturer should suffice for reducing the risk. We depict the ASIL ranking graph in Figure 1.

The ASIL methodology for risk assessment relies on the experts' judgments of the three risk parameters. The ISO 26262:2018 provides some general guidelines for assessing these parameters. However, the assessment is for the most part subjective and dependent on the experts who carry it out. Moreover, it is only capable of evaluating the risk of a single (hazardous) event within the context of a generic operational situation.

The alternative methodology proposed in STPA has the means for providing a quantitative risk assessment as it provides the means for connecting the hazard identification to a control system and its characteristics. However, this method skips the risk assessment entirely and does not offer any solutions.

We argue that as the automotive systems move towards higher automation levels, we require more formal methods for risk assessment. By quantifying risk assessment, we reduce the risk of subjective errors in the judgment. Risk quantification is a step towards run-time risk assessment for the autonomous systems.

The objective of this paper is to introduce a method for assessment and quantification of the risk of a driving scenario taking into account the entire operational situations and their relations. This is achieved by calculating the

Controllability C	Probability of Exposure (E)	Severity (S)			
		S0	S1	S2	S3
C1	E1	QM	QM	QM	QM
	E2	QM	QM	QM	QM
	E3	QM	QM	QM	A
	E4	QM	QM	A	B
C2	E1	QM	QM	QM	QM
	E2	QM	QM	QM	A
	E3	QM	QM	A	B
	E4	QM	A	B	C
C3	E1	QM	QM	QM	A
	E2	QM	QM	A	B
	E3	QM	A	B	C
	E4	QM	B	C	D

Figure 1: ASIL risk assessment graph.

Table 1: The terms and definitions.

Term	Definition
Severity	An estimate of the extent of harm to one or more individuals that can occur in a potentially hazardous event [5]
Exposure	The state of being in a driving scenario
Risk	The combination of the probability of occurrence of harm and the severity of that harm [5]
Condition	The constant parameters describing the environmental aspects of the operational design domain ¹
Actor	An element of a scenario acting on its own behalf [8]
Scenario	A quantitative description of the activities of the ego vehicle and other actors and the conditions from the static environment

probability of exposure to a certain scenario through analysis of real-world driving data. Next, we employ simulations to estimate the severity of the potential hazardous consequence of a scenario.

The paper is structured as follows. We first present the proposed method for estimating the risk quantitatively. Next, we perform a case study to illustrate the method using real-world data. We end the paper with a discussion and a conclusion.

PROPOSED RISK ESTIMATION METHOD

In the Hazard Analysis and Risk Assessment (HARA) required by the ISO 26262 standard, the estimation of Automotive Safety Integrity Level (ASIL) is calculated based on a so-called single specific hazardous event [5]. Although the operational situation in which this single event occurs as well as the operating mode are considered in the analysis, still the proceeding and successive events are not taken into account. In this paper, we propose a new method to estimate the risk of a certain scenario considering the whole chain of activities and conditions that constitute the scenario. The estimated risk is based on real-world driving data. To estimate the risk, we quantify the exposure and the severity. In Table 1, we present the definitions of the terms that are used in our proposed methodology.

As explained in Table 1, a scenario consist of a set of conditions and activities, denoted by A and C , respectively. We formulate the exposure as the average number of occurrences of the activities A under the conditions C , denoted by $\lambda_{A,C}$. The severity is the likelihood of the potential hazardous consequence R given the activities A and the conditions C , denoted by the conditional probability $P(R|A, C)$. The risk is computed as the multiplication of the exposure and the severity.

The proposed method is summarized in Figure 2. To compute the exposure, we calculate the likelihood of the conditions, denoted by $P(C)$, and the conditional likelihood of the activities, denoted by $P(A|C)$, based on real-world driving data. This is explained in detail in the next section. For the estimation of the severity, we consider all possible scenarios that are subject to a set of conditions C and consist of the activities A . Therefore, we parametrize the scenarios using the parameter vector θ . Based on the real-world driving data, the probability density function of the parameters, denoted by $P(\theta|A, C)$, is estimated. Next, using simulations, we estimate $P(R|\theta, A, C)$, the likelihood of a potential hazardous consequence R given a parametrized scenario. The details of the estimation of the severity are presented after the details of the estimation of the exposure. Finally, we describe how the risk is estimated based on the estimated exposure and severity.

Calculate exposure

The scenarios are subject to n_C conditions, denoted by C_1, \dots, C_{n_C} . For the sake of brevity, all conditions together are denoted by C , i.e., $P(C_1, \dots, C_{n_C}) = P(C)$. Many of these conditions might be based on the operational design domain of the AD system and might include conditions with respect to the infrastructure, weather conditions, lighting conditions, and geographical locations.

The first step is to compute the joint probability of the conditions, i.e., $P(C)$. In case these conditions are independent, the probability can be computed by simply multiplying the individual likelihoods for each condition, i.e.,

¹The operational design domain refers to the “operating conditions under which a given driving automation system or feature thereof is specifically designed to function” [7].

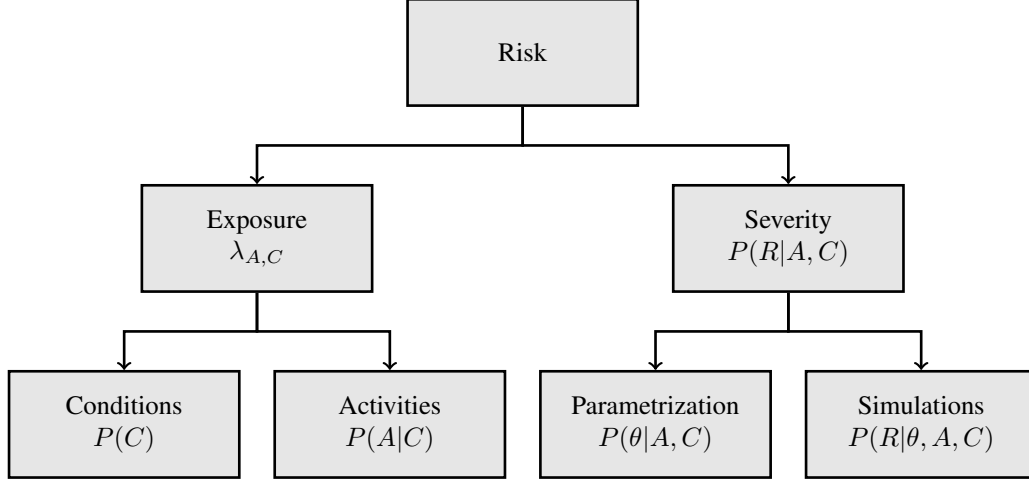


Figure 2: Proposed method for quantifying the risk. The risk is a multiplication of the exposure and the severity.

$P(C) = P(C_1) \cdot \dots \cdot P(C_{n_C})$. This, however, might not necessarily be the case, which requires either to compute the joint probability or to compute conditional probabilities. In some cases, it might also be reasonable to simply assume that the likelihood of certain conditions are independent.

Note that the defined conditions might not be the same as the conditions under which the data is collected that is used to compute $P(C)$. This might require additional assumptions, see our case study for examples.

To calculate the exposure, the average number of occurrences of the activities that constitute the scenarios within a certain time interval need to be estimated. Let n_A denote the number of activities, such that A_1, \dots, A_{n_A} denote the activities. For the sake of brevity, all activities together are denoted by A .

Without loss of generality, we assume that the time interval is an hour. To estimate the number occurrences of the activities, the data for which the conditions C are satisfied are analyzed. The average number of occurrences of the activities A for each hour of driving for which the conditions C are satisfied is denoted by $\lambda_{A|C}$. Next, we can calculate the average number of occurrences of the activities A under the conditions C for each hour of driving:

$$\lambda_{A,C} = \lambda_{A|C} \cdot P(C). \quad (1)$$

Regarding the scenarios consisting of conditions C and activities A , we assume the following:

- The occurrence of one scenario consisting of activities A and conditions C does not affect the probability that a second scenario consisting of activities A and conditions C occurs.
- The rate at which a scenario consisting of activities A and conditions C occurs is constant. I.e., $\lambda_{A,C}$ is constant.
- Two scenarios consisting of activities A and conditions C cannot occur at exactly the same time instant.

Based on these assumptions, the number of occurrences of scenarios consisting of activities A and conditions C is distributed according to the Poisson distribution:

$$P(k \text{ times } A, C \text{ in an hour}) = \exp\{-\lambda_{A,C}\} \frac{\lambda_{A,C}^k}{k!}. \quad (2)$$

Severity

The first step towards estimating the severity is to parametrize the scenarios with a parameter vector $\theta \in \mathbb{R}^d$. The parametrization enables the generation of infinitely many unique individual test cases that resemble the scenarios found in naturalistic driving [3], [9].

In case the parameters are dependent, which is often the case, it is important that the number of parameters is limited to avoid the curse of dimensionality [10]. This often requires some assumptions. An example is presented in our case study in the next section.

To estimate the probability density function (pdf) of the parameter vector θ , i.e., $P(\theta|A, C)$, either parametric models, non-parametric models, or a combination of the two can be used. In case of parametric models, a certain functional form of the pdf is assumed. For example, it might be assumed that the pdf can be modeled using a Gaussian distribution. In this paper, we present a non-parametric approach using Kernel Density Estimation (KDE) [11], [12]. Using KDE, there is no assumption on the functional form of the pdf because the shape of the pdf is automatically computed. With KDE, the estimated pdf is given by

$$P(\theta|A, C) = \frac{1}{nh^d} \sum_{i=1}^n K\left(\frac{\theta - \theta_i}{h}\right). \quad (3)$$

Here, $K(\cdot)$ is an appropriate kernel function and h denotes the bandwidth. From the data, n scenarios are extracted and each scenario is parametrized with θ_i . The choice of the kernel $K(\cdot)$ is not as important as the choice of the bandwidth h [13]. Often, a Gaussian kernel is used, which is given by

$$K(u) = \frac{1}{(2\pi)^{d/2}} \exp\left\{-\frac{1}{2}\|u\|^2\right\}, \quad (4)$$

where $\|u\|^2$ denotes the squared 2-norm of u , i.e., $u^T u$.

The bandwidth h controls the amount of smoothing. For the kernel of Eq. (4), the same amount of smoothing is applied in every direction, although this can easily be extended to a multi-dimensional bandwidth, see, e.g., [14], [15]. There are many different ways of estimating the bandwidth, ranging from simple reference rules like, e.g., Scott's rule of thumb [10] or Silverman's rule of thumb [16] to more elaborate methods; see [13], [17]–[19] for reviews of different bandwidth selection methods.

Let R denote a potential hazardous consequence of a scenario. We define the severity of a scenario with activities A and conditions C as the probability of R , given the activities A and C , i.e., $P(R|A, C)$. We cannot evaluate $P(R|A, C)$ directly, because the outcome of a scenario highly depends on the parametrization θ . Therefore, we estimate $P(R|\theta, A, C)$ through a simulation of the scenario with parameters θ . Using $P(\theta|A, C)$ from Eq. (3), we can compute

$$P(R, \theta|A, C) = P(R|\theta, A, C) \cdot P(\theta|A, C). \quad (5)$$

To obtain $P(R|A, C)$, we need to integrate Eq. (5) over θ , i.e.,

$$P(R|A, C) = \int_{\mathbb{R}^d} P(R|\theta, A, C) \cdot P(\theta|A, C) d\theta. \quad (6)$$

One approach to evaluate the integral of Eq. (6) is to perform Monte Carlo simulations. For sufficiently large N , we have

$$P(R|A, C) \approx \frac{1}{N} \sum_{k=1}^N P(R|\theta_k, A, C), \quad \theta_k \sim P(\theta|A, C). \quad (7)$$

To improve the accuracy of Eq. (7), importance sampling can be used where the parameters θ are drawn from another distribution with a focus on the critical scenarios, see, e.g., [9].

Calculating the risk

Analogous to the exposure, we define the risk as the number of occurrences of the hazardous consequence R in a scenario consisting of activities A and conditions C in a certain time interval. Let λ denote the average number of these occurrences in an hour of driving. The chain rule of probability tells us that this equals the product of $\lambda_{A,C}$ (i.e., the exposure) and $P(R|A, C)$ (i.e., the severity):

$$\lambda = \lambda_{A,C} \cdot P(R|A, C) \quad (8)$$

Analogous to the number of occurrences of a scenario consisting of activities A and conditions C , we assume that the number of occurrences of a harmful outcome R in a scenario consisting of activities A and conditions C can be modeled using a Poisson distribution:

$$P(k \text{ times } R, A, C \text{ in an hour}) = \exp\{-\lambda\} \frac{\lambda^k}{k!}. \quad (9)$$

Using Eq. (9), to calculate the probability of not having the harmful outcome R in a scenario consisting of activities A and conditions C we simply need to use $k = 0$:

$$P(\text{no } R, A, C \text{ in one hour}) = \exp \{-\lambda\}. \quad (10)$$

CASE STUDY

In this section, we present a case study to illustrate the method of quantifying the risk for a cut-in scenario. We will first describe the cut-in scenario and the use case. The actual system for which the risk is computed is presented in next. After these two steps, we will go through the steps of our proposed method.

The cut-in scenario and the use case

We want to quantify the risk for cut-in scenarios that are linguistically described as follows: while the ego vehicle drives at a moderate to high speed while staying in its lane, another vehicle cuts into the lane of the ego vehicle, such that this vehicle becomes the ego vehicle's lead vehicle. Furthermore, the ego vehicle needs to brake to prevent a collision.

For the quantification of the risk, 60 hours of data (see also [9]) are collected by driving a specific route in and between Eindhoven and Helmond, The Netherlands, with twenty different drivers, each driving the route twice. Therefore, it is assumed that the use case of the AD system is driving this route. We will use the data for the estimation of the risk. Hence, we will make use of the following assumption:

Assumption 1 *The recorded naturalistic driving data is representative for what a vehicle with the AD system might encounter along the same route.*

System-under-test

To reduce efforts for the assessment, often simulations are employed. However, even simulations can consume considerable time, as these simulations might run real-time [20] or slower when a higher level of detail is used [21]. For our method, we will simplify the simulations, such that the total required time on a common computer is in the order of minutes. Since we are interested in approximate results, a high level of detail is not required.

To simplify the system-under-test, it is assumed that the system's desired acceleration is similar to the adaptive cruise control defined in [9], i.e.,

$$u(t) = k_d(v(t))(d(t) - \tau_h v(t) - s_0) + k_v \left(\dot{d}(t) - ha(t) \right), \quad (11)$$

with

$$k_d(v(t)) = k_{d1} + (k_{d2} - k_{d1}) \exp \left\{ -\frac{v(t)^2}{2\sigma_d} \right\}. \quad (12)$$

Here, v is the speed of the ego vehicle, d denotes the clearance between the ego vehicle and its predecessor, i.e., the vehicle that performs the cut-in. The relative speed is denoted by \dot{d} and a refers to the acceleration of the ego vehicle. The ego vehicle is modeled using a first order model with a time delay, i.e.,

$$\tau \dot{a}(t) + a(t) = u(t - \theta). \quad (13)$$

Furthermore, the deceleration is limited at -6 ms^{-2} . A description of the constants of Eqs. (11) to (13) are listed in Table 2. The controller runs at 100 Hz.

Note that there is no intervention of a human:

Assumption 2 *The ego vehicle is fully controlled by the automation system as defined by Eqs. (11) and (12). Hence, there is no intervention of a human.*

Table 2: The constants used for the simple automation system of Eqs. (11) to (13).

Parameter	Description	Value
τ_h	Desired headway time	2.0 s
s_0	Safety distance	1.5 m
k_{d1}	Distance gain at high speed	0.7 s^{-2}
k_{d2}	Distance gain at low speed	2.0 s^{-2}
σ_d	Shaping coefficient of distance gain	5 ms^{-1}
k_v	Speed difference gain	0.35 s^{-1}
τ	Time constant of the vehicle model	0.1 s
θ	Delay of the vehicle response	0.2 s

Calculate exposure

The cut-in scenarios are subject to the following conditions:

- C_1 : The speed of the ego vehicle is within the range of 60 km/h and 130 km/h.
- C_2 : There are no restrictions on the weather conditions.
- C_3 : There are no restrictions on the lighting conditions.

Obviously, because there are no restrictions to the weather and lighting conditions, we have $P(C_2, C_3) = 1$. For the first condition, we can use the data to estimate the likelihood. The data, however, has been recorded during sunny weather at daylight. Therefore, we need to following assumption.

Assumption 3 Let C'_2 and C'_3 denote the conditions of having sunny weather and daylight, respectively. Then we have $P(C_1|C_2, C_3) = P(C_1|C'_2, C'_3)$.

From the data, it appeared that $P(C_1|C'_2, C'_3) = 0.20$. Using Assumption 3, we have

$$P(C) = P(C_1, C_2, C_3) = P(C_1|C'_2, C'_3) \cdot P(C_2, C_3) = 0.20. \quad (14)$$

The cut-in scenarios consist of the following activities:

- A_1 : The ego vehicle is lane following.
- A_2 : The target vehicle is driving in an adjacent lane in the same direction as the ego vehicle.
- A_3 : After activity A_2 , the target vehicle performs a lane change towards the lane of the ego vehicle, such that the ego vehicle needs to brake.
- A_4 : The automation system detects the cut-in.
- A_5 : After activity A_4 , the automation system activates the brakes of the ego vehicle.

The likelihood of the activities A_1 , A_2 , and A_3 can be estimated using the data. It is assumed that the ego vehicle needs to brake if the target vehicle is driving slower and the headway time is less than three seconds. In case of a slower target vehicle with a larger headway time, the scenario is referred to as a gap closing scenario [22], [23].

For simplicity, we assume the following:

Assumption 4 The automation system always detects the cut-in and activates the brakes after detecting the cut-in, such that $P(A_4, A_5|A_1, A_2, A_3, C) = 1$.

Using this assumption, we can compute $\lambda_{A|C}$ by detecting the number of occurrences of the activities A_1 , A_2 , and A_3 under the conditions C . Based on the dataset, we have $\lambda_{A|C} = 9.9 \text{ h}^{-1}$, i.e., in each hour that the ego vehicle is driving in a speed range of 60 km/h and 130 km/h, there are on average 9.9 cut-ins with the target vehicle driving slower than the ego vehicle, such that the headway time after the cut-in is less than three seconds. From this, it simply follows that

$$\lambda_{A,C} = \lambda_{A|C} \cdot P(C) = 2.0. \quad (15)$$

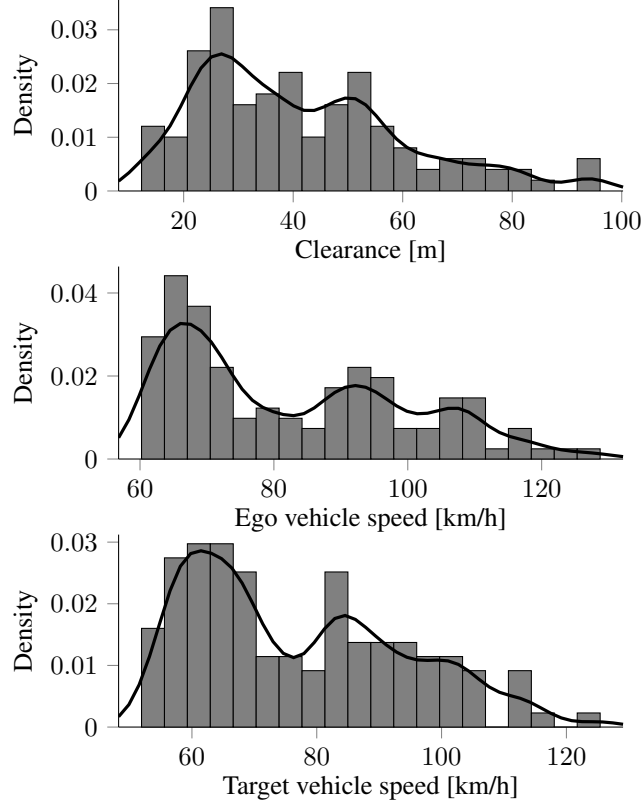


Figure 3: Histogram of the data of the parameters (bars) and their estimated marginal probabilities (lines).

Calculating severity

To limit the number of parameters, we assume the following:

Assumption 5 *The ego vehicle is driving at a constant speed at the moment of the cut-in of the target vehicle, i.e., the moment that the target vehicle enters the lane of the ego vehicle.*

Assumption 6 *The target vehicle is driving at a constant speed.*

Both assumptions can be justified using the data. In case of the ego vehicle, the average acceleration at the moment of the cut-in is -0.29 ms^{-2} and the standard deviation equals 0.50 ms^{-2} . In case of the target vehicle, the average deceleration at the moment of the cut-in is 0.05 ms^{-2} and the standard deviation equals 0.37 ms^{-2} . As a result, the scenario is parametrized using $d = 3$ parameters:

1. The clearance between the target vehicle and the ego vehicle at the moment of the cut-in, i.e., the moment than the target vehicle enters the lane of the ego vehicle.
2. The speed of the ego vehicle at the moment of the cut-in.
3. The speed of the target vehicle throughout the whole scenario.

A histogram of the data of the parameters is shown in Figure 3. The probability density function is estimated using the KDE of Eq. (3) with the Gaussian kernel of Eq. (4). Before applying KDE, the data is scaled, such that the standard deviation equals one for each parameter. We use leave-one-out cross validation to compute the bandwidth h (see also [24]) because this minimizes the Kullback-Leibler divergence between the real underlying pdf and the estimated pdf [13], [25]. The resulting bandwidth equals $h = 0.198$. The marginal probability distributions coming from the resulting joint distribution, i.e. the KDE, are shown in Figure 3 by the black lines.

Let R denote the result of having a collision. Given a certain parameter vector θ , we have $P(R|\theta, A, C) = 1$ if the outcome of the simulation is a collision and $P(R|\theta, A, C) = 0$ otherwise. For the simulation, we used the forward Euler method with a step size of 0.01 s, similar as the sample time of the controller. On a regular computer, approximately 2000 simulations are performed in a second. We performed a million simulations, i.e., $N = 10^6$. In total, 28 simulations ended with a collision, thus, according to Eq. (7), we have:

$$P(R|A, C) = 2.8 \cdot 10^{-5}. \quad (16)$$

Calculating the risk

Let λ denote the average number of collisions with a cut-in scenario as described earlier along the specified route for a vehicle with the automation system as described above. Using Eq. (8), we have:

$$\lambda = \lambda_{A,C} \cdot P(R|A, C) = 5.5 \cdot 10^{-5} \text{ h}^{-1}. \quad (17)$$

Using Eq. (10), the probability of having no collision in a cut-in scenario as described above during an hour of driving is

$$P(\text{no } R, A, C \text{ during an hour}) = 0.999945. \quad (18)$$

By solving the Poisson distribution of Eq. (9) for λ with $k = 0$, we can also conclude that with 95 % certainty, there will be no collision in a cut-in scenario as described earlier when driving 925 h.

DISCUSSION

We illustrated the applicability of our risk estimation method through an example in the previous section. However, our method has some limitations as well. As an example, many assumptions are made to simplify the calculation of estimated risk or because there are unknowns due to lack of data. These assumptions reduce the accuracy of the estimated risk. Another limitation is that we applied the proposed method for only one type of driving scenario, while the full potential can be better demonstrated by applying the method to a wider range of scenarios.

Despite the mentioned limitations, we believe that our proposed method takes an important step towards objective hazard and risk analysis as we summarize in the following points:

- All the assumptions that were made for estimating the risk are explicit and based on measured data. By making the assumptions explicit, it is much clearer why a certain risk is associated with — in this case — a certain/specific scenario.
- Because our proposed method explicates all the steps and assumptions that lead to a certain estimated risk, it is easily possible to update the risk when more information of the system is known or when more data is available.
- The systematic quantification of the risk provides additional objectified trust in the safety analysis that depends on the availability of data rather than experts' judgment.
- The method can be scaled up to be applied to multiple scenarios and operational situations with small modifications.

CONCLUSIONS

As automotive systems move towards higher automation levels, we require formal methods for risk assessment. Currently, however, measuring risk is often based on experts' judgments. Therefore, we propose a method for quantifying the risk assessment as to reduce the risk of subjective errors in the judgment. Our proposed method estimates the risk of a driving scenario while considering the entire operational situations and their relations through analysis of real-world driving data and simulations of the automation system. Our systematic approach for quantifying the risk provides additional trust in the safety analysis, as it depends on the available data rather than experts' judgment.

It remains future work to apply the method for different scenarios to show the full potential of the method. We also aim for extending our method by considering, next to the exposure and the severity, the controllability [26].

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