MONASH University

Eng. Math

ENG 1005

Week 12: Series 2

(MEC) Senior Lecturer: K.B. Goh, Ph.D.

Tutor: (a) Ian Keen & (b) Jack

Pass Leader: (i) Zi Wei and (ii) Yvonne

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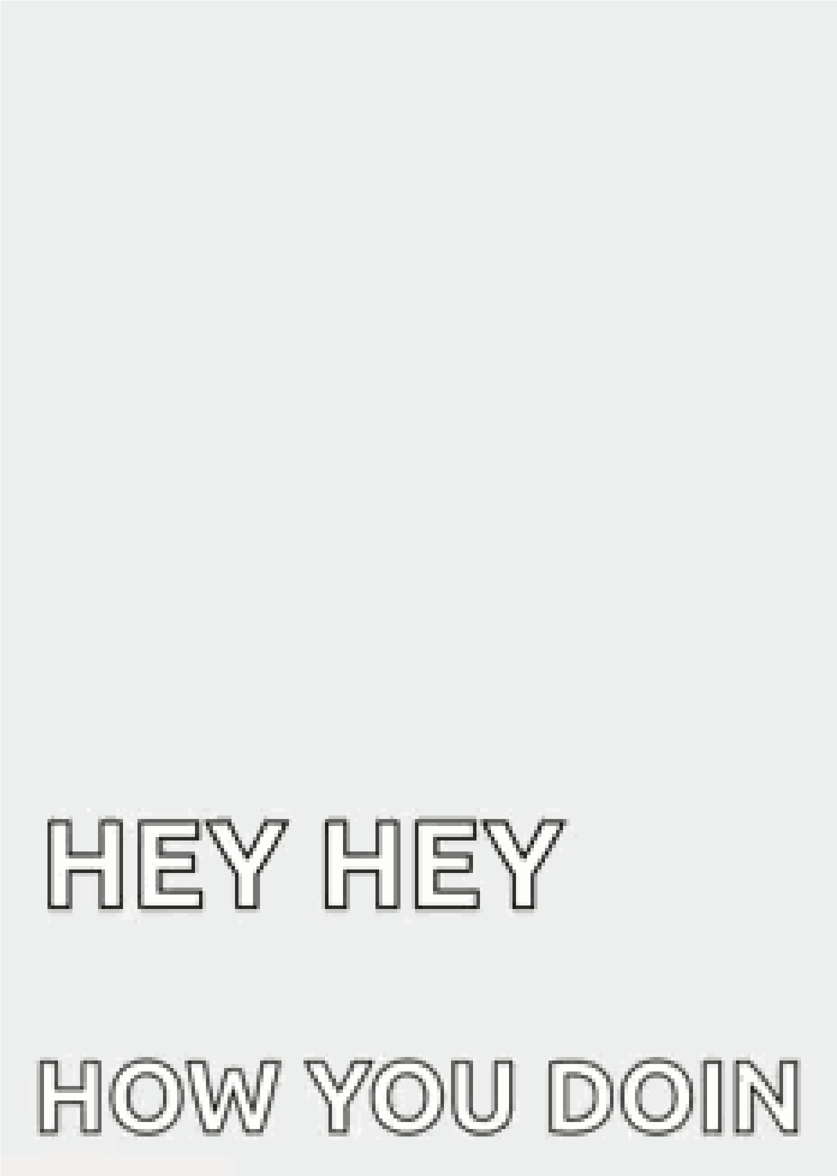

Check-In

Topics

Week	Topic
1	Vectors, Lines, and Planes
2	Systems of Linear Equations
3	Matrices
4	Eigenvalues & Eigenvectors
5	Multivariable Calculus 1
6	Multivariable Calculus 2
7	Integration techniques and hyperbolic functions
8	O.D.E 1
9	O.D.E 2
10	O.D.E 3
11	Series 1
12	Series 2

The Big Learning Outcomes for Week 12

After completing this week's task, you should be able to:

- Find Taylor series of given functions.
- Use truncated Taylor series to approximate functions.
- Understand linearisation.
- Understand the errors introduced in truncating Taylor series as finite order polynomials.
- Use l'Hopital's rule to find limits.
- Find Taylor series of given multivariable functions.

Attendance Codes (Week 12)

International students

Workshop	Thursday, 17 Oct	01	1:00PM	JTXF3
Workshop	Friday, 18 Oct	02	10:00AM	HRVMY

Tutorial	Wednesday, 16 Oct	02	8:00AM	JCHUN
Tutorial	Wednesday, 16 Oct	01	2:00PM	ZFTTB

Admin. Stuff (1)

1. Start your revision.

2. Submissions:

Summary

ASSESSMENT	DUE
Kick Starting Week 3: Lecture Quiz 3 (Total mark for all 12 weeks of lecture quizzes is 5%)	Tuesday, 6 August 2024, 11:55 PM **Due in 4 days**
Applied class quiz week 3 (Total mark for all 12 weeks of applied quizzes is 5%)	Wednesday, 7 August 2024, 11:55 PM **Due in 5 days**
Workshop 3 problem set (Total mark for all 12 weeks of workshop sets is 20%)	Sunday, 11 August 2024, 11:55 PM

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3. Consultation/Feedback hour

- Wed: 10 am till 11 am } Location: 9-4-01
- Fri: 8 am till 9 am } Location: 5-4-68

(every odd week, e.g. 3-Week, but msg me first at least a day before; appointment-based.)

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SWOTVAC	21/Oct	22/Oct	23/Oct	24/Oct	25/Oct	26/Oct
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10						
11	FAC MEET		PHD/MASTER	PHD/MASTER	PHD/MASTER	PHD/MASTER
12	FAC MEET	FYP DAY	PHD/MASTER	PHD/MASTER	PHD/MASTER	PHD/MASTER
13	FAC MEET	FYP DAY	ME TIME	PHD/MASTER	PHD/MASTER	PHD/MASTER
14		FYP DAY		ECN/MUMGRO		
15		FYP DAY		ECN/MUMGRO		

Resources

1. Additional: Videos

Week 11 Week 11: Series I	**Week 12** Week 12: Series II
Week 13 Final exam preparation	**Week 14** Communication
Week 15 Resources (MUM Pre-Recorded Videos)	**Week 16** Resources for developing the unit

2. Pass materials

Let us start!

What is Taylor Series?

Taylor Series

The Taylor series of a function $f(x)$ around a point $x = a$ is given by:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \dots$$

or equivalently in summation form:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n,$$

Try this Taylor Series: $\sin(x-3)$

a=0

To expand $f(x) = \sin(x-3)$ in a Taylor series around $x = 0$, we

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 +$$

Step-by-Step Calculation

- **Step 1: Calculate $f(0)$**

$$f(0) = \sin(0-3) = -\sin(3)$$

- **Step 2: First derivative**

$$f'(x) = \cos(x-3)$$
$$f'(0) = \cos(0-3) = \cos(3)$$

- **Step 3: Second derivative**

$$f''(x) = -\sin(x-3)$$
$$f''(0) = -\sin(0-3) = \sin(3)$$

- **Step 4: Third derivative**

$$f'''(x) = -\cos(x-3)$$
$$f'''(0) = -\cos(0-3) = -\cos(3)$$

- **Step 5: Fourth derivative**

$$f''''(x) = \sin(x-3)$$
$$f''''(0) = \sin(0-3) = -\sin(3)$$

- **Step 6: Fifth derivative**

$$f'''''(x) = \cos(x-3)$$
$$f'''''(0) = \cos(0-3) = \cos(3)$$

Taylor Expansion

Substituting these values into the Taylor series formula:

$$\begin{aligned} f(x) &= -\sin(3) + \cos(3)x + \frac{\sin(3)}{2!}x^2 - \frac{\cos(3)}{3!}x^3 - \frac{\sin(3)}{4!}x^4 + \frac{\cos(3)}{5!}x^5 + \cdots \\ &= -\sin(3) + x\cos(3) + \frac{1}{2}x^2\sin(3) - \frac{1}{6}x^3\cos(3) - \frac{1}{24}x^4\sin(3) + \frac{1}{120}x^5\cos(3) + \cdots \end{aligned}$$

$$\boxed{f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \ldots}$$

Today's Activity

1. Applied Problem Set

2. Applied Quiz

- ✓ Q1(b)
- ✓ Q1(c)
- ✓ Q3
- ✓ Q4

Question 1(b)

(b) Calculate the first four terms of the Taylor series for $f(x) = \sqrt{x}$ at $x = 1$.

(c) Not for credit: Write down a formula for the general term for part (b).

Learning Outcomes?

Taylor Series

- Taylor Series for $f(x) = \sqrt{x}$ about x=1

$$f(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2}(x-1)^2 + \frac{f''(1)}{6}(x-1)^3 + \cdots$$

- Derivative of $f(x) = \sqrt{x}$

$$f'(x) = \frac{1}{2}(1)^{-1/2} \qquad f''(x) = -\frac{1}{4}(1)^{-3/2} \qquad f'''(x) = \frac{3}{8}(1)^{-5/2}$$

- Solve $f(x)$ & its babies at x=1

$$f(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 + \cdots$$

Taylor Series

The Taylor series of a function $f(x)$ around a point $x = a$ is given by:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \dots$$

Question 1(c)

1. (a) Calculate the first four terms of the Taylor series for $f(x) = e^x$ about $x = 2$.

   (b) Calculate the first four terms of the Taylor series for $f(x) = \sqrt{x}$ at $x = 1$.

   (c) Not for credit: Write down a formula for the general term for part (b).

Learning Outcomes?

Taylor Series

What is Taylor Series?

Taylor Series

The Taylor series of a function $f(x)$ around a point $x = a$ is given by:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \dots$$

or equivalently in summation form:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n,$$

Let us see the trend: $\boldsymbol{f(x)}$ & its babies at x=1

$$f'(x) = \frac{1}{2}(x)^{-1/2}$$

$$f'''(x) = \frac{3}{8}(x)^{-5/2}$$

$$f''(x) = -\frac{1}{4}(x)^{-3/2}$$

$$f''''(x) = -\frac{15}{16}(x)^{-5/2}$$

Generalize these

- ✓ For each successive derivative, power x decreases by 1/2

Let us assume this:

$$f'(1) = \frac{1}{2}(1)^{-1/2} = \frac{1}{2}$$

$$f'''(1) = \frac{3}{8}(1)^{-5/2} = \frac{3}{8}$$

$$f''(1) = -\frac{1}{4}(1)^{-3/2} = -\frac{1}{4}$$

$$f''''(1) = -\frac{15}{16}(1)^{-5/2} = -\frac{15}{16}$$

When n=4, nominator is 15

$$f^{n}(1) = \frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2} \ldots . \times -\frac{(2n-3)}{2}$$

When n=4, denominator is 16

$$f^n(1) = \frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2} \ldots \times -\frac{(2n-3)}{2} = \prod_{\boxed{k=1}}^{n} -\frac{(2k-3)}{2}$$

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \ldots$$

or equivalently in summation form:

$$f(x) = \sum_{\boxed{n=0}}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n,$$

✓ Allowing the power series to start from n=1, instead of 0

$$f(x) = \frac{f^0(1)}{0!}(x-1)^0 + \sum_{n=1}^{\infty} \frac{f^n(1)}{n!}(x-1)^n$$

$$f(x) = \sqrt{x}$$

$$f(x) = \frac{f^0(1)}{0!}(x-1)^0 + \sum_{n=1}^{\infty} \frac{f^n(1)}{n!}(x-1)^n$$

$$f(x) = 1 + \sum_{n=1}^{\infty} \frac{\prod_{k=1}^{n} -\frac{(2k-3)}{2}(x-1)^n}{n!}$$

$$\prod_{k=1}^{n} -\frac{(2k-3)}{2} = \frac{(-1)^{n-1}(2n-2)!}{2^{2n-1}(n-1)!}$$

Rewrite the sum of product! Took me the whole day

✓ But lets do a part of the long solution!

$$\prod_{k=1}^{n}(2k-1) = \frac{(2n-1)!}{2^{n-1}(n-1)!}$$

- But let us do a part of the long solution!

$$\prod_{k=1}^{n}(2k-1)=\frac{(2n-1)!}{2^{n-1}(n-1)!}$$

- L.H.S

$$=(2\times 1-1)\times(2\times 2-1)\times(2\times 3-1)\times\cdots\ldots\times(2n-3)\times(2n-1)$$

$$=1\times 3\times 5\ldots(2n-3)\times(2n-1)$$

$$(2n-1)!=1\times 2\times 3\times 4\times 5\times 6\ldots(2n-1)$$

- R.H.S

$$\prod_{k=1}^{n}(2k-1)=\frac{(2n-1)!}{2^{n-1}(n-1)!}$$

Divide out the EVEN:

- ODD: $1\times 3\times 5\ldots(2n-1)$
- EVEN: $2\times 4\times 6\ldots(2n-2)=2^{n-1}(n-1)!$

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Simplifying $\prod_{k=1}^{n}(2k-1)$ in detailed

Add additional even terms in the numerator, but we can cancel them in the denominator

$$\prod_{k=1}^{n}(2k-1) = 1 \times 3 \times 5 \ldots\ldots \times (2n-3) \times (2n-1)$$

$$= \frac{1 \times 2 \times 3 \times 4 \times 5 \ldots\ldots \times (2n-4) \times (2n-3) \times (2n-2) \times (2n-1)}{2 \times 4 \ldots\ldots \times (2n-4) \times (2n-2)}$$

$$= \frac{(2n-1)!}{2(1) \times 2(2) \ldots\ldots \times 2(n-2) \times 2(n-1)}$$

$$= \frac{(2n-1)!}{\underbrace{[2 \times 2 \ldots\ldots \times 2 \times 2]}\underbrace{[1 \times 2 \times (n-2) \times (n-1)]}}$$

$n-1$ amount of terms

$$= \frac{(2n-1)!}{2^{n-1}(n-1)!}$$

$$\prod_{k=1}^{n}(2k-1) = \frac{(2n-1)!}{2^{n-1}(n-1)!}$$

Question 3(a)

3. Use L'Hôpital's rule to compute the following limits:

$$\text{(a)} \lim_{x\to 0} \frac{e^{3x}-1}{x} \qquad \text{and} \qquad \text{(b)} \lim_{x\to 0} \frac{e^{x}-1-x}{\sin^2(x)}\,.$$

Learning Outcomes?

L'Hopital!

goes 0

$$\lim_{x \to 0} \frac{e^{3x} - 1}{x} = 0$$

goes 0

L'Hopital's rule: $\frac{0}{0}; \frac{\infty}{\infty}$

$$= \lim_{x \to 0} \frac{d}{dx} \left(\frac{e^{3x} - 1}{x} \right)$$

$$= \lim_{x \to 0} \frac{3e^{3x}}{1}$$

$$= 3$$

Question 4

4. We can also compute complicated Maclaurin series using multiple simple ones.

   (a) Compute the Maclaurin series for $\ln(1+x)$ and $\ln(1-x)$.

   (b) Hence obtain a Maclaurin series for

$$f(x) = \ln\left(\frac{1+x}{1-x}\right).$$

Learning Outcomes?

Maclaurin

- Step 1: $\boxed{f(x) = \ln(1+x)}$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n,$$

$$\boxed{f(x) = \ln(1+x)}$$

$$f'(x) = \frac{1}{1+x}$$

$$f''(x) = -\frac{1}{(1+x)^2}$$

$$f'''(x) = \frac{2}{(1+x)^3}$$

$$f''''(x) = -\frac{6}{(1+x)^4}$$

$$\boldsymbol{f^n = \frac{(-1)^{n-1}(n-1)!}{(1)^n}}$$

Center around x= 0

Generalize it

$$f^n = \frac{(-1)^{n-1}(n-1)!}{(1+x)^n}$$

- We start calculate at 1

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)}(x)^{n+1}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$$

- n=0 is 1/0

$$= \sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{n}(x)^n$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^{n-1}(n-1)!}{n!}(x-a)^n$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^{n-1}\cancel{(n-1)!}}{\cancel{n!}\, n}(x)^n$$

- Center around 0

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n,$$

- Step 1.5: A Further Explanation!

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)} (x)^{n+1}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{n} (x)^n$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^{n-1}\cancel{(n-1)!}}{\cancel{n!}} (x)^n$$

$$\boxed{f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n,}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^{n-1}\cancel{(n-1)!}}{n\cancel{!}}(x)^n$$

$$\left.= \sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{n}(x)^n \quad\right\}$$

Hold on, n=0, cant work 1/0

Start n=1

$$\left\{\quad = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}(x)^n\right.$$

$$\ln(1+x) = \overbrace{\sum_{n=0}^{\infty}}^{\text{Move back to 0}} \underbrace{\frac{(-1)^{n}}{(n+1)}(x)^{n+1}}_{\text{Move forward to n+1}}$$

- Step 2: $\boxed{f(x) = \ln(1-x)}$

Repeat step 1

$$\ln(1-x) = \sum_{n=0}^{\infty} -\frac{(x)^{n+1}}{(\boldsymbol{n}+\mathbf{1})}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n,$$

- Step 3: $\boxed{f(x) = \ln(1+x) - \ln(1-x)}$

$$f(x) = \sum_{n=0}^{\infty} \left\{ \underbrace{(-1)^n \frac{(x)^{n+1}}{(n+1)} (x)^{n+1} + \frac{(x)^{n+1}}{(n+1)}} \right\}$$

$$\boxed{f(x) = \sum_{n=0}^{\infty} \frac{2x^{2n+1}}{(2n+1)}.}$$

- 2k change to n

$$= (-1)^n \frac{(x)^{n+1}}{(n+1)} + \frac{(x)^{n+1}}{(n+1)}$$

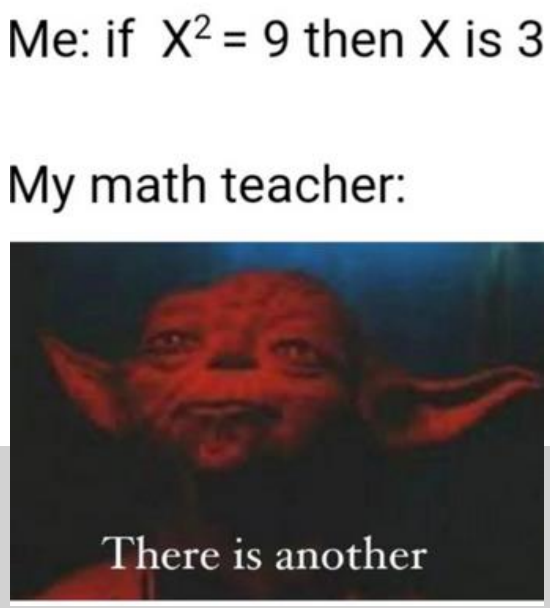
$$= \left(\frac{(x)^{n+1}}{(n+1)}\right) \underbrace{[(-1)^n + 1]} \rightarrow = \left(\frac{(x)^{2k+1}}{(2k+1)}\right) 2$$

- n=even; for it to work, & it's always 2
- n=2k

Today's Activity

1. Applied Problem Set

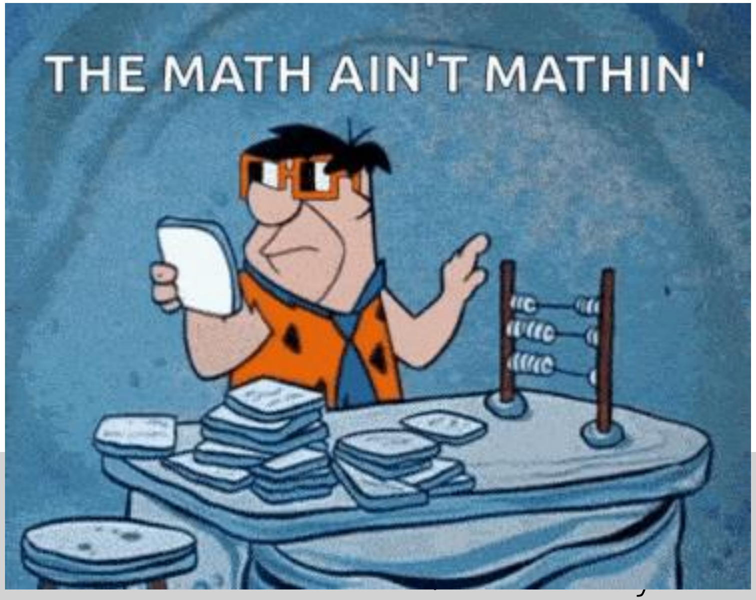
2. Applied Quiz



2. Applied Quiz

(9.30 am till 9.50 am)

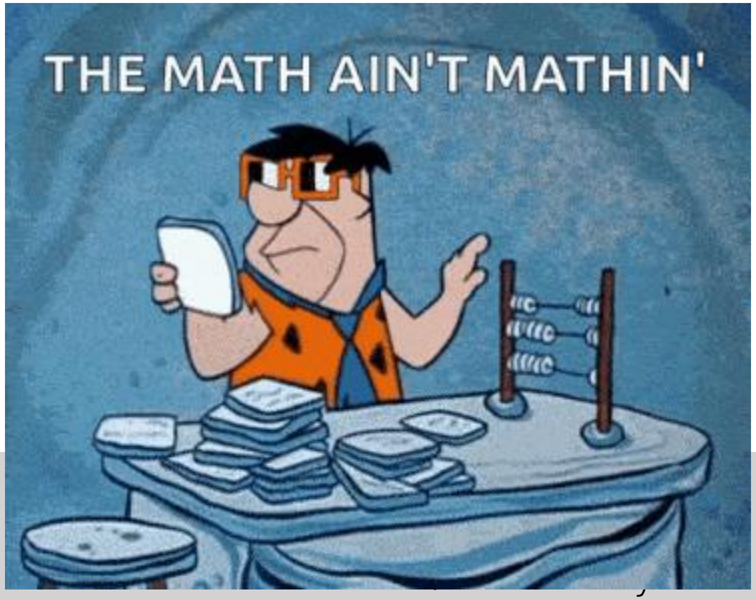
Password: MissingYouAlready!



2. Applied Quiz

(3.20 pm till 3.50 pm)

Password: MissingYouAlready!



Thank You

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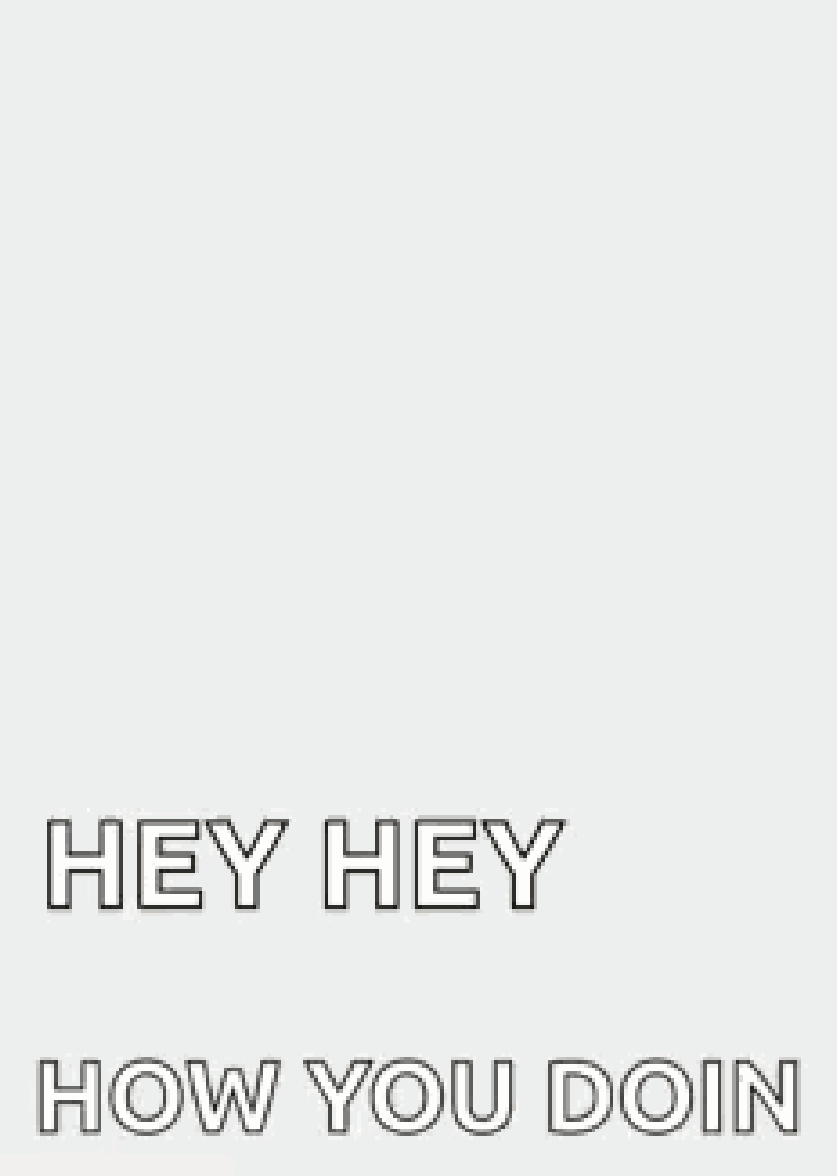
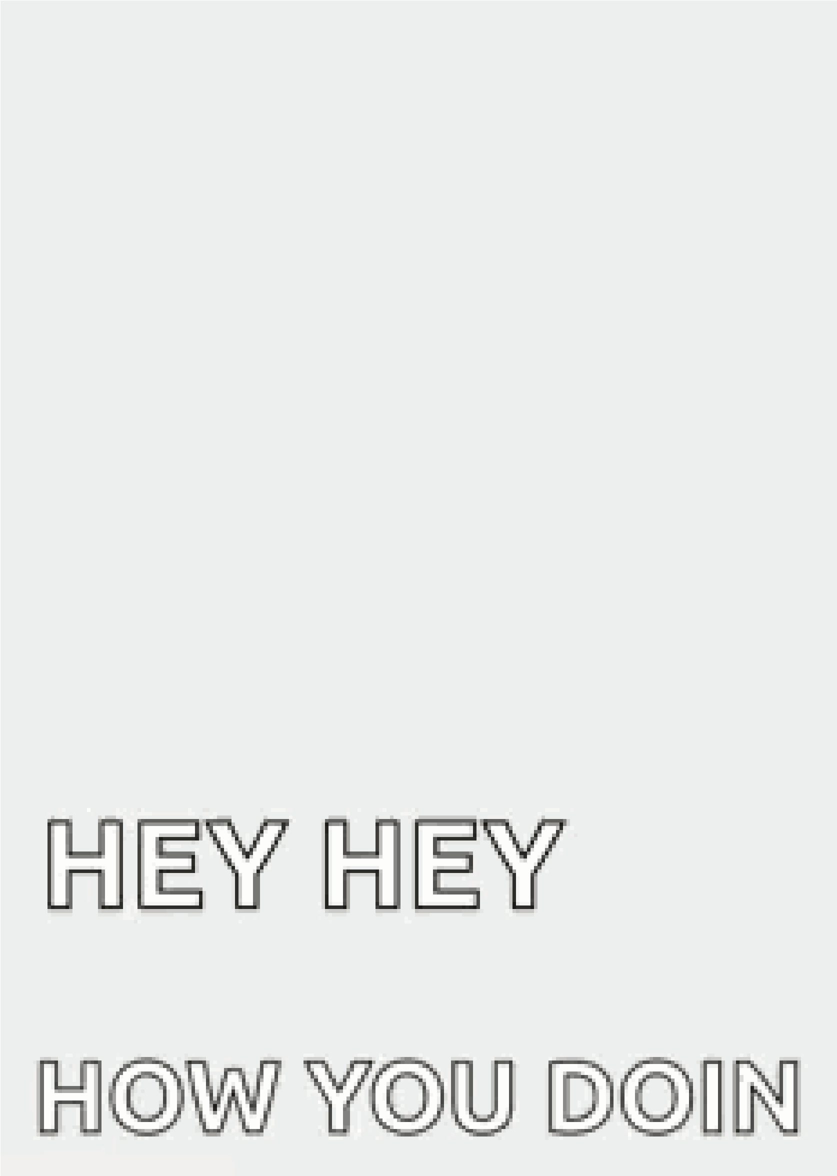


SETU!



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4. SWOTVAC: Consultation/Feedback hour (Fri 8am)

For Fairness, I will open it at 8am Friday on TEAM with AN EXCEL SHEET SIGNUP

Resources

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14		FYP DAY		ECN/MUMGRO	
15		FYP DAY		ECN/MUMGRO	

Let us start!

Today's Activity

1. Workshop Problem Set

Drag Force

In fluid dynamics, a body moving in a fluid encounters a drag force due to the turbulence created by the body's motion and the friction between the fluid and the surface of the body. The drag force is proportional to the square of the speed of the body and always opposes motion of the body. [1]

Suppose a submarine is moving at sea, its velocity at time t, $v(t)$, will satisfy a differential equation

$$v'(t) = f(t) - \kappa(v(t))^2$$

where $f(t)$ represents the acceleration generated by the submarine's turbine, and κ is a constant drag coefficient.

1. Suppose that the turbine generates a constant thrust of 1000, i.e., $f(t) = 1000$, and $\kappa = 0.1$. Is the resultant differential equation linear? Is it separable? **[1 mark]**

$$\frac{dv}{dt} = 1000 - 0.1v^2$$

- Non-Linear (see dependent variable v, power 2)

- Separable, like duh!

$$\frac{dv}{1000 - 0.1v^2} = dt$$

In fluid dynamics, a body moving in a fluid encounters a drag force due to the turbulence created by the body's motion and the friction between the fluid and the surface of the body. The drag force is proportional to the square of the speed of the body and always opposes motion of the body. [1]

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where $f(t)$ represents the acceleration generated by the submarine's turbine, and κ is a constant drag coefficient.

2. Solve the differential equation for $v(t)$ given the initial condition $v(0) = 0$. Hint: You may use partial fractions for your integral calculation. [5 marks]

You try with these guides (10mins):

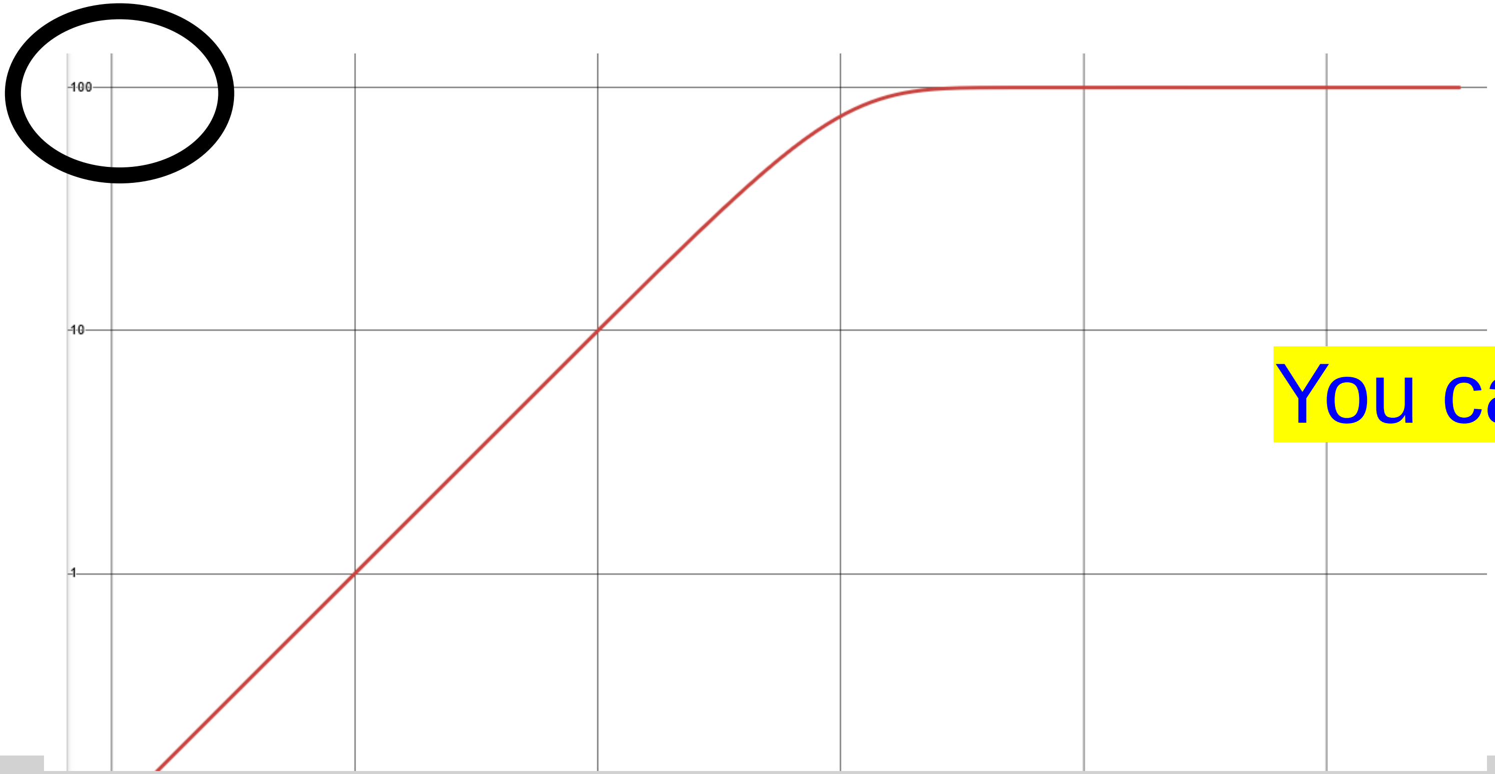
$$v(t) = \frac{100e^{20t} - 100}{e^{20t} + 1}$$

Integration trick

$$v(0) = 0$$

3. What is the behaviour of the submarine's velocity as t approaches infinity?

Desmos (5mins):



Relaxes at y=100

You can do L'hopital rule too!

$$v(t) = \frac{100e^{20t} - 100}{e^{20t} + 1}$$

We will now consider the situation where the submarine's thrust is not constant. For the rest of the workshop, assume $f(t) = t$, and for simplicity $\kappa = 1$. Hence the differential equation becomes

$$v'(t) = t - (v(t))^2$$

This is an example of a Riccati equation. For now we will not go into the deeper theory of Riccati equations, instead we will try to find a series solution for $v(t)$.

4. Is the resultant differential equation linear? Is it separable? **[1 mark]**

It is neither linear nor separable, and cannot be solved precisely by methods we have learnt in the unit.

5. We can write the Maclaurin series of $v(t)$ as

$$v(t) = a_0 + a_1 t + a_2 t^2 + a3t^3 + a_4 t^4 + \ldots$$

. Suppose the the Maclaurin series of $(v(t))^2$ is

$$(v(t))^2 = b_0 + b_1 t + b_2 t^2 + b3t^3 + b_4 t^4 + \ldots$$

Express b_0, b_1, b_2, b_3 and b_4 in terms of the a_i's. **[3 marks]**

Given:

$$v(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + \cdots$$

We calculate:

$$[v(t)]^2 = \left(a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + \cdots\right)^2$$

$$= a_0^2 + 2a_0 a_1 t + \left(2a_0 a_2 + a_1^2\right) t^2 + \left(2a_0 a_3 + 2a_1 a_2\right) t^3 + \left(2a_0 a_4 + 2a_1 a_3 + a_2^2\right) t^4 + \cdots$$

Compare the Maclaurin series with:

$$[v(t)]^2 = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + \cdots$$

The coefficients are:

$$b_0 = a_0^2$$

$$b_1 = 2a_0 a_1$$

$$b_2 = 2a_0 a_2 + a_1^2$$

$$b_3 = 2a_0 a_3 + 2a_1 a_2$$

$$b_4 = 2a_0 a_4 + 2a_1 a_3 + a_2^2$$

6. Hence find the series solution of $v(t)$, up to and including the t^4 term, to the differential equation $v'(t) = t - (v(t))^2$ with initial condition $v(0) = 1$. **[3 marks]**

That was pretty tedious! Using the theory of Taylor series, we can find the coefficients in another way.

You try with this guide (5mins):

$$v(t) = 1 - t + \frac{3}{2}t^2 - \frac{4}{3}t^3 + \frac{17}{12}t^4 + \dots$$

7. From the initial condition $v(0) = 1$, use the differential equation to find $v'(0)$.

$$v(t) = 1 - t + \frac{3}{2}t^2 - \frac{4}{3}t^3 + \frac{17}{12}t^4 + \dots$$

$$v'(0) = -1$$

6. Hence find the series solution of $v(t)$, up to and including the t^4 term, to the differential equation $v'(t) = t - (v(t))^2$ with initial condition $v(0) = 1$. **[3 marks]**

That was pretty tedious! Using the theory of Taylor series, we can find the coefficients in another way.

8. Implicitly differentiate the equation $v'(t) = t - (v(t))^2$ with respect to t to obtain a differential equation involving $v''(t)$, and hence fine $v''(0)$. **[3 marks]**

$$v'(t) = t - v(t)^2$$

$$v''(t) = 1 - 2v^1(t)\, v'(t)$$

$$v''(0) = 1 - 2v^1(0)\, v'(0) \;= 3$$

$$\boxed{v(0) = 1;\; v'(0) = -1}$$

9. Use the same method of differentiating the equation to find $v'''(0)$ and $v''''(0)$.

$$v'''(0) = -8$$

$$v''''(0) = 34$$

Hence write down the Taylor series for $v(t)$ at $t = 0$ up to and including the t^4 term.

$$v(t) = v(0) + v'(0)t + \frac{v''(0)}{2!}t^2 + \frac{v'''(0)}{3!}t^3 + \frac{v''''(0)}{4!}t^4$$

Recover Ans to Q6.

$$v(t) = 1 - t + \frac{3}{2}t^2 - \frac{4}{3}t^3 + \frac{17}{12}t^4 + \dots$$

The Big Learning Outcomes for Week 12

After completing this week’s task, you should be able to:

- Find Taylor series of given functions.
- Use truncated Taylor series to approximate functions.
- Understand linearisation.
- Understand the errors introduced in truncating Taylor series as finite order polynomials.
- Use l'Hopital's rule to find limits.
- Find Taylor series of given multivariable functions.

Topics

Week	Topic
1	Vectors, Lines, and Planes
2	Systems of Linear Equations
3	Matrices
4	Eigenvalues & Eigenvectors
5	Multivariable Calculus 1
6	Multivariable Calculus 2
7	Integration techniques and hyperbolic functions
8	O.D.E 1
9	O.D.E 2
10	O.D.E 3
11	Series 1
12	Series 2

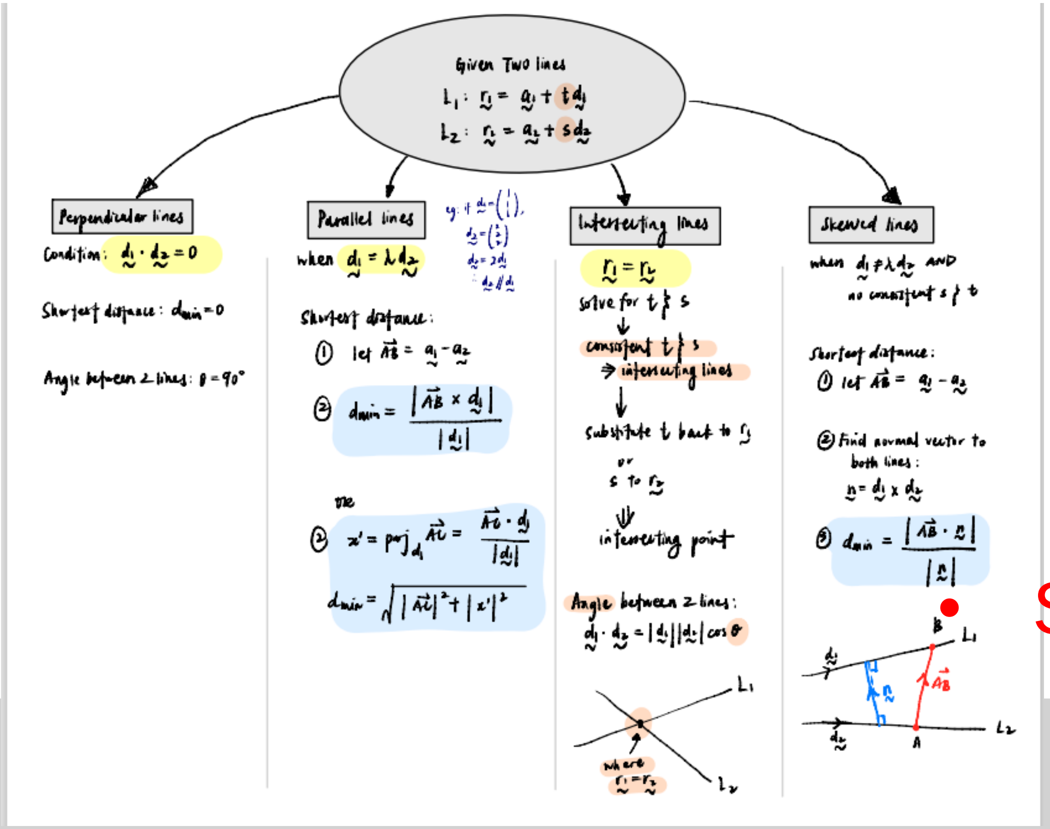
Time to Recap

In the beginning!

But, after 12 Weeks!

Final: How to Kill It?

- Step 1: RoadMap given in the Pass Sessions (MUM campus only).



- Step 2: Textbook Examples.
- Step 3: MUM recorded Vids.
- Step 4: Mock Exams/Revision Quiz
- Step 5: Questions attempted in class/pass

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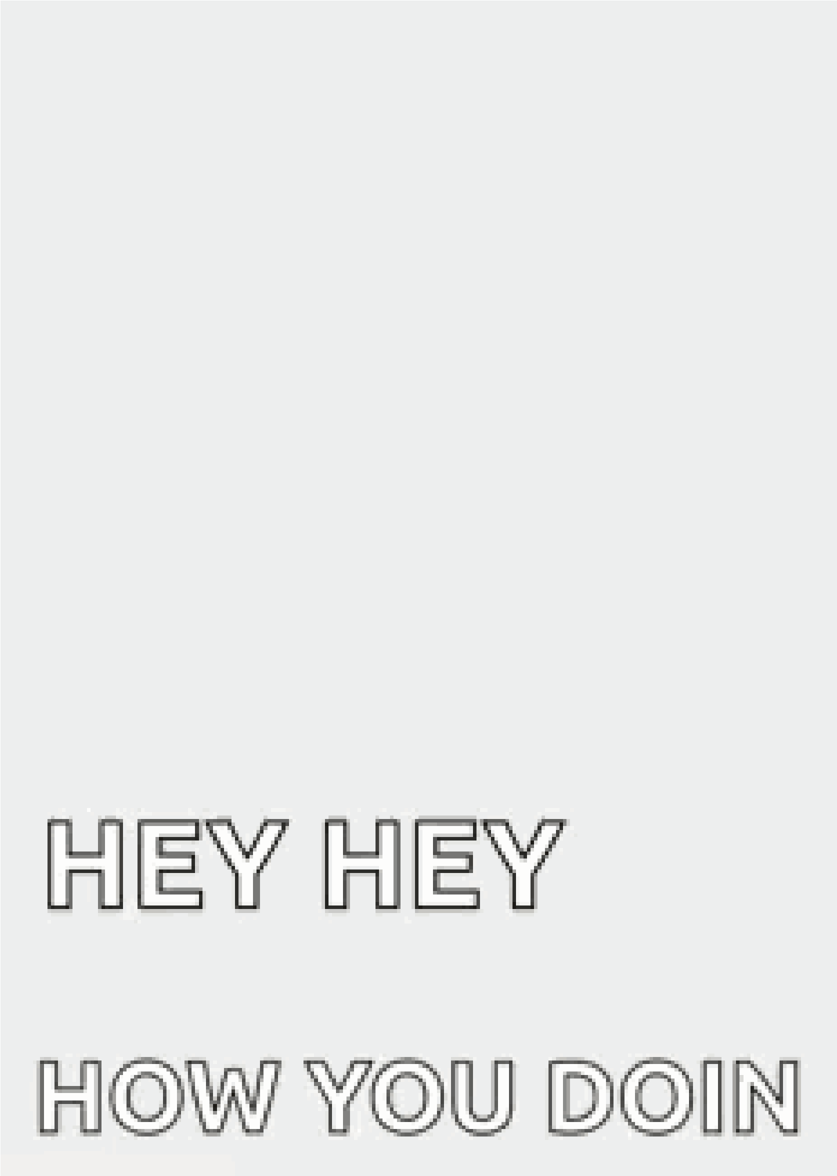

Thank You Very Much!