# ENG1005 S1 2024 Workshop 7
# Pursuit Problems

24 marks total

This problem set is intended for you to apply the mathematical skills you are learning. It is also designed to practice communicating your work clearly.

It is expected that you will use the workshop to develop (rough) solutions. During the workshop, you should discuss the problems with your peers and the academic staff who are there to assist you. In particular, if you are uncertain about what the problems are asking or you are stuck on a particular point, this is the time to get assistance. The time between the end of the workshop and when the solutions are due is only meant to be for writing up your solutions and for this you should not need more than an hour or two at most.

**General submission information:**

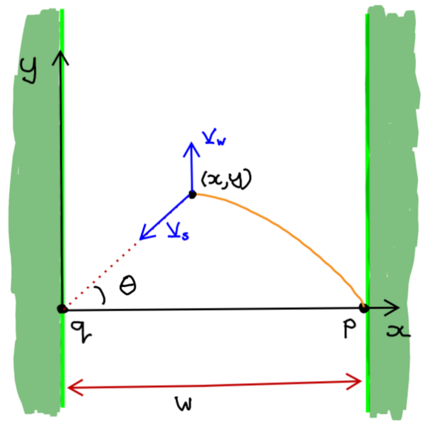
1. Electronic submission of your solutions is due on Moodle by **11:55 pm (Melbourne time) on Friday of the same week**.
2. **Your solutions should include a description/explanation of what you are doing at each step and relevant working.** Without these you will receive limited marks. The description should be in complete English sentences. All mathematics should be appropriately laid out and with appropriate notation. Your writing should be similar in style to the worked solutions from the Applied Class problem sheets, not the annotations from the videos. For more information and advice, please read the "Guidelines for writing in mathematics" document posted under the "Additional information and resources" section of the ENG1005 Moodle page.
3. Your solutions may be typed or handwritten and scanned (the latter is encouraged). The **final document should be submitted as a single pdf file that is clearly and easily legible**. If the marker is unable to read it (or any part of it) you may lose marks.

**Academic integrity:**

You can (and should!) discuss your solutions with the other students, but **you must write up your solutions by yourself**. Copying solutions is serious academic misconduct and will be penalised according to Monash University guidelines. Other examples of academic misconduct include asking a personal tutor to do any of your assessments and posting your assessments to a "homework" website. Please refer back to your Academic Integrity module if you are in any doubt about what constitutes academic misconduct. **Your integrity is an important part of who you are. It is much more important than any grade you could receive.**

**Pursuit Problems**

Pursuit problems involve determining the trajectory needed by one object to intercept another. This could be a rocket carrying astronauts to the international space space station, a missile launched at an aircraft, or a police car in pursuit of a fleeing criminal. In this workshop, you will explore the following pursuit problem: consider a canal of width $w > 0$, see diagram below.




Relative to the $xy$-coordinate system indicated on the diagram, assume that the water in the canal is flowing in the positive $y$-direction with a speed $s \geq 0$ and that a swimmer enters the canal at the point $p = (w, 0)$. The swimmer then swims towards the point $q = (0, 0)$ always facing in the direction of $q$. Letting $(x, y) = (x(t), y(t))$ denote the position of the swimmer at time $t$ and $\mathbf{v}(t) = \left(\frac{dx}{dt}, \frac{dy}{dt}\right)$ their velocity, your objective is to determine the trajectory of swimmer as they move through the canal and attempt to get to the point $q$ on the other side. You may assume that swimmer can swim at a constant speed $c > 0$ in still water and swims at this speed in the canal. In the above diagram, $\mathbf{v}_s$ denotes the velocity of the swimmer in still water and $\mathbf{v}_w$ is the velocity of the water in the canal, so that the total velocity of the swimmer is $\mathbf{v} = \mathbf{v}_s + \mathbf{v}_w$.

1. Express the velocity vector $\mathbf{v}$ in terms of $x, y$ and the constants $s, c$. **[2 marks]**

2. Use the formula for $\mathbf{v}$ and the chain rule to calculate $\dfrac{dy}{dx}$. This will yield a first order differential equation of the form
$$\frac{dy}{dx} = f(x, y)$$
for an appropriate function $f(x, y)$ that also contains the constants $s$ and $c$. **[3 marks]**

3. Is this differential equation linear? Is it separable? Make sure you explain your answers. **[2 marks]**

4. Show that the differential equation can be written as
$$\frac{dy}{dx} = g\left(\frac{y}{x}\right)$$
where $g$ is the function $g(u) = u - \frac{s}{c}\sqrt{1 + u^2}$. **[1 mark]**

5. To solve the differential equation we make the substitution $u = \frac{y}{x}$. Show that with this substitution, your original first order differential equation of $y$ and $x$ turns into a *separable* differential equation of $u$ and $x$. **[3 marks]**

6. Solve the separable equation and use the initial condition $y(w) = 0$ to show that the trajectory of the swimmer is given by

$$y = y(x) = \frac{w}{2}\left(\left(\frac{x}{w}\right)^{1-\frac{s}{c}} - \left(\frac{x}{w}\right)^{1+\frac{s}{c}}\right)$$

   Make sure you explain the main steps in the derivation of the solution. **[7 marks]**

7. Is it always possible for the swimmer to reach the point $q$ for any choice of $s \geq 0$ and $c > 0$? If not, then determine the set of speeds $(s, c)$ for which the swimmer is able to reach $q$. Can you provide an intuitive explanation of your findings? **[4 marks]**

**There is also 1 additional mark given for the quality of the English and 1 additional mark for correct mathematical notation.** These marks are easy to obtain but the markers will be instructed to be strict in awarding these marks.