ENG1005 Week 7: Applied class problem sheet

This problem sheet is intended for you to work through in your Applied Class in a small-group setting with the help of your instructor and your peers. **At the end of the applied class you will be asked to complete a quiz for credit. The quiz questions are based on the questions on this problem sheet. You may (and should!) ask your group members and your instructor for guidance if needed.**

1. Use integration by parts twice to find

$$\int_0^{2\pi} \sin(4x)\cos(5x)\,dx.$$

   As an extension, can you use the same approach to find the value of

$$\int_0^{2\pi} \sin(mx)\cos(nx)\,dx$$

   for all integers m and n?

2. Use integration by parts (potentially combined with a substitution), to find
   (a) $\int \sin(x)\,\ln(\cos(x))\,dx$
   (b) $\int \sin^{-1}(x)\,dx$

3. If $\tanh(x) = -15/17$, then find the value of the other hyperbolic functions at x.

4. Evaluate

$$\frac{d}{dx}(\cosh^{-1} x)$$

   [Hint: you might find implicit differentiation useful.]

5. Use a substitution and integration by parts to evaluate

   (a) $\int x^3 e^{-x^2}\,dx$

   (b) $\int \sin[\ln(x)]\,dx$

6. Show that

   (a) $\sinh(-x) = -\sinh(x)$

   (b) $\cosh(-x) = \cosh(x)$

   (c) $\cosh(x+y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$

7. Find an explicit expression for inverse sinh and inverse tanh. (These expressions will likely have logarithms and square roots.)

8. This question explores why sinh and cosh are called hyperbolic functions. Let $x = \cosh t$ and $y = \sinh t$. As t changes, what is the curve that is drawn in space? Can you eliminate the parameter t using some of the relationships between hyperbolic functions to create an implicit representation of this curve?

9. Use the identity $\cosh^2(x) - \sinh^2(x) = 1$ and integration by parts to express

$$I_n(x) = \int \cosh^n(x)\,dx$$

   in terms of $I_{n-2}(x)$. Using this relationship with $n = -2$, find

$$\int_0^{\ln(2)} \frac{1}{\cosh^4(x)}\,dx.$$

10. (For interest) The Mercator projection of the world is nearly 500 years old, but is still one of the most common. It translates the longitude (the east-west angle) λ in degrees into an x value through $x = R\frac{2\pi}{360}\lambda$ and the latitude (the north-south angle) ϕ into a y value through $y = R\tanh^{-1}(\sin\phi)$, where $R \approx 6400\,\text{km}$ is the radius of the Earth. The latitudinal extent of various countries/regions is given below. What are their real north-south extents and what are their apparent north-south extents on a Mercator projection?

    - Australia extends from 12 to 43 degrees South.
    - The UK extends from 49 to 59 degrees North.
    - Greenland extends from 61 to 69 degrees North.