ENG1005: Week 6 Applied class problem sheet

This problem sheet is intended for you to work through in your Applied Class in a small-group setting with the help of your instructor and your peers. **At the end of the applied class you will be asked to complete a quiz for credit. The quiz questions are based on the questions on this problem sheet. You may (and should!) ask your group members and your instructor for guidance if needed.**

1. Find and classify all the critical points of $f(x, y) = x^3/3 - x + (y^3/3 - y)$.

2. Which of the following guarantees a saddle point of the function $f(x, y)$ at a critical point (x_0, y_0)?

   (a) $\partial^2 f/\partial x^2$ and $\partial^2 f/\partial y^2$ have the same sign at (x_0, y_0)

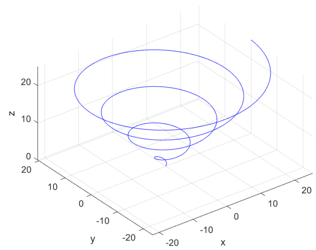
   (b) $\partial^2 f/\partial x^2$ and $\partial^2 f/\partial y^2$ have opposite signs at (x_0, y_0)

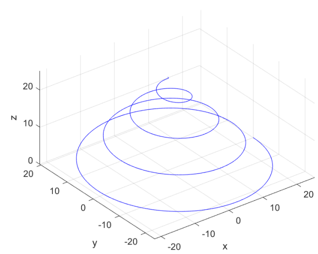
   (c) $\partial^2 f/\partial x \partial y$ is negative at (x_0, y_0)

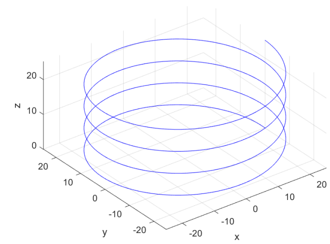
   (d) none of the above

3. Match the following parametric representations to the correct curve:

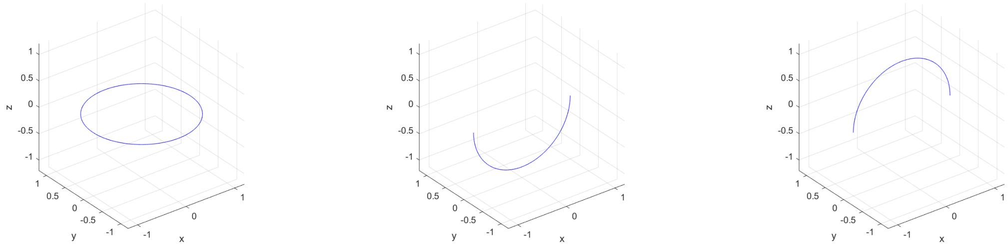
   (a) $\mathbf{x}(t) = (t\cos(t), t\sin(t), t)$ for $0 \le t < 8\pi$







(b) $\mathbf{x}(t) = (\cos(t), 0, -\sin(t))$ for $0 \le t < \pi$



4. Use Lagrange multipliers to determine the point on the sphere $x^2 + y^2 + z^2 = 1$ that is furthest from the point $(1, 2, 2)$. *Hint: To solve the system of equations, you can first write x, y, z all in terms of λ, and then substitute into the constraint $x^2 + y^2 + z^2 = 1$ to find λ.*

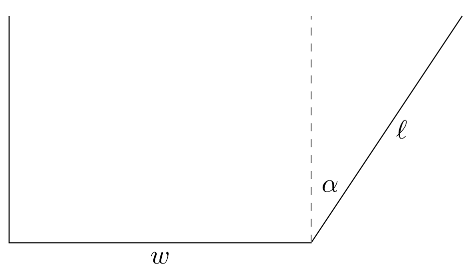
5. Consider the function $f(x, y) = x^3 - 3xy^2$. Find the critical point(s) of the function. What happens if your try to classify the critical point(s) using the usual test we perform? In such cases, we must resort to other means to classify the critical point. Using your favourite program, plot a selection of both positive and negative contours. Why do you think this function might be called a "monkey saddle"?

6. Find the absolute maximum and absolute minimum of the function $f(x, y) = x^2 + y^2 - 2x$ inside the triangular region with vertices at $(2, 0)$, $(0, 2)$ and $(0, -2)$.

7. Find a parameterisation for the ellipse
$$4x^2 + y^2 = 16$$
and calculate its tangent vector. Hence show that the equation of the tangent line to the ellipse passing through the point (x_0, y_0) can be given by
$$4x_0x + y_0y = 16.$$

8. Find a parametric representation of the elliptical cone
$$z = \sqrt{9x^2 + y^2}\ .$$

9. (Challenging!) You are asked to make a rain gutter for a house. You have a sheet of metal that is $L = 15 + 10\sqrt{3}\,$cm wide and you can bend it to have a profile as shown below where w, l and α are values you can choose.



Write down an expression for the cross-sectional area of the gutter in terms of w, α and ℓ. Now write down a constraint connecting L, w, α and ℓ. Using Lagrange multipliers, if your aim is to maximise the capacity of the gutter, what should ℓ and α be? *Hint: You can assume that there is a single interesting solution and this doesn't have $\alpha = 0$ or $\alpha = \pi/2$.*