# ENG1005 S1 2024 Workshop 8
# Pollution Modelling

24 marks total

This problem set is intended for you to apply the mathematical skills you are learning. It is also designed to practice communicating your work clearly.

It is expected that you will use the workshop to develop (rough) solutions. During the workshop, you should discuss the problems with your peers and the academic staff who are there to assist you. In particular, if you are uncertain about what the problems are asking or you are stuck on a particular point, this is the time to get assistance. The time between the end of the workshop and when the solutions are due is only meant to be for writing up your solutions and for this you should not need more than an hour or two at most.

**General submission information:**

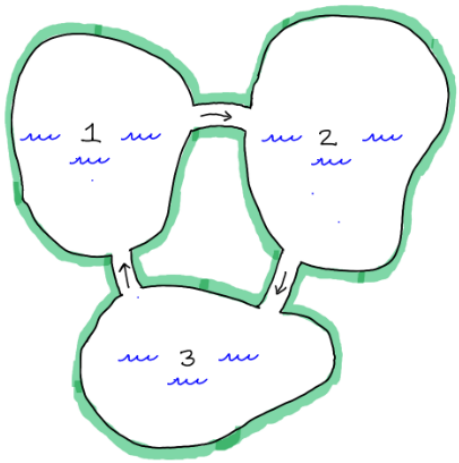
1. Electronic submission of your solutions is due on Moodle by **11:55 pm (Melbourne time) on Friday of the same week**.
2. **Your solutions should include a description/explanation of what you are doing at each step and relevant working.** Without these you will receive limited marks. The description should be in complete English sentences. All mathematics should be appropriately laid out and with appropriate notation. Your writing should be similar in style to the worked solutions from the Applied Class problem sheets, not the annotations from the videos. For more information and advice, please read the "Guidelines for writing in mathematics" document posted under the "Additional information and resources" section of the ENG1005 Moodle page.
3. Your solutions may be typed or handwritten and scanned (the latter is encouraged). The **final document should be submitted as a single pdf file that is clearly and easily legible**. If the marker is unable to read it (or any part of it) you may lose marks.

**Academic integrity:**

You can (and should!) discuss your solutions with the other students, but **you must write up your solutions by yourself**. Copying solutions is serious academic misconduct and will be penalised according to Monash University guidelines. Other examples of academic misconduct include asking a personal tutor to do any of your assessments and posting your assessments to a "homework" website. Please refer back to your Academic Integrity module if you are in any doubt about what constitutes academic misconduct. **Your integrity is an important part of who you are. It is much more important than any grade you could receive.**

**Pollution Modelling**

In this workshop, we will use the systems of differential equations to model the spread of pollution in three connected lakes. Consider three lakes connected by rivers as in the diagram below.




Suppose lake 1 holds 8000 megalitres of water, lake 2 holds 24000 megalitres and lake 3 holds 3000 megalitres. Three rivers connect the lakes, flowing in the directions indicated on the diagram. All three rivers have a constant flow rate of 24000 megalitre per day. Suppose at time 0, 10kg of pollutant is discharged into lake 1. You may assume that any amount of pollutant would instantaneously dissolve and spread evenly throughout each lake.

1. Show that the amount of water in each lake stays constant. **[1 mark]**
2. Let $x_i$ denote the amount of pollutant in lake $i$ for $i = 1, 2, 3$. Calculate the rate of change $\frac{dx_1}{dt}$ of lake 1 in terms of the $x_i$'s. **[2 marks]**
3. Repeat the calculation for lakes 2 and 3, and hence write down a system of first order differential equations in the form of $\frac{d\vec{x}}{dt} = A\vec{x}$ where $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. **[2 marks]**
4. Find all the eigenvalues and corresponding eigenvectors of the matrix $A$. **[4 marks]**
5. Use the eigenvalue/eigenvector method to write down the general solution of the system of ODE's. **[2 marks]**
6. Hence solve the system of ODE's with initial condition. **[3 marks]**
7. What is the long term behaviour of the distribution of pollutant in the lakes? Can you offer a physical explanation? **[2 marks]**
8. A filtering plant is set up at the middle of the river between lake 3 and lake 1 to remove the pollutant from the water. As water passes through the plant, the concentration of pollutant is halved. Write down a system of first order differential equations in the form of $\frac{d\vec{x}}{dt} = B\vec{x}$ where $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ to model this situation. **[2 marks]**

9. Find the eigenvalues of $B$. **[2 marks]**

10. Without any further calculation, describe the long term behaviour of the solution $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. Does it match your expectation? **[2 marks]**

**There is also 1 additional mark given for the quality of the English and 1 additional mark for correct mathematical notation.** These marks are easy to obtain but the markers will be instructed to be strict in awarding these marks.