ENG1005: Week 9 Applied class problem sheet

This problem sheet is intended for you to work through in your Applied Class in a small-group setting with the help of your instructor and your peers. **At the end of the applied class you will be asked to complete a quiz for credit. The quiz questions are based on the questions on this problem sheet. You may (and should!) ask your group members and your instructor for guidance if needed.**

1. Find the homogeneous solution of the ODE

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = r(x).$$

For each of the following functions $r(x)$, write down an appropriate guess for $y_p(x)$

(a) $r(x) = 2 + 3x + 6e^{-x}$
(b) $r(x) = x\sin(x)$
(c) $r(x) = 6xe^{3x}$. For this case, also find the particular solution. *This one is tricky - think carefully about your guess.*

2. Find the solution of the ODE

$$y'' - y' - 2y = 0$$

that satisfies $y(0) = 1$ and y is bounded (its magnitude stays less than a finite value) as $x \to \infty$.

3. The general solution of the ODE

$$y'' - 7y' + 12y = -24$$

is

$$y(x) = Ae^{3x} + Be^{4x} - 2.$$

Find the solution satisfying the initial conditions $y(0) = 1$ and $y'(0) = 1$.

4. The general solution of an ODE is

$$y(x) = Ae^{x} + Be^{2x} + x^2$$

What is the ODE that this solution satisfies, given that the coefficient for d^2y/dx^2 is 1?

5. How might you try finding the general solution of the third order ODE

$$y''' - 2y' + y = x?$$

Bonus: find the general solution.

6. Consider a beam of length L with a load on top P. The vertical coordinate is z and the horizontal deflection of the beam from straight is w. The deflection of the beam w satisfies the ODE

$$Bw'' + Pw = 0$$

with boundary conditions $w(0) = 0$ and $w(L) = 0$. The constant B is the bending stiffness of the beam.

Let $B = 1$. Find the particular solution of the ODE above satisfying the boundary conditions given. Can you find more than the trivial solution $w = 0$ for some values of P? What does this tell you about when the column buckles under the load?

7. The ODE
$$ax^2y'' + bxy' + cy = 0$$
where a, b and c are constants is known as the Cauchy–Euler equation. It can be solved in a similar way to constant-coefficient linear ODEs except the appropriate form to try for the homogeneous ODE is
$$y = Ax^n$$
where n is an exponent that we need to find. Using this information, find the general solution of the ODE
$$x^2y'' + 4xy' + 2y = 0.$$

8. Sometimes, a constant-coefficient, linear, second order differential equation has solutions of the form $y(t) = e^{\lambda t}$ and $y(t) = te^{\lambda t}$. Can you construct a constant-cofficient, linear, **third-order** differential equation such that
$$y(t) = e^{2t}, \quad y(t) = te^{2t}, \quad \text{and} \quad y(t) = t^2e^{2t}$$
are all solutions?