ENG1005 S2 2024 Workshop 1
Concentrated solar thermal (CST)

18 marks total

This problem set is intended for you to apply the mathematical skills you are learning. It is also designed to practice communicating your work clearly.

It is expected that you will use the workshop to develop (rough) solutions. During the workshop, you should discuss the problems with your peers and the academic staff who are there to assist you. In particular, if you are uncertain about what the problems are asking or you are stuck on a particular point, this is the time to get assistance. The time between the end of the workshop and when the solutions are due is only meant to be for writing up your solutions and for this you should not need more than an hour or two at most.

General submission information:

1. Electronic submission of your solutions is due on Moodle by **11:55 pm on Sunday of the same week**.
2. **Your solutions should include a description/explanation of what you are doing at each step and relevant working.** Without these you will receive limited marks. The description should be in complete English sentences. All mathematics should be appropriately laid out and with appropriate notation. Your writing should be similar in style to the worked solutions from the Applied Class problem sheets, not the annotations from the videos. For more information and advice, please read the "Guidelines for writing in mathematics" document posted under the "Additional information and resources" section of the ENG1005 Moodle page.
3. Your solutions may be typed or handwritten and scanned (the latter is encouraged). The **final document should be submitted as a <u>single pdf file</u> that is clearly and easily legible**. If the marker is unable to read it (or any part of it) you may lose marks.

Academic integrity:

You can (and should!) discuss your solutions with the other students, but **you must write up your solutions by yourself**. Copying solutions is serious academic misconduct and will be penalised according to Monash University guidelines. Other examples of academic misconduct include asking a personal tutor to do any of your assessments and posting your assessments to a "homework" website. Please refer back to your Academic Integrity module if you are in any doubt about what constitutes academic misconduct. **Your integrity is an important part of who you are. It is much more important than any grade you could receive.**

Concentrated solar thermal (CST)

Concentrated solar thermal is one approach to combat the problem of intermittency in solar power generation. The idea is to focus the sun's rays from a broad area onto a single collector to heat molten salt (or another material). This heat can then be stored for later use to generate steam and drive turbines when the sun isn't shining, complementing the use of photovoltaics. In this workshop, you will explore how the sun's rays can be focused onto a collector using an array of plane mirrors (as shown in the image below; a similar plant is being developed near Port Augusta, SA, and others are planned around Australia)

By National Renewable Energy Laboratory https://commons.wikimedia.org/w/index.php?curid=1455806

The physical laws for reflection in a mirror are the following:

- The reflected light ray is in the same plane as the incident (i.e. incoming) ray and the normal to the mirror.
- The angle between the reflected ray and the normal to the mirror (the angle of reflection) is the same as that between the incident ray and the normal to the mirror (the angle of incidence). This is usually stated as "the angle of reflection is equal to the angle of incidence".

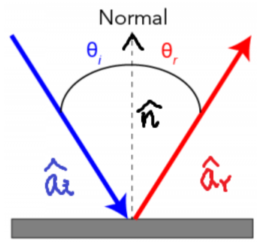
If $\hat{\mathbf{a}}_i$ is a unit vector in the direction of an *incoming* light ray onto a plane mirror with normal $\hat{\mathbf{n}}$, then the mirror reflection law says that the direction of the reflected ray $\hat{\mathbf{a}}_r$ is given by

$$\hat{\mathbf{a}}_r = \hat{\mathbf{a}}_i - 2(\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}.$$

Here both $\hat{\mathbf{a}}_i$ and $\hat{\mathbf{a}}_r$ are pointing in the direction that the light ray is travelling towards, and $\hat{\mathbf{n}}$ is the unit normal vector pointing away from the surface of the mirror.

Q1: Make a diagram that illustrates the physical setup and includes the mirror, the light ray before and after reflection. Clearly label the vectors $\hat{\mathbf{n}}$, $\hat{\mathbf{a}}_i$ and $\hat{\mathbf{a}}_r$ on your diagram. **[2 marks]**

Solution:



Q2: On the same diagram, clearly label the angle of incidence θ_i and the angle of reflection θ_r. **[1 mark]**

Solution: See previous diagram.

Q3: Verify that if $\hat{\mathbf{a}}_r$ is given by the above formula, then the two physical laws are satisfied. **[4 marks]**

Solution: We need to check both rules individually. Since $\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}}$ is a scalar, $\hat{\mathbf{a}}_r$ is the sum of $\hat{\mathbf{a}}_i$ and a scalar multiple of $\hat{\mathbf{n}}$. By the definition of vector addition, the sum of two vectors lies on the same plane as the original two vectors. This also works if we take scalar multiples of of the vectors. Therefore $\hat{\mathbf{a}}_r$ is on the same plane as $\hat{\mathbf{a}}_i$ and $\hat{\mathbf{n}}$.

For the second rule, notice that θ_r is the angle between $\hat{\mathbf{a}}_r$ and $\hat{\mathbf{n}}$, and θ_i is the angle betweeen $-\hat{\mathbf{a}}_i$ and $\hat{\mathbf{n}}$. The negative sign on $-\hat{\mathbf{a}}_i$ is introduced because the angle between vectors is measured when their tails are at the same point, so we need to reverse $\hat{\mathbf{a}}_i$.

From the property of the dot product we know

$$\cos(\theta_r) = \frac{\hat{\mathbf{a}}_r \cdot \hat{\mathbf{n}}}{|\hat{\mathbf{a}}_r||\hat{\mathbf{n}}|}, \qquad \cos(\theta_i) = \frac{-\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}}}{|-\hat{\mathbf{a}}_r||\hat{\mathbf{n}}|}$$

Since $\hat{\mathbf{a}}_i$, $\hat{\mathbf{a}}_r$ and $\hat{\mathbf{n}}$ are all unit vectors, we have

$$\cos(\theta_r) = \hat{\mathbf{a}}_r \cdot \hat{\mathbf{n}}, \qquad \cos(\theta_i) = -\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}}$$

Using the formula for $\hat{\mathbf{a}}_r$, we have

$$\begin{aligned}
\cos(\theta_r) &= \hat{\mathbf{a}}_r \cdot \hat{\mathbf{n}} \\
&= (\hat{\mathbf{a}}_i - 2(\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}) \cdot \hat{\mathbf{n}} \\
&= \hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}} - 2(\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}}) \cdot \hat{\mathbf{n}} \\
&= \hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}} - 2(\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}})|\hat{\mathbf{n}}|^2 \\
&= \hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}} - 2(\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}}) \\
&= -\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}} \\
&= \cos(\theta_i)
\end{aligned}$$

Therefore $\theta_r = \theta_i$.

As the sun travels across the sky, the mirrors need to be rotated to ensure that they continue to reflect the light onto the collector. You will now investigate how to adjust the position of an individual mirror.

Choose the coordinate system such that the x-axis is aligned due East, the y-axis due North, the z-axis vertically upwards, and the centre of the mirror is located at the origin. In its initial resting position, the mirror has its normal pointing due East. Suppose that in this coordinate system, the sun appears to be in the direction of the vector $(10, 2, 11)$.

Q4 Write down the unit normal vector $\hat{\mathbf{n}}$ to the mirror in its resting position. **[1 mark]**

Solution: The unit vector in the East direction is $(1, 0, 0)$.

Q5 Write down the unit vector $\hat{\mathbf{a}}_i$ of the *incoming* sunlight. **[1 mark]**

Solution: $\hat{\mathbf{a}}_i$ is in the direction $(-10, -2, -11)$, the negative sign representing the *incoming* light direction instead of the sun's position from the mirror. Hence $\hat{\mathbf{a}}_i = \dfrac{(-10, -2, -11)}{\sqrt{100 + 4 + 121}} = \dfrac{(-10, -2, -11)}{15}$.

Q6 Calculate the unit vector $\hat{\mathbf{a}}_r$ of the reflected light ray. **[2 marks]**

Solution: Using the formula we get

$$\begin{aligned}\hat{\mathbf{a}}_r &= \frac{(-10, -2, -11)}{15} - 2\left(\frac{(-10, -2, -11)}{15} \cdot (1, 0, 0)\right)(1, 0, 0) \\ &= \frac{(-10, -2, -11)}{15} - \frac{(-20, 0, 0)}{15} \\ &= \frac{(10, -2, -11)}{15}\end{aligned}$$

Q7 Suppose the collector is located at the position $(-100, 100, 50)$, will the reflected light ray hit the collector? Justify your answer. **[1 mark]**

Solution: If the reflected light would hit $(-100, 100, 50)$, then $(-100, 100, 50)$ would be in the same direction as $\hat{\mathbf{a}}_r$. However this is not the case, since $(-100, 100, 50) = 50(-2, 2, 1)$ is clearly in a different direction to $\dfrac{(10, -2, -11)}{15}$.

Q8 The mirror is rotated so that the reflected light ray will hit the collector. Calculate the unit normal vector $\hat{\mathbf{n}}$ to the mirror in this position. **[4 marks]**

Solution: Since we want the reflected light to hit the collector, $\hat{\mathbf{a}}_r$ should be in the direction of the vector $(-100, 100, 50)$. Therefore $\hat{\mathbf{a}}_r = \dfrac{(-2, 2, 1)}{3}$.

We can rewrite the equation $\hat{\mathbf{a}}_r = \hat{\mathbf{a}}_i - 2(\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}$ as

$$2(\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}})\hat{\mathbf{n}} = \hat{\mathbf{a}}_i - \hat{\mathbf{a}}_r$$

Observe that $\hat{\mathbf{n}}$ is a unit vector. From the diagram it is also clear that the dot product $\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}}$ is a negative numbers, since $\hat{\mathbf{a}}_i$ is coming into the mirror, but $\hat{\mathbf{n}}$ is pointing away from the mirror. It follows that $\hat{\mathbf{n}}$ is a unit vector in the negative direction of $\hat{\mathbf{a}}_i - \hat{\mathbf{a}}_r$.

We know both $\hat{\mathbf{a}}_i$ and $\hat{\mathbf{a}}_r$, so all we need to do is calculate the unit vector in the direction of $-(\hat{\mathbf{a}}_i - \hat{\mathbf{a}}_r)$.

$$\hat{\mathbf{a}}_i - \hat{\mathbf{a}}_r = \frac{(-10, -2, -11)}{15} - \frac{(-2, 2, 1)}{3} = \frac{(0, -12, -16)}{15} = \frac{4}{15}(0, -3, -4)$$

Therefore $\hat{\mathbf{n}} = \dfrac{(0, 3, 4)}{\sqrt{9 + 16}} = \dfrac{(0, 3, 4)}{5}$.

There is also 1 additional mark given for the quality of the English and 1 additional mark for correct mathematical notation. These marks are easy to obtain but the markers will be instructed to be strict in awarding these marks.