ENG1005 S1 2024 Workshop 10
Drag Force

25 marks total

This problem set is intended for you to apply the mathematical skills you are learning. It is also designed to practice communicating your work clearly.

It is expected that you will use the workshop to develop (rough) solutions. During the workshop, you should discuss the problems with your peers and the academic staff who are there to assist you. In particular, if you are uncertain about what the problems are asking or you are stuck on a particular point, this is the time to get assistance. The time between the end of the workshop and when the solutions are due is only meant to be for writing up your solutions and for this you should not need more than an hour or two at most.

General submission information:

1. Electronic submission of your solutions is due on Moodle by **11:55 pm (Melbourne time) on Friday of the same week**.
2. **Your solutions should include a description/explanation of what you are doing at each step and relevant working.** Without these you will receive limited marks. The description should be in complete English sentences. All mathematics should be appropriately laid out and with appropriate notation. Your writing should be similar in style to the worked solutions from the Applied Class problem sheets, not the annotations from the videos. For more information and advice, please read the "Guidelines for writing in mathematics" document posted under the "Additional information and resources" section of the ENG1005 Moodle page.
3. Your solutions may be typed or handwritten and scanned (the latter is encouraged). The **final document should be submitted as a single pdf file that is clearly and easily legible**. If the marker is unable to read it (or any part of it) you may lose marks.

Academic integrity:

You can (and should!) discuss your solutions with the other students, but **you must write up your solutions by yourself**. Copying solutions is serious academic misconduct and will be penalised according to Monash University guidelines. Other examples of academic misconduct include asking a personal tutor to do any of your assessments and posting your assessments to a "homework" website. Please refer back to your Academic Integrity module if you are in any doubt about what constitutes academic misconduct. **Your integrity is an important part of who you are. It is much more important than any grade you could receive.**

Drag Force

In fluid dynamics, a body moving in a fluid encounters a drag force due to the turbulence created by the body's motion and the friction between the fluid and the surface of the body. The drag force is proportional to the square of the speed of the body and always opposes motion of the body. [1]

Suppose a submarine is moving at sea, its velocity at time t, $v(t)$, will satisfy a differential equation

$$v'(t) = f(t) - \kappa(v(t))^2$$

where $f(t)$ represents the acceleration generated by the submarine's turbine, and κ is a constant drag coefficient.

1. Suppose that the turbine generates a constant thrust of 1000, i.e., $f(t) = 1000$, and $\kappa = 0.1$. Is the resultant differential equation linear? Is it separable? **[1 mark]**
2. Solve the differential equation for $v(t)$ given the initial condition $v(0) = 0$. Hint: You may use partial fractions for your integral calculation. **[5 marks]**
3. What is the behaviour of the submarine's velocity as t approaches infinity? **[1 mark]**

We will now consider the situation where the submarine's thrust is not constant. For the rest of the workshop, assume $f(t) = t$, and for simplicity $\kappa = 1$. Hence the differential equation becomes

$$v'(t) = t - (v(t))^2$$

This is an example of a Riccati equation. For now we will not go into the deeper theory of Riccati equations, instead we will try to find a series solution for $v(t)$.

4. Is the resultant differential equation linear? Is it separable? **[1 mark]**
5. We can write the Maclaurin series of $v(t)$ as

   $$v(t) = a_0 + a_1 t + a_2 t^2 + a3t^3 + a_4 t^4 + \dots$$

   . Suppose the the Maclaurin series of $(v(t))^2$ is

   $$(v(t))^2 = b_0 + b_1 t + b_2 t^2 + b3t^3 + b_4 t^4 + \dots$$

   Express b_0, b_1, b_2, b_3 and b_4 in terms of the a_i's. **[3 marks]**
6. Hence find the series solution of $v(t)$, up to and including the t^4 term, to the differential equation $v'(t) = t - (v(t))^2$ with initial condition $v(0) = 1$. **[3 marks]**

That was pretty tedious! Using the theory of Taylor series, we can find the coefficients in another way.

7. From the initial condition $v(0) = 1$, use the differential equation to find $v'(0)$. **[1 mark]**
8. Implicitly differentiate the equation $v'(t) = t-(v(t))^2$ with respect to t to obtain a differential equation involving $v''(t)$, and hence fine $v''(0)$. **[3 marks]**
9. Use the same method of differentiating the equation to find $v'''(0)$ and $v''''(0)$. **[3 marks]**
10. Hence write down the Taylor series for $v(t)$ at $t = 0$ up to and including the t^4 term. **[2 marks]**

Compare your answer with the previous method, you should have the same series for $v(t)$!

There is also 1 additional mark given for the quality of the English and 1 additional mark for correct mathematical notation. These marks are easy to obtain but the markers will be instructed to be strict in awarding these marks.

[1] For low speeds, the drag force is actually proportional to just the speed itself, not the square, but we will ignore this in this workshop for simplicity.