# Eng. Math

## ENG 1005

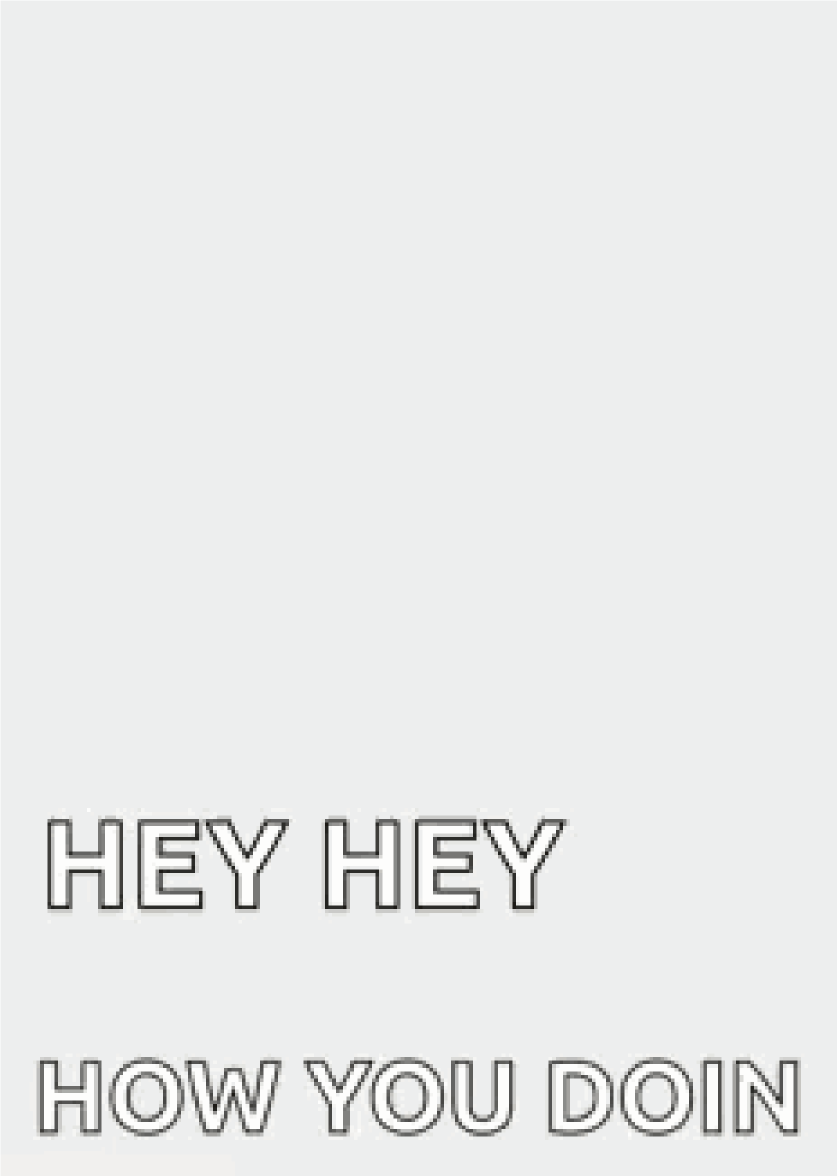
## Week 11: Series 1


**(MEC) Senior Lecturer: K.B. Goh, Ph.D.**

**Tutor: (a) Ian Keen & (b) Jack**

**Pass Leader: (i) Zi Wei and (ii) Yvonne**

**kekboon.goh@monash.edu**




***Check-In***

***50%***

# Topics

| Week | Topic |
| --- | --- |
| 1 | Vectors, Lines, and Planes |
| 2 | Systems of Linear Equations |
| 3 | Matrices |
| 4 | Eigenvalues & Eigenvectors |
| 5 | Multivariable Calculus 1 |
| 6 | Multivariable Calculus 2 |
| 7 | Integration techniques and hyperbolic functions |
| 8 | O.D.E 1 |
| 9 | O.D.E 2 |
| 10 | O.D.E 3 |
| 11 | Series 1 |
| 12 | Series 2 |

# The Big Learning Outcomes for Week 11

***After completing this week's task, you should be able to:***

- Do basic manipulations of sequences and series.
- Use the ratio test to determine convergence of series and radii of convergence.
- Manipulate general power series.
- Use power series to solve ODEs.
- Find Maclaurin series of given functions.

# Attendance Codes (Week 11)

## *International students*

| | | | | |
|---|---|---|---|---|
| Tutorial | Wednesday, 9 Oct | 02 | 8:00AM | 4QWEG |
| Tutorial | Wednesday, 9 Oct | 01 | 2:00PM | 4L7QX |

| | | | | |
|---|---|---|---|---|
| Workshop | Thursday, 10 Oct | 01 | 1:00PM | 2YTN2 |
| Workshop | Friday, 11 Oct | 02 | 10:00AM | 8UPUK |

# Admin. Stuff (1)

1. Start your revision.

2. Submissions:

## Summary

| ASSESSMENT | DUE |
|---|---|
| Kick Starting Week 3: Lecture Quiz 3 (Total mark for all 12 weeks of lecture quizzes is 5%) | Tuesday, 6 August 2024, 11:55 PM **Due in 4 days** |
| Applied class quiz week 3 (Total mark for all 12 weeks of applied quizzes is 5%) | Wednesday, 7 August 2024, 11:55 PM **Due in 5 days** |
| Workshop 3 problem set (Total mark for all 12 weeks of workshop sets is 20%) | Sunday, 11 August 2024, 11:55 PM |

# Admin. Stuff (2)

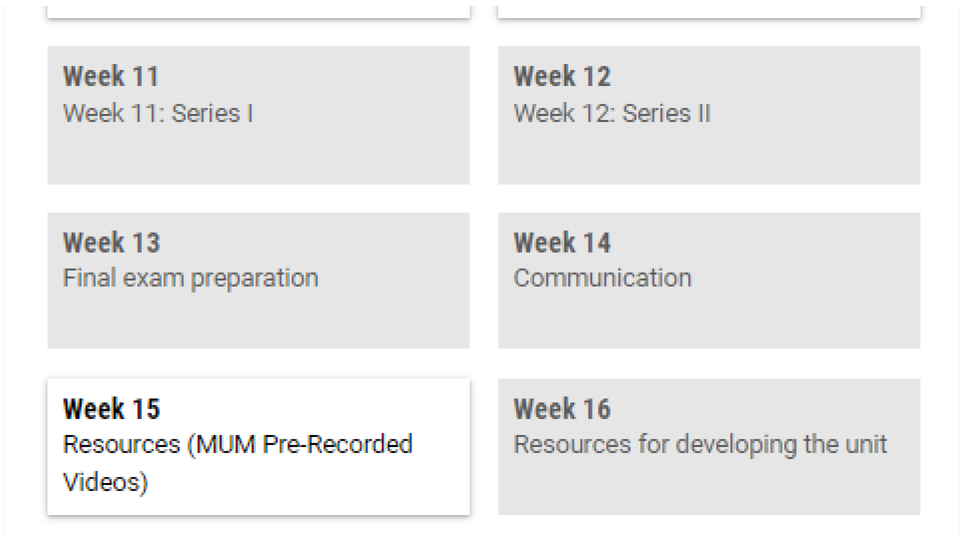
## 3. Consultation/Feedback hour

- Wed: 10 am till 11 am } Location: 9-4-01
- Fri: 8 am till 9 am } Location: 5-4-68

(every odd week, e.g. 3-Week, but msg me first at least a day before; appointment-based.)

# Resources

1. Additional: Videos




2. Pass materials

# Let us start!

# What is Taylor Series?

## Taylor Series

The Taylor series of a function $f(x)$ around a point $x = a$ is given by:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \dots$$

or equivalently in summation form:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n,$$

# Try this Taylor Series: $\sin(x-3)$ (5min)

a=0

To expand $f(x) = \sin(x-3)$ in a Taylor series around $x = 0$, we

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 +$$

Step-by-Step Calculation

- **Step 1: Calculate $f(0)$**

$$f(0) = \sin(0-3) = -\sin(3)$$

- **Step 2: First derivative**

$$f'(x) = \cos(x-3)$$
$$f'(0) = \cos(0-3) = \cos(3)$$

- **Step 3: Second derivative**

$$f''(x) = -\sin(x-3)$$
$$f''(0) = -\sin(0-3) = \sin(3)$$

- **Step 4: Third derivative**

$$f'''(x) = -\cos(x-3)$$
$$f'''(0) = -\cos(0-3) = -\cos(3)$$

- **Step 5: Fourth derivative**

$$f''''(x) = \sin(x-3)$$
$$f''''(0) = \sin(0-3) = -\sin(3)$$

- **Step 6: Fifth derivative**

$$f'''''(x) = \cos(x-3)$$
$$f'''''(0) = \cos(0-3) = \cos(3)$$

Taylor Expansion

Substituting these values into the Taylor series formula:

$$\begin{aligned} f(x) &= -\sin(3) + \cos(3)x + \frac{\sin(3)}{2!}x^2 - \frac{\cos(3)}{3!}x^3 - \frac{\sin(3)}{4!}x^4 + \frac{\cos(3)}{5!}x^5 + \cdots \\ &= -\sin(3) + x\cos(3) + \frac{1}{2}x^2\sin(3) - \frac{1}{6}x^3\cos(3) - \frac{1}{24}x^4\sin(3) + \frac{1}{120}x^5\cos(3) + \cdots \end{aligned}$$

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \ldots$$

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# Today's Activity

1. Applied Problem Set

2. Applied Quiz

- ✓ Q2
- ✓ Q3
- ✓ Q4
- ✓ Q6
- ✓ Q8

# Question 2

2. Consider the sequence

$$\frac{15}{2}, \quad \frac{45}{8}, \quad \frac{135}{32}, \quad \frac{405}{128}, \quad \frac{1215}{512}, \quad \ldots$$

(a) What is the expression for the $n$th term in the sequence $a_n$, assuming the sequence starts at $a_0$?

(b) Does the *series* obtained by adding the terms of the sequence, $\sum_{n=0}^{\infty} a_n$, converge or diverge?

Learning Outcomes?

Series trend!

1

$$\underbrace{\frac{15}{2}, \frac{45}{8}, \frac{135}{32}, \frac{405}{128}, \frac{1215}{512}, \ldots =}$$

Let's find a trend

2

$$\left(\frac{15}{2}\right)\left(1, \frac{3}{4}, \frac{9}{16}, \frac{27}{64}, \ldots\right)$$

Can you see it yet?

3

$$\left(\frac{15}{2}\right)\left\{\left(\frac{3}{4}\right)^0, \left(\frac{3}{4}\right)^1, \left(\frac{3}{4}\right)^2, \left(\frac{3}{4}\right)^3 \ldots\right\} \underbrace{=}$$

4

$$\sum_{n=0}^{\infty}\left(\frac{15}{2}\right)\left(\frac{3}{4}\right)^n$$

Write in series format

$$\frac{15}{2}, \quad \frac{45}{8}, \quad \frac{135}{32}, \quad \frac{405}{128}, \quad \frac{1215}{512}, \quad \ldots$$

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# Diverge or Converge?

$$\sum_{n=0}^{\infty}\left(\frac{15}{2}\right)\underbrace{\left(\frac{3}{4}\right)^{n}}$$

Smaller than unity/1

Converge!

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# Question 3

3. Consider the IVP

$$y'' - xy' + y^2 = 1$$

subject $y(0) = 1$ and $y'(0) = 6$. Find a series solution up to and including $x^4$.

## Learning Outcomes?

Series to solve ODE

# Let us assume this:

$$y(x) = A + Bx + Cx^2 + Dx^3 + Ex^4 + \cdots$$

But why 5 individual terms?

- Rule of thumb, odd number term
- So, it can be 1, 3 , 5,….

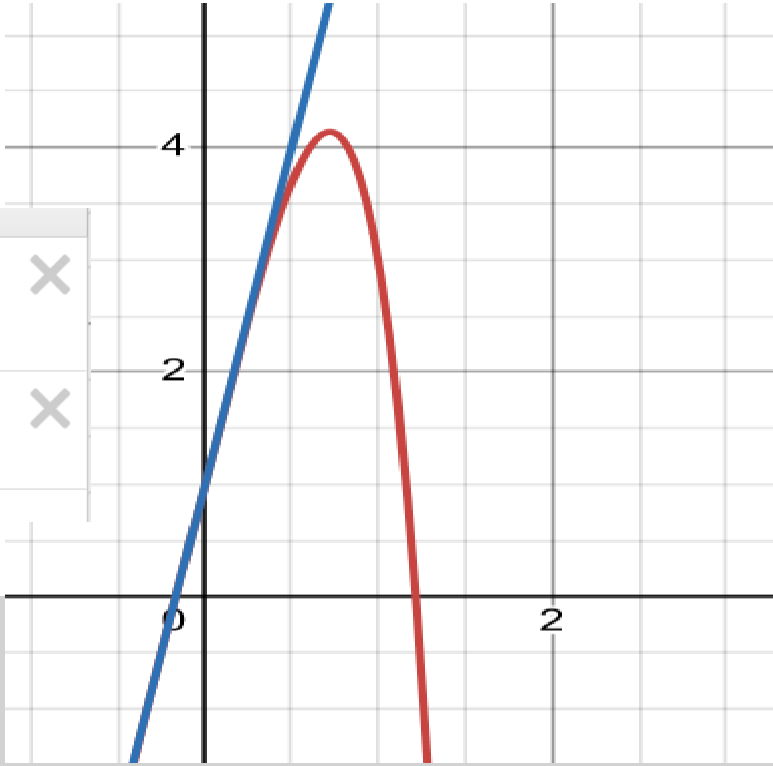
- You Try It! (10 mins)

➢ Lets us compare: 5 vs. 3 terms

$$y(x) = A + Bx + Cx^2 + Dx^3 + Ex^4$$

$$y(x) = A + Bx + Cx^2$$

1. $y = 1 + 6x - x^3 - 3x^4$
2. $y = 1 + 6x$
3.




$$A = 1, \quad B = 6, \quad C = 0, \quad D = -1, \quad E = -3$$

# Why don’t we try 7, 9 or 11 terms?

$$y(x) = A + Bx + Cx^2 + Dx^3 + Ex^4 + \cdots$$

# Question 4

4. (a) Show that the series expansions for $\sin(x)$ and $\cos(x)$ are

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

Learning Outcomes?

Series Trend

- You Try!

# Question 6

Find the radius of convergence of the following power series

$$g(x) = \sum_{n=0}^{\infty} \frac{n!(x-1)^n}{2^n n^n} \quad \left[\text{Hint: use } \lim_{n\to\infty}\left(1+\frac{t}{n}\right)^n = e^t\right]$$

$$\boxed{L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|} \quad \begin{cases} \text{series converges if } L < 1, \\ \\ \text{diverges if } L > 1 \end{cases}$$

- You Try!

❑ Useful

$$\frac{(n+1)!}{n!} = \frac{(n+1)\cdot n!}{n!}$$

# Question 8(a)

8. This question considers the series solution about $x = 0$ of the ODE

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \lambda y = 0$$

where $\lambda$ is a constant.

(a) By expanding as $y = \hat{a}_0 + \hat{a}_1 x + \hat{a}_2 x^2 + \hat{a}_3 x^3 + \hat{a}_4 x^4 + \ldots + \hat{a}_n x^n + \ldots$ and substituting into the ODE, find expressions for $\hat{a}_2$ and $\hat{a_4}$ in terms of $\hat{a}_0$ and $\hat{a_3}$ in terms of $\hat{a}_1$.

(b) By equating coefficients of terms in $x^{n-2}$, find an expression for $\hat{a_n}$ in terms of $\hat{a}_{n-2}$.

- Step 1: Compute the derivatives

- Step 2: Sub. them into the ODE

- Step 3: Equate the same x power

- Step 4: General Solutions.

Click Me!

# Question 8(b)

8. This question considers the series solution about $x = 0$ of the ODE

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \lambda y = 0$$

where $\lambda$ is a constant.

(a) By expanding as $y = \hat{a}_0 + \hat{a}_1 x + \hat{a}_2 x^2 + \hat{a}_3 x^3 + \hat{a}_4 x^4 + \ldots + \hat{a}_n x^n + \ldots$ and substituting into the ODE, find expressions for $\hat{a}_2$ and $\hat{a_4}$ in terms of $\hat{a}_0$ and $\hat{a_3}$ in terms of $\hat{a}_1$.

(b) By equating coefficients of terms in $x^{n-2}$, find an expression for $\hat{a_n}$ in terms of $\hat{a}_{n-2}$.

- Back to the QR Code and See Step 2

$$n(n-1)\hat{a}_n - 2(n-2)\hat{a}_{n-2} + \lambda\hat{a}_{n-2} = 0 \implies \hat{a}_n = \frac{2(n-2)-\lambda}{n(n-1)}\hat{a}_{n-2}$$

# Thank You

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# Admin. Stuff (1)

1. Revision, Revision, Revision, Please, Please, Please!

2. Feel Free SPEC. CON.

3. Submissions:

## Summary

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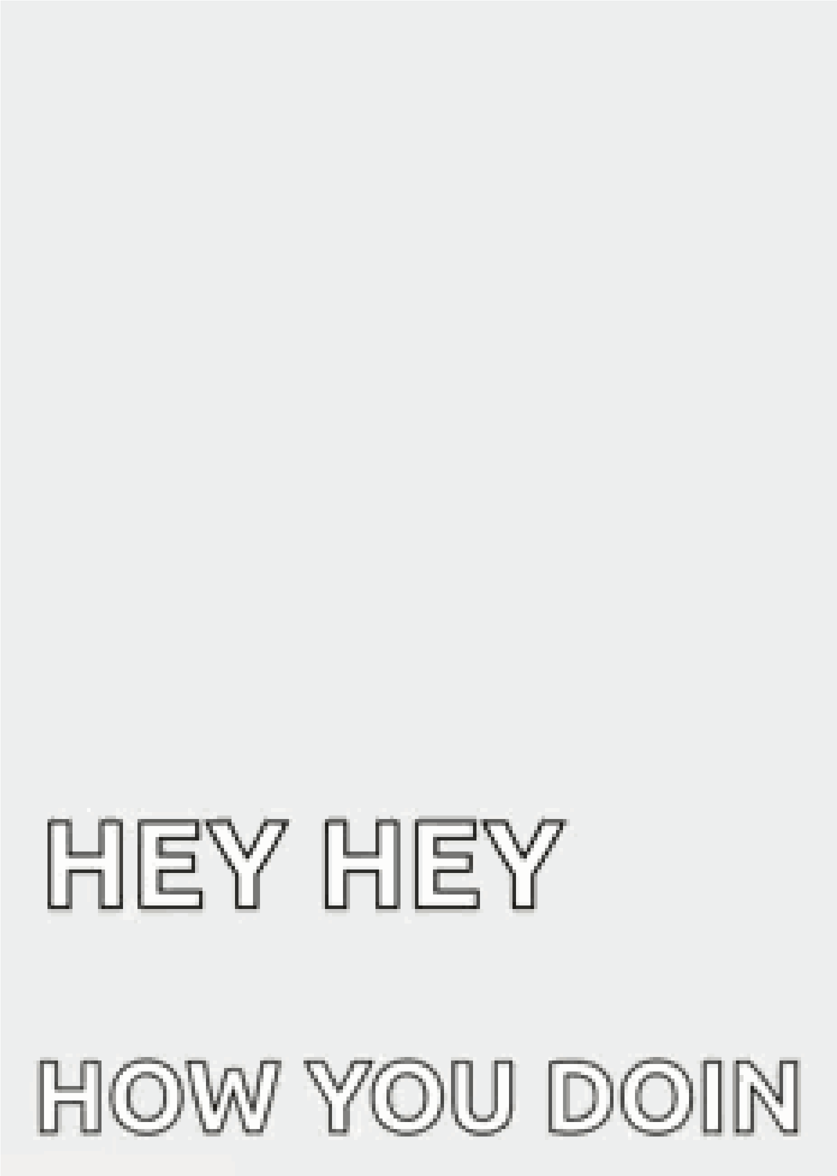
# Resources

## 1. Additional: Videos

| | |
|---|---|
| **Week 11**<br>Week 11: Series I | **Week 12**<br>Week 12: Series II |
| **Week 13**<br>Final exam preparation | **Week 14**<br>Communication |
| **Week 15**<br>Resources (MUM Pre-Recorded Videos) | **Week 16**<br>Resources for developing the unit |

## 2. Pass materials




# *Check-In*

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Let us start!

# The Big Learning Outcomes for Week 11

***After completing this week's task, you should be able to:***

- Do basic manipulations of sequences and series.
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# Today’s Activity

1. Workshop Problem Set

## Bacteria Growth

Taylor and Maclaurin series are very power tools in the study of differential equations. In this workshop we will use ODE's to find a formula for the $n$-th term in a recurrence relation.

A bacteria culture test is commonly used to identify the bacteria responsible for an infection. A small sample containing bacteria cells is allowed to grow and multiply until there are enough cells to make an accurate identification. A particular bacteria divides once per day when a mature cell splits off a new cell in a process called mitosis. This particular bacteria has a maturation delay of one day, meaning that a newly divided out cell will not immediately start splitting off a cell the next day, but instead waits a day and only starts to split two days later.

Suppose there is 1 immature cell in a Petri dish in day 0. Then in day 1, the cell would mature but would not divide yet, so there is still only 1 cell in the dish. In day 2, the mature cell will divide and now there are 2 cells, 1 mature and 1 immature.


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1. Find the numbers of cells in the dish in day 3, 4 and 5. Can you see a pattern in the sequence?

**[2 marks]**

| Day | Immature Cell | Mature Cell | Total Cell |
|---|---|---|---|
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 2 | 1 | 1 | 2 |
| 3 | 1 | 1+1=2 | 3 |
| 4 | 2 | 2+1=3 | 5 |
| 5 | 3 | 3+2=5 | 8 |

Suppose there is 1 immature cell in a Petri dish in day 0. Then in day 1, the cell would mature but would not divide yet, so there is still only 1 cell in the dish. In day 2, the mature cell will divide and now there are 2 cells, 1 mature and 1 immature.

Let $M_n$ denote the number of mature cells in day $n$, and $I_n$ the number of immature cells. Express $M_{n+1}, M_{n+2}, I_{n+1}$ and $I_{n+2}$ in terms of $M_n$ and $I_n$. **[2 marks]**

You know that:

| Day | Immature Cell | Mature Cell | Total Cell |
|---|---|---|---|
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 2 | 1 | 1 | 2 |
| 3 | 1 | 1+1=2 | 3 |
| 4 | 2 | 2+1=3 | 5 |
| 5 | 3 | 3+2=5 | 8 |

Mature Trend:

$$M_{n+1} = M_n + I_n$$

$$M_{n+2} = 2M_n + I_n$$

Immature Trend:

$$I_{n+1} = M_n$$

$$I_{n+2} = M_n + I_n$$

3. Hence show that the total number of cells in day $n$ satisfies the recurrence relation

$$T_{n+2} = T_{n+1} + T_n$$

$$T_{n+2} = M_{n+2} + I_{n+2} = (2M_n + I_n) + (M_n + I_n)$$

$$=$$

$$T_{n+1} = M_{n+1} + I_{n+1} = (M_n + I_n) + (M_n)$$

$$+$$

$$T_n = M_n + I_n$$

$$M_{n+1} = M_n + I_n$$
$$I_{n+1} = M_n$$

$$M_{n+2} = 2M_n + I_n$$
$$I_{n+2} = M_n + I_n$$

$\boxed{\text{Show that } T_{n+1} \leq 2T_n.}$

From the previous slide, you know that:

$$T_{n+1} = 2M_n + I_n$$

$$T_n = M_n + I_n$$

$$\boxed{2M_n + I_n \leq 2(M_n + I_n)}$$

# Q5: Lesson Learned

Get More Sleep!

From the sequence of numbers $T_0, T_1, T_2, \ldots$ we construct a power series

$$f(x) = \sum_{i=0}^{\infty} \frac{T_i}{i!} x^i$$

Find the radius of convergence of $f(x)$.

$f(x)$ is known as the *exponential generating function* of the sequence $\{T_i, i = 0, 1, 2, \ldots\}$.

Let us do ratio test!

$$a_{n+1} = \frac{T_{n+1}}{(n+1)!} x^{n+1}$$

$$a_n = \frac{T_n}{n!} x^n$$

$$\frac{a_{n+1}}{a_n} = \frac{T_{n+1} x^{\cancel{n}+1}}{(n+1)\cancel{!}} \times \frac{\cancel{n}!}{T_n \cancel{x^n}}$$

$$= \frac{x}{n+1} \frac{T_{n+1}}{T_n} = \frac{2x}{n+1}$$

The radius is inf.

$$x \lim_{n \to \infty} \frac{2}{n+1} = 0 = \frac{a_{n+1}}{a_n}$$

(very small) $a_{n+1} \ll a_n$ (very big)

Show that $T_{n+1} \leq 2T_n$.

6. Show that $f(x)$ satisfies the differential equation

$$f''(x) - f'(x) - f(x) = 0$$

**Step 1: Define the Function**

Let:

$$f(x) = \sum_{i=0}^{\infty} \frac{T_i}{i!} x^i$$

Write the function in expanded form:

$$f(x) = T_0 + T_1 x + \frac{1}{2} T_2 x^2 + \frac{1}{6} T_3 x^3 + \frac{1}{24} T_4 x^4 + \cdots$$

You try with this guide (5mins):

7. What are the values of the initial conditions $f(0)$ and $f'(0)$?

**$f(0)$**

**To find $f(0)$:**

Substitute $x = 0$ into the series expansion of $f(x)$:

$f(x) = T_0 + T_1(0) + \frac{1}{2}T_2(0^2) + \frac{1}{6}T_3(0^3) + \frac{1}{24}T_4(0^4) + \ldots$

This simplifies to:

$f(0) = T_0$

**Using the formula $T_n = M_n + I_n$:**

For $T_0$:

$T_0 = M_0 + I_0 = 1 + 0 = 1$

**Initial Condition:**

$f(0) = 1$

**$f'(0)$**

**To find $f'(0)$:**

Substitute $x = 0$ into the series expansion of $f'(x)$:

$f'(x) = T_1 + T_2(0) + \frac{1}{2}T_3(0^2) + \frac{1}{6}T_4(0^3) + \ldots$

This simplifies to:

$f'(0) = T_1$

**Using the formula $T_n = M_n + I_n$:**

For $T_1$:

$T_1 = M_1 + I_1 = 0 + 1 = 1$

**Initial Condition:**

$f'(0) = 1$

8. Solve the differential equation $f''(x) - f'(x) - f(x) = 0$ with the initial conditions to find $f(x)$.

[4 m

## You try with this guide (10mins):

$$f(x) = \left(\frac{1}{2} + \frac{1}{2\sqrt{5}}\right) e^{\lambda_1 x} + \left(\frac{1}{2} - \frac{1}{2\sqrt{5}}\right) e^{\lambda_2 x}$$

$$\lambda_1 = \frac{1+\sqrt{5}}{2}, \quad \lambda_2 = \frac{1-\sqrt{5}}{2}$$

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9. Use the closed-form solution of $f(x)$ to write down the Maclaurin series of $f(x)$. You may use the series $e^t = \sum_0^\infty \frac{t^i}{i!}$. **[3 marks]**

$$f(n) = \frac{1}{2}\left(1+\frac{1}{\sqrt{5}}\right)e^{\left(\frac{1+\sqrt{5}}{2}\right)x} + \frac{5-\sqrt{5}}{10}e^{\left(\frac{1-\sqrt{5}}{2}\right)x}$$

$$\frac{1}{2}\left(1 \pm \frac{1}{\sqrt{5}}\right) = \frac{5 \pm \sqrt{5}}{10}$$

$$e^t = \sum_{i=0}^{\infty} \frac{t^i}{i!}$$

**When** $t = \left(\frac{1+\sqrt{5}}{2}\right)x$: $e^{\left(\frac{1+\sqrt{5}}{2}x\right)} = \sum_{i=0}^{\infty} \frac{\left(\frac{1+\sqrt{5}}{2}x\right)^i}{i!}$

**When** $t = \left(\frac{1-\sqrt{5}}{2}\right)x$: $e^{\left(\frac{1-\sqrt{5}}{2}x\right)} = \sum_{i=0}^{\infty} \frac{\left(\frac{1-\sqrt{5}}{2}x\right)^i}{i!}$

$$f(x) = \sum_{i=0}^{\infty}\left[\frac{1}{2}\left(1+\frac{1}{\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right)^i + \frac{5-\sqrt{5}}{10}\left(\frac{1-\sqrt{5}}{2}\right)^i\right]\frac{x^i}{i!}$$

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10. Hence find a formula for $T_n$ in terms of $n$.

$$f(x) = \sum_{i=0}^{\infty} \underbrace{\left[ \frac{1}{2}\left(1 + \frac{1}{\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right)^i + \frac{5-\sqrt{5}}{10}\left(\frac{1-\sqrt{5}}{2}\right)^i \right]}_{T_n} \frac{x^i}{i!}$$

From the sequence of numbers $T_0, T_1, T_2, \ldots$ we construct a power series

$$f(x) = \sum_{i=0}^{\infty} \frac{T_i}{i!} x^i$$

$f(x)$ is known as the *exponential generating function* of the sequence $\{T_i, i = 0, 1, 2, \ldots\}$.

# 11. What is the behaviour of $T_n$ as $n$ approaches infinity?

$$\left[\frac{1}{2}\left(1+\frac{1}{\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right)^i + \frac{5-\sqrt{5}}{10}\left(\frac{1-\sqrt{5}}{2}\right)^i\right]$$

$$T_{n\to\infty} \to \infty$$

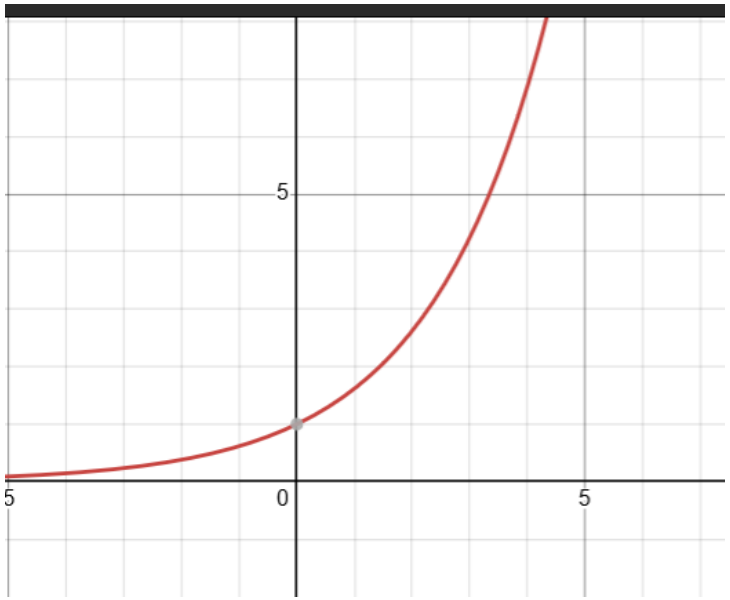
Replace i=n

goes inf.

goes kaput ?

$$y_1 = \left(\frac{\left(1+5^{0.5}\right)}{2}\right)^x$$

$$y_2 = \left(\frac{\left(1-5^{0.5}\right)}{2}\right)^x$$

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12. If you need 10000 cells to identify the bacteria, how many days do you have to wait? You should use the (very accurate) approximation from previous question. [1 marks]

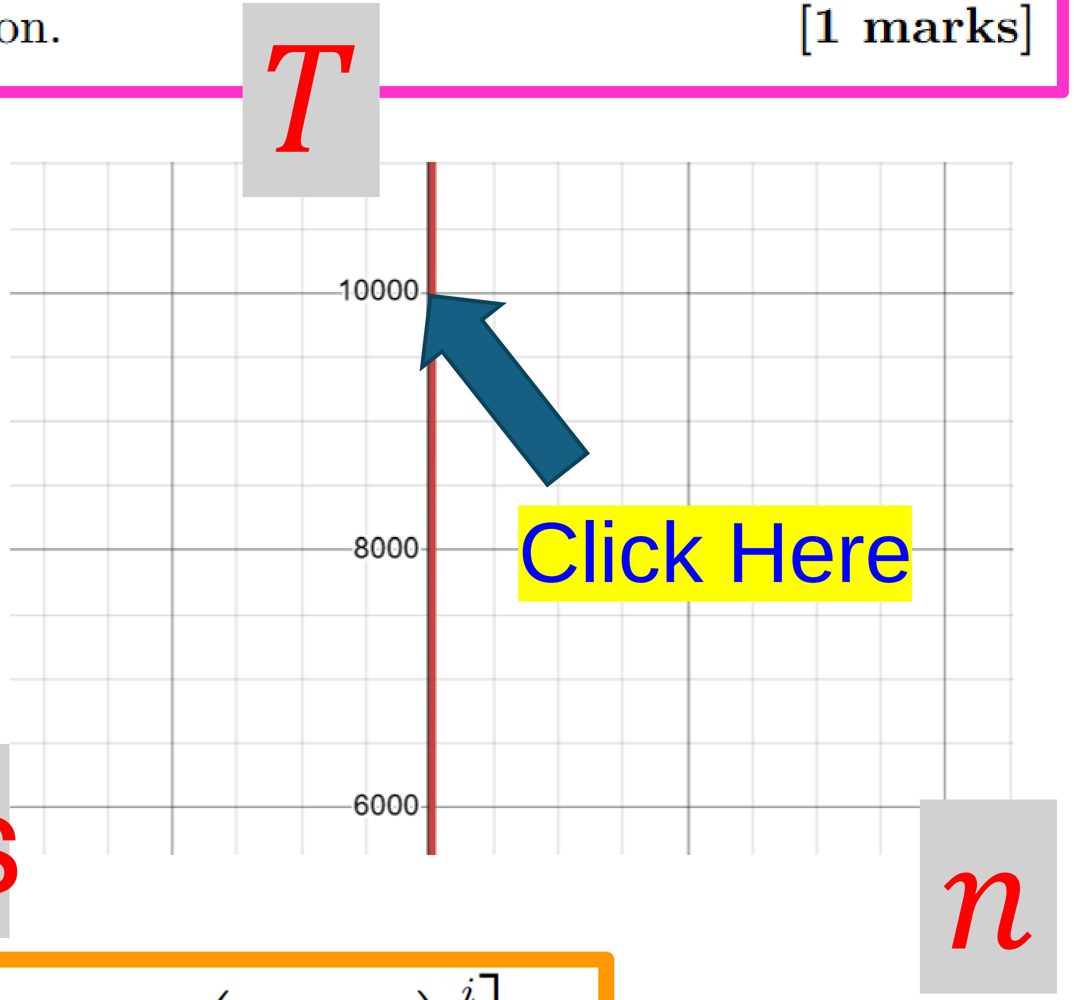
$$T_n = \frac{1}{2}\left(1+\frac{1}{\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right)^n = 10000$$

1

$$T = \frac{1}{2}\left(1+\frac{1}{\sqrt{5}}\right)\left(\frac{(1+\sqrt{5})}{2}\right)^n$$

2

$$n = 19.8 \approx 20 \text{ days}$$




$$f(x) = \sum_{i=0}^{\infty}\left[\frac{1}{2}\left(1+\frac{1}{\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right)^i + \frac{5-\sqrt{5}}{10}\left(\frac{1-\sqrt{5}}{2}\right)^i\right]\frac{x^i}{i!}$$

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# Thank You