MONASH University

Eng. Math

ENG 1005

Week 5: Multivariable Calculus 1

(MEC) Senior Lecturer: K.B. Goh, Ph.D.

Tutor: (a) Ian Keen & (b) Jack

Pass Leader: (i) Zi Wei and (ii) Yvonne

kekboon.goh@monash.edu

Check-In

Topics

Week	Topic
1	Vectors, Lines, and Planes
2	Systems of Linear Equations
3	Matrices
4	Eigenvalues & Eigenvectors
5	Multivariable Calculus 1
6	Multivariable Calculus 2
7	Integration techniques and hyperbolic functions
8	O.D.E 1
9	O.D.E 2
10	O.D.E 3
11	Series 1
12	Series 2

The Big Learning Outcomes for Week 5

After completing this week's task, you should be able to:

- Find contours of functions of two variables.
- Understand and calculate partial/directional derivatives and gradient of functions of several variables.
- Calculate tangent planes.
- Calculate higher derivatives of functions of several variables.

Attendance Codes (Week 5)

International students

Tutorial	Wednesday, 21 Aug	02	8:00AM	RLBQV
Tutorial	Wednesday, 21 Aug	01	2:00PM	8GMZF

Workshop	Thursday, 22 Aug	01	1:00PM	WB3EN
Workshop	Friday, 23 Aug	02	10:00AM	DQDF5

Admin. Stuff (1)

1. Feedback on Workshop Submission: On Thur/Fri

2. Submissions:

Summary

ASSESSMENT	DUE
Kick Starting Week 3: Lecture Quiz 3 (Total mark for all 12 weeks of lecture quizzes is 5%)	Tuesday, 6 August 2024, 11:55 PM **Due in 4 days**
Applied class quiz week 3 (Total mark for all 12 weeks of applied quizzes is 5%)	Wednesday, 7 August 2024, 11:55 PM **Due in 5 days**
Workshop 3 problem set (Total mark for all 12 weeks of workshop sets is 20%)	Sunday, 11 August 2024, 11:55 PM

Admin. Stuff (2)

3. Consultation/Feedback hour

- Wed: 10 am till 11 am
- Fri: 8 am till 9 am
- Sat: 1030 am till 1130 am

Location: 5-4-68

Appointment Based

(every odd week, e.g. 3-Week, but msg me first at least a day before; appointment-based.)

Resources

1. PASS with Yvonne and Zi Wei
2. Mid-Term Mock Exams! Week 6
3. Additional: Videos (Mid-Term Prep.)

Resources (update comes every Fri.)

3. Additional: Videos (Mid-Term Prep)

Let us start!

Challenging topic! (Q8)

Today’s Activity

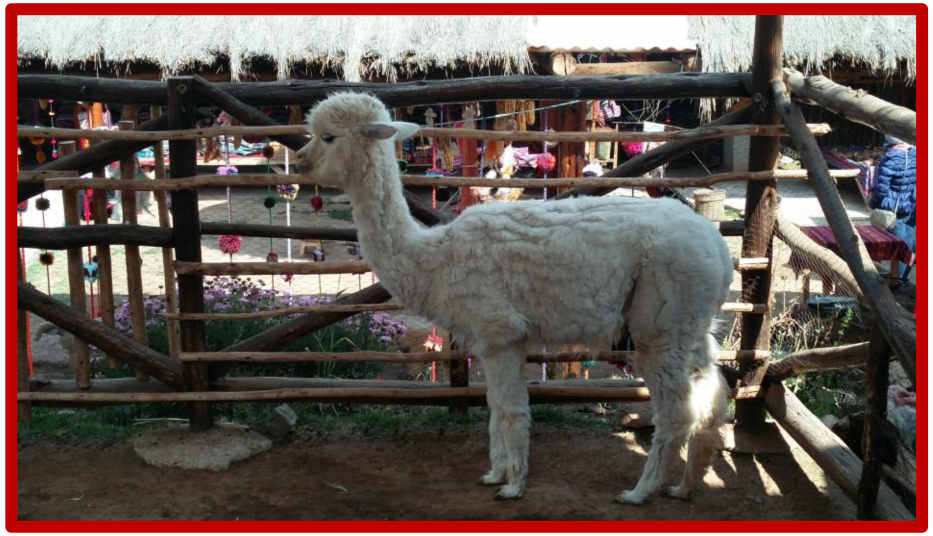
0. Refresher!

1. Applied Problem Set

2. Applied Quiz

Some historical structures!

Inca Terrance: Peru (2018)

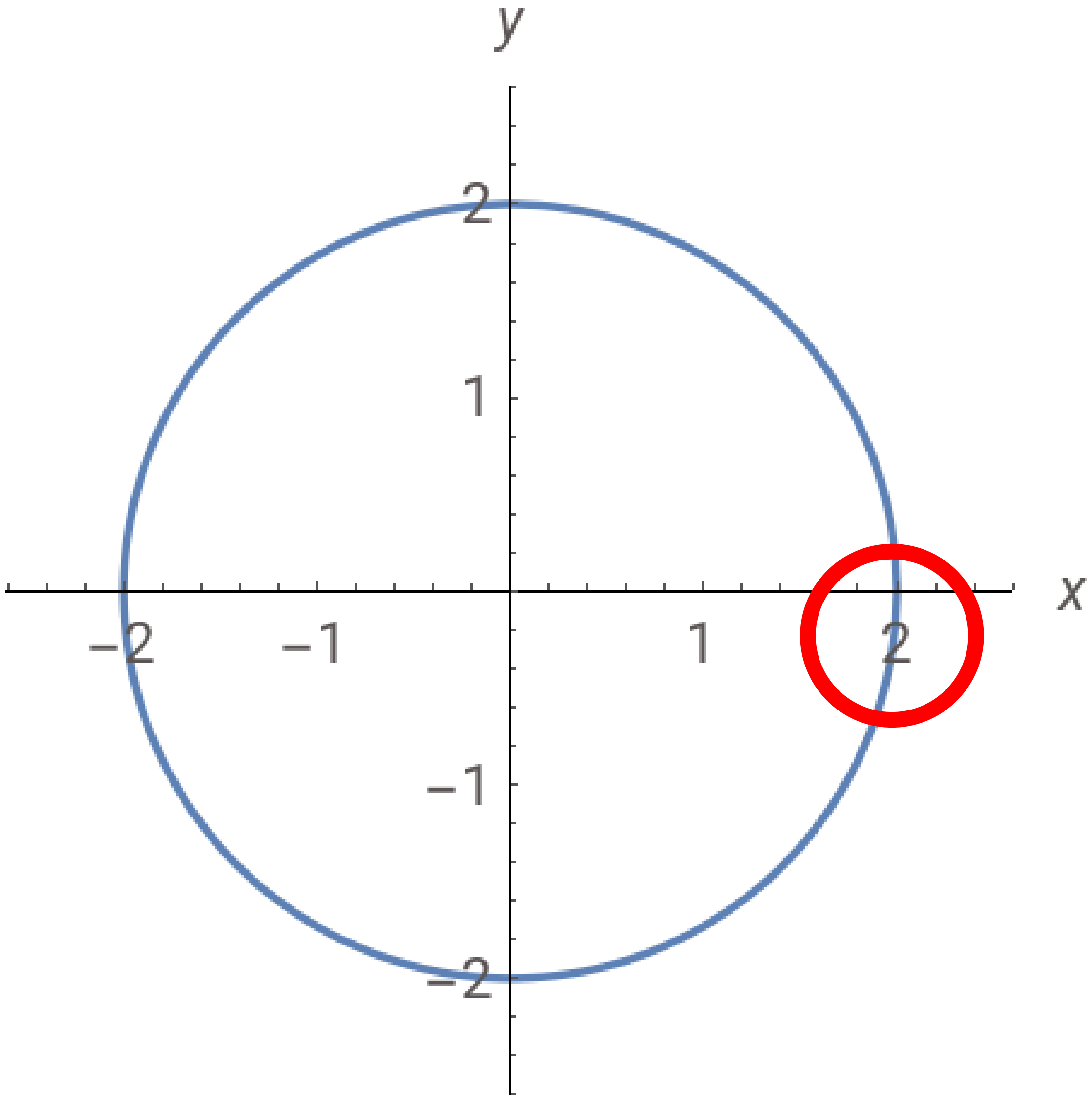


Ephesus: Türkiye (2019)

Let us start with the Maths!

Refresher! Can you sketch this expression?

$$x^2 + y^2 = 2^2$$



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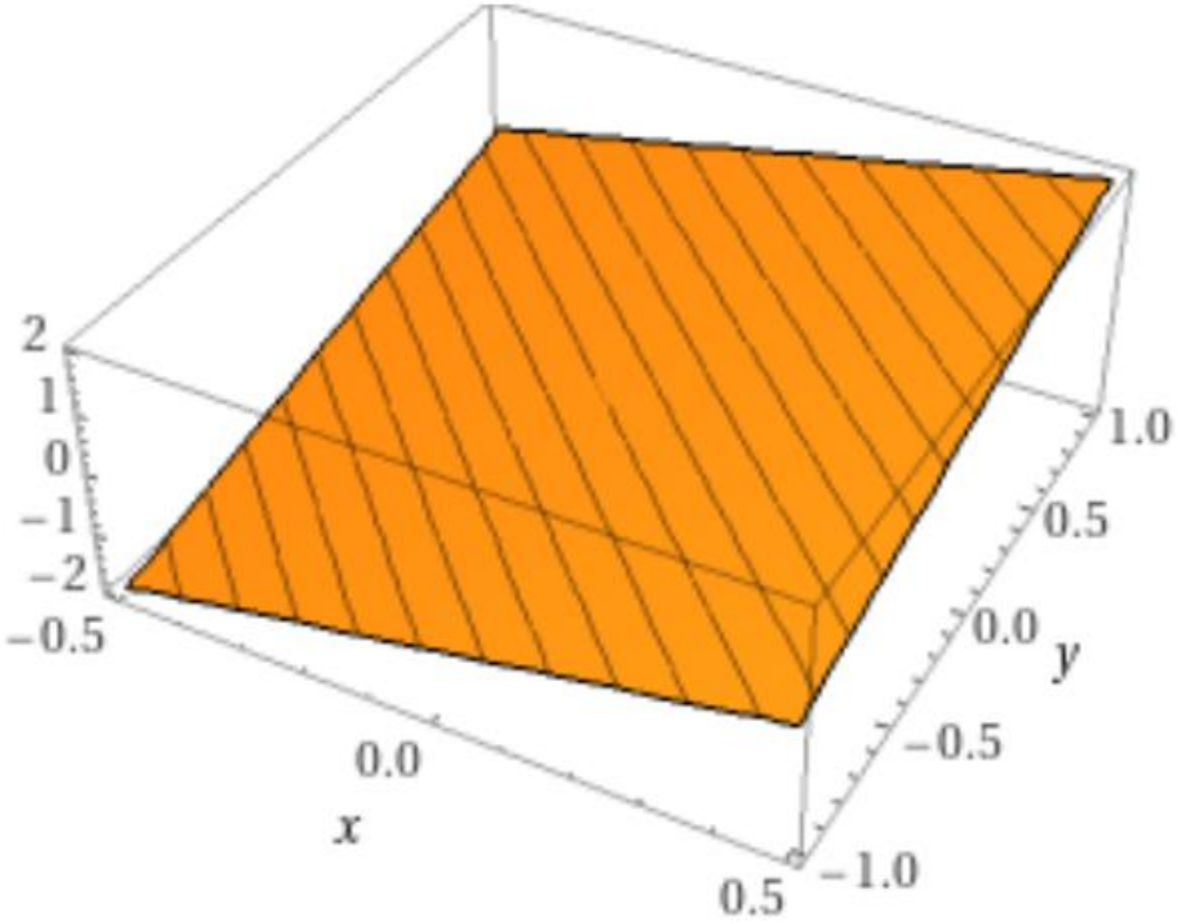
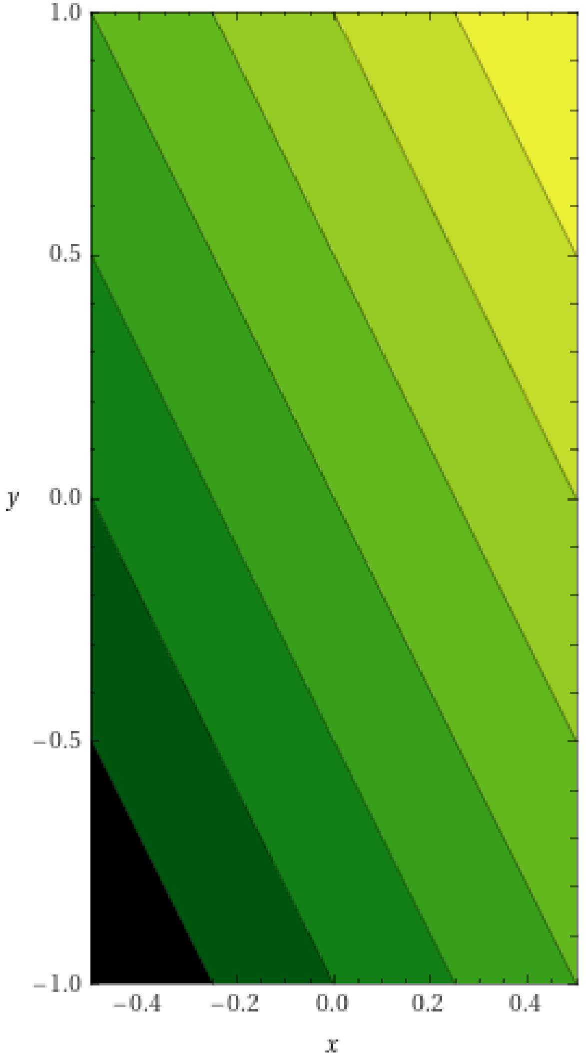
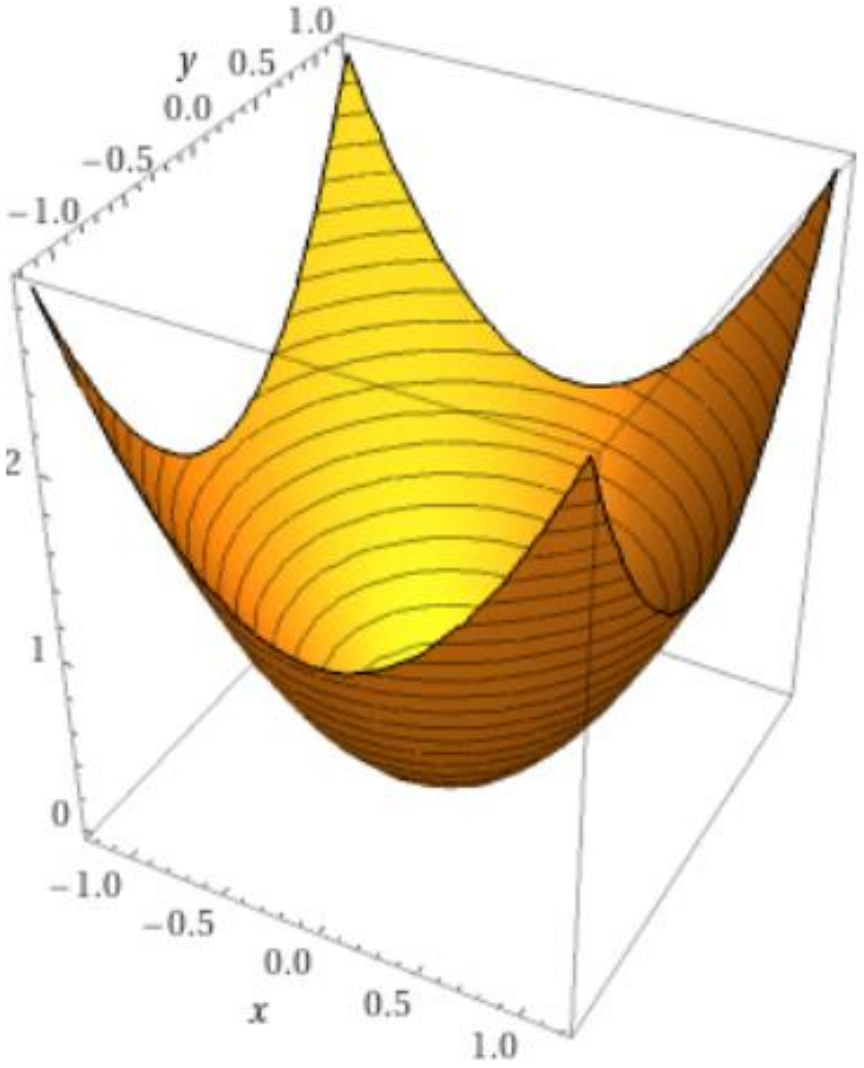
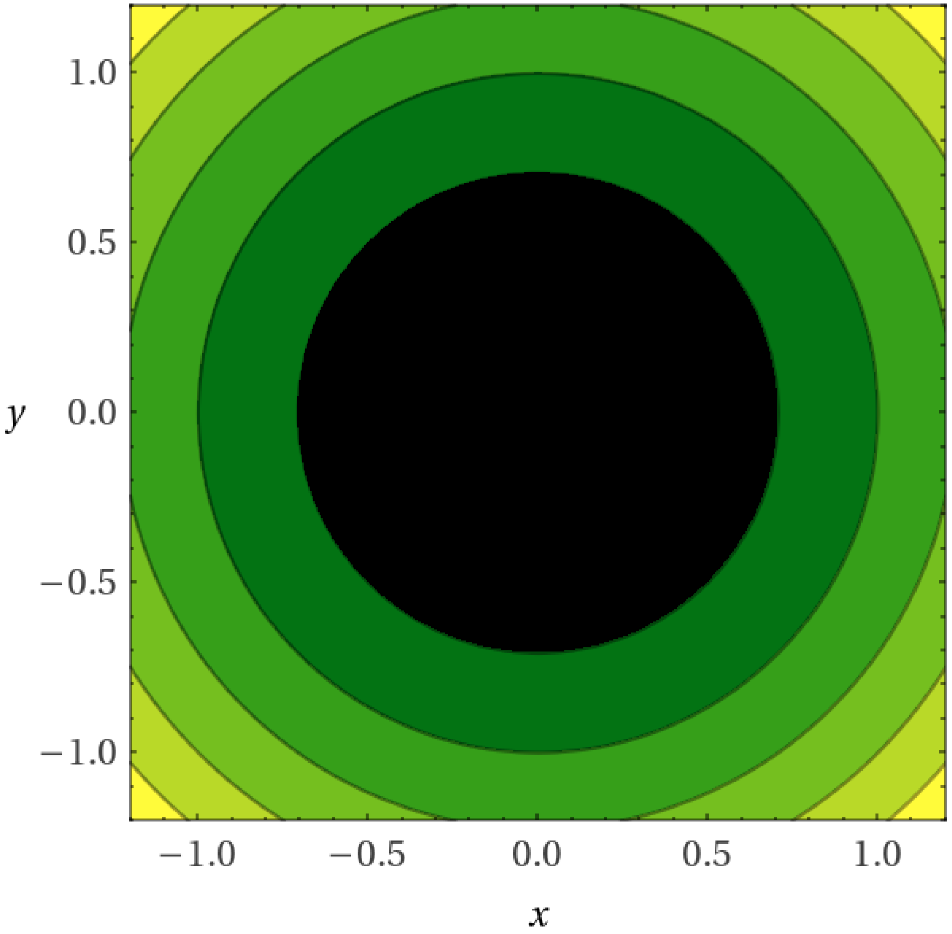
$$z = 2x - y$$



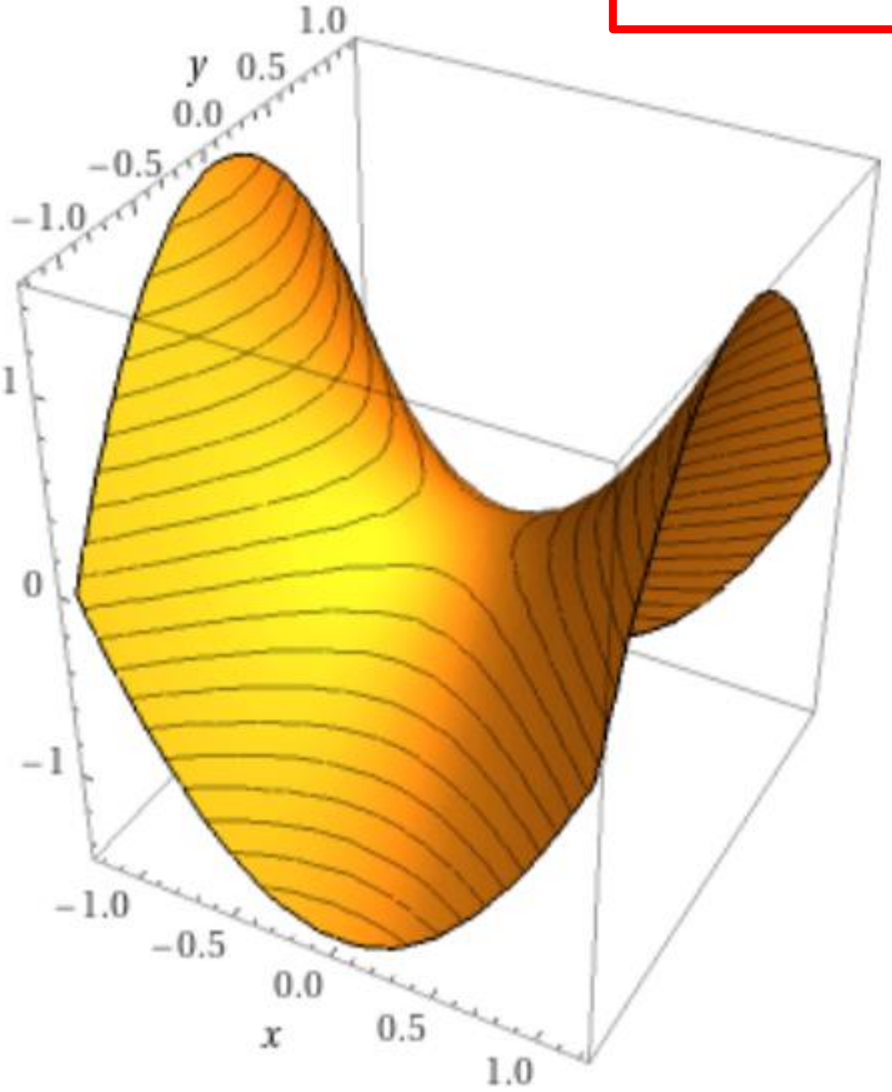


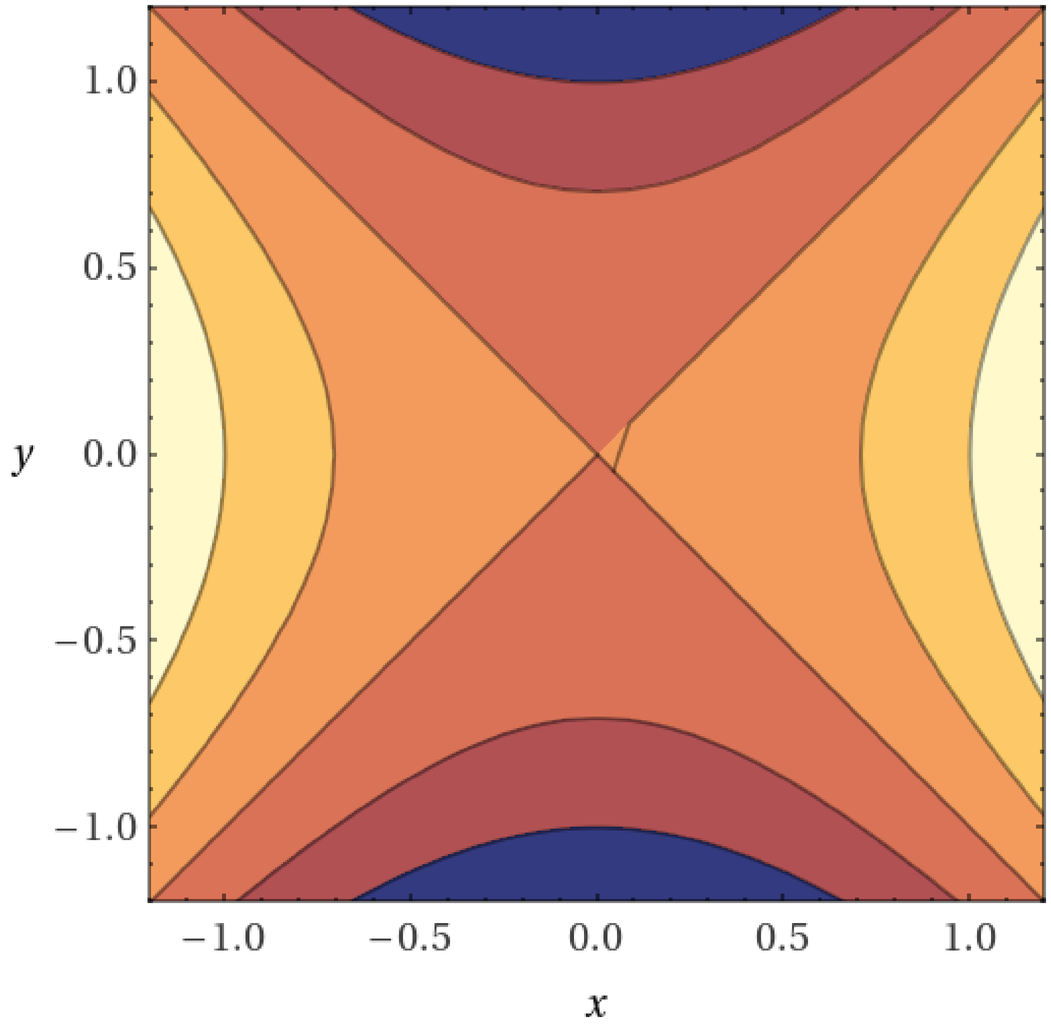
$$z = x^2 + y^2$$





$$z = x^2 - y^2$$





Saddle Plot!

Today's Activity

0. Refresher!

1. Applied Problem Set

2. Applied Quiz

Question 1

1. Consider the function

$$f(x,y) = \sin\sqrt{x^2+y^2}$$

   (a) Sketch the contours with levels 0, ± 1 for this function
   (b) Calculate the partial derivatives of the function, $\partial f/\partial x$ and $\partial f/\partial y$
   (c) Calculate the gradient vector at the point $(\pi/\sqrt{2}, \pi/\sqrt{2})$
   (d) Indicate the direction of the gradient vector at this point on your contour sketch

Learning Outcomes?

1. Draw the contours.
2. Calculate partial/directional derivatives and gradients.

1. Consider the function

$$f(x,y) = \sin\sqrt{x^2+y^2}$$

(a) Sketch the contours with levels 0, ± 1 for this function

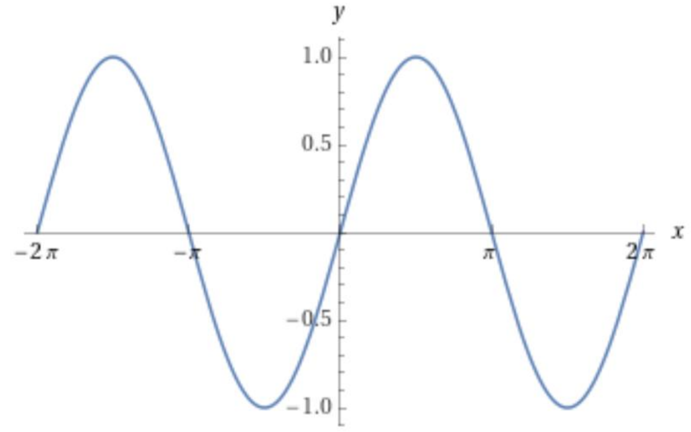
For: $f(x,y) = \sin\left(\sqrt{x^2+y^2}\right)$

We want: $f(x,y) = 0, \pm 1$

Let's start with $f(x,y) = 0$

We can write: $\sin\left(\sqrt{x^2+y^2}\right) = 0$

For 0, pi, 2pi, 3pi,....



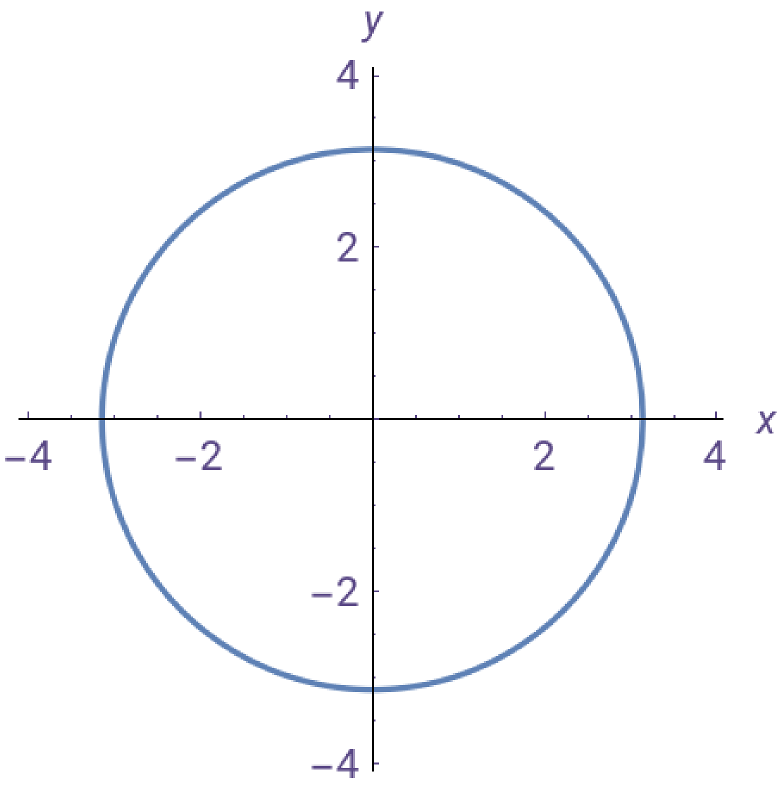
$$\sqrt{x^2+y^2} = n\pi,$$

$$n = 0, 1, 2, \ldots$$

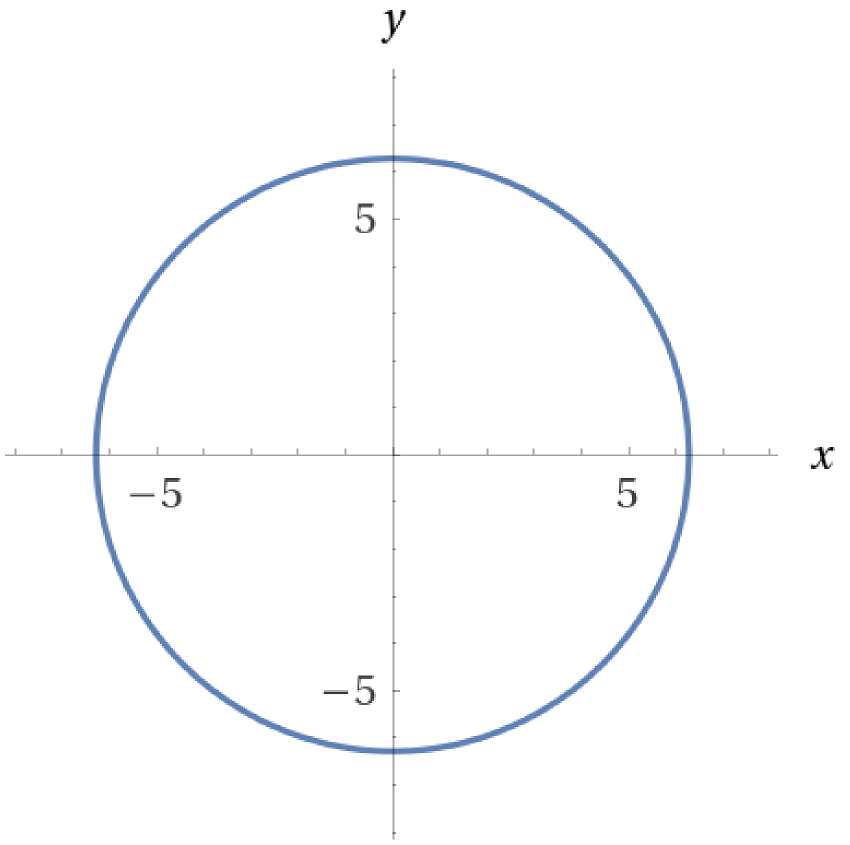
You draw next

$$\boxed{x^2 + y^2 = (n\pi)^2}$$

Let n=1: $x^2 + y^2 = (\pi)^2$



Let n=2: $x^2 + y^2 = (2\pi)^2$



Try at home for:

$$f(x, y) = \pm 1$$

Question 1(b)

1. Consider the function

$$f(x,y) = \sin\sqrt{x^2+y^2}$$

   (a) Sketch the contours with levels 0, ± 1 for this function

   (b) Calculate the partial derivatives of the function, $\partial f/\partial x$ and $\partial f/\partial y$

Let's start with x first, follow by y last.

Are You Camera ready?

With x (i).

Step 1: Invoke Chain Rule: (5 min)

$$\frac{d}{dx}\left(\sin\left(\sqrt{x^2+y^2}\right)\right) = \boxed{\frac{d}{du}}(\sin(u)) \cdot \boxed{\frac{du}{dx}}$$

$\boxed{\text{where } u = \sqrt{x^2+y^2} \text{ and } \frac{d}{du}(\sin(u)) = \cos(u)\text{:}}$

$$= \cos\left(\sqrt{x^2+y^2}\right) \cdot \boxed{\frac{d}{dx}\left(\sqrt{x^2+y^2}\right)}$$

Step 2: Invoke Chain Rule Again (2 mins):

$$\boxed{\frac{d}{dx}\left(\sqrt{x^2+y^2}\right)} = \frac{d}{du}\left(\sqrt{u}\right) \cdot \frac{du}{dx}$$

where $u = x^2+y^2$ and $\frac{d}{du}\left(\sqrt{u}\right) = \frac{1}{2\sqrt{u}}$:

$$= \frac{\frac{d}{dx}\left(x^2+y^2\right)}{2\sqrt{x^2+y^2}} \cdot \cos\left(\sqrt{x^2+y^2}\right)$$

$$\boxed{f(x,y) = \sin\sqrt{x^2+y^2}}$$

With x (ii).

Step 3: Differentiate the sum term by term:

$$= \left(\frac{d}{dx}\left(x^2\right) + \underbrace{\frac{d}{dx}\left(y^2\right)}_{\text{Vanishes}} \right) \cdot \frac{\cos\left(\sqrt{x^2+y^2}\right)}{2\sqrt{x^2+y^2}}$$

Step 4: Simplifying it:

Answer: $$\frac{x \cdot \cos\left(\sqrt{x^2+y^2}\right)}{\sqrt{x^2+y^2}}$$

$$= \frac{\frac{d}{dx}\left(x^2+y^2\right)}{2\sqrt{x^2+y^2}} \cdot \cos\left(\sqrt{x^2+y^2}\right)$$

With y (i).

Step 1: Invoke Chain Rule:

$$\frac{d}{dy}\left(\sin\left(\sqrt{x^2+y^2}\right)\right) = \frac{d}{du}\left(\sin(u)\right)\cdot\frac{du}{dy}$$

where $u=\sqrt{x^2+y^2}$ and $\frac{d}{du}\left(\sin(u)\right)=\cos(u)$:

$$=\cos\left(\sqrt{x^2+y^2}\right)\cdot \boxed{\frac{d}{dy}\left(\sqrt{x^2+y^2}\right)}$$

Step 2: Invoke Chain Rule Again:

$$\boxed{\frac{d}{dy}\left(\sqrt{x^2+y^2}\right)} = \frac{d}{du}\left(\sqrt{u}\right)\cdot\frac{du}{dy}$$

where $u=x^2+y^2$ and $\frac{d}{du}\left(\sqrt{u}\right)=\frac{1}{2\sqrt{u}}$:

$$=\frac{\frac{d}{dy}\left(x^2+y^2\right)}{2\sqrt{x^2+y^2}}\cdot\cos\left(\sqrt{x^2+y^2}\right)$$

$$f(x,y)=\sin\sqrt{x^2+y^2}$$

With y (ii).

Step 3: Differentiate the sum term by term:

$$= \Bigg(\underbrace{\frac{d}{dy}\left(x^2\right)}_{\text{Vanishes}} + \frac{d}{dy}\left(y^2\right)\Bigg) \cdot \frac{\cos\left(\sqrt{x^2+y^2}\right)}{2\sqrt{x^2+y^2}}$$

Step 4: Simplifying it:

Answer: $\dfrac{y \cdot \cos\left(\sqrt{x^2+y^2}\right)}{\sqrt{x^2+y^2}}$

$$= \frac{\frac{d}{dy}\left(x^2+y^2\right)}{2\sqrt{x^2+y^2}} \cdot \cos\left(\sqrt{x^2+y^2}\right)$$

Question 1(c)

1. Consider the function

$$f(x,y) = \sin\sqrt{x^2+y^2}$$

   (a) Sketch the contours with levels 0, ± 1 for this function
   (b) Calculate the partial derivatives of the function, $\partial f/\partial x$ and $\partial f/\partial y$
   (c) Calculate the gradient vector at the point $(\pi/\sqrt{2}, \pi/\sqrt{2})$

$$\frac{\partial f}{\partial x} = \frac{x\cos\left(\sqrt{x^2+y^2}\right)}{\sqrt{x^2+y^2}}.$$

$$\frac{\partial f}{\partial y} = \frac{y\cos\left(\sqrt{x^2+y^2}\right)}{\sqrt{x^2+y^2}}.$$

Question 1(c)

(c) Calculate the gradient vector at the point $(\pi/\sqrt{2}, \pi/\sqrt{2})$

$$\frac{\partial f}{\partial x} = \frac{x\cos\left(\sqrt{x^2+y^2}\right)}{\sqrt{x^2+y^2}}.$$

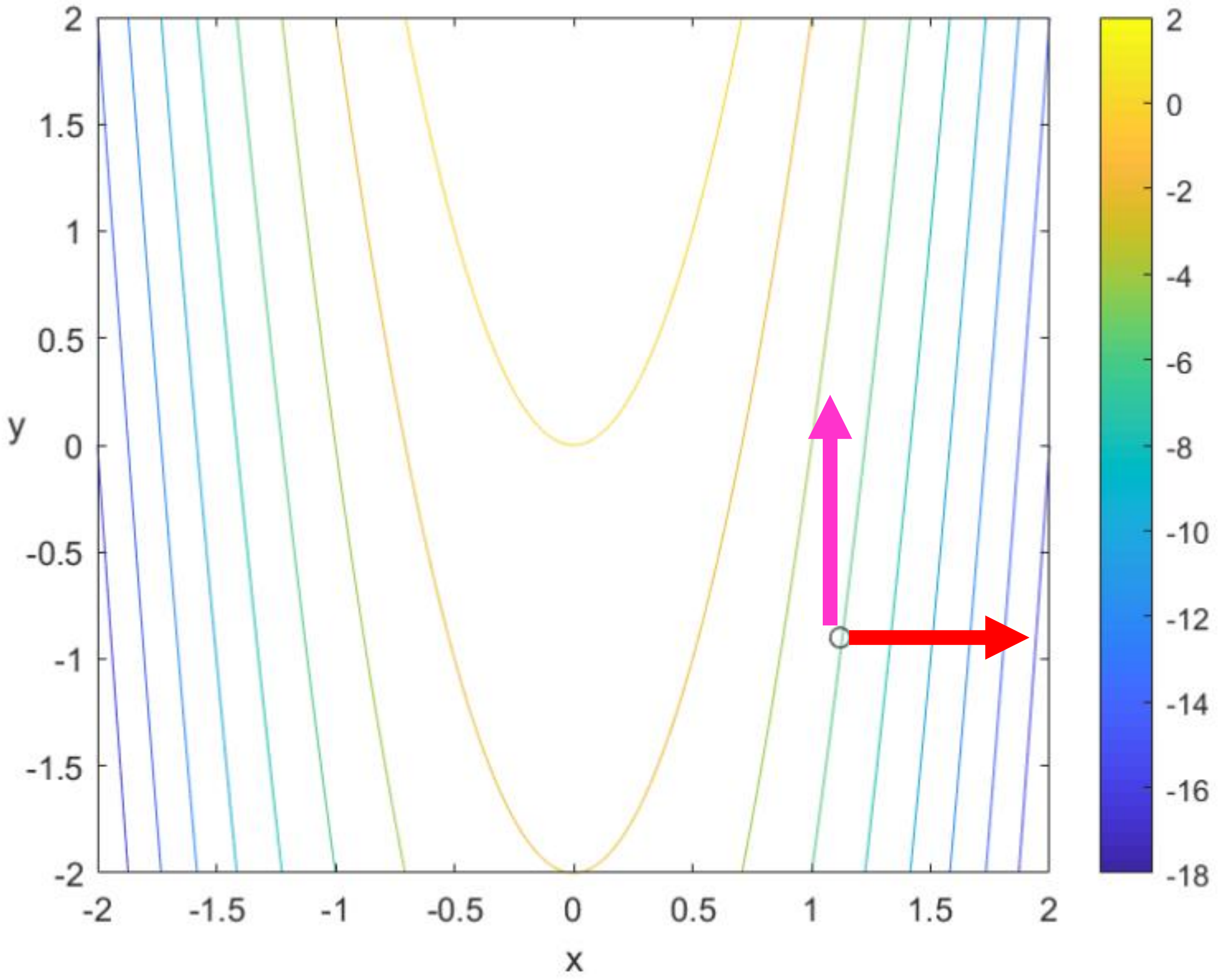
$$\frac{\partial f}{\partial y} = \frac{y\cos\left(\sqrt{x^2+y^2}\right)}{\sqrt{x^2+y^2}}.$$

$$\nabla f(x,y) = \left(\underbrace{\frac{x\cos\left(\sqrt{x^2+y^2}\right)}{\sqrt{x^2+y^2}}}_{\partial f/\partial x}, \underbrace{\frac{y\cos\left(\sqrt{x^2+y^2}\right)}{\sqrt{x^2+y^2}}}_{\partial f/\partial y}\right)$$

$$\nabla f\left(\frac{\pi}{\sqrt{2}}, \frac{\pi}{\sqrt{2}}\right) = \left(\frac{\pi\cos\left(\sqrt{\pi^2}\right)}{\sqrt{2}\sqrt{\pi^2}}, \frac{\pi\cos\left(\sqrt{\pi^2}\right)}{\sqrt{2}\sqrt{\pi^2}}\right)$$

$$\nabla f\left(\frac{\pi}{\sqrt{2}}, \frac{\pi}{\sqrt{2}}\right) = \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right).$$

Question 2: Which is true?



$$\frac{\text{change in f}}{\text{change in x}} < 0$$

$$\frac{\text{change in f}}{\text{change in y}} > 0$$

(a) $\partial f/\partial x > 0$ and $\partial f/\partial y > 0$

(b) $\partial f/\partial x > 0$ and $\partial f/\partial y < 0$

(c) $\partial f/\partial x < 0$ and $\partial f/\partial y > 0$

(d) $\partial f/\partial x < 0$ and $\partial f/\partial y < 0$

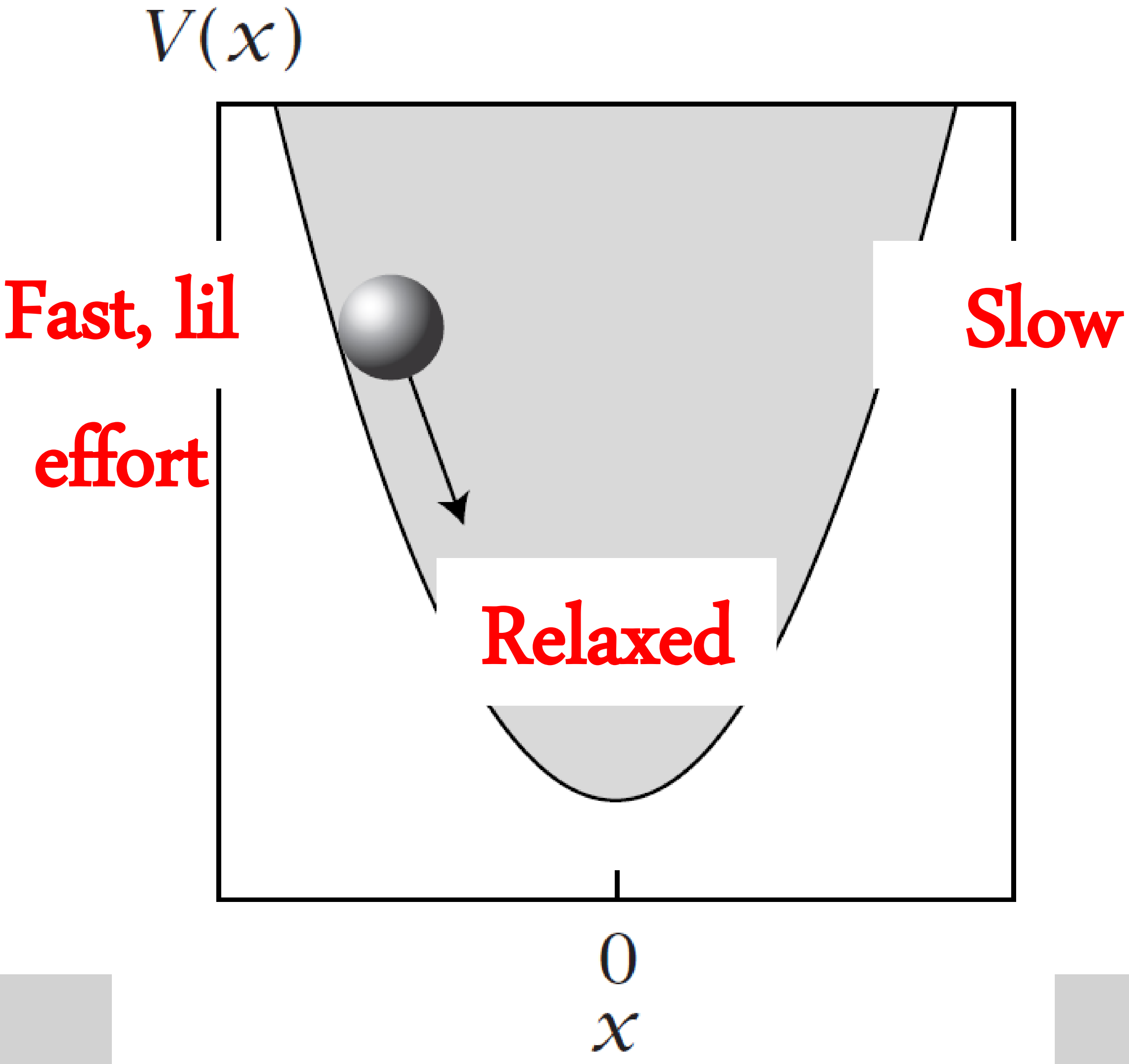
Question 3

3. Consider the surface $z = -xy^2 \cos(\pi x)$. In which direction will a ball begin to roll if it is placed on the surface at the point $x = y = 1$?

Learning Outcomes?

Directional Derivative?

Pitstop: The story of a ball!



- The ball moves the fastest when the slope has $\boxed{\text{a negative gradient.}}$

- Mathematically, we need $-\nabla$, allowing your ball to move without external cues.

If you want to know more about this physical system, come to talk to me after class!

Step 1: The gradient of a function z in Cartesian coordinates (x, y) is:

$$\boxed{z = -xy^2\cos(\pi x).}$$

$$\nabla z = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) = \left(\frac{\partial(-xy^2\cos(\pi x))}{\partial x}, \frac{\partial(-xy^2\cos(\pi x))}{\partial y}\right)$$

$$y^2\left(-\cos(\pi x) + \pi x\sin(\pi x)\right) \qquad\qquad -2xy\cos(\pi x)$$

$$\boxed{\nabla z = \left(y^2\left(-\cos(\pi x) + \pi x\sin(\pi x)\right), -2xy\cos(\pi x)\right)}$$

$$\boxed{\nabla z = \left(y^2\left(-\cos(\pi x) + \pi x \sin(\pi x)\right), -2xy\cos(\pi x)\right)}$$

Step 2: At Cartesian coordinates (1, 1) is

$$\cos(0) = 1$$
$$\cos(\pi) = -1$$
$$\sin(0) = 0$$
$$\sin(\pi) = 0$$

$$\nabla z = (1, 2)$$

Step 3: Direction it begins to roll:

$$-\nabla z = -(1, 2) = (-1, -2)$$

Question 4

Consider the function $f(x,y) = x^y$ for $x > 0$. Calculate the mixed partial derivatives of this function.

Learning Outcomes?

Performing mixed derivatives

Do it with x first, then y next.

Step 1: First derivative w.r.t x

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^y) = y \cdot x^{y-1}$$

Step 2: Second derivative w.r.t y

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left(y \cdot x^{y-1}\right)$$

$$\boxed{\frac{\partial}{\partial y}\left(y \cdot x^{y-1}\right) = x^{y-1} \boxed{+\, y \cdot x^{y-1} \ln(x)} = x^{y-1}\left(1 + y \ln(x)\right)}$$

Pitstop: A Trick! $\frac{d}{dy}\left(x^{y-1}\right) \boxed{= x^{y-1}\log(x)}$

Step a: Chain rulez!

$\frac{d}{dy}\left(x^u\right) = \frac{d}{du}\left(x^u\right) \cdot \frac{du}{dy}$, where $u = y - 1$ and $\frac{d}{du}\left(x^u\right) = x^u \log(x)$:

$$\frac{d}{dy}\left(x^{y-1}\right) = x^{y-1}\log(x) \cdot \frac{d}{dy}(y-1)$$

Step b: Differentiate the expression y – 1 term by term

$$\frac{d}{dy}(y-1) = \frac{d}{dy}(y) - \frac{d}{dy}(1) = 1 - 0$$

Do it with y first, then x next.

Step 1: First derivative w.r.t y

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^y) = x^y \ln(x)$$

Step 2: Second derivative w.r.t x

$$\boxed{\frac{\partial}{\partial x}\left(x^y \ln(x)\right) = yx^{y-1}\ln(x) + x^y \cdot \frac{1}{x} = x^{y-1}\left(y\ln(x) + 1\right)}$$

In short!

The mixed partial derivatives are equals!

Question 5

5. Which of the following are true and which are false?

   (a) If all the contours of a function $f(x, y)$ are parallel lines, then the surface given by $z = f(x, y)$ is a plane.

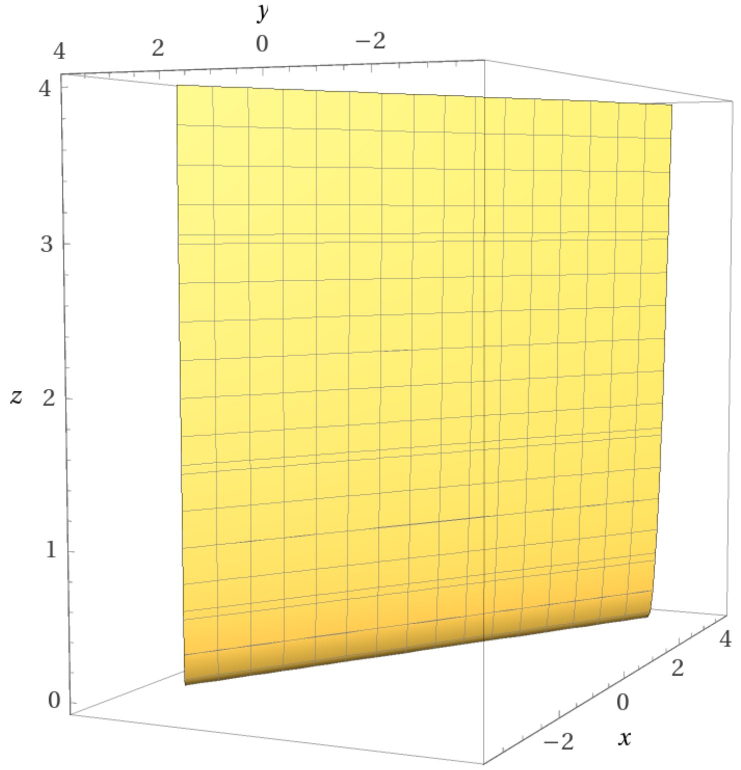
Learning Outcomes?

Contours and Derivatives

5. Which of the following are true and which are false?

(a) If all the contours of a function $f(x, y)$ are parallel lines, then the surface given by $z = f(x, y)$ is a plane.

$$\boxed{z = (x + y)^2}$$



A myth! False

Question 6

6. Consider the function

$$V(x, y, z) = 2y^2 - 3xy + yz^2$$

(a) Find the rate of change of V at the point $(1, 1, 1)$ in the direction of the vector $\mathbf{v} = (1, 1, -1)$.

(b) In which direction does V increase most rapidly at $(1, 1, 1)$?

(c) What is the maximum rate of change of V as $(1, 1, 1)$?

Learning Outcomes?

The rate of change

Pens down!

Full attention is appreciated.

To find the directional derivative, follow these generic steps

$$f(x, y, z) = Ax + By + Cz$$

$$\mathbf{v} = (D, E, F)$$

Step 1: Compute the Gradient

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$\nabla f = (A, B, C)$$

Step 2: Compute the Unit Vector in the Direction of v

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \left(\frac{D}{\sqrt{D^2+E^2+F^2}}, \frac{E}{\sqrt{D^2+E^2+F^2}}, \frac{F}{\sqrt{D^2+E^2+F^2}} \right)$$

Step 3: Compute the dot product; that is the directional derivative

$$\nabla f \cdot \mathbf{u} = \frac{AD+BE+CF}{\sqrt{D^2+E^2+F^2}}$$

$$f(x,y,z) = Ax + By + Cz$$

$$\mathbf{v} = (D, E, F)$$

Step 4: Direction in which f increases most rapidly at (D, E, F) is?

Direction of the gradient, evaluated at that point: $\nabla f = (A, B, C)$

Step 5: What is the maximum rate of change of f as (D, E, F)

$$D_{\mathbf{u}} f(x, y, z) = \nabla f(x, y, z) \cdot \mathbf{u} = \|\nabla f(x, y, z)\| \cdot \cancelto{1}{\|\mathbf{u}\|} \cdot \cancelto{1}{\cos(\theta)}$$

The maximum rate of change is equal to the magnitude of the gradient

$$f(x, y, z) = Ax + By + Cz$$

$$\mathbf{v} = (D, E, F)$$

Question 6

6. Consider the function

$$V(x, y, z) = 2y^2 - 3xy + yz^2$$

(a) Find the rate of change of V at the point $(1, 1, 1)$ in the direction of the vector $\mathbf{v} = (1, 1, -1)$.

(b) In which direction does V increase most rapidly at $(1, 1, 1)$?

(c) What is the maximum rate of change of V as $(1, 1, 1)$?

Step 1: Compute the Gradient

$$\nabla V = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z} \right)$$

$$= (-3y, 4y - 3x + z^2, 2yz)$$

Step 2: Compute the Unit Vector in the Direction of v (1, 1, -1)

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$$

Step 3: Compute the dot product; that is the directional derivative

$$\nabla V \cdot \mathbf{u} = (-3y, 4y - 3x + z^2, 2yz) \cdot \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$$

Step 4: Direction in which V increases most rapidly at (1, 1, 1) is?

The gradient, evaluated at that point, gives the direction
in which the function increases most rapidly/steepest ascend

$$\nabla V = (-3y, 4y - 3x + z^2, 2yz)$$

$$\boxed{\nabla V(1, 1, \quad 1) = (-3, 2, \quad 2)}$$

Step 5: What is the maximum rate of change of V as (1, 1, -1)

$$\|\nabla V\| = \sqrt{\left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2}$$

$$f(x, y, z) = Ax + By + Cz$$

$$\mathbf{v} = (D, E, F)$$

$$\boxed{\|\nabla V\| = \sqrt{9 + 4 + 4} = \sqrt{17}}$$

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Question 7

Consider the point in three-dimensional space $\mathbf{x} = (x, y, z)$ and a constant vector $\mathbf{a} = (a_1, a_2, a_3)$. Let $r = |\mathbf{x}|$. Show that

$$\boldsymbol{\nabla}(r^3) = 3r\mathbf{x} \quad \text{and} \quad \boldsymbol{\nabla}(\mathbf{a} \cdot \mathbf{x}) = \mathbf{a}$$

Learning Outcomes?

Find the ways to satisfy the constraints

To prove this expression:

$$\nabla(r^3) = 3r\mathbf{x}$$

Step 1: Recognize that when using Cartesian and radial spaces, we have

$$\mathbf{x} = (x, y, z)$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

Consider the point in three-dimensional space $\mathbf{x} = (x, y, z)$ and a constant vector $\mathbf{a} = (a_1, a_2, a_3)$. Let $r = |\mathbf{x}|$. Show that

$$\boldsymbol{\nabla}(r^3) = 3r\mathbf{x} \quad \text{and} \quad \boldsymbol{\nabla}(\mathbf{a} \cdot \mathbf{x}) = \mathbf{a}$$

Step 1: Recognize that when using Cartesian and radial spaces,

we have $\mathbf{x} = (x, y, z) \qquad r = \sqrt{x^2 + y^2 + z^2}$

Step 2: Let us express r^3

$$r^3 = \left(\sqrt{x^2 + y^2 + z^2}\right)^3$$

$$\boxed{\nabla(r^3) = 3r\mathbf{x}.}$$

Step 3: Compute the Gradient $\nabla(r^3)$

$$\nabla(r^3) = \left(\frac{\partial}{\partial x}\left(\left(x^2+y^2+z^2\right)^{3/2}\right), \frac{\partial}{\partial y}\left(\left(x^2+y^2+z^2\right)^{3/2}\right), \frac{\partial}{\partial z}\left(\left(x^2+y^2+z^2\right)^{3/2}\right)\right),$$

$$\nabla(r^3) = \left(3x\left(x^2+y^2+z^2\right)^{1/2}, 3y\left(x^2+y^2+z^2\right)^{1/2}, 3z\left(x^2+y^2+z^2\right)^{1/2}\right),$$

$$\boxed{\nabla(r^3) = \left(3x\left(x^2+y^2+z^2\right)^{1/2}, 3y\left(x^2+y^2+z^2\right)^{1/2}, 3z\left(x^2+y^2+z^2\right)^{1/2}\right)}$$

Factorize it out!

$$= 3(\sqrt{x^2+y^2+z^2})(x, y, z)$$

$$\boxed{\nabla(r^3) = 3r\mathbf{x}.}$$

$$\mathbf{x} = (x, y, z)$$

$$r = \sqrt{x^2+y^2+z^2}$$

For the second one,
you try at home! Should only take
you a few mins

Question 8

The error function $\mathrm{erf}(z)$ is given by

$$\mathrm{erf}(z) = \frac{2}{\pi^{0.5}} \int_0^z e^{-y^2}\, dy.$$

Show that

$$u(x,t) = 1 - \mathrm{erf}(x/2\sqrt{t})$$

is a solution of the equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

(*i.e.*, substitute this function into the equation given and show that the equation holds).

This equation is known as the one-dimensional *heat equation* and it is a mathematical model for how heat diffuses. The solution given has $u(0,t) = 1$ and $u \to 0$ as $x \to \infty$. It could describe how heat diffuses in a semi-infinite rod that is initially at temperature zero and the end at $x = 0$ is kept at temperature 1 for times $t \geq 0$.

Learning Outcomes?

Lets us do a simple prove!

Could you compute the integral of the function $y = x^2$ with respect to x from the lower limit 0 to the upper limit A?

Could you then differentiate the resulting expression with respect to????? To What???

Lets us do a simple prove! Integration followed by differentiation

To evaluate the integral of the function $y = x^2$ from 0 to A, we compute:

$$\int_0^A x^2\,dx$$

The antiderivative of x^2 is given by $\frac{x^3}{3}$. Applying the Fundamental Theorem of Calculus, we find:

$$\int_0^A x^2\,dx = \left.\frac{x^3}{3}\right|_0^A = \frac{A^3}{3} - \frac{0^3}{3} = \frac{A^3}{3}$$

Herein you replace x=A for the limit;

Next, we differentiate this result with respect to A:

$$\frac{d}{dA}\left(\frac{A^3}{3}\right)$$

Consequently, derivative wrt A now

Using the power rule for differentiation:

$$\frac{d}{dA}\left(\frac{A^3}{3}\right) = \frac{1}{3}\cdot 3A^2 = A^2$$

We recover the same, but turn into the limit variable!

Road Map for this Question!

$$\frac{\partial u}{\partial t}$$

$$\frac{\partial u}{\partial x}$$

Let us try this first!

$$\frac{\partial^2 u}{\partial x^2}$$

To prove this expression:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

$$\boxed{z = x/2t^{0.5}}$$

We are given:

$$u(x,t) = 1 - \mathrm{erf}(x/2\sqrt{t})$$

where:

$$\mathrm{erf}(z) = \frac{2}{\pi^{0.5}} \int_0^z e^{-y^2}\, dy.$$

To find this first: $\dfrac{\partial u}{\partial x}$

$$\boxed{u(x,t) = 1 - \operatorname{erf}(x/2\sqrt{t})}$$

$$\boxed{\operatorname{erf}(z) = \frac{2}{\pi^{0.5}}\int_0^z e^{-y^2}\,dy.}$$

Step 1: Differentiate the sum term by term

$$\frac{\partial u}{\partial x} = \frac{d}{dx}(1) - \frac{d}{dx}\left(\operatorname{erf}\left(\frac{x}{2\sqrt{t}}\right)\right) = -\frac{d}{dx}\left(\operatorname{erf}\left(\frac{x}{2\sqrt{t}}\right)\right).$$

Step 2: Using chain rule!

$$\frac{d}{dx}\left(\operatorname{erf}\left(\frac{x}{2\sqrt{t}}\right)\right) = \frac{d}{du}(\operatorname{erf}(u)) \cdot \frac{du}{dx},$$

Anyone get this?

where $u = \frac{x}{2\sqrt{t}}$ and $\boxed{\frac{d}{du}(\operatorname{erf}(u)) = \frac{2e^{-u^2}}{\sqrt{\pi}}}$;

Some Explaining: $\frac{d}{dx}\mathrm{erf}(x)$

$$\boxed{\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt}$$

Step 2.1: Amalgamate it! Combine it! Couple it!

$$\frac{d}{dx}\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \frac{d}{dx} \int_0^x e^{-t^2}\, dt$$

Anyone can solve this?

Step 2.2: The cancellation effect! Integration followed by diferentation

$$\frac{d}{dx}\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} e^{-x^2}$$

Note this is a different example, illustrating the derivative of a ERF only

Step 3: Merging!

$$\frac{\partial u}{\partial x} = -\underbrace{\frac{d}{dx}\left(\operatorname{erf}\left(\frac{x}{2\sqrt{t}}\right)\right)}_{\textit{Step 1}} = -\underbrace{\frac{d}{du}\left(\operatorname{erf}(u)\right)\cdot\frac{du}{dx}}_{\textit{Step 2}},$$

Step 4: Concluding

$$\frac{\partial u}{\partial x} = -\frac{2e^{-\frac{x^2}{4t}}}{\sqrt{\pi}}\cdot\frac{d}{dx}(x)\cdot\frac{1}{2\sqrt{t}}. = -\frac{e^{-\frac{x^2}{4t}}}{\sqrt{\pi}\sqrt{t}}.$$

where $u = \frac{x}{2\sqrt{t}}$ and $\frac{d}{du}(\operatorname{erf}(u)) = \frac{2e^{-u^2}}{\sqrt{\pi}}$:

Road Map for this Question!

$$\frac{\partial u}{\partial t} \Rightarrow \text{DIY}$$

$$\frac{\partial u}{\partial x} \Rightarrow \text{Done!}$$

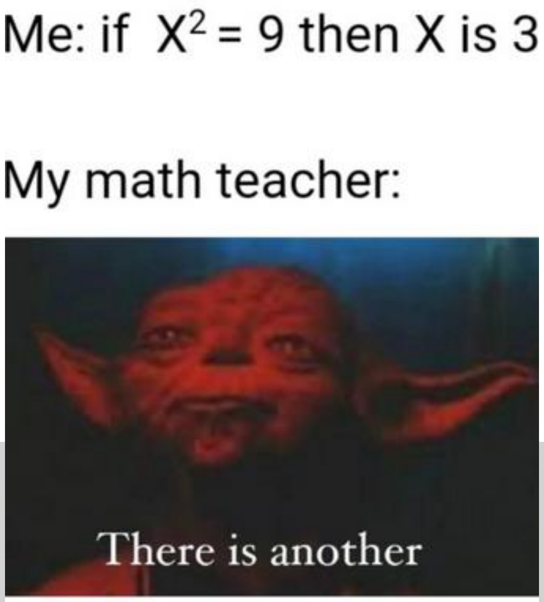
$$\frac{\partial^2 u}{\partial x^2} \Rightarrow \text{DIY}$$

$$\overbrace{\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}}^{\text{Left=Right}}$$

Today's Activity

1. Applied Problem Set

2. Applied Quiz



Thank You

Eng. Math

(Workshop) ENG 1005

Week 5: Multivariable Calculus 1

(MEC) Senior Lecturer: K.B. Goh, Ph.D.

Tutor: (a) Ian Keen & (b) Jack

Pass Leader: (i) Zi Wei and (ii) Yvonne

kekboon.goh@monash.edu

Check-In

*Please bear with me, attending

to your messages soon*

STUDENT FEEDBACK: ESSFS (5 min)

Take the survey,
get your voice heard.

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Resources

1. PASS with Yvonne and Zi Wei

2. Mid-Term Mock Exams! Week 6

3. Additional: Videos (Mid-Term Prep.)

Resources (updated)

3. Additional: Videos (Mid-Term Prep.)

Topics

Week	Topic
1	Vectors, Lines, and Planes
2	Systems of Linear Equations
3	Matrices
4	Eigenvalues & Eigenvectors
5	Multivariable Calculus 1
6	Multivariable Calculus 2
7	Integration techniques and hyperbolic functions
8	O.D.E 1
9	O.D.E 2
10	O.D.E 3
11	Series 1
12	Series 2

Assessments breakdown

Task description	*Value*	*Due date*
Lecture quizzes	5%	Weekly 11:55pm the night before your applied class (except Monday classes) except in Week 1 when the due date is Friday at 11:55 pm (Malaysia time)
Applied class quizzes	5%	Weekly during your applied class
Workshop problems	20%	Weekly at 11:55pm Sunday, except for Weeks 7 (midsem)
Mid-semester exam	20%	During your workshop in Week 7

The Big Learning Outcomes for Week 5

After completing this week's task, you should be able to:

- Find contours of functions of two variables.
- Understand and calculate partial/directional derivatives and gradient of functions of several variables.
- Calculate tangent planes.
- Calculate higher derivatives of functions of several variables.

Admin. Stuff (1)

1. Feedback on Workshop Submission: On Thur/Fri

2. Submissions:

Summary

ASSESSMENT	DUE
Kick Starting Week 3: Lecture Quiz 3 (Total mark for all 12 weeks of lecture quizzes is 5%)	Tuesday, 6 August 2024, 11:55 PM **Due in 4 days**
Applied class quiz week 3 (Total mark for all 12 weeks of applied quizzes is 5%)	Wednesday, 7 August 2024, 11:55 PM **Due in 5 days**
Workshop 3 problem set (Total mark for all 12 weeks of workshop sets is 20%)	Sunday, 11 August 2024, 11:55 PM

Admin. Stuff (2)

3. Consultation/Feedback hour

- Wed: 10 am till 11 am
- Fri: 8 am till 9 am
- Sat: 1030 am till 1130 am

Location: 5-4-68

Library

(every odd week, e.g. 3-Week, but msg me first at least a day before; appointment-based.)

Today’s Activity

0. Some Discussion

1. Workshop Problem Set

Pit-Stop:0

Error Function

To find this first: $\dfrac{\partial u}{\partial x}$

$$\boxed{u(x,t) = 1 - \operatorname{erf}(x/2\sqrt{t})}$$

$$\boxed{\operatorname{erf}(z) = \frac{2}{\pi^{0.5}} \int_0^z e^{-y^2}\, dy.}$$

Step 1: Differentiate the sum term by term

$$\frac{\partial u}{\partial x} = \boxed{} - \frac{d}{dx}\left(\operatorname{erf}\left(\frac{x}{2\sqrt{t}}\right)\right)$$

Step 2: Using chain rule!

$$\frac{d}{dx}\left(\operatorname{erf}\left(\frac{x}{2\sqrt{t}}\right)\right) = \frac{d}{du}(\operatorname{erf}(u)) \boxed{\frac{du}{dx}},$$

Anyone get this?

where $u = \dfrac{x}{2\sqrt{t}}$

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Some Explaining: $\frac{d}{dx}\operatorname{erf}(x)$

$$\boxed{\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt}$$

Step 2.1: Amalgamate it! Combine it! Couple it!

$$\boxed{\frac{d}{dx}}\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \boxed{\frac{d}{dx}} \int_0^x e^{-t^2} \boxed{dt}$$

Anyone can solve this?

Step 2.2: The cancellation effect! Integration followed by diferentation

$$\frac{d}{dx}\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} e^{-x^2}$$

Swap the limit x with the variable t!

Note this is a different example, illustrating the derivative of a ERF only

Step 3: Merging!

$$\frac{\partial u}{\partial x} = -\underbrace{\boxed{\frac{d}{du}\left(\operatorname{erf}(u)\right)}\,\boxed{\frac{du}{dx},}}_{Step\ 2}$$

Step 4: Concluding

$$\frac{\partial u}{\partial x} = \boxed{-\frac{2e^{-\frac{x^2}{4t}}}{\sqrt{\pi}}}\cdot\boxed{\frac{d}{dx}(x)\cdot\frac{1}{2\sqrt{t}}}\cdot = -\frac{e^{-\frac{x^2}{4t}}}{\sqrt{\pi}\sqrt{t}}.$$

$$\boxed{z = x/2t^{0.5}}$$

where $u = \frac{x}{2\sqrt{t}}$ and $\frac{d}{du}(\operatorname{erf}(u)) = \frac{2e^{-u^2}}{\sqrt{\pi}}$:

$$\operatorname{erf}(z) = \frac{2}{\pi^{0.5}}\int_0^z e^{-y^2}\,dy.$$

Road Map for this Question!

$\frac{\partial u}{\partial t}$ $\Rightarrow$ DIY

$\frac{\partial u}{\partial x}$ $\Rightarrow$ Done!

$\frac{\partial^2 u}{\partial x^2}$ $\Rightarrow$ DIY

$$\overbrace{\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}}^{\text{Left=Right}}$$

Pit-Stop:1

Physical Meaning: Directional Derivative!

Directional Derivative:
(i) Positive= Ascending
(ii) Negative=Descending

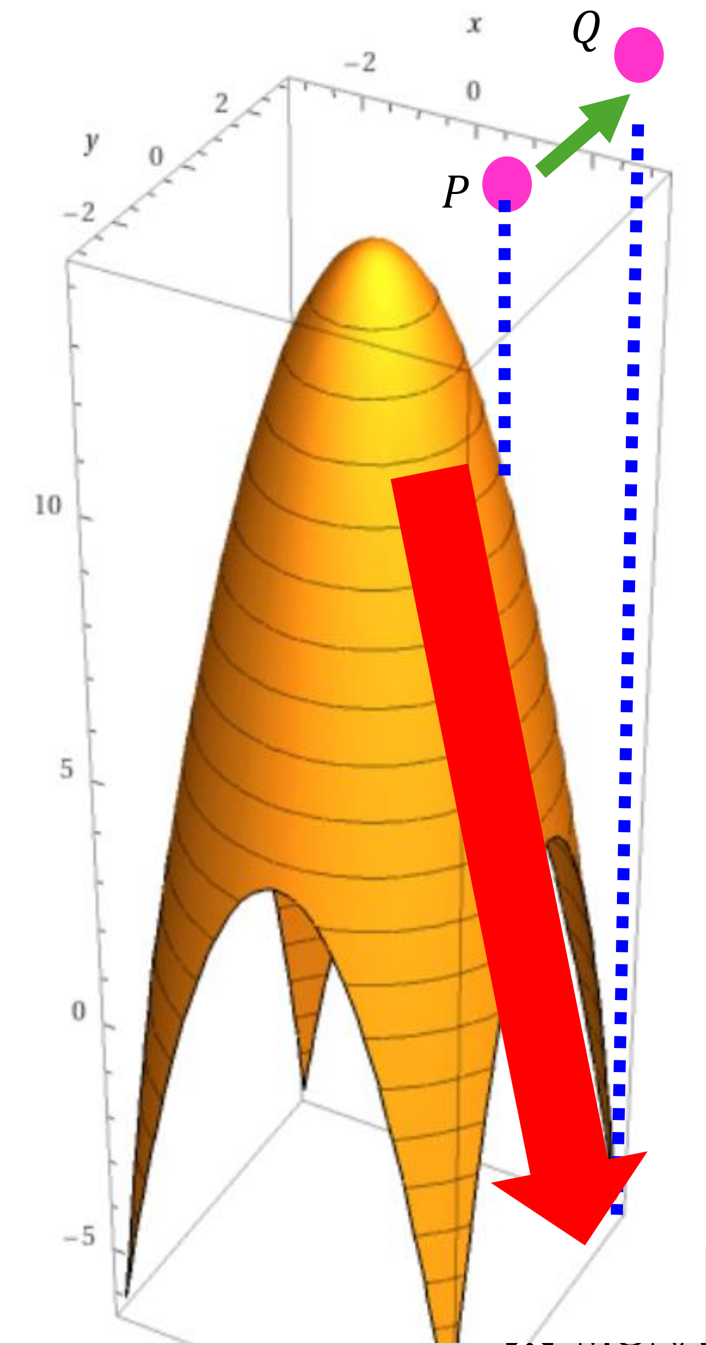
Let $z = 14 - x^2 - y^2$

Question: Find the directional derivative

toward Q from P

When we move in that general direction, to Q from P, we technically doing down the hill!

Directional Derivative:
(i) Positive= Ascending
(ii) Negative=Descending



Let $z = 14 - x^2 - y^2$

$P = (1,2) \quad Q = (3,4)$

Step 1

$\overrightarrow{PQ} = (2,2)$

Step 2

$$\text{Unit vector} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

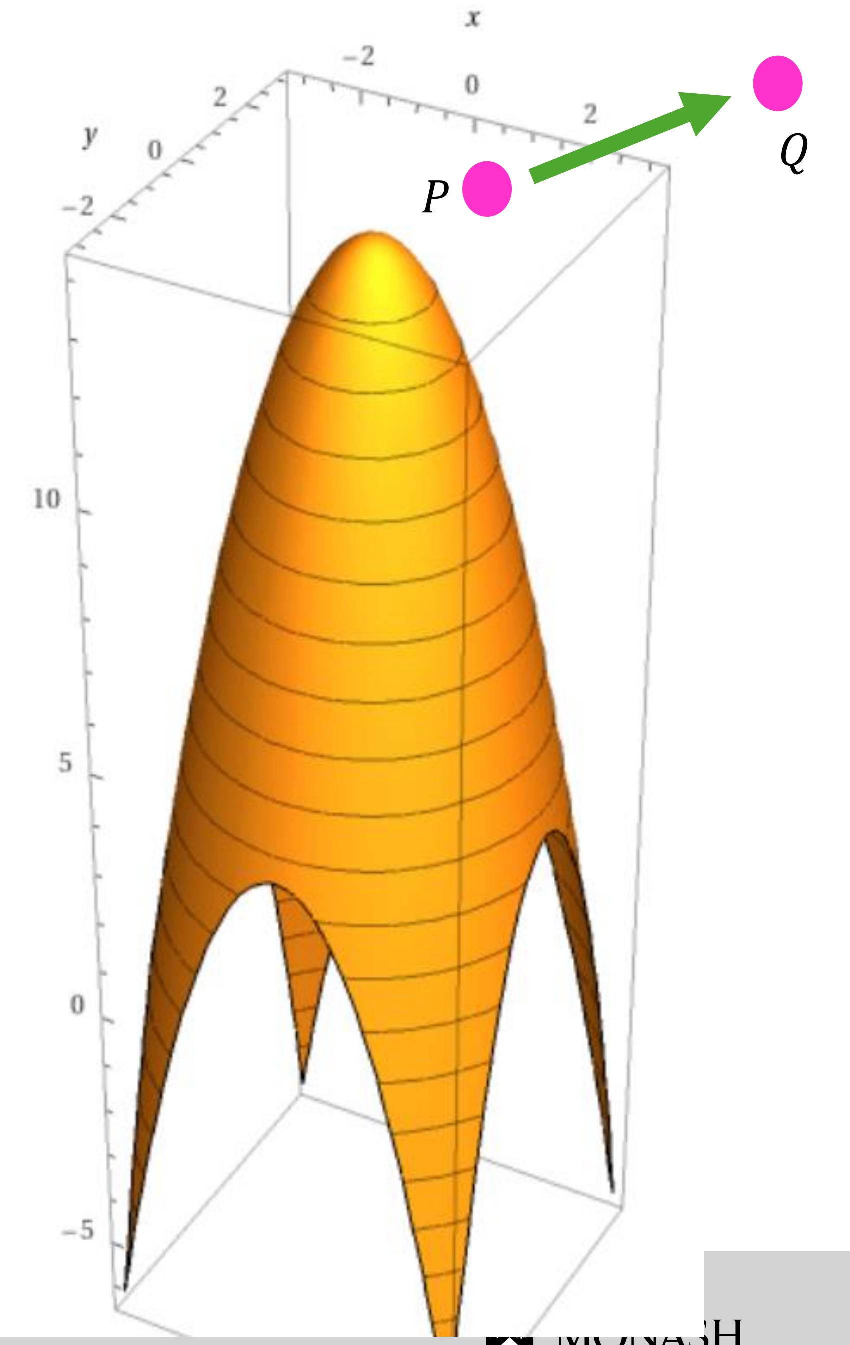
Step 3 (x,y) = (1,2)

$$\nabla z = \begin{pmatrix} -2x \\ -2y \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$$

Step 4: Dir. Der.

$$= \begin{pmatrix} -2 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\approx -4.24$$

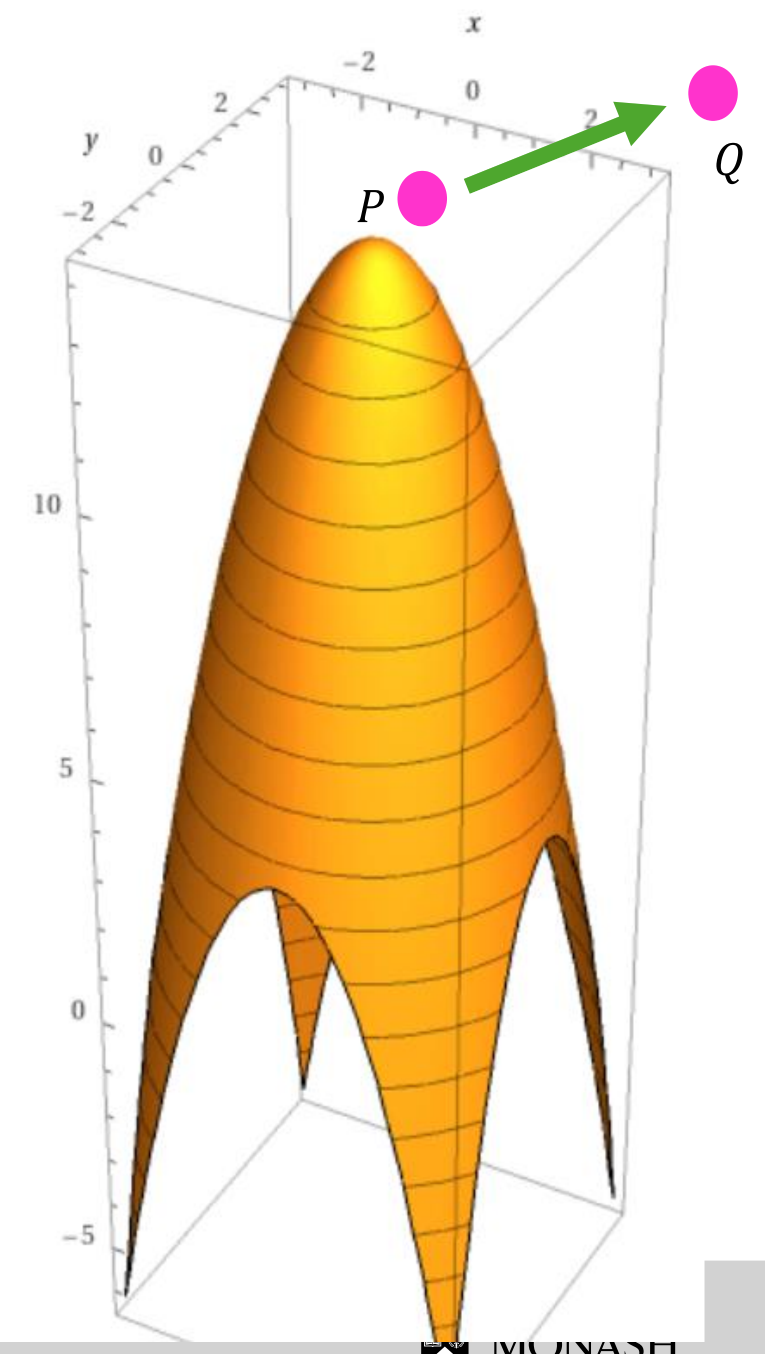


Let $z = 14 - x^2 - y^2$

$P = (1,2) \quad Q = (3,4)$

"Therefore, the instantaneous rate of change when moving from the point $(1, 2, 9)$ on the surface in the direction of the vector $\mathbf{u}_1$ (which points toward the point Q) is approximately -4.24. Moving in this direction results in a steep descent."

$$\frac{\text{change in } z}{\text{change in } PQ} < 0$$



Additional Questions

Try At Home!

(Take home) Pit-Stop:2

Why the gradient is always perpendicular to the contour line?

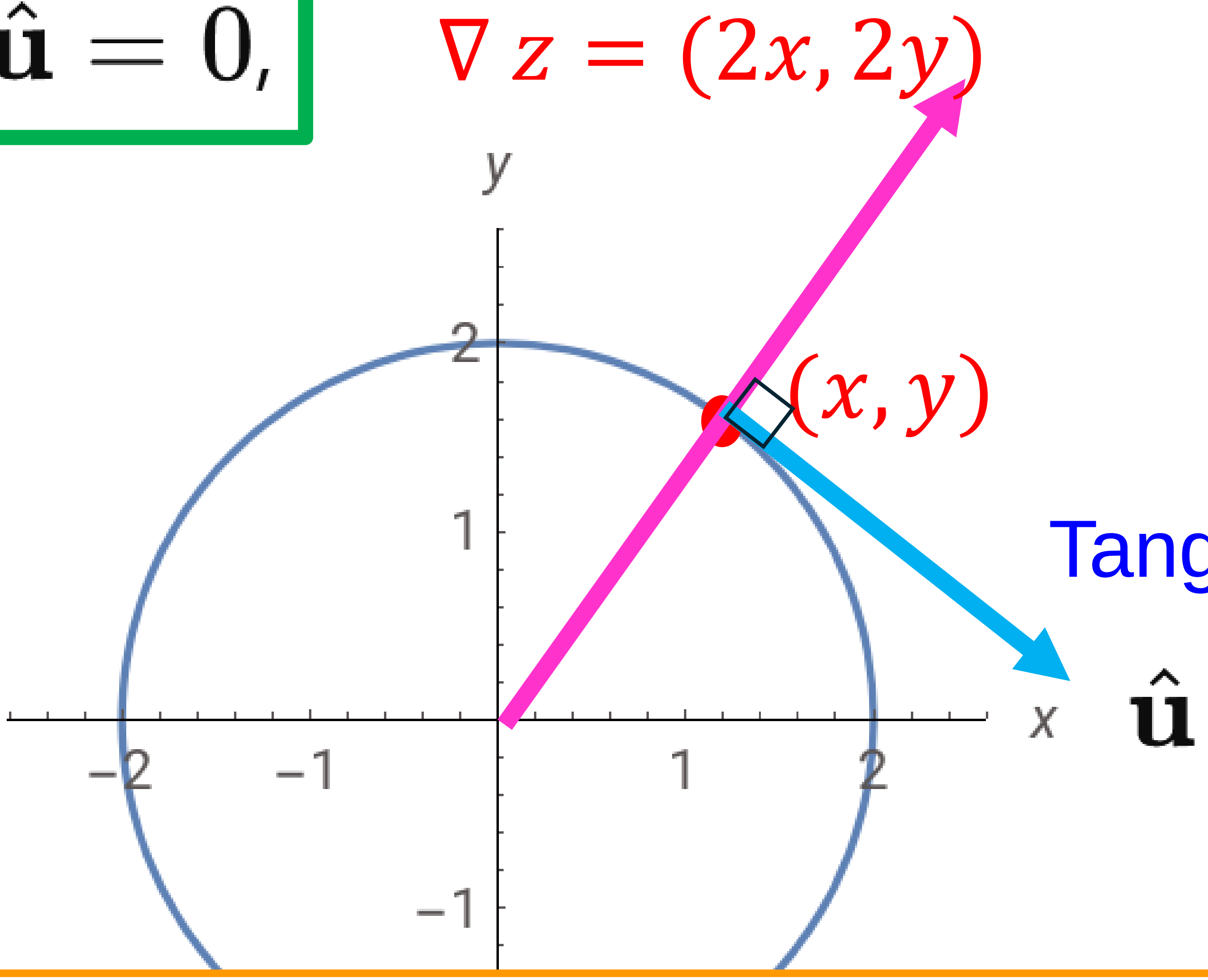
Refresher!

$$\boxed{\nabla z \cdot \hat{\mathbf{u}} = 0,}$$

$$z = x^2 + y^2 = 2^2$$

$$\nabla z = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right)$$

$$\nabla z = (2x, 2y)$$



Yes, the gradient ∇z is always perpendicular (normal) to the contour lines of the function $z(x, y)$.

(Take home) Pit-Stop:3

How to find a tangent plane to a surface?

An Alternative Treatment

Find the tangent plane to this surface at (2,1,1):

$$-z^2 + x^2 + y^2 = 4$$

Step 1: Recognize that we are a constant level set; namely w = 4

$$w = -z^2 + x^2 + y^2 = 4$$

Step 2: Let us express the grad.

$$\nabla w = < 2x, 2y, -2z >$$

$$\nabla w\ (2,1,1) = < 4, 2, -2 >$$

Normal to the tangent/surface plane! See Pit-Stop 2

Step 3: Recognize that a plane formula can be written as such,

4.3.3 Vector equation of a plane

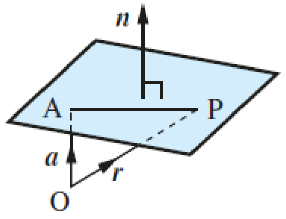


Figure 4.52
Equation of a plane; $\boldsymbol{n}$ is perpendicular to the plane.

To obtain the equation of a plane, we use the result that the line joining any two points in the plane is perpendicular to the normal to the plane, as illustrated in Figure 4.52. The vector $\boldsymbol{n}$ is perpendicular to the plane, $\boldsymbol{a}$ is the position vector of a given point A in the plane and $\boldsymbol{r}$ is the position vector of any point P on the plane. The vector $\overrightarrow{AP} = \boldsymbol{r} - \boldsymbol{a}$ is perpendicular to $\boldsymbol{n}$, and hence

$$(\boldsymbol{r} - \boldsymbol{a})\cdot\boldsymbol{n} = 0$$

so that

$$\boldsymbol{r}\cdot\boldsymbol{n} = \boldsymbol{a}\cdot\boldsymbol{n} \quad \text{or} \quad \boldsymbol{r}\cdot\boldsymbol{n} = p \qquad (4.19)$$

Step 4: Complete the Eq 4.19 at pt (2,1,1)

$$\overbrace{\begin{pmatrix}x\\y\\z\end{pmatrix}}^{\text{r}}\cdot\overbrace{\begin{pmatrix}4\\2\\-2\end{pmatrix}}^{\text{n}}=\overbrace{\begin{pmatrix}4\\2\\-2\end{pmatrix}}^{\text{n}}\cdot\overbrace{\begin{pmatrix}2\\1\\1\end{pmatrix}}^{\text{a}}$$

$$\boxed{4x + 2y - 2z = 8}$$

$$\boxed{-z^2 + x^2 + y^2 = 4}$$

$$\boxed{\nabla w = < 4, 2, -2 >}$$

Today’s Activity

0. Some Discussion

1. Workshop Problem Set

Wifi Access Point

As you can no doubt imagine, functions of several variables and multivariable calculus are everywhere in our lives. In this workshop we will study an application of in the placement of a WiFi access point.

WiFi signals are eletromagnetic radiations. As you move away from the source of the radiation, the signal decreases in strength. In fact the power of the signal obeys the Inverse-square Law:

$$\boxed{P \propto \frac{1}{r^2}}$$

where P is power of the signal (measured in milliwatts), $\propto$ is the mathematical symbol for proportionality, and r is the distance from the source of the signal (measured in metres).

Abby, Ben and Carol want to install a WiFi access point to provide WiFi coverage in their house. The power consumption T of the access point is directly proportional to its output signal power P measured 1 metre away. The new WiFi access point will negotiate its signal power with the receiver so that the signal received by the receiver is always the same strength no matter how far away they are.

1. Show that the power consumption of the access point with a user l metres away is $T = kl^2$ for some constant k. **[1 mark]**

We know that:

$$T \propto P(1\text{m}) \Rightarrow T = k_1 P(1)$$

The power consumption T of the access point is directly proportional to its output signal power P measured 1 meter away.

$$P \propto \frac{1}{r^2} \Rightarrow P = \frac{k_2}{r^2}$$

The inverse law!

$$P(l) = A$$

The new WiFi access point will negotiate its signal power with the receiver so that the signal received by the receiver is always the same strength no matter how far away they are.

Let us merge these to find

$$P(l) = A$$

step1

$$P = \frac{k_2}{r^2} \rightarrow P(l) = \frac{k_2}{l^2}$$

$$P(l) = \frac{k_2}{l^2} = A$$

$$k_2 = Al^2$$

Step 2

$$T = k_1 P(1)$$

$$T = k_1 \frac{k_2}{1^2}$$

Step 3

$$T = k_1 \frac{Al^2}{1^2}$$

Step 4

$$T = k \frac{l^2}{1}$$

$k_1 A = k$

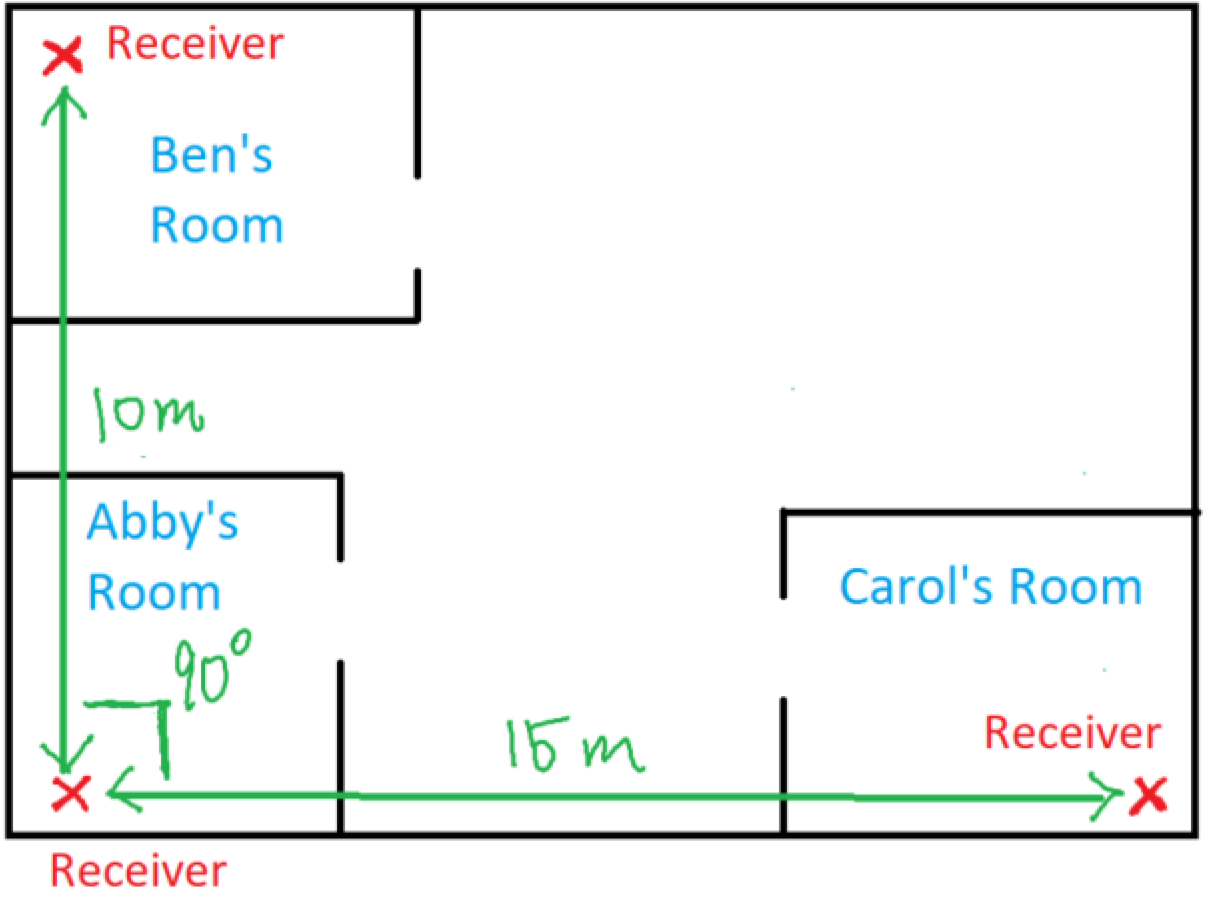
2. Standing 5 metres away from the access point, Abby measures that the access point is consuming 0.5 watts of power. Use this to determine the constant in the previous question. **[1 mark]**

$$T = kl^2$$

$$0.5 = k(5)^2$$

$$k = 0.02 \text{ or } 1/50$$

3. Set up a coordinate system to represent the house and write down the coordinates of the three receivers in your system. **[2 marks]**



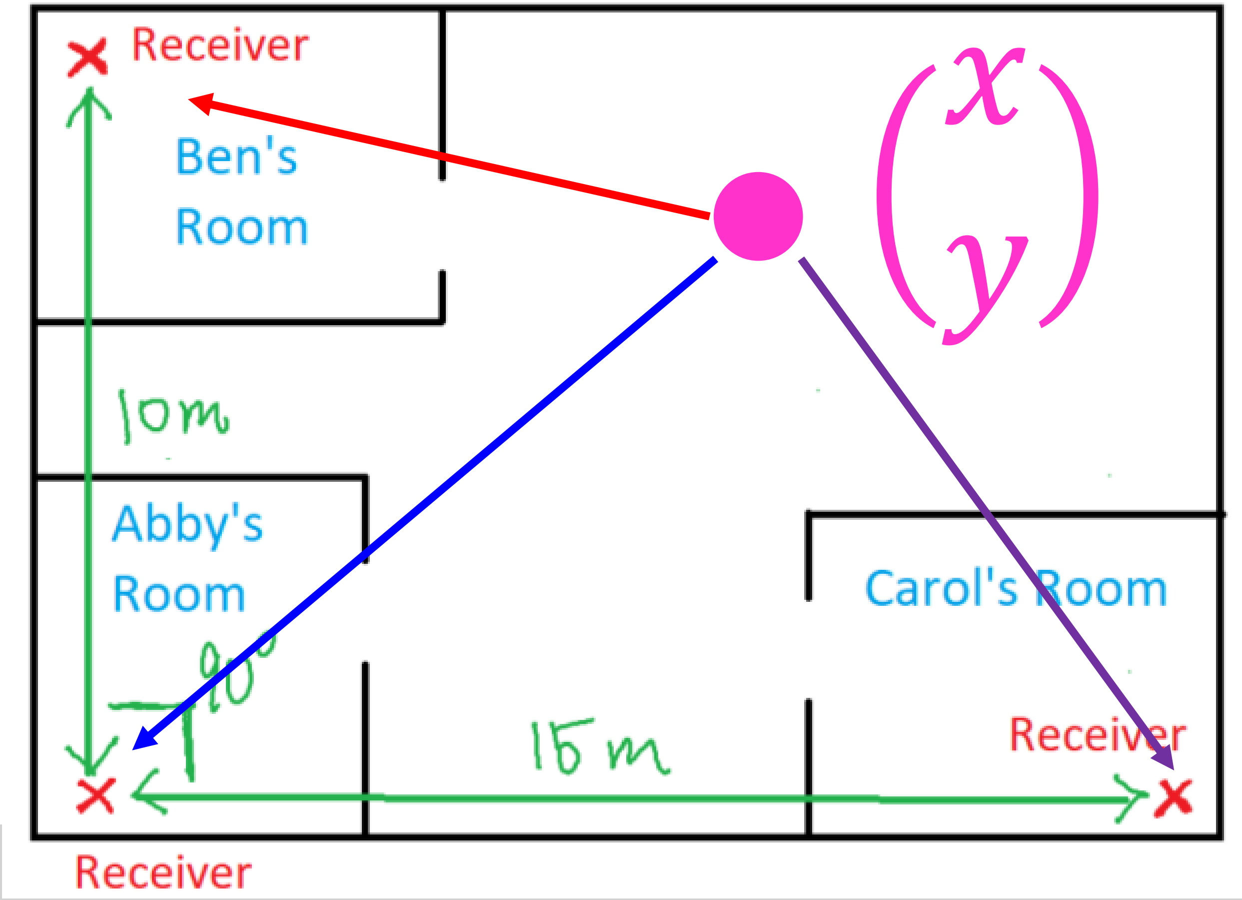
Abby $(0,0)$

Ben $(0,10)$

Carol $(15,0)$

4. Let (x, y) be the location of the access point. Write down a formula for power consumption T of the access point in terms of x and y. You may assume the three signals to the receivers are independent, and the power consumption is the sum of the three power consumptions required to maintain a good connection with each receiver. **[2 marks]**

$$\boxed{T = kl^2}$$



Step 1: Find the position vector

Abby $\begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}$

Ben $\begin{pmatrix} 0 \\ 10 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}$

Carol $\begin{pmatrix} 15 \\ 0 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}$

Step 2: Magnitude of these vectors

$$l_{Abby} = \sqrt{(-x)^2 + (-y)^2}$$

$$l_{Ben} = \sqrt{(-x)^2 + (10-y)^2}$$

$$l_{Carol} = \sqrt{(15-x)^2 + (-y)^2}$$

Abby $\begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}$

Ben $\begin{pmatrix} 0 \\ 10 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}$

Carol $\begin{pmatrix} 15 \\ 0 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}$

Step 3: Compute the Individual Power Consumption

$$T = kl^2$$

$$k = 0.02$$

$$T_{Abby} = 0.02\left(\sqrt{(-x)^2 + (-y)^2}\right)^2$$

$$T_{Ben} = 0.02\left(\sqrt{(-x)^2 + (10 - y)^2}\right)^2$$

$$T_{Carol} = 0.02\left(\sqrt{(15 - x)^2 + (-y)^2}\right)^2$$

$$l_{Abby} = \sqrt{(-x)^2 + (-y)^2}$$

$$l_{Ben} = \sqrt{(-x)^2 + (10 - y)^2}$$

$$l_{Carol} = \sqrt{(15 - x)^2 + (-y)^2}$$

Step 4: Compute the Total Power Consumption

$$T = T_{Abby} + T_{Ben} + T_{carol}$$

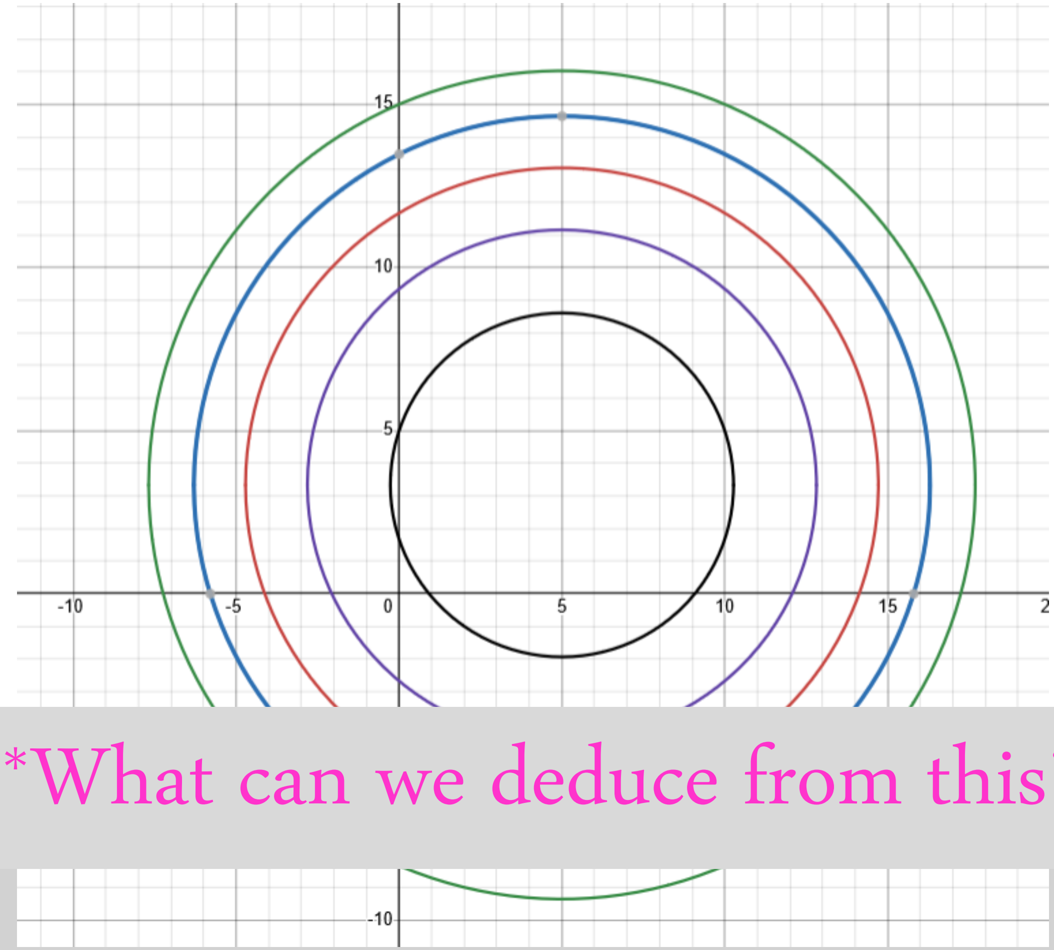
$$= 0.02(3x^2 + 3y^2 - 30x - 20y + 325)$$

5. With the help of Matlab or CAS, sketch three different contours of the function $T(x, y)$. **[2 marks]**

Can you go: www.desmos.com/calculator

Equate to a constant, namely

1. $0.02\left(3x^2 + 3y^2 - 30x - 20y + 325\right) = 6$
2. $0.02\left(3x^2 + 3y^2 - 30x - 20y + 325\right) = 8$
3. $0.02\left(3x^2 + 3y^2 - 30x - 20y + 325\right) = 10$
4. $0.02\left(3x^2 + 3y^2 - 30x - 20y + 325\right) = 12$
5. $0.02\left(3x^2 + 3y^2 - 30x - 20y + 325\right) = 14$



Note the corresponding colours

What can we deduce from this?

$$T = 0.02(3x^2 + 3y^2 - 30x - 20y + 325)$$

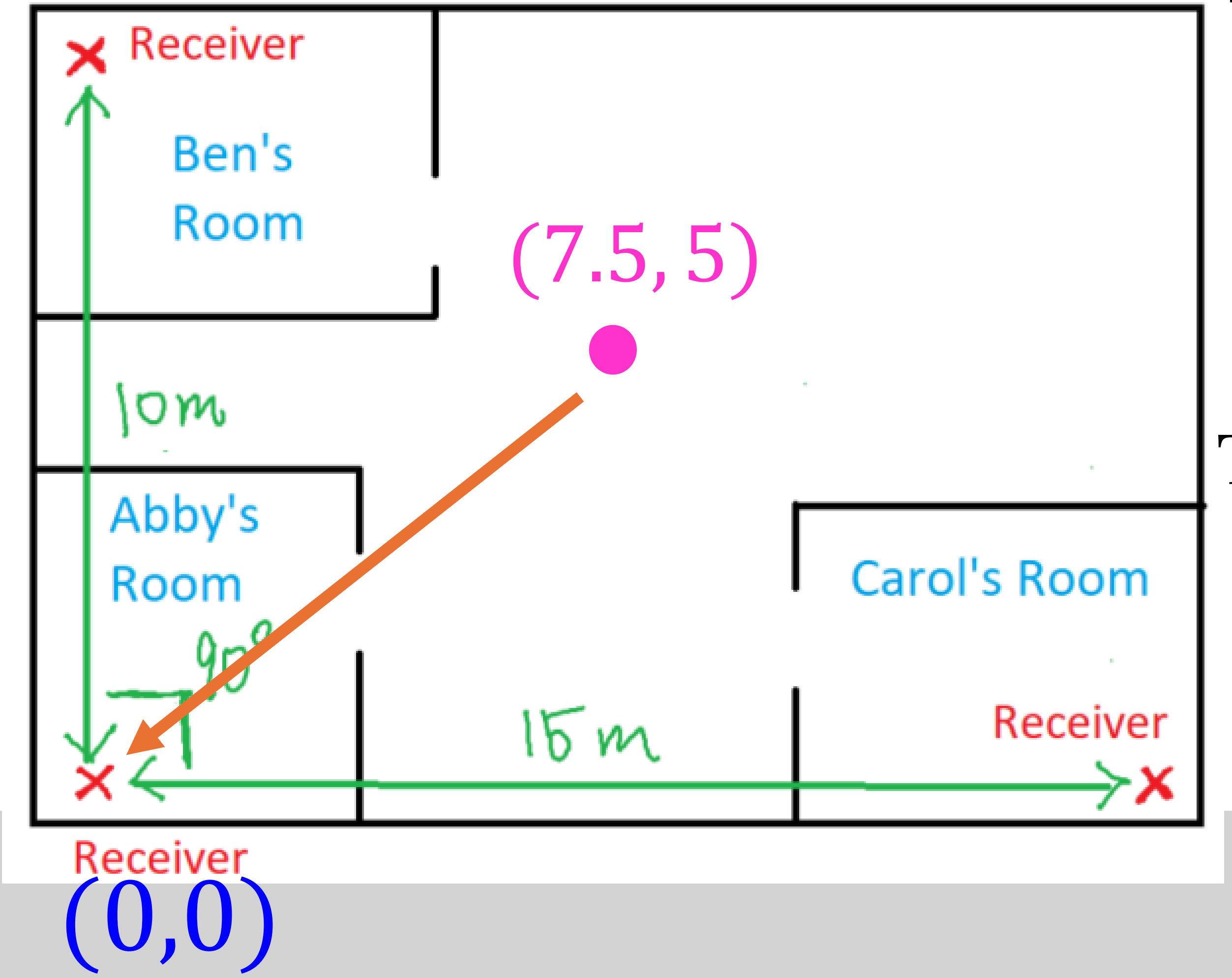
6. Calculate the gradient vector of T.

$$T = 0.02(3x^2 + 3y^2 - 30x - 20y + 325)$$

$$\nabla T = \begin{bmatrix} \frac{1}{50}(6x - 30) \\ \frac{1}{50}(6y - 20) \end{bmatrix}$$

7. Suppose the WiFi access point is initially placed at the centre of the house. Will the power consumption increase or decrease if the access point is moved directly towards the receiver in Abby's room?

[2 marks]



The vector: $= \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 7.5 \\ 5.0 \end{pmatrix}$

$= - \begin{pmatrix} 7.5 \\ 5.0 \end{pmatrix}$

The unit vector:

$$= \left(-\frac{7.5}{\sqrt{81.25}}, -\frac{5}{\sqrt{81.25}} \right)$$

$$\boxed{\nabla T\,(7.5, 5) \cdot \hat{u}} = \left(-\frac{7.5}{\sqrt{81.25}}, -\frac{5}{\sqrt{81.25}}\right)$$

$$\nabla T = \begin{bmatrix} \frac{1}{50}(6x - 30) \\ \frac{1}{50}(6y - 20) \end{bmatrix}$$

Now, you calculate this directional derivative?

- You will get negative values!
- This is clearly negative, therefore the power consumption will decrease

But will it decrease most rapidly?

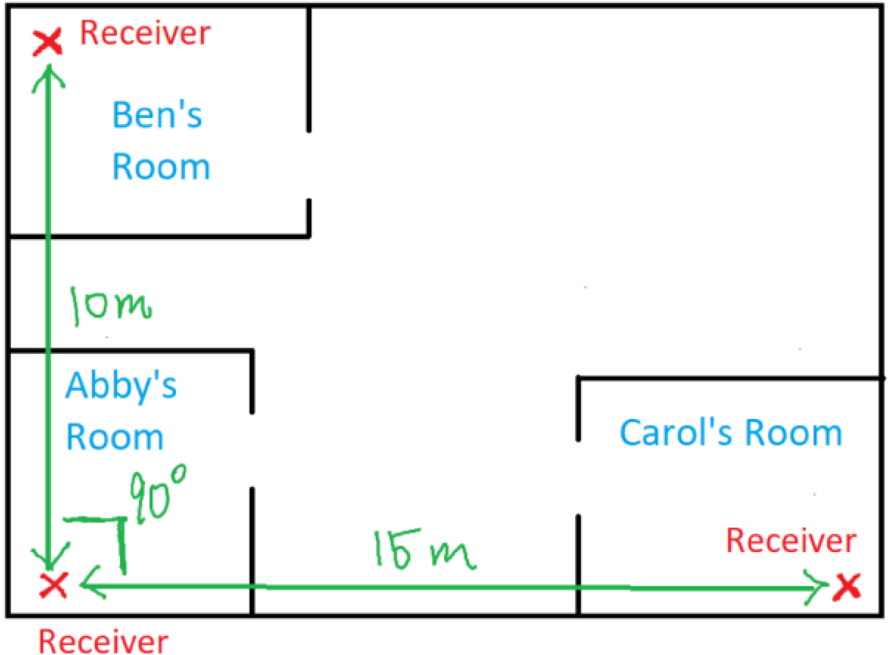
8. Find the direction to move the access point to reduce power consumption most rapidly.

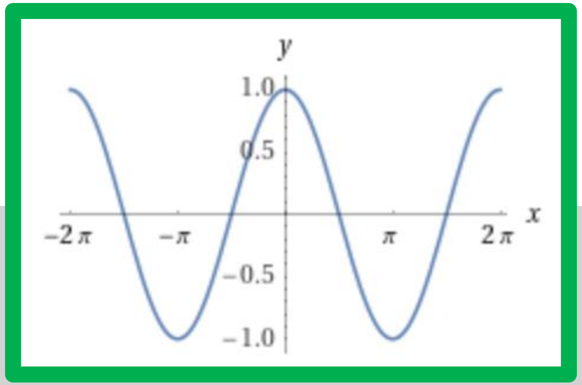
$$\overbrace{\nabla f(x,y,z)\cdot\mathbf{u}}^{\text{Direction Derivative}} = \|\nabla f(x,y,z)\|\cdot\|\underbrace{\mathbf{u}}_{1}\|\cdot\underbrace{\cos(\theta)}_{-1}$$

- We want to descend/decrease most quickly; that is consume less power, but how?

❑ Directional Derivative=most negative, but how?

❑ Negative grad!



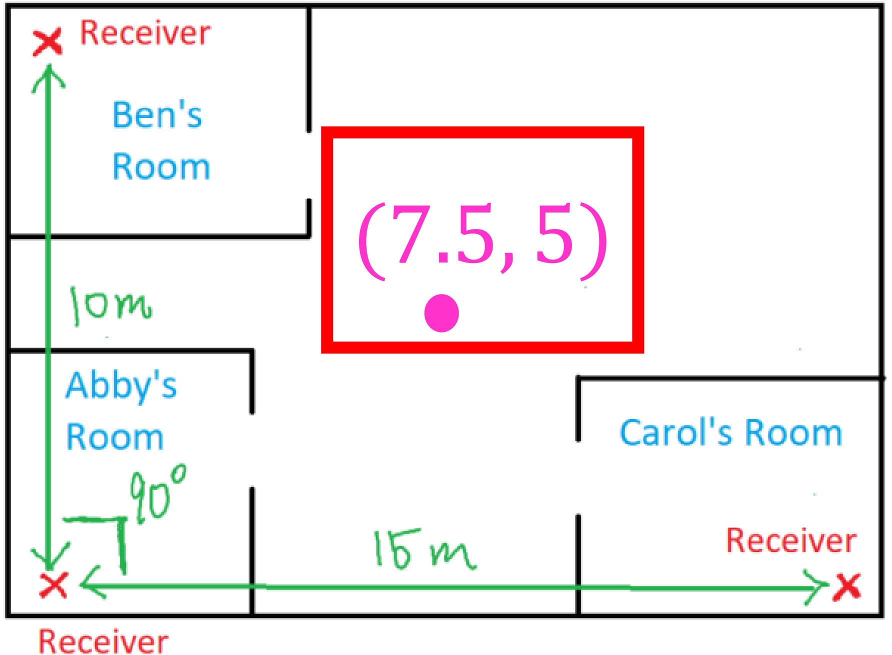


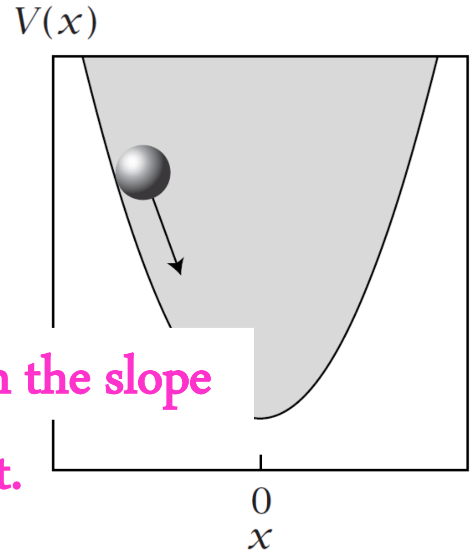
Directional Derivative:
(i) Positive= Ascending
(ii) Negative=Descending

8. Find the direction to move the access point to reduce power consumption most rapidly.

Inverse the increase in power most rapidly at $(x = 7.5, y = 5)$

$$-\nabla T = -\begin{bmatrix} \frac{1}{50}(6x - 30) \\ \frac{1}{50}(6y - 20) \end{bmatrix}$$

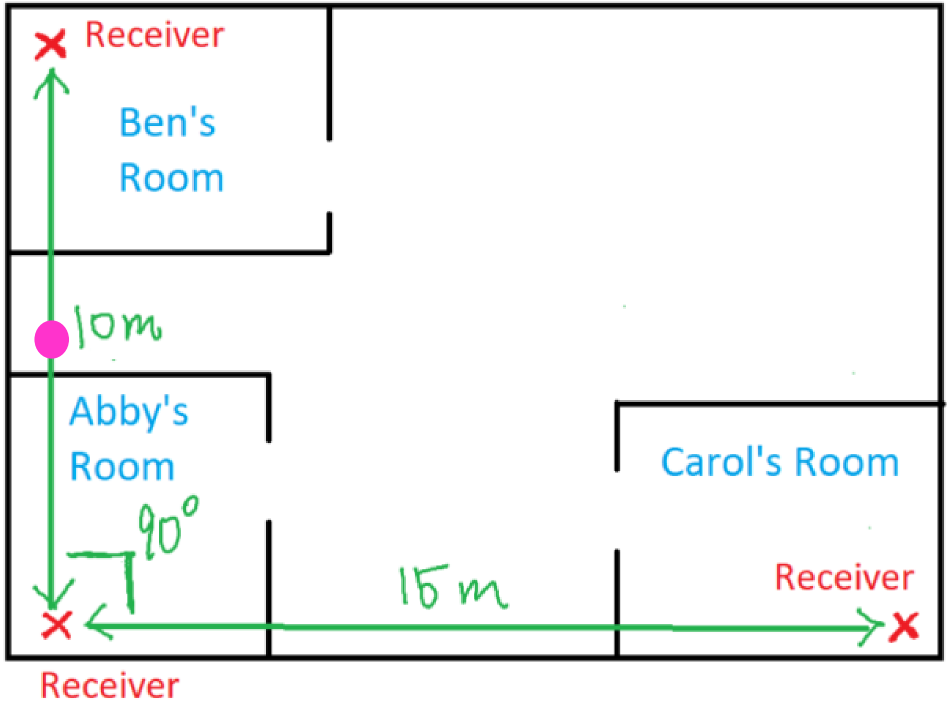




The ball moves the fastest when the slope has a negative gradient.

9. Suppose the access point is moved to the midpoint between the receivers in Abby and Ben's rooms. From that point, in which direction should be move the receiver to reduce power consumption most rapidly? **[2 marks]**

$(0, 5)$



The access point has been relocated to the midpoint between the receivers in Abby's and Ben's rooms.

It's important to note that we need to reverse the direction of the gradient vector, as we're seeking the direction of the greatest decrease in power consumption, not increase.

$$-\nabla T = -(3/25x - 3/5, 3/25y - 2/5)$$

$$-\nabla T|(0,5) = -(3/25(0) - 3/5, 3/25(5) - 2/5)$$

$$= (3/5, -1/5)$$

Unit vector: $= \left(\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}}\right)$

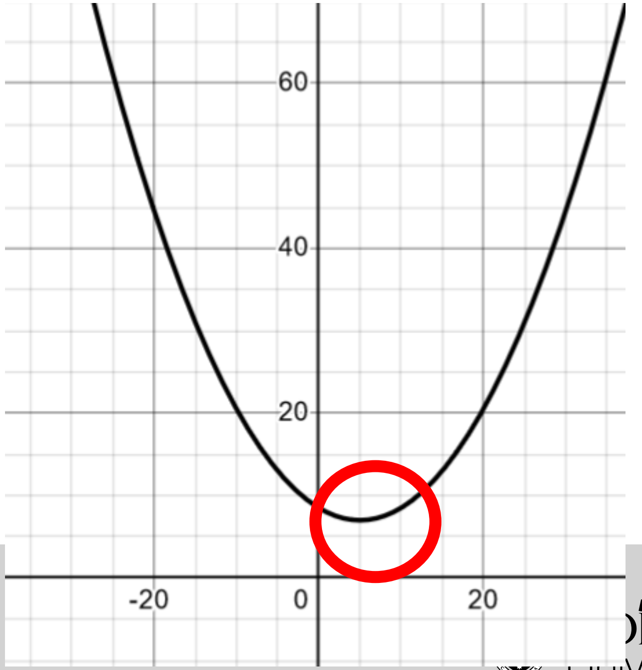
$$\boxed{\nabla T = \begin{bmatrix} \frac{1}{50}(6x - 30) \\ \frac{1}{50}(6y - 20) \end{bmatrix}}$$

10. Abby, Ben and Carol decide to alway move the WiFi access point in the direction that reduces its power consumption most rapidly, until they arrive at the most power efficient position for the access point (this is called the method of gradient flow or gradient descend). What will the gradient vector be at the most power efficient position? Make sure you explain your answer. **[2 marks]**

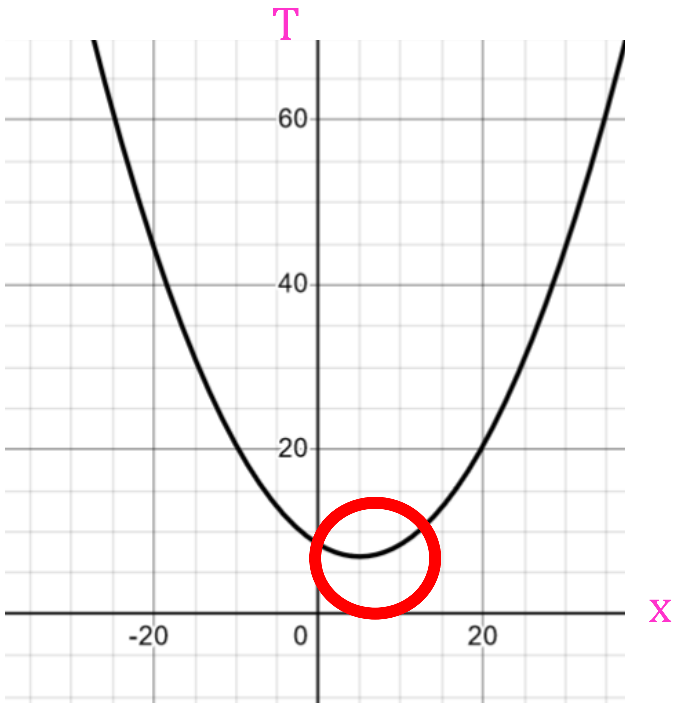
- To find the gradient vector when the access point is in its most power-efficient spot, we need to understand where a gradient is at the lowest point of a function.

➢ For example, if we look at the function T with respect to x and keep y constant, let's say y=10, we'll see a parabolic graph with a minimum value, as shown in the diagram below.

T



$$\mathbf{T = 0.02(3x^2 + 3(100) - 30x - 20(10) + 325)}$$



- When the gradient is zero, this point is a turning point on the graph, meaning it could be either a minimum or a maximum.

- So, for Abby, Ben, and Carol to find the most power-efficient position, their gradient vector needs to be zero. This approach, known as the gradient descent method, is used to find the local minimum of a function for optimization.

- T min= consume less energy?

11. Using the answer to the previous question, find the most power efficient position to place the WiFi access point. **[2 marks]**

$$\nabla T = \begin{bmatrix} \frac{1}{50}(6x - 30) \\ \frac{1}{50}(6y - 20) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{25}x - \frac{3}{5} \\ \frac{3}{25}y - \frac{2}{5} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$(x, y) = (5, 10/3)$

The Big Learning Outcomes for Week 5

After completing this week's task, you should be able to:

- Find contours of functions of two variables.
- Understand and calculate partial/directional derivatives and gradient of functions of several variables.
- Calculate tangent planes.
- Calculate higher derivatives of functions of several variables.

STUDENT FEEDBACK: ESSFS (5 min)

Take the survey,
get your voice heard.

Thank You