ENG1005 S2 2024 Workshop 3
Missile Trajectory

27 marks total

This problem set is intended for you to apply the mathematical skills you are learning. It is also designed to practice communicating your work clearly.

It is expected that you will use the workshop to develop (rough) solutions. During the workshop, you should discuss the problems with your peers and the academic staff who are there to assist you. In particular, if you are uncertain about what the problems are asking or you are stuck on a particular point, this is the time to get assistance. The time between the end of the workshop and when the solutions are due is only meant to be for writing up your solutions and for this you should not need more than an hour or two at most.

General submission information:

1. Electronic submission of your solutions is due on Moodle by **11:55 pm on Sunday of the same week**.
2. **Your solutions should include a description/explanation of what you are doing at each step and relevant working.** Without these you will receive limited marks. The description should be in complete English sentences. All mathematics should be appropriately laid out and with appropriate notation. Your writing should be similar in style to the worked solutions from the Applied Class problem sheets, not the annotations from the videos. For more information and advice, please read the "Guidelines for writing in mathematics" document posted under the "Additional information and resources" section of the ENG1005 Moodle page.
3. Your solutions may be typed or handwritten and scanned (the latter is encouraged). The **final document should be submitted as a single pdf file that is clearly and easily legible**. If the marker is unable to read it (or any part of it) you may lose marks.

Academic integrity:

You can (and should!) discuss your solutions with the other students, but **you must write up your solutions by yourself**. Copying solutions is serious academic misconduct and will be penalised according to Monash University guidelines. Other examples of academic misconduct include asking a personal tutor to do any of your assessments and posting your assessments to a "homework" website. Please refer back to your Academic Integrity module if you are in any doubt about what constitutes academic misconduct. **Your integrity is an important part of who you are. It is much more important than any grade you could receive.**

Missile Trajectory

Radars have long been used to track flying objects. However it is typically not possible to continuously track a object in real time. Instead, locations of the object are taken at different times. An interpolation process is then used to estimate the trajectory and make predictions about the future location of the object. In this workshop, we will study the methods of polynomial interpolation and least square approximation.

Suppose a missile is fired from sea level and travels directly east. Three radar stations are situated 1km, 2km and 3km due east of the launch site. As the missile passes directly overhead, each radar station measures the altitude of the missile. A short time after launch, the missile engine is switched off. For simplicity, ignore any aerodynamic forces, and assume that without any thrust from its engine, the only force acting on the missile is gravity. From Newtonian mechanics, we know that the missile will follow a parabolic trajectory, i.e., if we let x be the horizontal distance of the missile from the launch site, and y be its altitude, then we have

$$y(x) = Ax^2 + Bx + C$$

where A, B, C are constants.

From the three radar station measurements, we have the follow data about the location of the missile, with both x and y in kilometres.

x	1	2	3
y	6	7	5

We will try to find the constants A, B and C such that the three points $(1, 6)$, $(2, 7)$, and $(3, 5)$ all lie on the parabola $y = Ax^2 + Bx + C$.

1. Using the given measurements from the table, write down a **matrix equation** for the unknown variables A, B, and C. **[1 marks]**
2. Calculate the determinant of the matrix in your equation. It should be non-zero. What does this tell you about whether you can fit a unique quadratic curve through the three data points? **[2 marks]**
3. Calculate the **inverse of the matrix**, and hence find the values of A, B and C. **[3 marks]**
4. Where do you predict the missile will land (i.e. hit sea level)? **[1 mark]**

Usually having more data will give a more accurate prediction. Suppose a fourth radar station located 4km east also made a measurement, giving the following table:

x	1	2	3	4
y	6	7	5	4

5. Without doing any calculation, do you expect to be able to find a parabola through those four data points? Explain your answer. **[1 marks]**

We will now try to find a cubic $y = Ax^3 + Bx^2 + Cx + D$ that goes through the four data points.

6. Using the given measurements from the new table, write down a matrix equation for the unknown variables A, B, C and D. **[1 marks]**
7. Using appropriate row and column operations and properties of determinants with those operations, find the determinant of the matrix in your equation. What does this tell you about whether there is a unique cubic through the four data points? **[4 marks]**
8. Using Matlab or CAS, find A, B, C and D. You might find the commands on the last page useful. **[2 marks]**

9. Plot your data points and the graph of the interpolating polynomial through them. Be sure to include axis labels.

[1 mark]

10. Where does the missile land? Does this plot seem a realistic trajectory for a missile? **[1 mark]**

In practice, measurements are never exact and almost always contain some error. For example, you may find the data points don't fit a parabola precisely. In theses situations, the best we can do is to find the curve that best fits the data points. For simplicity, we will investigate the (straight) line of best fit, but similar ideas can be applied to find the parabola or any curve of best fit.

Suppose the missile did not turn of its engine, so that it flies in a straight line instead of having a parabolic trajectory. Let's return to the first set of measurements.

x	1	2	3
y	6	7	5

11. Show that the three observed points do not lie on a straight line. **[1 mark]**

Let $y(x) = Sx + T$ denote a straight line. We allow S and T to vary over all real numbers to cover all possible straight lines. For each pair (S, T), we let the triple $(y(1), y(2), y(3))$ be the *predicted value* of the radar station measurements. The actual *observed value* from the radar stations is $(6, 7, 5)$. In the method of least squares, the idea is to minimize the distance (as points in $\mathbf{R}^3$) between the predicted value and the observed value. The particular S and T that achieve this minimum will give us the line of best fit.

12. Express $(y(1), y(2), y(3))$ in terms of S and T. **[1 mark]**

13. Explain that the set of all predicted values, as S and T vary, form a plane in $\mathbf{R}^3$, the space of all triples. **[1 marks]**

14. Using ideas from week 1, find the minimum distance between the plane of all predicted values and $(6, 7, 5)$. **[3 marks]**

15. Find the line of best fit for the given data set. **[2 marks]**

There is also 1 additional mark given for the quality of the English and 1 additional mark for correct mathematical notation. These marks are easy to obtain but the markers will be instructed to be strict in awarding these marks.

Some Matlab commands

You may find these helpful in your calculations.

Try the following matrix-related commands:

1. Enter a matrix A in Matlab/Octave using

   ```
   A = [1 2 3; 6 7 8; 1 1 1]}
   ```

2. Enter a column vector **b** in Matlab/Octave using

   ```
   b = [2; 3; 5]
   ```

3. Solve the system of equations $\mathsf{A}\mathbf{x} = \mathbf{b}$ using

   ```
   x = A\b
   ```

4. Create a matrix whose columns are powers of the vector **b**

   ```
   A = vander(b)
   ```

5. Calculate the determinant of a matrix A

   ```
   det(A)
   ```

For plotting, if you have your values of a_i in a vector **a** you can use

```
ti = [1 2 3]
di = [3 1 1]
t = [1:0.01:3];
d = polyval(a,t);
plot(ti,di,'bo')
hold on
plot(t,d,'r-')
hold off
```