ENG1005 S2 2024 Workshop 5
Wifi Access Point

24 marks total

This problem set is intended for you to apply the mathematical skills you are learning. It is also designed to practice communicating your work clearly.

It is expected that you will use the workshop to develop (rough) solutions. During the workshop, you should discuss the problems with your peers and the academic staff who are there to assist you. In particular, if you are uncertain about what the problems are asking or you are stuck on a particular point, this is the time to get assistance. The time between the end of the workshop and when the solutions are due is only meant to be for writing up your solutions and for this you should not need more than an hour or two at most.

General submission information:

1. Electronic submission of your solutions is due on Moodle by **11:55 pm on Sunday of the same week**.
2. **Your solutions should include a description/explanation of what you are doing at each step and relevant working.** Without these you will receive limited marks. The description should be in complete English sentences. All mathematics should be appropriately laid out and with appropriate notation. Your writing should be similar in style to the worked solutions from the Applied Class problem sheets, not the annotations from the videos. For more information and advice, please read the "Guidelines for writing in mathematics" document posted under the "Additional information and resources" section of the ENG1005 Moodle page.
3. Your solutions may be typed or handwritten and scanned (the latter is encouraged). The **final document should be submitted as a single pdf file that is clearly and easily legible**. If the marker is unable to read it (or any part of it) you may lose marks.

Academic integrity:

You can (and should!) discuss your solutions with the other students, but **you must write up your solutions by yourself**. Copying solutions is serious academic misconduct and will be penalised according to Monash University guidelines. Other examples of academic misconduct include asking a personal tutor to do any of your assessments and posting your assessments to a "homework" website. Please refer back to your Academic Integrity module if you are in any doubt about what constitutes academic misconduct. **Your integrity is an important part of who you are. It is much more important than any grade you could receive.**

Wifi Access Point

As you can no doubt imagine, functions of several variables and multivariable calculus are everywhere in our lives. In this workshop we will study an application of in the placement of a WiFi access point.

WiFi signals are eletromagnetic radiations. As you move away from the source of the radiation, the signal decreases in strength. In fact the power of the signal obeys the Inverse-square Law:

$$P \propto \frac{1}{r^2}$$

where P is power of the signal (measured in milliwatts), $\propto$ is the mathematical symbol for proportionality, and r is the distance from the source of the signal (measured in metres).

Abby, Ben and Carol want to install a WiFi access point to provide WiFi coverage in their house. The power consumption T of the access point is directly proportional to its output signal power P measured 1 metre away. The new WiFi access point will negotiate its signal power with the receiver so that the signal received by the receiver is always the same strength no matter how far away they are.

1. Suppose the power consumption is T_0 to keep a user 4 metres away connected. Express in terms of T_0 the power consumption required to keep a user 6 metres away connected. Show that in general, the power consumption of the access point with a user l metres away is $T = kl^2$ for some constant k.

   [2 marks]

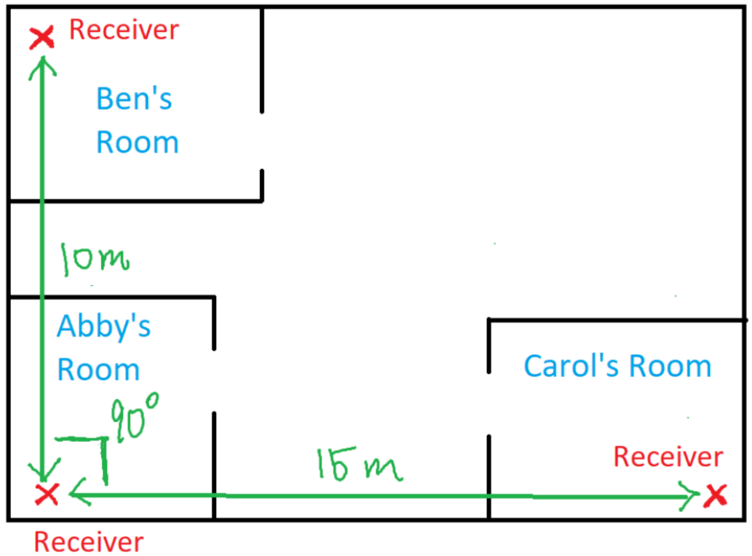
   Solution: Since signal power decreases by the inverse square law, for the same power consumption, the signal strength at 6 metres will be $\frac{4^2}{6^2} = \frac{4}{9}$ of the signal strength at 4 metres. Therefore to make sure the user at 6 metres receives the same signal strength, power consumption will have to increase by a factor of $\frac{9}{4}$. Hence the power consumption is $\frac{9}{4}T_0$.

   In general let $P(r)$ be the signal power at a distance of r. We know $T = k_1 P(1)$ for some constant of proportionality. From the inverse square law we also know $P(r) = \dfrac{k_2}{r^2}$, and that $P(l) = A$ for constants k_2 and A. Therefore $A = P(l) = \dfrac{k_2}{l^2}$ and $k_2 = Al^2$. It follows that $T = k_1 P(1) = k_1 \dfrac{Al^2}{1^2} = kl^2$ for some constant k.

2. Standing 5 metres away from the access point, Abby measures that the access point is consuming 0.5 watts of power. Use this to determine the constant in the previous question. **[1 mark]**

   Solution: We have $0.5 = k(5^2)$, hence $k = \frac{1}{50} = 0.02$. It is also OK to use milliwatts as the unit, in which case $500 = k(5^2)$ and $k = 20$. From now on we will use $k = \frac{1}{50}$ but all calculations would be the same for $k = 20$.

Three receivers are placed in the house as shown:



3. Set up a coordinate system to represent the house and write down the coordinates of the three receivers in your system. **[2 marks]**

   Solution: We set up the natural coordinate system where Abby's receiver is at the origin $(0, 0)$, Ben's receiver is on the y-axis at $(0, 10)$, and Carol's receiver is on the x-axis at$(15, 0)$.

4. Let (x, y) be the location of the access point. Write down a formula for power consumption T of the access point in terms of x and y. You may assume the three signals to the receivers are independent, and the power consumption is the sum of the three power consumptions required to maintain a good connection with each receiver. **[2 marks]**

   Solution: Let the coordinate of the access point be (x, y). The distance from the access point to Ben's receiver is $l = \sqrt{x^2 + (y - 10)^2}$. Hence the power consumption by Ben's signal is

$$\frac{l^2}{50} = \frac{x^2 + (y - 10)^2}{50}$$

   Similarly the power consumptions from Abby's and Carol's signals are

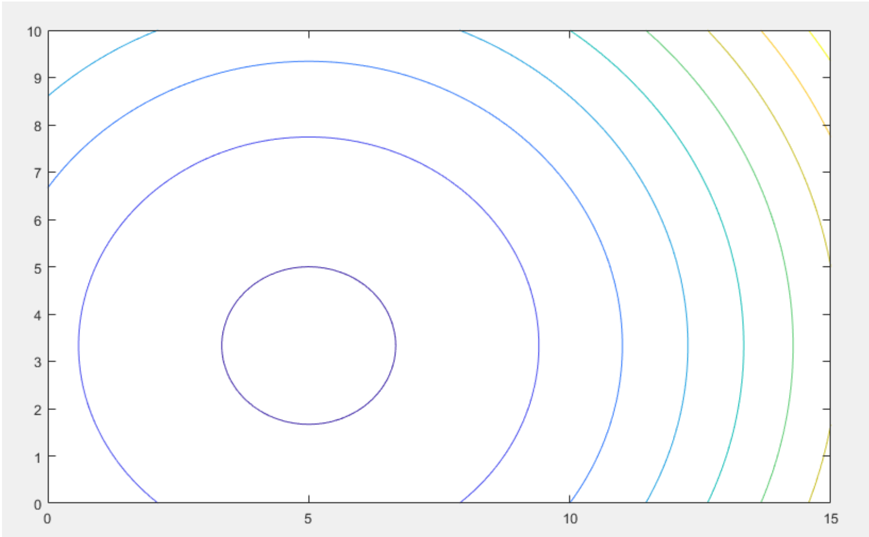
$$\frac{x^2 + y^2}{50} \quad \text{and} \quad \frac{(x - 15)^2 + y^2}{50}$$

   Hence the total power consumption is

$$\begin{aligned} T(x, y) &= \frac{1}{50}(x^2 + (y - 10)^2 + x^2 + y^2 + (x - 15)^2 + y^2) \\ &= \frac{1}{50}(3x^2 + 3y^2 - 30x - 20y + 325) \end{aligned}$$

5. With the help of Matlab or CAS, sketch three different contours of the function $T(x, y)$. **[2 marks]**

   Solution: Using Matlab, you should get a contour plot similar to this:



6. Calculate the gradient vector of T. **[3 marks]**

   Solution: We compute the partial derivatives of $T(x, y)$:

$$\frac{\partial T}{\partial x}(x, y) = \frac{1}{50}(6x - 30)$$
$$\frac{\partial T}{\partial y}(x, y) = \frac{1}{50}(6y - 20)$$

   Therefore the gradient vector is

$$\nabla T = \begin{bmatrix} \frac{1}{50}(6x - 30) \\ \frac{1}{50}(6y - 20) \end{bmatrix}$$

7. Suppose the WiFi access point is initially placed at the centre of the house. Will the power consumption increase or decrease if the access point is moved directly towards the receiver in Abby's room? **[2 marks]**

   Solution: The centre of the house is $(7.5, 5)$. The direction from $(7.5, 5)$ to $(0, 0)$ is $(-7.5, -5)$. To compute the directional derivative at $(7.5, 5)$ in the direction of $(-7.5, -5)$ we first turn the direction into a unit vector $u = \dfrac{1}{7.5^2 + 5^2}(-7.5, -5)$, and then take the dot product with the gradient vector at $(7.5, 5)$.

$$D_u(7.5, 5) = \nabla T(7.5, 5) \cdot u = \frac{1}{50}(15, 10) \cdot \frac{1}{\sqrt{7.5^2 + 5^2}}(-7.5, -5)$$

   This is clearly negative, therefore the power consumption will decrease.

8. Find the direction to move the access point to reduce power consumption most rapidly. **[2 marks]**

   Solution: The direct of most rapid decrease is $-\nabla T$. At $(7.5, 5)$, $-\nabla T = \frac{1}{50}(-15, -10)$. Hence the unit direction vector is $\frac{1}{\sqrt{13}}(-3, -2)$.

9. Suppose the access point is moved to the midpoint between the receivers in Abby and Ben's rooms. From that point, in which direction should be move the receiver to reduce power consumption most rapidly? **[2 marks]**

   Solution: This is similar to the previous question except we have a different starting point. At $(0, 5)$, $-\nabla T = \frac{1}{50}(30, -10)$. Hence the unit direction vector is $\frac{1}{\sqrt{10}}(3, -1)$.

10. Abby, Ben and Carol decide to alway move the WiFi access point in the direction that reduces its power consumption most rapidly, until they arrive at the most power efficient position for the access point (this is called the method of gradient flow or gradient descend). What will the gradient vector be at the most power efficient position? Make sure you explain your answer. **[2 marks]**

    Solution: At the most power efficient position, the gradient vector must be $(0, 0)$. If it is not, then just like the previous two questions, we can move a little in the negative gradient vector direction, and reduce the power consumption further, contradicting the fact that we are already at the minimum.

11. Using the answer to the previous question, find the most power efficient position to place the WiFi access point. **[2 marks]**

    Solution: Setting $\nabla T = (0, 0)$, we get $(6x - 30, 6y - 20) = (0, 0)$. The only solution is $(x, y) = (5, \frac{10}{3})$. This must be the most power efficient position!

There is also 1 additional mark given for the quality of the English and 1 additional mark for correct mathematical notation. These marks are easy to obtain but the markers will be instructed to be strict in awarding these marks.