Eng. Math

ENG 1005

Week 1: Vectors; Lines; Planes

(MEC) Senior Lecturer: K.B. Goh, Ph.D.

Tutor: (a) Ian Keen & (b) Jack

Pass Leader: (i) Zi Wei and (ii) Yvonne

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Topics

Week	Topic
1	Vectors, Lines, and Planes
2	Systems of Linear Equations
3	Matrices
4	Eigenvalues & Eigenvectors
5	Multivariable Calculus 1
6	Multivariable Calculus 2
7	Integration techniques and hyperbolic functions
8	O.D.E 1
9	O.D.E 2
10	O.D.E 3
11	Series 1
12	Series 2

The Big Learning Outcomes for Week 1

After completing this week's task, you should be able to:

- Calculate cross-products and dot-products and know their use.
- Find equations of lines in 3D.
- Find equations of planes in 3D.
- Find points of intersection.

Attendance Codes (Week 1)

International students

Tutorial	Wednesday, 24 Jul	02	8:00AM	3B5NS
Tutorial	Wednesday, 24 Jul	01	2:00PM	MYX7R

Workshop	Thursday, 25 Jul	01	1:00PM	B4FFF
Workshop	Friday, 26 Jul	02	10:00AM	77JQT

Admin. Stuff (1)

1. Unit's mechanics: more in our workshop

2. Pls join our MS TEAM group: see your email

3. Submissions:

Summary

ASSESSMENT	DUE
Applied class quiz week 1	Thursday, 25 July 2024, 9:55 PM **Due in 2 days**
Lecture Quiz 1	Friday, 26 July 2024, 11:55 PM **Due in 3 days**
Workshop 1 problem set	Sunday, 28 July 2024, 11:55 PM **Due in 5 days**

Admin. Stuff (2)

4. Consultation hour

- Wed: 10 am till 11 am

- Fri: 8 am till 9 am

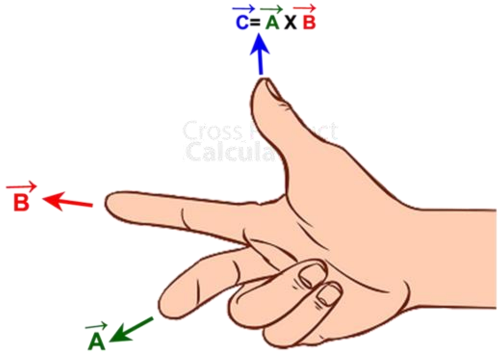
Location: 5-4-68

Today's Activity

1. Ice Breaker

2. Applied Problem Set

3. Applied Quiz



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Ice-Breaker

Icebreaker Linemaker

This exercise will help you practice some skills working with vectors and, more importantly, help you get to know your study group a bit better. To each person in the group, we will associate a line

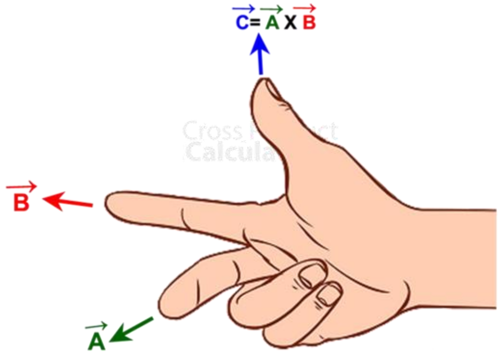
$$(a, b, c) + t(d, e, f)$$

where t is a free parameter. This is the line passing through (a, b, c) pointing in the direction of (d, e, f). For your personal line, the numbers a though f are determined as follows:

- a is the number of musical instruments that you play,
- b is the number of pets you had as a child,
- c is how you rate the Harry Potter books/movies on a scale from -5 (thoroughly hated them) to $+5$ (absolutely loved them),
- d is the number of languages that you speak,
- e is how many eggs you have eaten so far today,
- f is four minus the episode number of your favorite Star Wars movie.

Today's Activity

1. Ice Breaker
2. Applied Problem Set
3. Applied Quiz



Question 1

1. Do any two members of your group have the exact same personal line? Note that two people might have the same line without having the same values of a through f.

Learning Outcomes?

Understanding the basic property of inter- and intra-lines

1. Do any two members of your group have the same personal line? (*compare to the person on your left*)

- Step 1: Check direction vectors; *Multiple of each other?*

$$\boxed{\mathbf{r}_1(t) = (a_1, b_1, c_1) + t(d_1, e_1, f_1)}$$

$$\boxed{\mathbf{r}_2(s) = (a_2, b_2, c_2) + s(d_2, e_2, f_2)}$$

position vectors

direction vectors

- Step 1: Check direction vectors; *Multiple of each other?*

$$\boxed{\mathbf{r}_1(t) = (a_1, b_1, c_1) + t(d_1, e_1, f_1)}$$

$$\boxed{\mathbf{r}_2(s) = (a_2, b_2, c_2) + s(d_2, e_2, f_2)}$$

reduce the 1st component of the direction vector to its smallest full number form

$$t(3, 6, 9) = 3t(1, 2, 3)$$

$$\boxed{(d_1, e_1, f_1)}$$

Two Possible Outcomes:

$(d_1, e_1, f_1) \neq (d_2, e_2, f_2)$ *Not*: Skew or intersect

$(d_1, e_1, f_1) = (d_2, e_2, f_2)$ *Condition 1*: Same line or just parallel

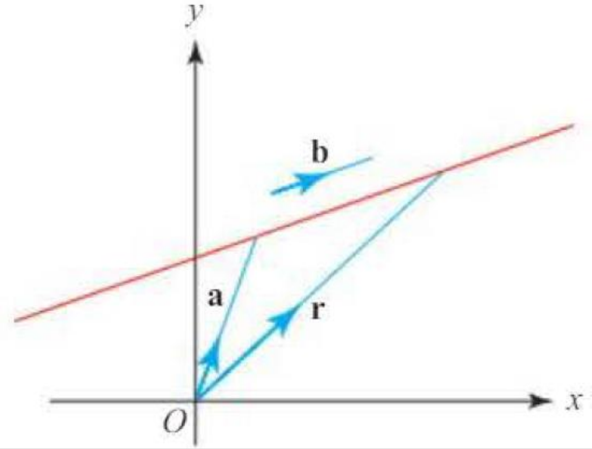
- Step 2_Check for common point: *Equate the* ***entire vector of*** *the one line to the* ***position vector*** *of the second line*

$$(a_2, b_2, c_2) = (a_1, b_1, c_1) + t(d_1, e_1, f_1)$$

- Step 3_Check for t for each **i**, **j**, **k** components

$$\boxed{\mathbf{r}_1(t) = (a_1, b_1, c_1) + t(d_1, e_1, f_1)}$$

$$\boxed{\mathbf{r}_2(s) = (a_2, b_2, c_2) + s(d_2, e_2, f_2)}$$



- Step 3_Check for t for each components

$$(a_2, b_2, c_2) = (a_1, b_1, c_1) + t(d_1, e_1, f_1)$$

Let's write in a simpler form:

$$a_2 = a_1 + td_1$$
$$b_2 = b_1 + te_1$$
$$c_2 = c_1 + tf_1$$

Condition 2: If the t values are the same for these 3 Eqs., the two lines, in principle, are the same ones.

Question 2

2. Ignoring any lines that are the same, which two personal lines are closest together? (Try to distribute the calculation load so that you all have at most two calculations to do.)

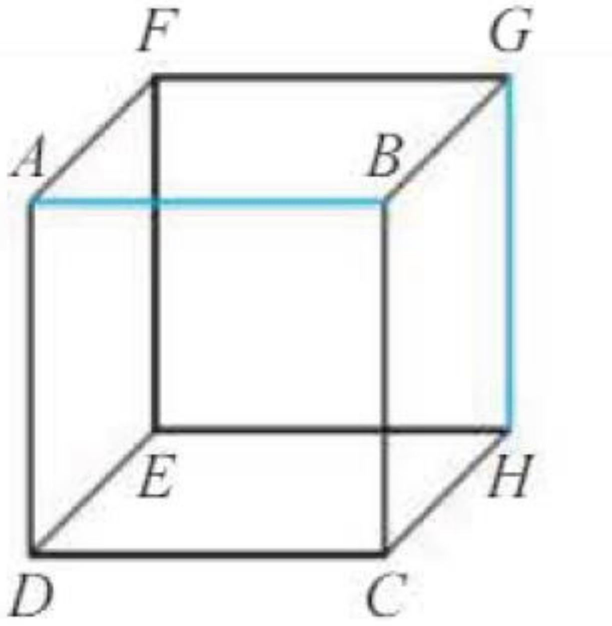
Learning Outcomes?

Calculate the min. distance between two skew lines

2. Ignoring any lines that are the same, which two personal lines are closest together?

- Step 1_ Check the type of 3D lines: *Parallel; Intersect; Skew*

- Step 2_ Watch the lecture video, topic 7



$$\lambda = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot \hat{\mathbf{n}}|$$

- Step 3_ Ascertain $(\mathbf{a}_2 - \mathbf{a}_1)$ and $\mathbf{n}$

- Step 4_ Mix-well according to the formula

Step 3.1: Physical meaning of $(\mathbf{a}_2 - \mathbf{a}_1)$

A vector connecting these 2 points

$$\boxed{\begin{aligned}\mathbf{r}_1(t) &= \mathbf{a}_1 + t\mathbf{d}_1 \\ \mathbf{r}_2(s) &= \mathbf{a}_2 + s\mathbf{d}_2\end{aligned}}$$

$$\mathbf{a}_2 - \mathbf{a}_1 = (a_2 - a_1, b_2 - b_1, c_2 - c_1)$$

$$\boxed{\lambda = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot \hat{\mathbf{n}}|}$$

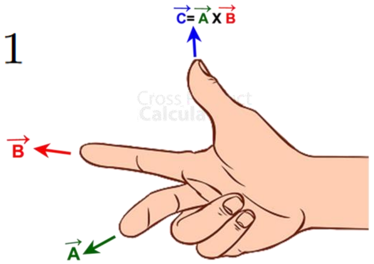
2. Ignoring any lines that are the same, which two personal lines are closest together?

Step 3.2: Physical meaning of $\mathbf{n}$

$$\boxed{\mathbf{n} = \mathbf{d}_1 \times \mathbf{d}_2}$$

A vector normal to these 2 lines

$$\mathbf{d}_1 \times \mathbf{d}_2 \neq \mathbf{d}_2 \times \mathbf{d}_1$$



$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d_1 & e_1 & f_1 \\ d_2 & e_2 & f_2 \end{vmatrix}$$

$$= (e_1 f_2 - e_2 f_1)\hat{i} - (d_1 f_2 - d_2 f_1)\hat{j} + (d_1 e_2 - d_2 e_1)\hat{k}$$

then find the unit vector

$$\lambda = \left| (\mathbf{a}_2 - \mathbf{a}_1) \cdot \hat{\mathbf{n}} \right|$$

2. Ignoring any lines that are the same, which two personal lines are closest together?

See lecture video, topic 7

Step 4: Solved the expression below

$$\lambda = \left|(\mathbf{a}_2 - \mathbf{a}_1) \cdot \hat{\mathbf{n}}\right|$$

Question 3

Which of the following statements are true in three-dimensional space? (Select all that apply.)

- Two lines that don't intersect must be parallel
- It's possible to find two lines that neither intersect nor are parallel
- Two lines that are not parallel must intersect
- Two lines that are parallel will never intersect

Learning Outcomes?

Difference between 2D and 3D lines

- Step 1_ For 2-D

2 Possible Outcomes:

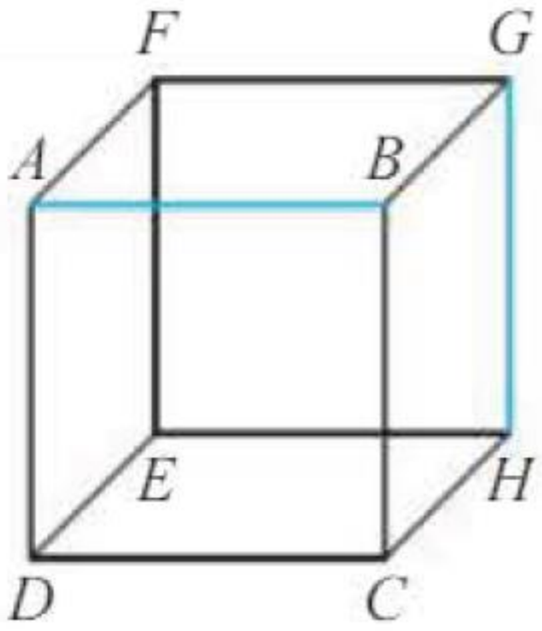
  - Parallel; or
  - Intersect

- Two lines that don't intersect must be parallel
- It's possible to find two lines that neither intersect nor are parallel
- Two lines that are not parallel must intersect
- Two lines that are parallel will never intersect

- Step 2_ For 3-D

3 Possible Outcomes:

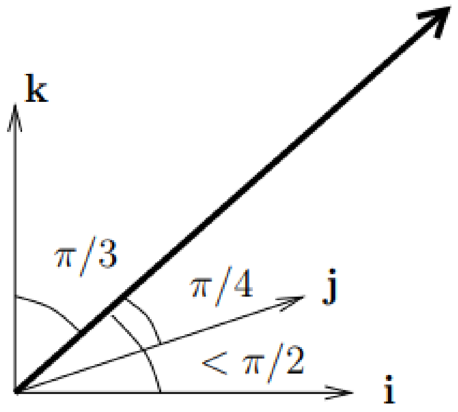
- Parallel; or
- Intersect; or
- Skew



- Two lines that don't intersect must be parallel
- It's possible to find two lines that neither intersect nor are parallel
- Two lines that are not parallel must intersect
- Two lines that are parallel will never intersect

Question 4

4. A vector is given as sketched below. It has length 2 and it has angles with the unit vectors **i**, **j** and **k** as shown. Find the vector in component form.



Learning Outcomes?

Understanding dot product

- Step 1_ Dot product definition:

$$\mathbf{b}\cdot\mathbf{i} = |\mathbf{b}||\mathbf{i}|\cos\alpha = b_1$$
$$\mathbf{b}\cdot\mathbf{j} = |\mathbf{b}||\mathbf{j}|\cos\beta = b_2$$
$$\mathbf{b}\cdot\mathbf{k} = |\mathbf{b}||\mathbf{k}|\cos\gamma = b_3$$

- Step 2_ Ingredients given

$$\alpha = [0, \pi/2]$$
$$\beta = \pi/4$$
$$\gamma = \pi/3$$

&

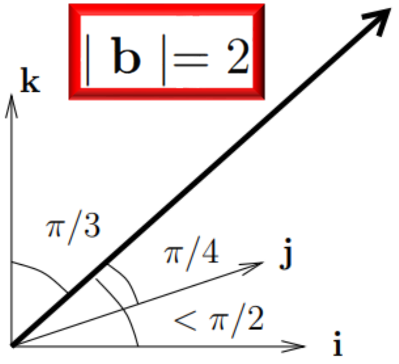
$$|\mathbf{b}| = 2$$

Recall that the modulus of a unit vector is 1

- Step 3_ Recall Pythagoras Theorem

$$r^2 = x^2 + y^2 + z^2$$
$$|\mathbf{b}|^2 = b_1^2 + b_2^2 + b_3^2$$

$$|\mathbf{b}| = 2$$



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Question 5

5. Which of the following statements are true (select all that apply)

- $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$ for all vectors $\mathbf{a}$ and $\mathbf{b}$
- $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$ for all vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$

Learning Outcomes?

Understanding cross product

5. Which of the following statements are true (select all that apply)

- $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$ for all vectors $\mathbf{a}$ and $\mathbf{b}$
- $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$ for all vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$

- Step 1_ cross product: Anti-commutative law

$$\boldsymbol{a} \times \boldsymbol{b} = -(\boldsymbol{b} \times \boldsymbol{a})$$

- Step 2_ cross product: Distributive law over addition

$$\boldsymbol{a} \times (\boldsymbol{b} + \boldsymbol{c}) = (\boldsymbol{a} \times \boldsymbol{b}) + (\boldsymbol{a} \times \boldsymbol{c})$$

Question 6

6. Consider a plane defined by the algebraic equation

$$3x + 2y - 4z = 5.$$

Find two vectors in the plane. From this, write a parametric vector equation for the plane.

Learning Outcomes?

How to convert from Cartesian to parametric format

- Step 1_ Find 3 arbitrary points on the plane; for instance,

$$P1\ (0, 0, z);\ P2\ (0, y, 0);\ \text{and}\ P3\ (x, 0, 0)$$

- Step 2_ Construct two vectors on the plane, i.e.,

$$\text{(i)}\ P2 - P1;\ \text{and}\ \text{(ii)}\ P3 - P1$$

- Step 3_ Mix-well to construct your plane expressions

$$\mathbf{r} = P1 + s*(P2 - P1) + t*(P3 - P1)$$

- Step 4_ Alternative (Q5: lecture quiz)

$$\mathbf{r} = P1 + s*(P2 - P1) + t*(P3 - P2)$$

Question 7

7. Explain why d in the vector equation of the plane

$$\mathbf{x} \cdot \hat{\mathbf{n}} = d,$$

where $\hat{\mathbf{n}}$ is a unit normal to the plane, is the perpendicular distance of the plane to the origin. This is a useful result and you are encouraged to note it.

Learning Outcomes?

Understanding the physical meaning of the dot product

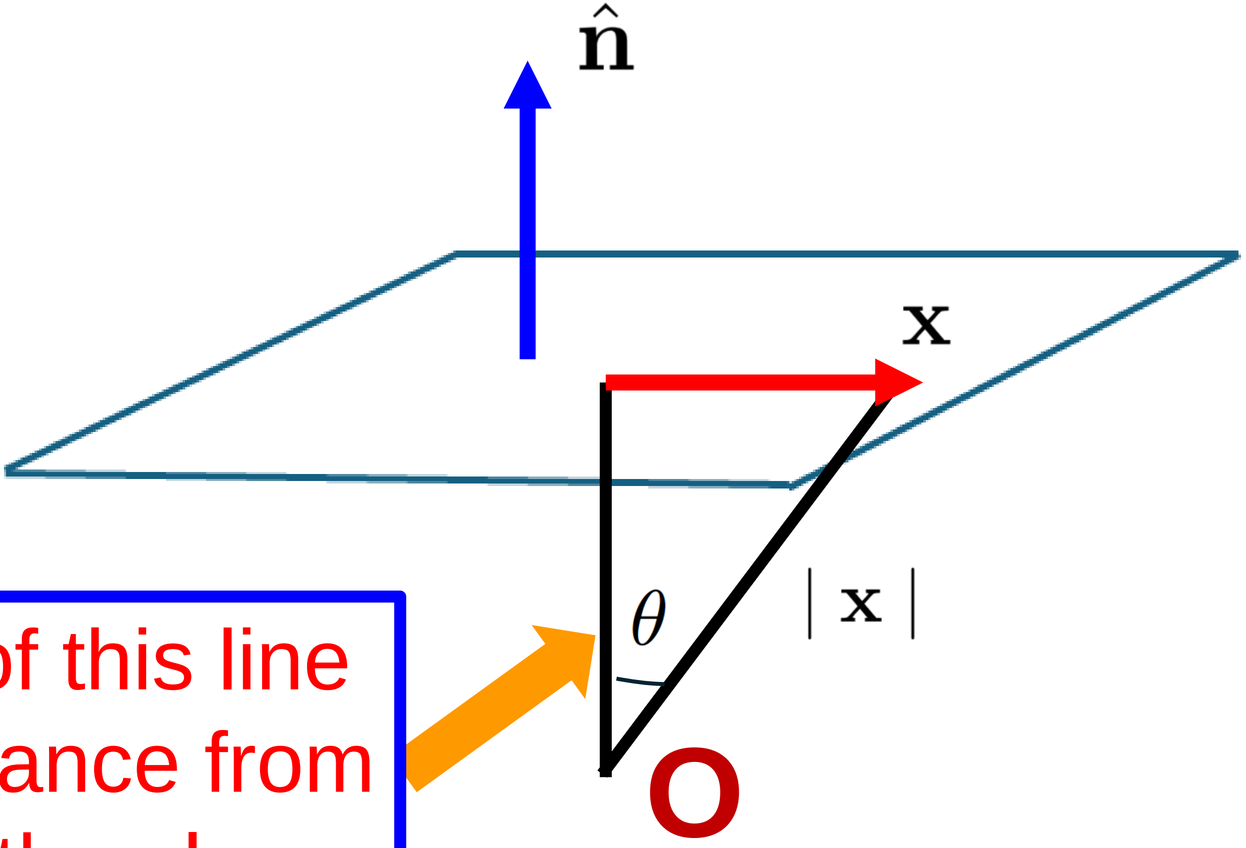
- Step 1_ Recall dot product expression:

$$\mathbf{x} \cdot \hat{\mathbf{n}} = |\mathbf{x}||\hat{\mathbf{n}}|\cos\theta$$

$$\downarrow \qquad |\hat{\mathbf{n}}| = 1$$

$$\mathbf{x} \cdot \hat{\mathbf{n}} = |\mathbf{x}|\cos\theta$$

- Step 2_ Visualize this $\mathbf{x} \cdot \hat{\mathbf{n}} = |\,\mathbf{x}\,| \cos\theta$



The length of this line is the, *d*: distance from the origin to the plane, perpendicularly

$$\boxed{\mathbf{x} \cdot \hat{\mathbf{n}} = d}$$

Question 8

8. What is the area of the triangle with corners at the three points $(1, 2, -1)$, $(0, 2, -3)$ and $(5, 4, 3)$?
Note: this is a variation of a popular exam question!

Learning Outcomes?

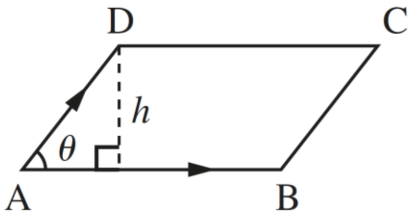
Using cross-product operation to find area

- Step 1_ Develop your two vectors from these points

P1 (1,2,-1); P2(0,2,-3) and P3(5,4,3)

AD= P2 –P1; and **AB=**P3-P1

- Step 2_ Execute the triangle formula below



$$\frac{1}{2}|\overrightarrow{AD} \times \overrightarrow{AB}|$$

Question 9

Find an expression for the distance between a point $\mathbf{a}_P$ and a line $\mathbf{x}(s) = \mathbf{a}_L + s\mathbf{b}_L$.

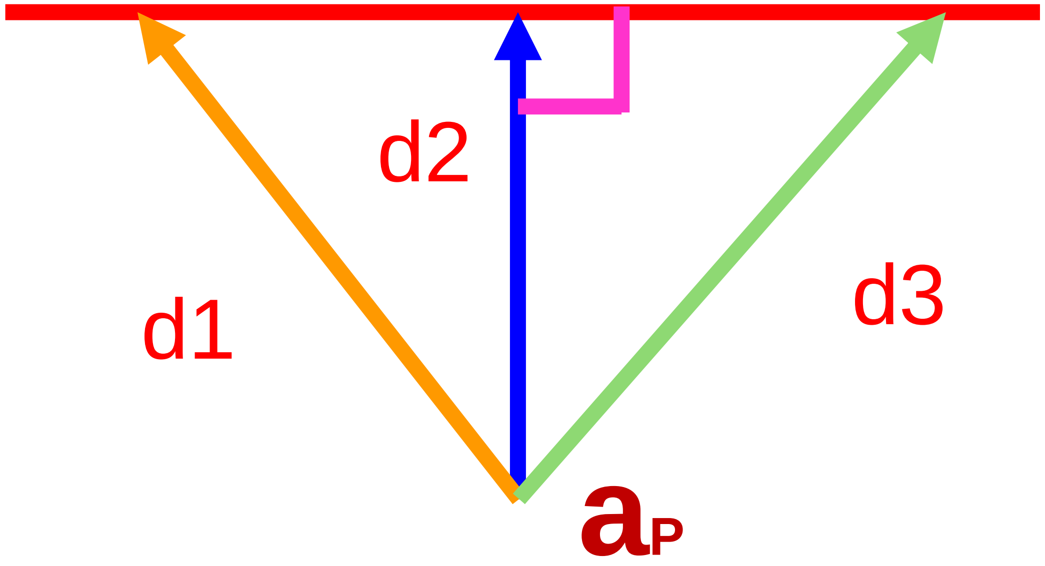
Learning Outcomes?

Previously: Min distance between 2 skew lines (Q2)

Currently: Min distance between a point and a line

Find the shortest distance, d

- Step 1: Min Distance (d)?

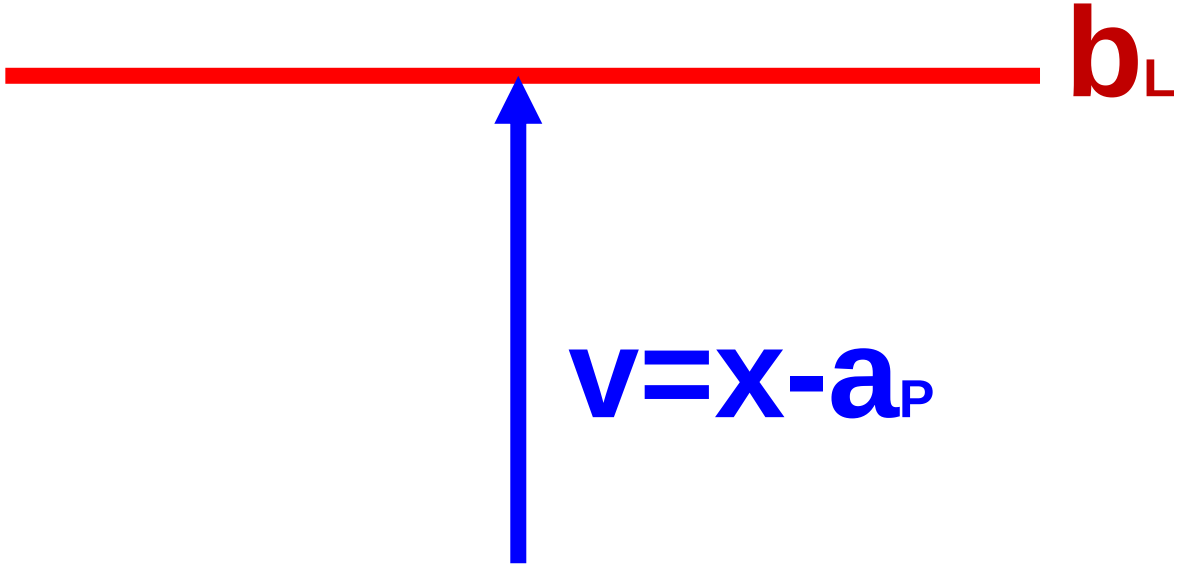


- Step 2: Blue line is the shortest

- Step 3: Finding the blue vector: between an arbitrary location on the line and the point

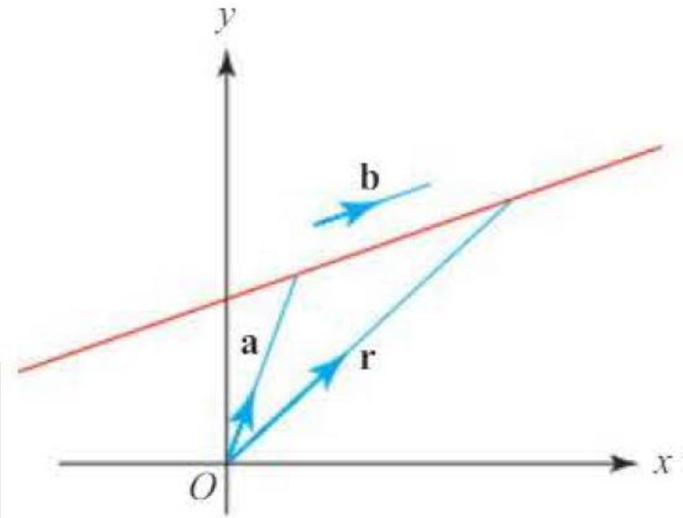
$$\mathbf{v}=\mathbf{x}-\mathbf{a}_P \quad \longrightarrow \quad \mathbf{v}=\mathbf{a}_L-\mathbf{a}_P+s\mathbf{b}_L$$

- Step 4: Express this relationship mathematically



$\Rightarrow$ $\mathbf{v}\cdot\mathbf{b}_L=0$

$\Rightarrow$ *Obtain an expression for* S_{min}



- Step 5: With the *Smin* expression we can write

$$\mathbf{v}_{min} = \mathbf{a}_L - \mathbf{a}_P + S_{min}\mathbf{b}_L$$

- Step 6: For the numerical distance, we perform

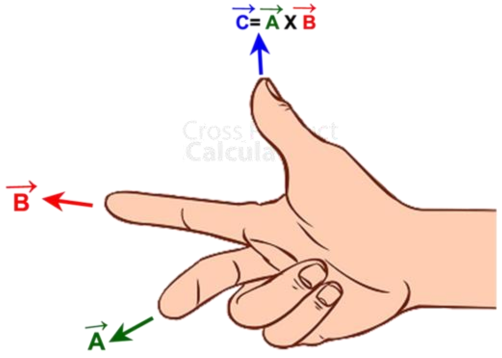
$$\sqrt{\mathbf{v}_{min} \cdot \mathbf{v}_{min} = |\mathbf{v}_{min}|\,|\mathbf{v}_{min}| \cos 0}$$

Today's Activity

1. Ice Breaker

2. Applied Problem Set

3. Applied Quiz



Thank You

Eng. Math

(Workshop) ENG 1005

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Admin. Stuff (1)

1. ENG 1090 Option: before the end of Week 2

2. Pls join our MS TEAM group: see your email

3. Submissions:

Summary

ASSESSMENT	DUE
Applied class quiz week 1	Thursday, 25 July 2024, 9:55 PM **Due in 2 days**
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Assessments breakdown

Task description	*Value*	*Due date*
Lecture quizzes	5%	Weekly 11:55pm the night before your applied class (except Monday classes) except in Week 1 when the due date is Friday at 11:55 pm (Malaysia time)
Applied class quizzes	5%	Weekly during your applied class
Workshop problems	20%	Weekly at 11:55pm Sunday, except for Weeks 7 (midsem)
Mid-semester exam	20%	During your workshop in Week 7

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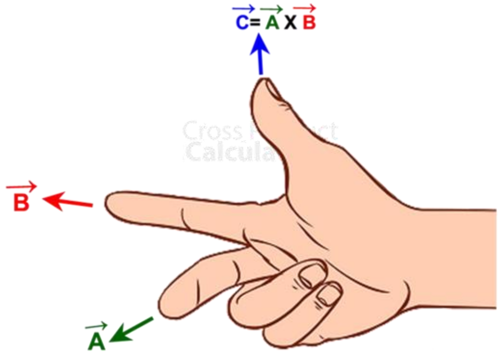
Today's Activity

1. Recap of Q9

2. Workshop Problem Set



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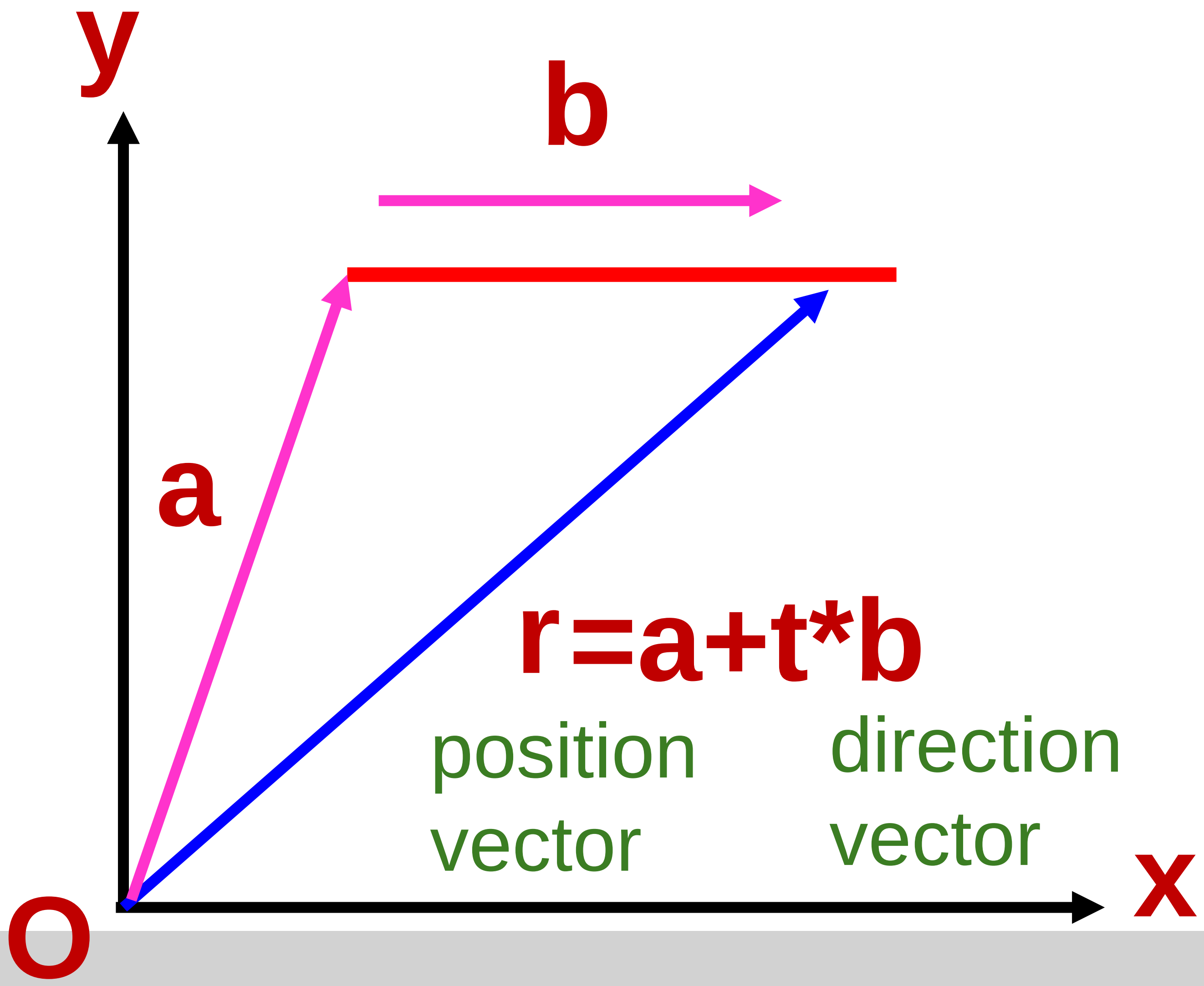
Find an expression for the distance between a point $\mathbf{a}_P$ and a line $\mathbf{x}(s) = \mathbf{a}_L + s\mathbf{b}_L$.

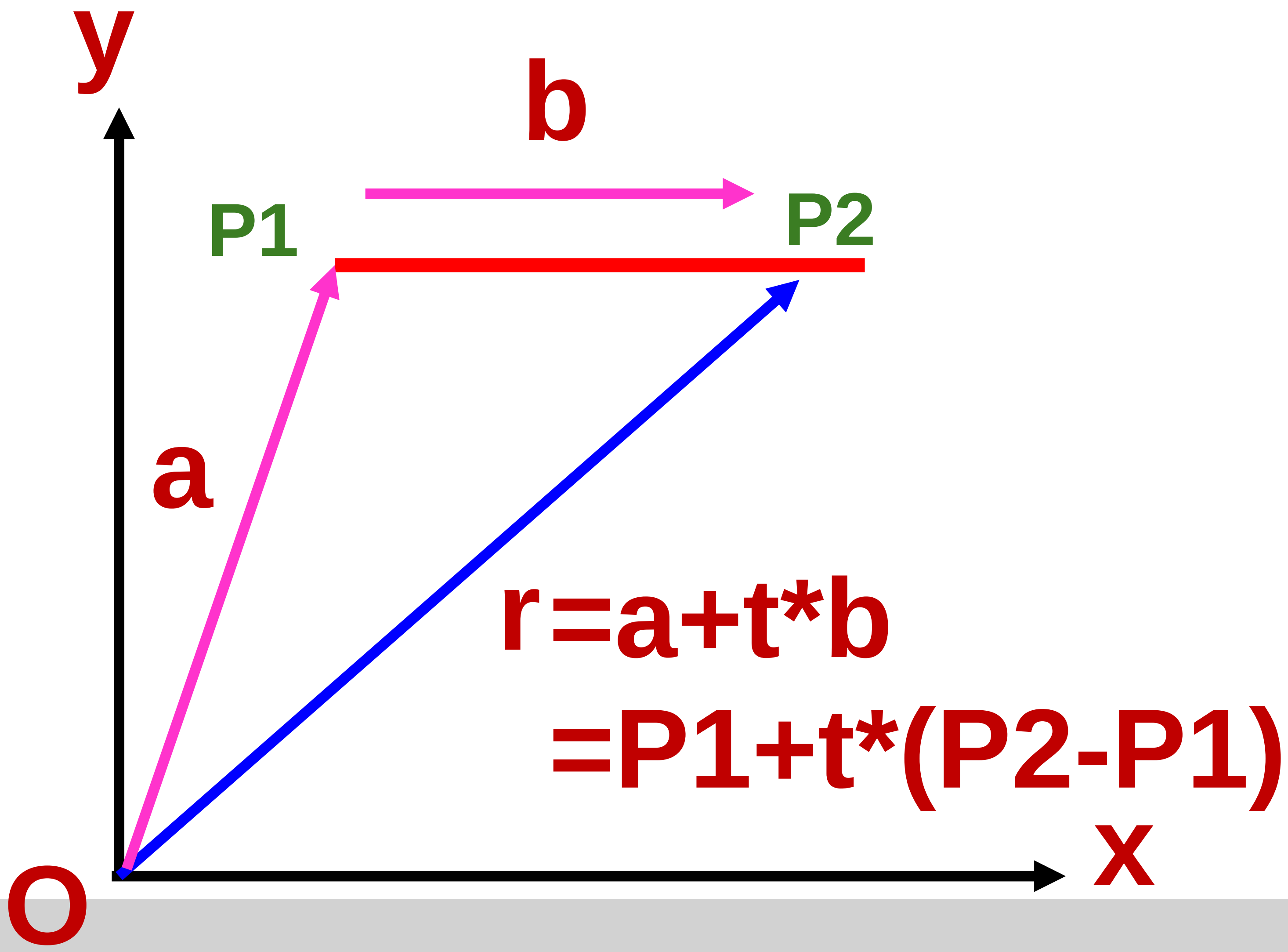
Learning Outcomes?

Min distance between a point and a line

Basics of a line (1)



Basics of a line (2)



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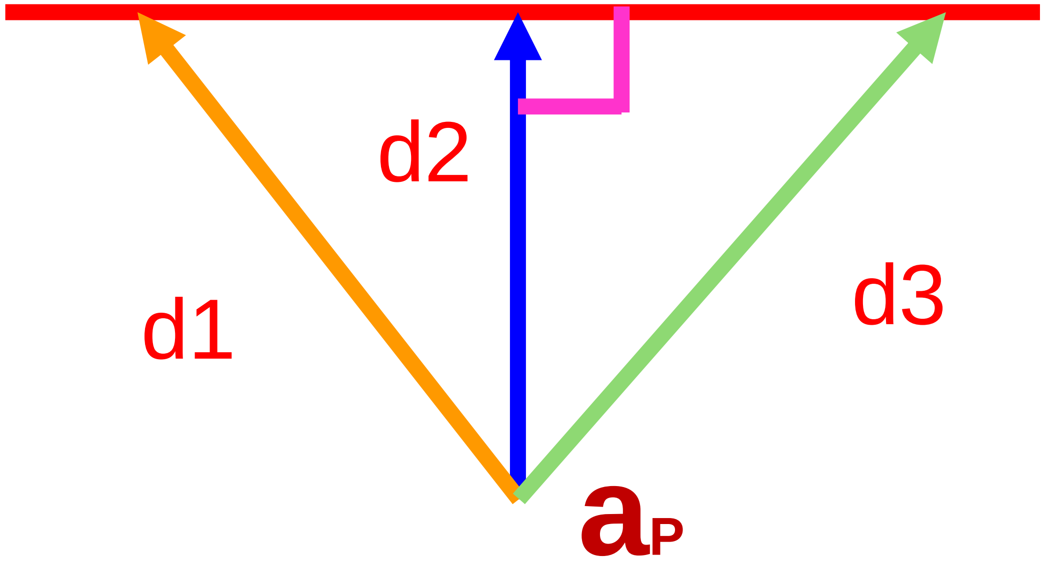
Find the shortest distance, d

- Step 1: Min Distance (d)?

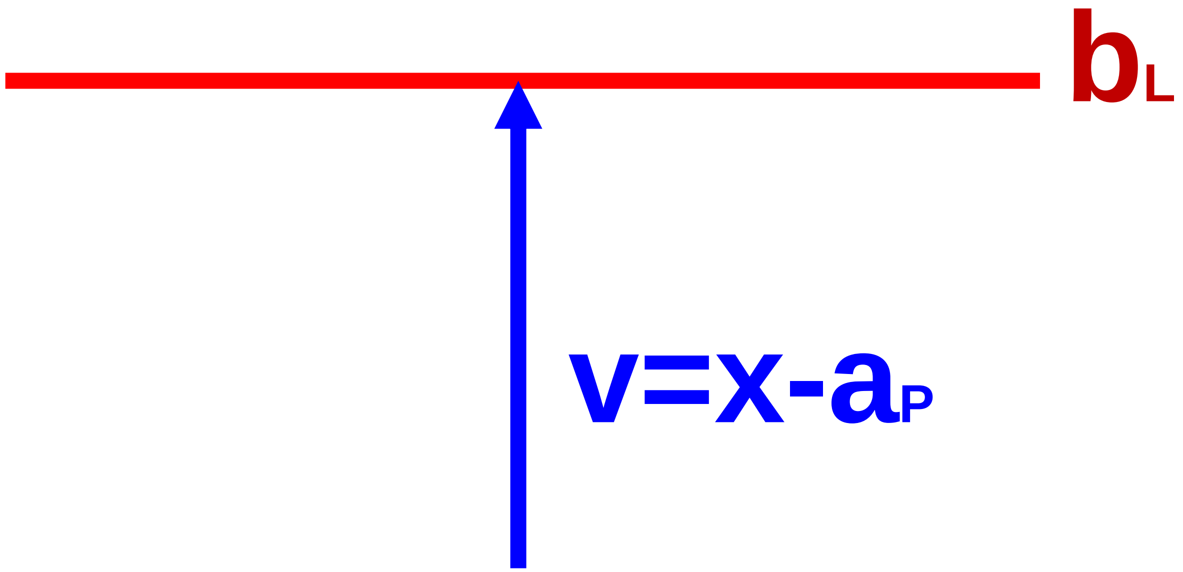


- Step 2: Blue line is the shortest

- Step 3: Finding the blue vector: between an arbitrary location on the line and the point

$$\mathbf{v}=\mathbf{x}-\mathbf{a}_P \Longrightarrow \mathbf{v}=\mathbf{a}_L-\mathbf{a}_P+s\mathbf{b}_L$$

- Step 4: Express this relationship mathematically



$\Rightarrow$ $\mathbf{v}\cdot\mathbf{b}_L=0$

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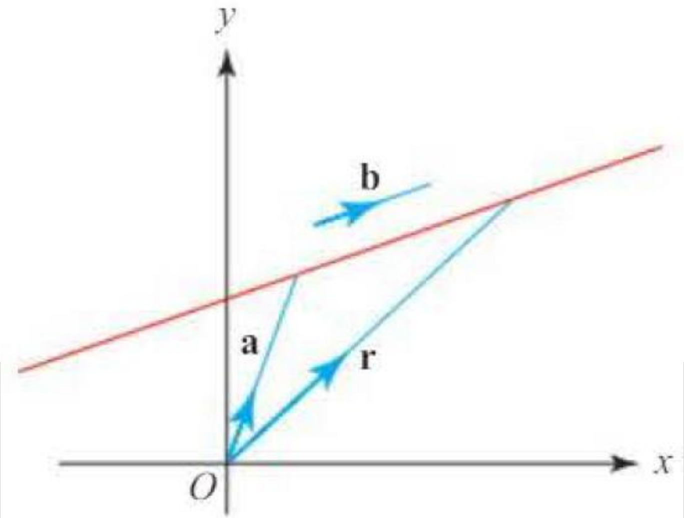

- Step 5: With the *Smin* expression we can write

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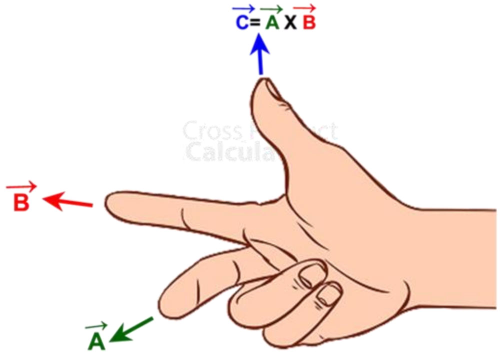
- Step 6: For the numerical distance, we perform

$$\sqrt{\mathbf{v}_{min} \cdot \mathbf{v}_{min} = |\mathbf{v}_{min}|\,|\mathbf{v}_{min}| \cos 0}$$

Today's Activity

1. Recap of Q9

2. ==Workshop Problem Set (Group Work)==



Concentrated solar thermal is one approach to combat the problem of intermittency in solar power generation. The idea is to focus the sun's rays from a broad area onto a single collector to heat molten salt (or another material). This heat can then be stored for later use to generate steam and drive turbines when the sun isn't shining, complementing the use of photovoltaics. In this workshop, you will explore how the sun's rays can be focused onto a collector using an array of plane mirrors (as shown in the image below; a similar plant is being developed near Port Augusta, SA, and others are planned around Australia)

By National Renewable Energy Laboratory https://commons.wikimedia.org/w/index.php?curid=1455806

The physical laws for reflection in a mirror are the following:

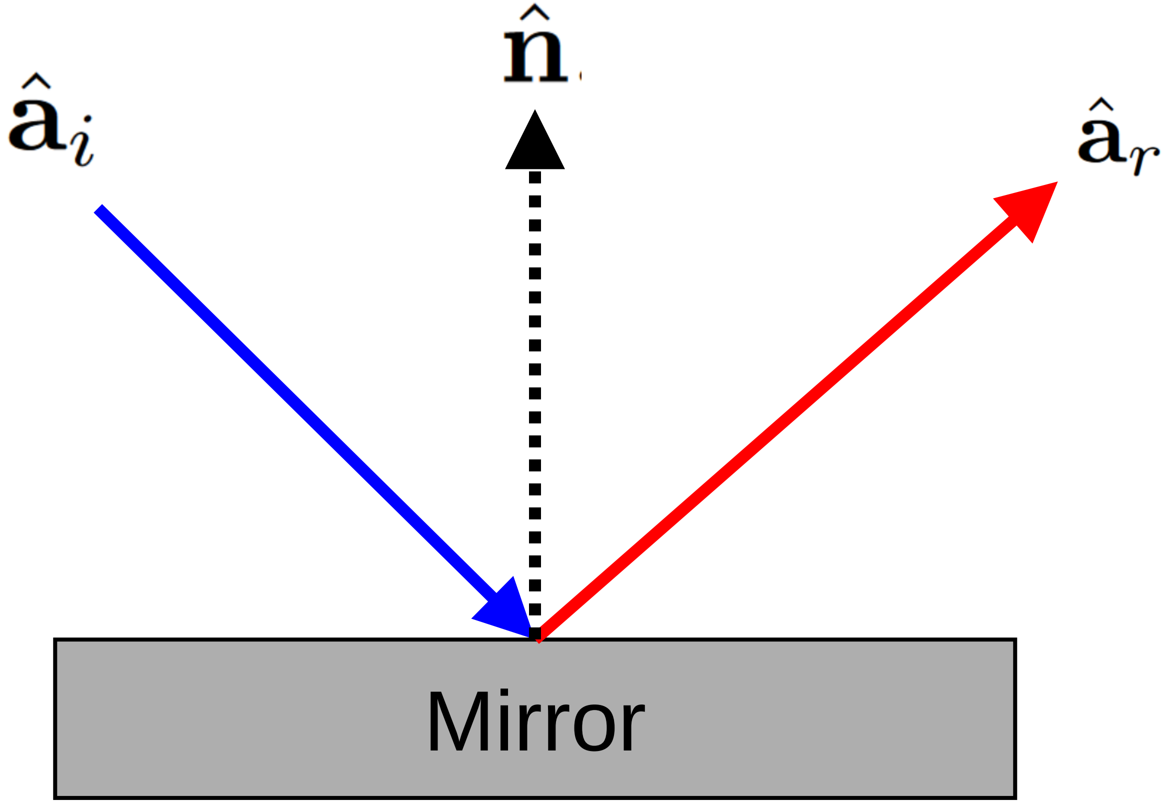
- The reflected light ray is in the same plane as the incident (i.e. incoming) ray and the normal to the mirror.
- The angle between the reflected ray and the normal to the mirror (the angle of reflection) is the same as that between the incident ray and the normal to the mirror (the angle of incidence). This is usually stated as "the angle of reflection is equal to the angle of incidence".

If $\hat{\mathbf{a}}_i$ is a unit vector in the direction of an *incoming* light ray onto a plane mirror with normal $\hat{\mathbf{n}}$, then the mirror reflection law says that the direction of the reflected ray $\hat{\mathbf{a}}_r$ is given by

$$\hat{\mathbf{a}}_r = \hat{\mathbf{a}}_i - 2(\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}.$$

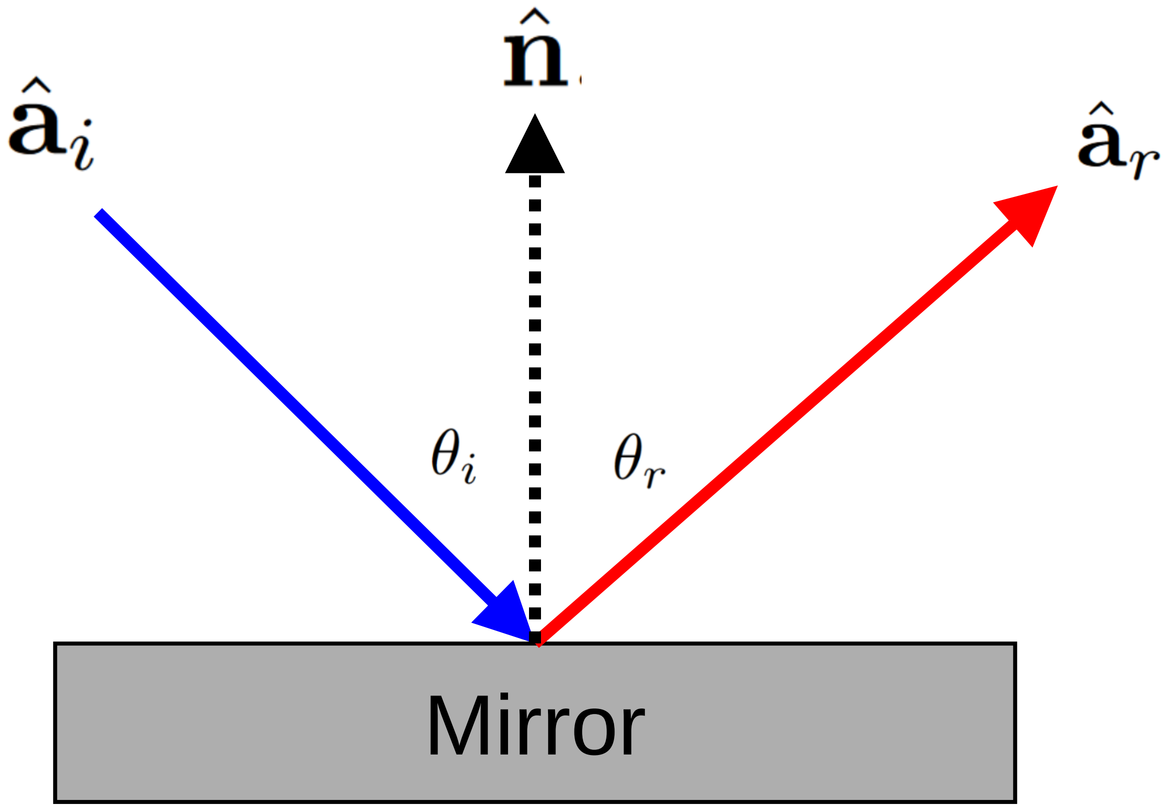
Here both $\hat{\mathbf{a}}_i$ and $\hat{\mathbf{a}}_r$ are pointing in the direction that the light ray is travelling towards, and $\hat{\mathbf{n}}$ is the unit normal vector pointing away from the surface of the mirror.

Q1: Make a diagram that illustrates the physical setup which includes the mirror, the light ray before and after reflection. Clearly label the vectors $\hat{\mathbf{n}}$, $\hat{\mathbf{a}}_i$ and $\hat{\mathbf{a}}_r$ on your diagram. **[2 marks]**



Here both $\hat{\mathbf{a}}_i$ and $\hat{\mathbf{a}}_r$ are pointing in the direction that the light ray is travelling towards, and $\hat{\mathbf{n}}$ is the unit normal vector pointing away from the surface of the mirror.

Q2: On the same diagram, clearly label the angle of incidence θ_i and the angle of reflection θ_r. [**1 mark**]



Q3: Verify that if $\hat{\mathbf{a}}_r$ is given by the above formula, then the two physical laws are satisfied.

$$\boxed{\hat{\mathbf{a}}_r = \hat{\mathbf{a}}_i - 2(\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}.}$$

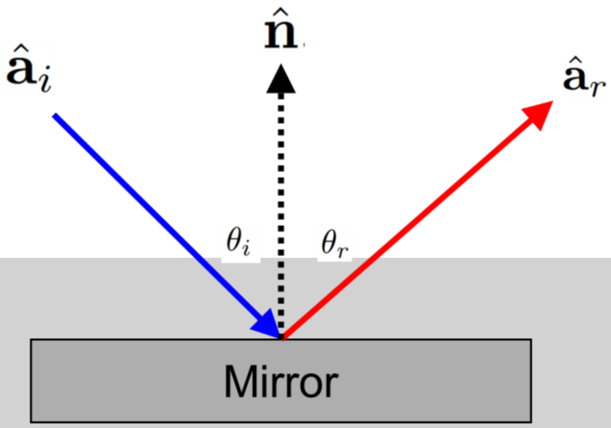
- Step 3.1: (see diagram) Dot Product?
  - Obtain $\cos(\theta_r)$

$$\cos(\theta_r) = \frac{\hat{\mathbf{a}}_r \cdot \hat{\mathbf{n}}}{|\hat{\mathbf{a}}_r||\hat{\mathbf{n}}|}$$

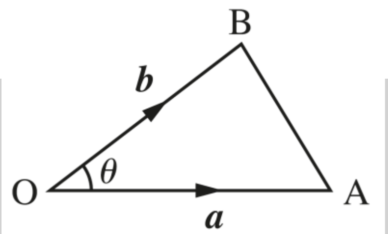
  - How about $\cos(\theta_i)$?

$$\cos(\theta_i) = \frac{-\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}}}{|-\hat{\mathbf{a}}_r||\hat{\mathbf{n}}|}$$

Recall that the modulus of a unit vector is 1



$$\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}|\cos\theta,$$



Arrow pointing away from each other

Q3: Verify that if $\hat{\mathbf{a}}_r$ is given by the above formula, then the two physical laws are satisfied.

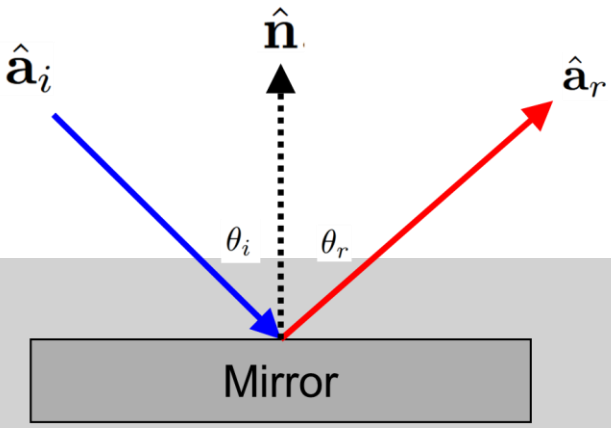
- Step 3.2: Mix-well

$$\cos(\theta_r) = \hat{\mathbf{a}}_r \cdot \hat{\mathbf{n}}$$

$$\cos(\theta_r) = (\hat{\mathbf{a}}_i - 2(\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}) \cdot \hat{\mathbf{n}}$$

$$\hat{\mathbf{a}}_r = \hat{\mathbf{a}}_i - 2(\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}.$$

"the angle of reflection is equal to the angle of incidence"

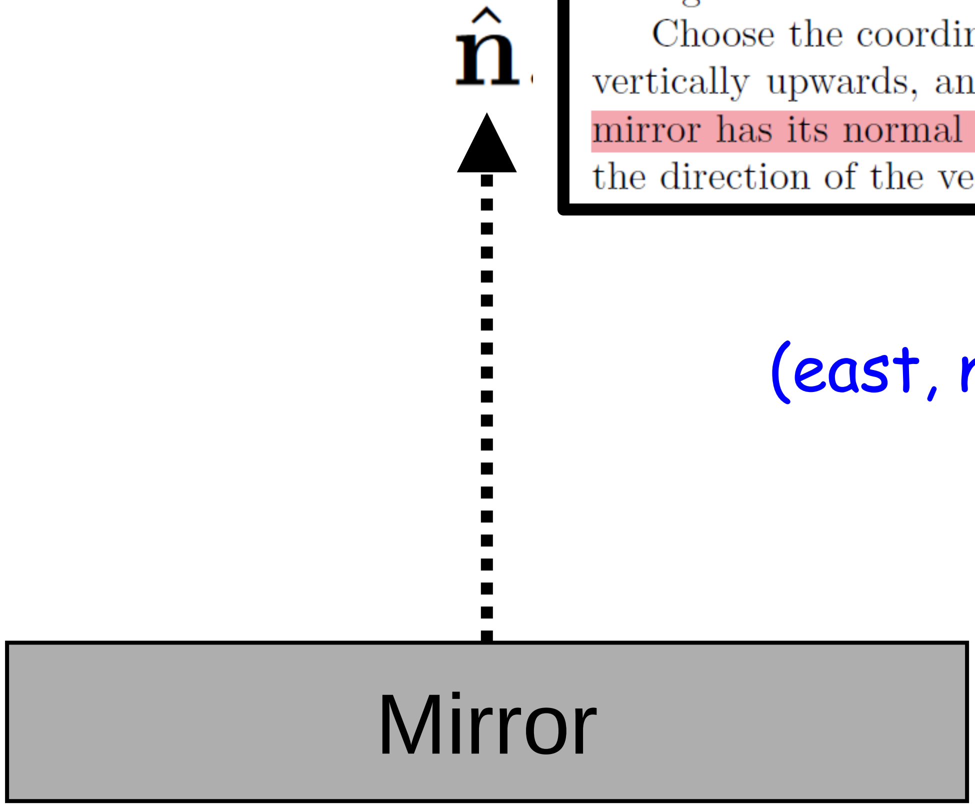


$$\cos(\theta_i) = \frac{-\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}}}{|-\hat{\mathbf{a}}_r||\hat{\mathbf{n}}|}$$

Q4 Write down the unit normal vector $\hat{\mathbf{n}}$ to the mirror in its resting position.

As the sun travels across the sky, the mirrors need to be rotated to ensure that they continue to reflect the light onto the collector. You will now investigate how to adjust the position of an individual mirror.

Choose the coordinate system such that the x-axis is aligned due East, the y-axis due North, the z-axis vertically upwards, and the centre of the mirror is located at the origin. In its initial resting position, the mirror has its normal pointing due East. Suppose that in this coordinate system, the sun appears to be in the direction of the vector $(10, 2, 11)$.

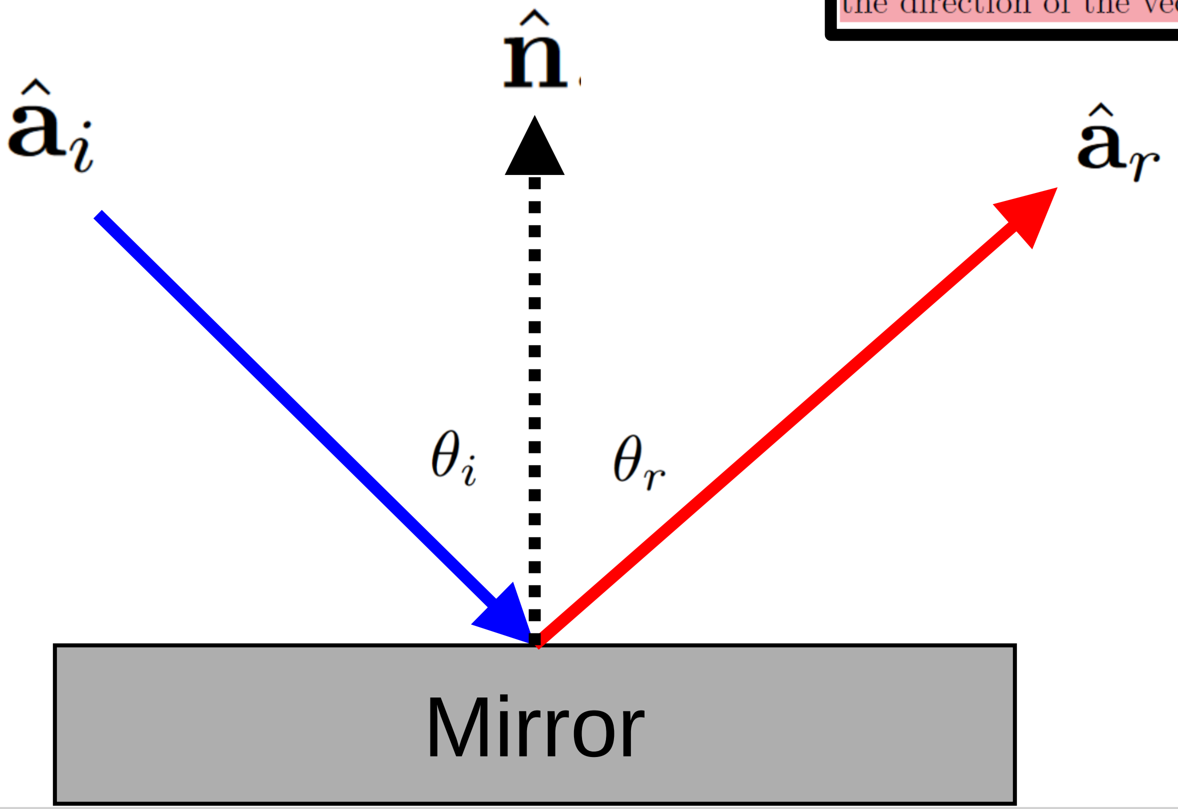


(east, north, upwards) $\longrightarrow$ (1, 0, 0)

Q5 Write down the unit vector $\hat{\mathbf{a}}_i$ of the *incoming* sunlight.

As the sun travels across the sky, the mirrors need to be rotated to ensure that they continue to reflect the light onto the collector. You will now investigate how to adjust the position of an individual mirror.

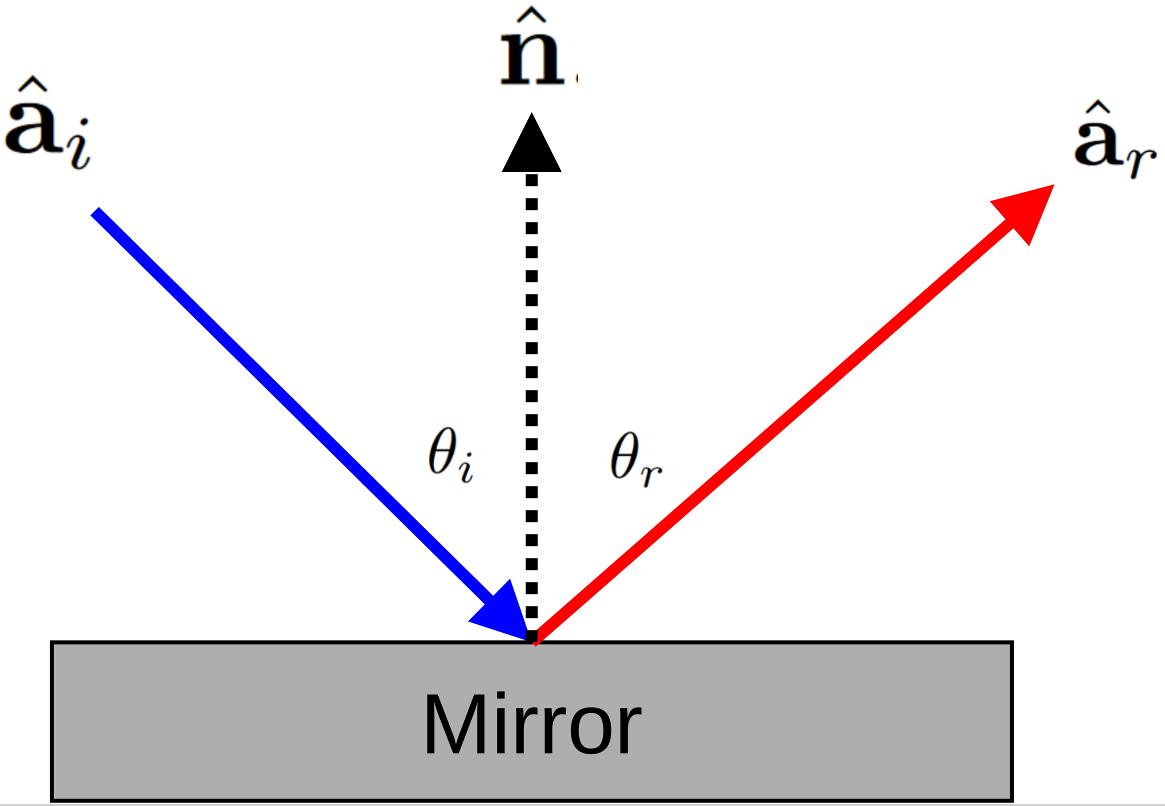
Choose the coordinate system such that the x-axis is aligned due East, the y-axis due North, the z-axis vertically upwards, and the centre of the mirror is located at the origin. In its initial resting position, the mirror has its normal pointing due East. Suppose that in this coordinate system, the sun appears to be in the direction of the vector $(10, 2, 11)$.



Don't forget the negative

$$\hat{\mathbf{a}} = \frac{-\mathbf{a}}{|\mathbf{a}|}$$

Q6 Calculate the unit vector $\hat{\mathbf{a}}_r$ of the reflected light ray.

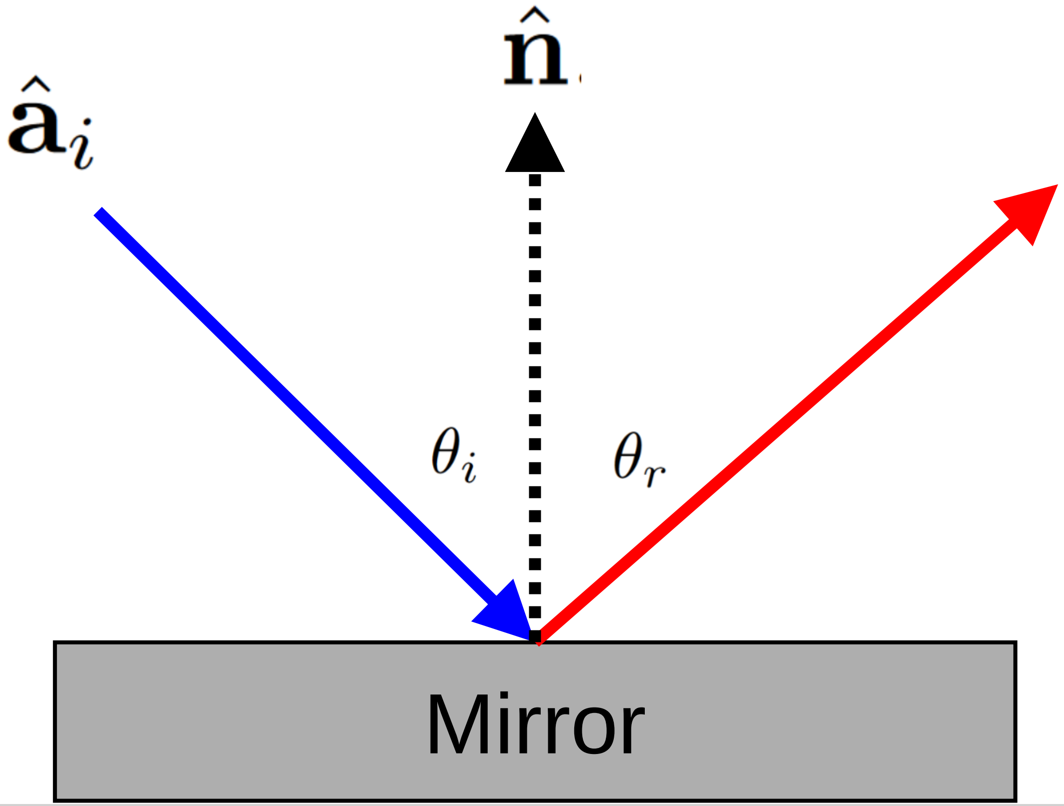


- Given, we have:

$$\boxed{\hat{\mathbf{a}}_r = \hat{\mathbf{a}}_i - 2(\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}.}$$

- We ascertain (Q4): $\hat{\mathbf{n}}$
- We determined (Q5): $\hat{\mathbf{a}}_i$

Q7 Suppose the collector is located at the position $(-100, 100, 50)$, will the reflected light ray hit the collector? Justify your answer. **[1 mark]**



- Check the direction:

$$(-100, 100, 50)$$

vs.

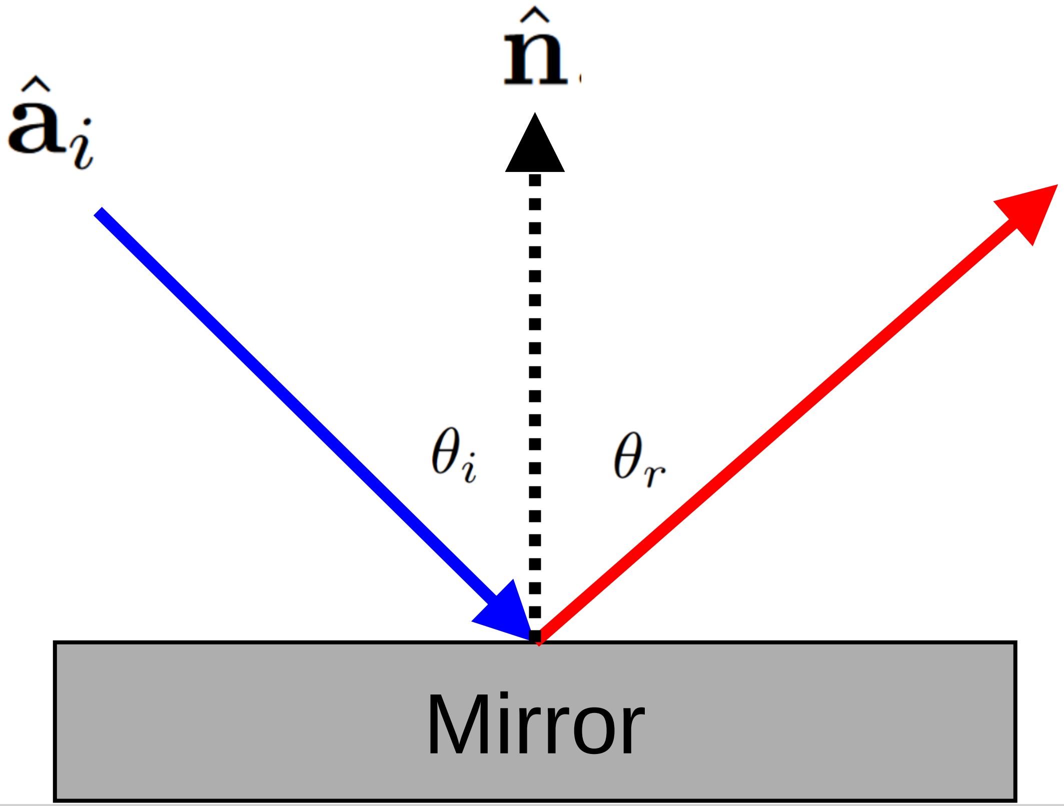
Your calculated $\hat{\mathbf{a}}_r$

reduce the vectors to their smallest full number form before comparing

Q8 The mirror is rotated so that the reflected light ray will hit the collector. Calculate the unit normal vector $\hat{\mathbf{n}}$ to the mirror in this position. **[4 marks]**

$\boxed{(-100, 100, 50)}$

Understand the expression



$$\underbrace{2(\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}})}\hat{\mathbf{n}} = \hat{\mathbf{a}}_i - \hat{\mathbf{a}}_r$$

Pre-factor, recalling (negative) (scalar) dot product

$$\hat{n} \propto -(\widehat{a_i} - \widehat{a_r})$$

$$\boldsymbol{a} \cdot \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\boxed{\hat{\mathbf{a}} = \frac{\boldsymbol{a}}{|\boldsymbol{a}|}}$$

$$\boxed{\hat{\mathbf{a}}_r = \hat{\mathbf{a}}_i - 2(\hat{\mathbf{a}}_i \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}.}$$

The Big Learning Outcomes for Week 1

After completing this week's task, you should be able to:

- Calculate cross-products and dot-products and know their use.
- Find equations of lines in 3D.
- Find equations of planes in 3D.
- Find points of intersection.

Thank You

VECTORS

Vector

A quantity that has both magnitude and direction is called a vector. The distance between the initial and terminal points of a vector is called its **magnitude**. Magnitude of vector $\overrightarrow{AB}$ is $|\overrightarrow{AB}|$.

Direction ratios and direction cosines

The scalar components of a vector are its direction ratios, and represent its projections along the respective axes. The **magnitude** (r), **direction ratios** (a,b,c) and **direction cosines** (l,m,n) of vector $a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ are related as:

$$l = \frac{a}{r}, m = \frac{b}{r}, n = \frac{c}{r}$$

eg: if $\overrightarrow{AB} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, then r = $\sqrt{1+4+9} = \sqrt{14}$
Direction ratios are (1,2,3)
and direction cosines are $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

Position of Vectors

Position vector of a point P (x, y, z) is $x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and its **magnitude** is $OP(r) = \sqrt{x^2 + y^2 + z^2}$.

eg: Position vector of P(2,3,5) is $2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ and its magnitude is $\sqrt{2^2 + 3^2 + 5^2} = \sqrt{38}$.

Position Vector

The **position vector** of a pointing R **dividing a line segment** joining P and Q whose position vectors are $\vec{a}, \vec{b}$ resp., in the ratio m : n
(i) Internally is $\frac{n\vec{a} + m\vec{b}}{m+n}$, (ii) externally is $\frac{m\vec{b} - n\vec{a}}{m-n}$

Scalar product of two vectors

$\vec{a}$, $\vec{b}$ are the vectors and 'θ', angle between them, then their **scalar product** $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$$\Longrightarrow \cos\theta = \frac{\vec{a}\ \vec{b}}{|\vec{a}||\vec{b}|}$$

Cross product of two vector

$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta\, \hat{n},$$

$\hat{n}$ is a unit vector perpendicular to line joining a, b.

Unit Vector

For a given vector $\vec{a}$, the vector $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$ gives the **unit vector in the direction of $\vec{a}$**.
eg: if $\vec{a} = 5\,\hat{\imath}$, then $\hat{a} = \frac{5\hat{\imath}}{|5|} = \hat{\imath}$, which is a unit vector.

Types of Vectors

(i) **Zero** vector (*initial and terminal points coincide*)
(ii) **Unit** vector (*magnitude is unity*)
(iii) **Coinitial** vectors (*same initial points*)
(iv) **Collinear** vectors (*parallel to the same line*)
(v) **Equal** vectors (*same magnitude and direction*)
(vi) **Negative** of a vector (*same magnitude, opp. direction*)

Properties of Vector

If we have two vectors
$\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$, $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ and
λ is any scalar, then-

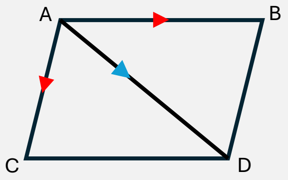
(i) $\vec{a} \pm \vec{b} = (a_1 \pm b_1)\hat{\imath} + (a_2 \pm b_2)\hat{\jmath} + (a_3 \pm b_3)\hat{k}$

(ii) $\lambda\,\vec{a} = (\lambda a_1)\hat{\imath} + (\lambda a_2)\hat{\jmath} + (\lambda a_3)\hat{k}$

(iii) $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$ and

(iv) $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

The **vector sum** of two **co-initials vector** is given by the diagonal of the parallelogram whose adjacent sides are given vectors.



If $\overline{AB}, \overline{AC}$ are the given vectors,
Then $\overline{AB} + \overline{AC} = \overline{AD}$

The **vector sum** of the three sides of a **triangle** taken in order is $\vec{0}$.

i.e. If ABC is given triangle, then
$\overline{AB} + \overline{BC} + \overline{CA} = \vec{0}$.