MONASH University

Eng. Math

ENG 1005

Week 3: Matrices

(MEC) Senior Lecturer: K.B. Goh, Ph.D.

Tutor: (a) Ian Keen & (b) Jack

Pass Leader: (i) Zi Wei and (ii) Yvonne

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Topics

Week	Topic
1	Vectors, Lines, and Planes
2	Systems of Linear Equations
3	Matrices
4	Eigenvalues & Eigenvectors
5	Multivariable Calculus 1
6	Multivariable Calculus 2
7	Integration techniques and hyperbolic functions
8	O.D.E 1
9	O.D.E 2
10	O.D.E 3
11	Series 1
12	Series 2

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The Big Learning Outcomes for Week 3

After completing this week's task, you should be able to:

- Perform matrix algebra.
- Understand how matrices represent transformations of space.
- Understand what a matrix determinant represents and how to calculate it.
- Calculate matrix inverses.

Attendance Codes (Week 3)

International students

Tutorial	Wednesday, 7 Aug	02	8:00AM	YZ95W
Tutorial	Wednesday, 7 Aug	01	2:00PM	LJPLZ

Workshop	Thursday, 8 Aug	01	1:00PM	ACQ9T
Workshop	Friday, 9 Aug	02	10:00AM	BTKTR

Admin. Stuff (1)

1. Feedback on Workshop Submission: On Thur/Fri

2. Pls join our MS TEAM group: (90%)_Email me

3. Submissions:

Summary

ASSESSMENT	DUE
Kick Starting Week 3: Lecture Quiz 3 (Total mark for all 12 weeks of lecture quizzes is 5%)	Tuesday, 6 August 2024, 11:55 PM **Due in 4 days**
Applied class quiz week 3 (Total mark for all 12 weeks of applied quizzes is 5%)	Wednesday, 7 August 2024, 11:55 PM **Due in 5 days**
Workshop 3 problem set (Total mark for all 12 weeks of workshop sets is 20%)	Sunday, 11 August 2024, 11:55 PM

Admin. Stuff (2)

4. Consultation/Feedback hour

- Wed: 10 am till 11 am
- Fri: 8 am till 9 am
- Sat: 1030 am till 1130 am

(every odd week, e.g. 3-Week, but msg me first at least a day before; appointment-based.)

Location: 5-4-68

Resources

1. PASS with Yvonne and Zi Wei

2. Math Centre

Pass Session: Monday or Wednesday

Monday: 12 pm – 2 pm (6302)

Wednesday: 10 am – 12 pm (LT6008)

Even if it is full in allocate+, feel free to attend!

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School of Engineering

• MATHEMATICS & COMPUTING LEARNING
• CENTRE (MCLC)

- **What**
  - The Mathematics and computing Learning Centre is a drop-in centre for one-to-one help for students enrolled in any mathematic units (ENG1090, ENG1005, ENG2005, ENG 1013, ENG1014, and MEC3456)

When

The MCLC is open on Mondays to Fridays (except public holidays) ***including*** the mid-semester break and SwotVac

Monday - Friday -- 12:00 noon - 2:00 pm

Where

5-5-01

MATHEMATICS & COMPUTING LEARNING CENTRE (MCLC)

Semester 2, 2024 Tutors

Min Khant

Engineering Postgraduate Student

Python and MatLab

Nader Kamrani

Lecturer

Mathematics

Siow Kaei Te (Dantes)

Year 3 Mechanical Engineering Student

Python, Mathematics

Joshua Nah

PHD, Sessional Lecturer

Math, Python and MatLab

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Today's Activity

1. Applied Problem Set

2. Applied Quiz

Question 1

1. Consider the following matrices

$$\mathsf{A}=\begin{bmatrix}-8 & 0\\ -1 & 2\end{bmatrix},\ \mathsf{B}=\begin{bmatrix}0\\ 7\\ 8\end{bmatrix},\ \mathsf{C}=\begin{bmatrix}4 & 1 & 4\\ 1 & 4 & 1\\ 0 & 1 & -1\end{bmatrix},\ \mathsf{D}=\begin{bmatrix}5 & 2 & 1\\ 3 & 5 & 2\\ 4 & 3 & 5\end{bmatrix},\ \mathsf{E}=\begin{bmatrix}3\\ 4\end{bmatrix}$$

(a) Which pairs can you add together?

(b) Which pairs can you multiply?

Learning Outcomes?

Understanding the basic operations of matrices

Systems of Eqs.

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}$$

Augmented Matrix

$$\left(\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right)$$

The vertical bar separates the coefficient matrix from the constants

Coefficient Matrix (Let's call it *A*)

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

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(a) Which pairs can you add together?

Matrices can only be added together if they have the same dimensions.

Ans: **C+D** or **D+C**

(both are 3 by 3 matrices)

1. Consider the following matrices

$$\mathsf{A} = \begin{bmatrix} -8 & 0 \\ -1 & 2 \end{bmatrix}, \mathsf{B} = \begin{bmatrix} 0 \\ 7 \\ 8 \end{bmatrix}, \mathsf{C} = \begin{bmatrix} 4 & 1 & 4 \\ 1 & 4 & 1 \\ 0 & 1 & -1 \end{bmatrix}, \mathsf{D} = \begin{bmatrix} 5 & 2 & 1 \\ 3 & 5 & 2 \\ 4 & 3 & 5 \end{bmatrix}, \mathsf{E} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

(a) Which pairs can you add together?

(b) Which pairs can you multiply?

(b) Which pairs can you multiply together?

Matrices can only be multiplied together if the **number of columns in the first matrix** is the same as the **number of rows in the second one.**

For instance; between 3X2 and 2X3

3 (row) X 2 (column)   2 (row) X 3 (column)

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \\ 1 & 3 \end{bmatrix} \quad \& \quad \begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

1. Consider the following matrices

$$A = \begin{bmatrix} -8 & 0 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 7 \\ 8 \end{bmatrix}, C = \begin{bmatrix} 4 & 1 & 4 \\ 1 & 4 & 1 \\ 0 & 1 & -1 \end{bmatrix}, D = \begin{bmatrix} 5 & 2 & 1 \\ 3 & 5 & 2 \\ 4 & 3 & 5 \end{bmatrix}, E = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

(a) Which pairs can you add together?
(b) Which pairs can you multiply?

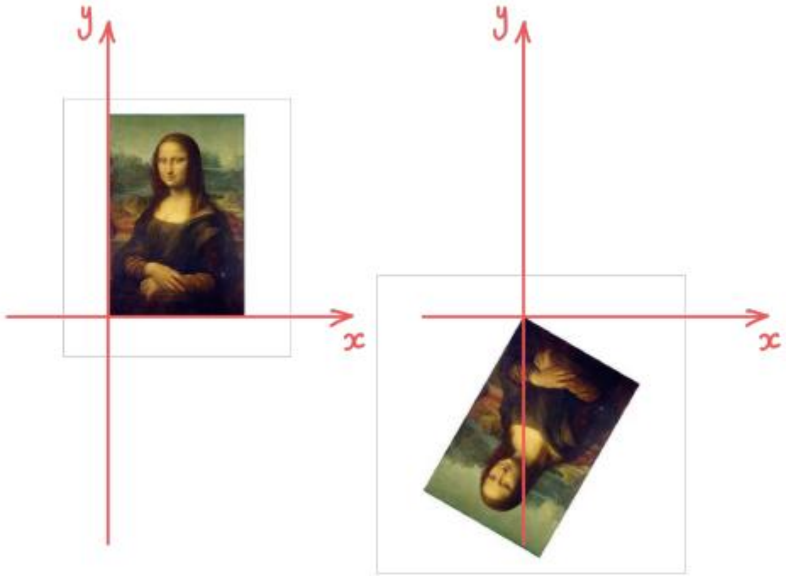
Ans: AE, CD, …, …, …

Question 2

You are given the following four matrices that represent basic transformations of space

$$A = \begin{bmatrix} \cos(30°) & -\sin(30°) \\ \sin(30°) & \cos(30°) \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} \frac{9}{25} & \frac{12}{25} \\ \frac{12}{25} & \frac{16}{25} \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

You are asked to transform the Mona Lisa as shown below.



This can be done by doing some of the linear transformations above, one after the other. Write down the matrix multiplication you need to do to create a transformation matrix able to do the transformation shown. **You should think very carefully about the order in which you do the transformations and what the resulting order of matrix multiplication should be.**

Learning Outcomes? Transformation

Let's understand the matrices first!

For the following transformation matrices

$$A = \begin{bmatrix} \cos(30°) & -\sin(30°) \\ \sin(30°) & \cos(30°) \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} \frac{9}{25} & \frac{12}{25} \\ \frac{12}{25} & \frac{16}{25} \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

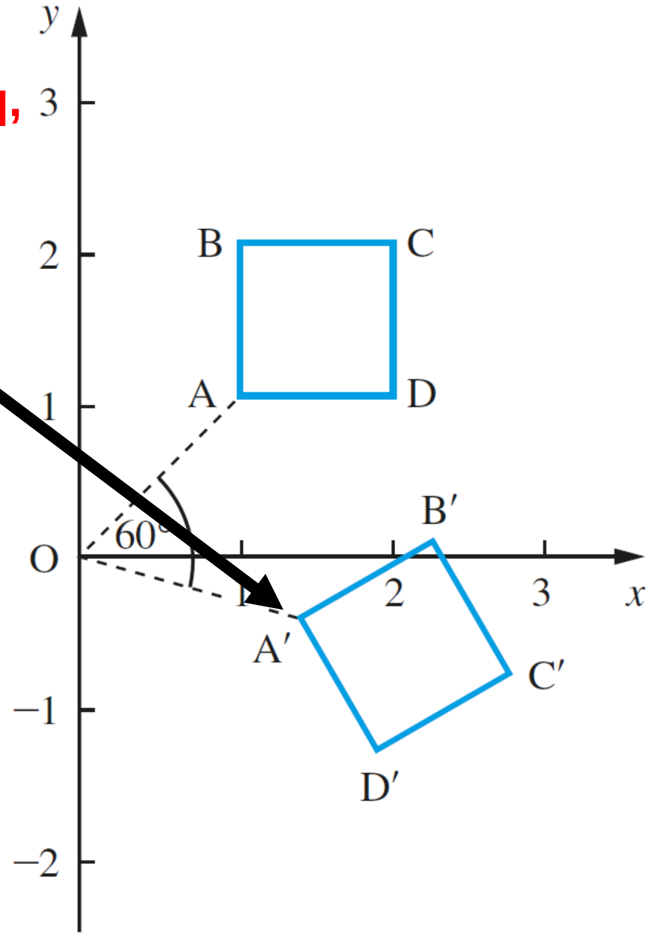
- A: rotation
  - B: shear [Solid Mechanics (SM)]
    - C: projection, but why?
      - D: reflection (about x-axis)

Rotation (a generic example!)

$$\underbrace{\begin{bmatrix} x' \\ y' \end{bmatrix}}_{\text{New (location)}} = \underbrace{\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}}_{\text{Transformation Matrix}} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\text{Old (location)}}$$

For example, we take point A, (x=1,y=1) and the theta as 60 deg

By solving the eq, we get A’, (x’=1.37,y=-0,37)



***This is not matrix A, note the difference, using this above eq. to demonstrate the physical meaning only ***

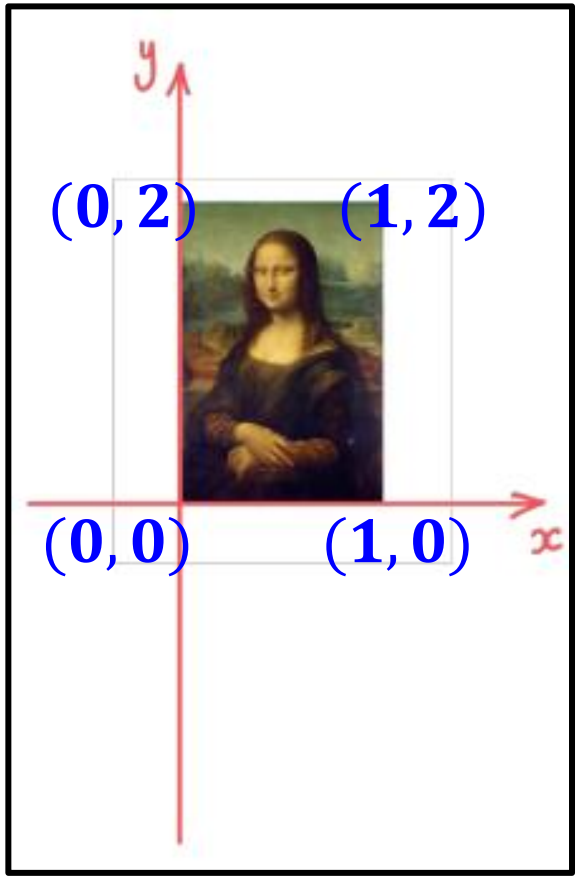
Use this trick to visualize the mapping from the old to the new location!

$$\underbrace{\begin{bmatrix} x' \\ y' \end{bmatrix}}_{\text{New (location)}} = \underbrace{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}}_{\text{Transformation Matrix}} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\text{Old (location)}}$$

Let's return to our Q2

For example:

- Wide/Thicc: 1
- Length/Height: 2



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

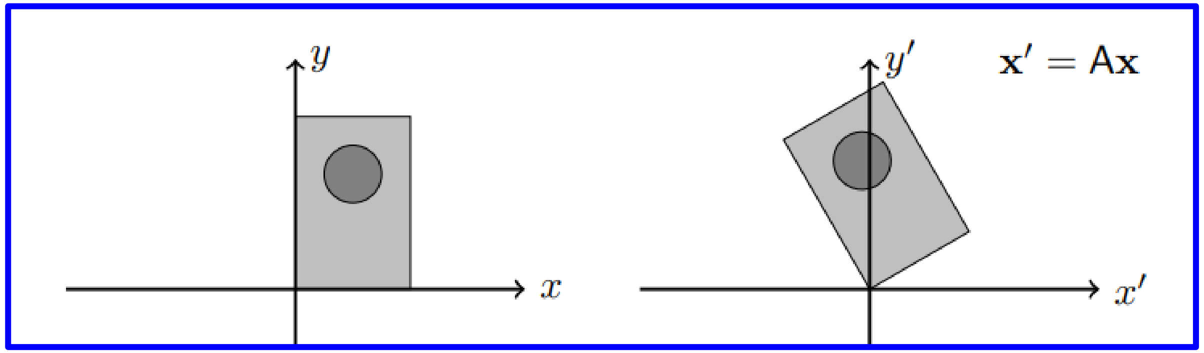
You are given the following four matrices that represent basic transformations of space

$$\mathsf{A} = \begin{bmatrix} \cos(30°) & -\sin(30°) \\ \sin(30°) & \cos(30°) \end{bmatrix}, \quad \mathsf{B} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad \mathsf{C} = \begin{bmatrix} \frac{9}{25} & \frac{12}{25} \\ \frac{12}{25} & \frac{16}{25} \end{bmatrix}, \quad \mathsf{D} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

- ❑ Since Miss Mona is angled w.r.t. to the horizontal axis; we can?

  - ➢ Do it with A first!

- ❑ We get:

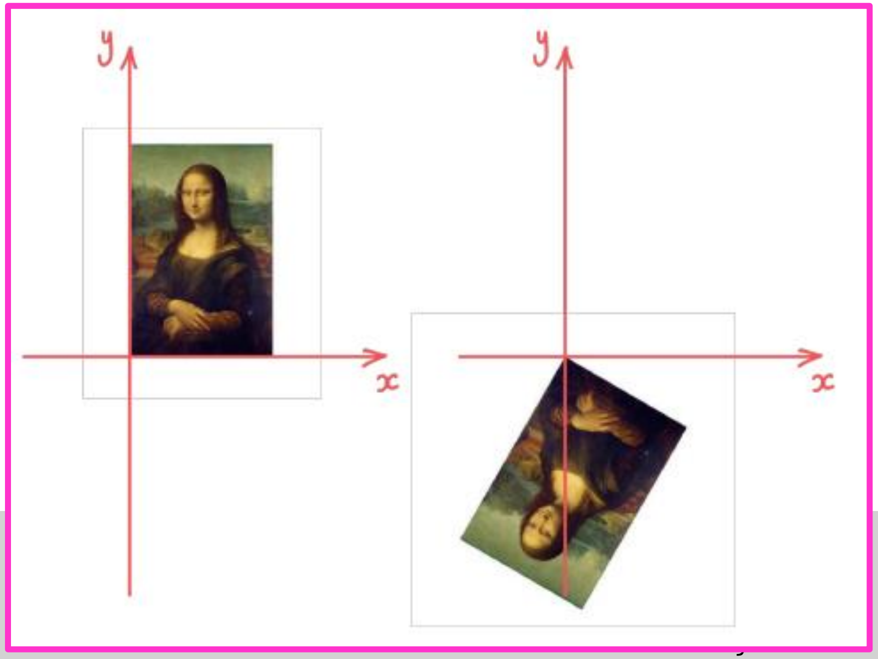


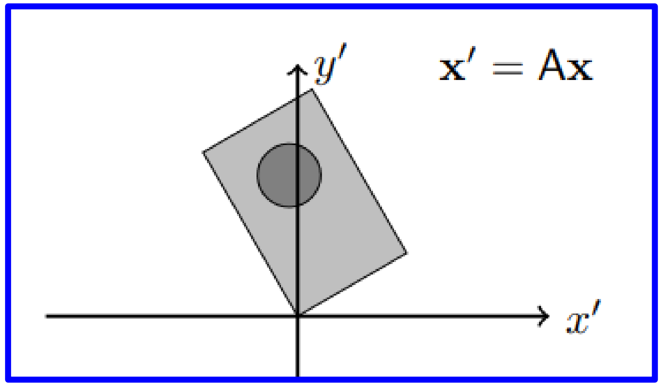
What is next?

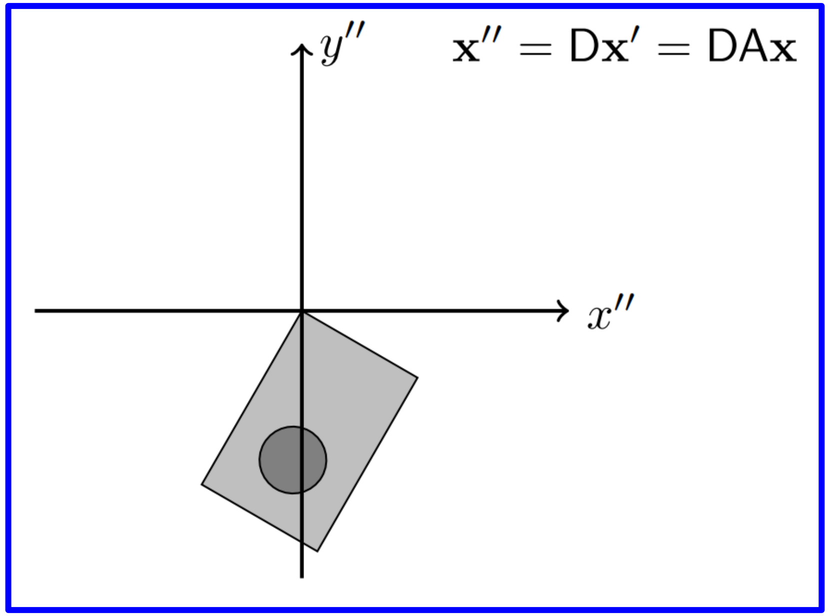
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

You are given the following four matrices that represent basic transformations of space

$$\mathsf{A} = \begin{bmatrix} \cos(30^\circ) & -\sin(30^\circ) \\ \sin(30^\circ) & \cos(30^\circ) \end{bmatrix}, \quad \mathsf{B} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad \mathsf{C} = \begin{bmatrix} \frac{9}{25} & \frac{12}{25} \\ \frac{12}{25} & \frac{16}{25} \end{bmatrix}, \quad \mathsf{D} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

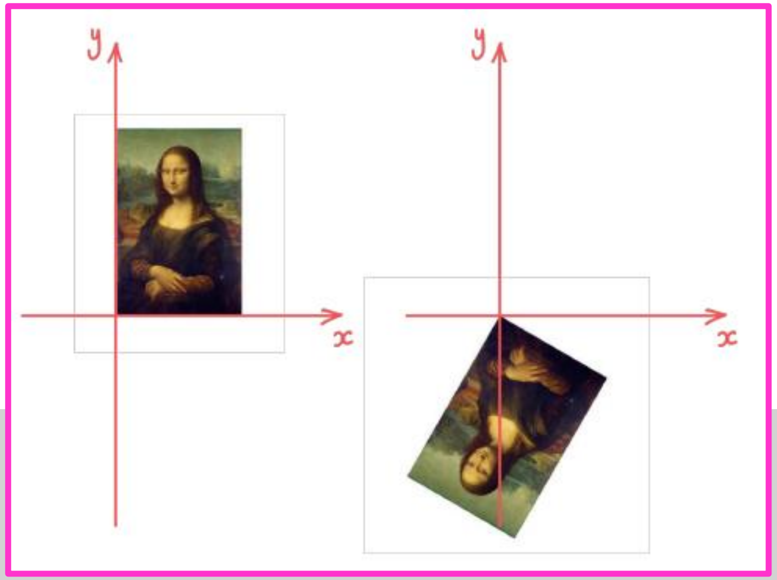






How can we do this?

- Do it with D next!



You are given the following four matrices that represent basic transformations of space

$$\mathsf{A} = \begin{bmatrix} \cos(30^\circ) & -\sin(30^\circ) \\ \sin(30^\circ) & \cos(30^\circ) \end{bmatrix}, \quad \mathsf{B} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad \mathsf{C} = \begin{bmatrix} \frac{9}{25} & \frac{12}{25} \\ \frac{12}{25} & \frac{16}{25} \end{bmatrix}, \quad \mathsf{D} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Question 3

The matrix T represents a linear transformation that satisfies the following two equations

$$\mathsf{T}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}2\\4\end{bmatrix} \quad \text{and} \quad \mathsf{T}\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}0\\-2\end{bmatrix}$$

Find T.

Learning Outcomes?

Matrix + Sys. Of Linear Eqs

- Let's figure out the order of the T matrix first!
  - 2 by 2 or 1 by 2?
    - We firstly denote **T** as (a general form)

$$\boxed{\mathrm{T} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}}$$

- Put **T** into the two matrices

$$\boxed{\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}}$$

$$\boxed{\mathrm{T}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \quad \text{and} \quad \mathrm{T}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}}$$

- Put **T** into the two matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

- Solve them simultaneously (last week's topic)

  - Can you solve it? (5 mins)

$$\mathrm{T} = \begin{bmatrix} 2 & 2 \\ 4 & 2 \end{bmatrix}$$

Question 4

Let

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

Given that the $\boxed{\text{determinant of A is 4,}}$ find the determinants of the following four matrices *without direct calculation*.

$$B = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & -1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -1 \\ 2 & -1 & 1 \end{bmatrix}, \quad E = \begin{bmatrix} 4 & -1 & 1 \\ -2 & 2 & -1 \\ 2 & -1 & 2 \end{bmatrix}$$

Learning Outcomes?

Tricks with determinants

❑ Determinant: A vs B

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & -1 & 2 \end{bmatrix}$$

- To get matrix **B**, the second row of **A** is replaced with the sum of the second and third rows of **A**.

- If we add one row to another row, the determinant does not change!

$$\det(B) = 4$$

❑ Determinant: A vs C

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

- To get matrix **C**, the first column of **A** is replaced with the sum of the first and second columns of **A**.
- If we add one column to another column, the determinant does not change!

$$\det(C) = 4$$

❑ Determinant: A vs D

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$

- To get matrix **D**, the first and third rows of **A** are interchanged.
- If we swap between two rows, the determinant changes its sign.

$$\det(D) = -4$$

❑ Determinant: A vs E

$$A = \begin{bmatrix} \boxed{\begin{matrix} 2 \\ -1 \\ 1 \end{matrix}} & \begin{matrix} -1 \\ 2 \\ -1 \end{matrix} & \begin{matrix} 1 \\ -1 \\ 2 \end{matrix} \end{bmatrix}$$

$$E = \begin{bmatrix} \boxed{\begin{matrix} 4 \\ -2 \\ 2 \end{matrix}} & \begin{matrix} -1 \\ 2 \\ -1 \end{matrix} & \begin{matrix} 1 \\ -1 \\ 2 \end{matrix} \end{bmatrix}$$

- To get matrix **E**, multiply the 1st column of **A** with 2.

- Multiplying a column by a scalar multiplies the determinant by the same scalar

$$\det(E) = 8$$

In short!

- Find these tricks in Topic 7 Vids.
    - If all fails, resort to our basics

$$\boxed{\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}}$$

$$\boxed{\det(A) = a(ei - fh) - b(di - fg) + c(dh - eg)}$$

Question 5

Consider the matrix

$$\mathsf{A} = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}.$$

Calculate A^2 and A^3.

What would you guess happens for A^n for arbitrary n?

Learning Outcomes?

Find a pattern for a diagonal matrix!

Consider the matrix

$$\mathsf{A} = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}.$$

Calculate A^2 and A^3.

What would you guess happens for A^n for arbitrary n?

Since A is a diagonal matrix, raising A to the power n involves raising each of the diagonal elements to the power n. This property of diagonal matrices makes the computation straightforward.

$$A^n = \begin{pmatrix} 3^n & 0 \\ 0 & (-2)^n \end{pmatrix}$$

For matrix, one can use [] or ()

$$A = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$$

$$A^2 = A \times A = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix} \times \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} (3 \cdot 3 + 0 \cdot 0) & (3 \cdot 0 + 0 \cdot -2) \\ (0 \cdot 3 + -2 \cdot 0) & (0 \cdot 0 + -2 \cdot -2) \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix}$$

$$A^3 = A^2 \times A = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} \times \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} (9 \cdot 3 + 0 \cdot 0) & (9 \cdot 0 + 0 \cdot -2) \\ (0 \cdot 3 + 4 \cdot 0) & (0 \cdot 0 + 4 \cdot -2) \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 27 & 0 \\ 0 & -8 \end{pmatrix}$$

Question 6

Consider the matrix

$$\mathsf{A} = \begin{bmatrix} 0 & -2 & 7 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

Calculate A^2 and A^3.

What would you guess happens for an $n \times n$ upper triangular matrix with zeros on the leading diagonal? Why?

Learning Outcomes?

Find a pattern for a 0, 0, 0 diagonal matrix!

$$A^2 = A \times A = \begin{pmatrix} 0 & -2 & 7 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & -2 & 7 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 0 & 0 & -6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A^3 = A^2 \times A = \begin{pmatrix} 0 & 0 & -6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & -2 & 7 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Question 7

Given

$$A = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$$

Compute A^2, A^3 and hence write down A^n for $n \geq 1$.

Learning Outcomes?

Understanding the multiplication of matrices

- When $n \geq 0$

Try this!

$$\boxed{\mathsf{A} = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}, \quad \mathsf{A}^2 = \begin{bmatrix} 1 & 2k \\ 0 & 1 \end{bmatrix}, \quad \mathsf{A}^3 = \begin{bmatrix} 1 & 3k \\ 0 & 1 \end{bmatrix}, \quad \mathsf{A}^n = \begin{bmatrix} 1 & nk \\ 0 & 1 \end{bmatrix}.}$$

$$\boxed{A = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}}$$

Question 8

For any $n \times n$ matrices A and B, the following applies

$$\det(\mathsf{AB}) = \det(\mathsf{A})\det(\mathsf{B})$$

Explain why you might expect this. Using this result, show that $\det(\mathsf{A}^{-1}) = 1/\det(\mathsf{A})$.

Learning Outcomes?

Property of determinants

Pens down!

Full attention is appreciated.

Take photos if you want too

❑ Let's prove the relationship first, letting: $\boxed{\det(\mathrm{AB}) = \det(\mathrm{A})\det(\mathrm{B})}$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$. $\Rightarrow$ $AB = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$

➢ This allows us to find the determinant of ***AB***, namely

$$\det(AB) = (ae+bg)(cf+dh) - (af+bh)(ce+dg)$$

$$\Downarrow$$

$$= \cancel{aecf} + aedh + bgcf + \cancel{bgdh} - \cancel{afce} - afdg - bhce - \cancel{bhdg}$$

- After some housekeeping, we have

$$\det(AB) = aedh - adfg + bcfg - bceh$$

- Now, let's calculate the determinants of ***A*** and ***B***

$$\det(A) = ad - bc$$
$$\det(B) = eh - fg$$

$$\boxed{\det(\mathrm{AB}) = \det(\mathrm{A}) \det(\mathrm{B})}$$ ✓

- Eh, they are equal to each other! Yeaaaaa

$$\det(AB) = aedh - adfg + bcfg - bceh = \det(A)\det(B)$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ and } B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}.$$

Up for a mini challenge?

Prove it using 3 by 3 matrices.

(Optional, submit with P.S 3)

For any $n \times n$ matrices A and B, the following applies

$$\boxed{\det(AB) = \det(A)\det(B)}$$

Explain why you might expect this. Using this result, show that $\boxed{\det(A^{-1}) = 1/\det(A)}$.

- Let's do some re-arranging:

$$\boxed{\det(A)\det(A^{-1}) = 1}$$

versus

$$\boxed{\det(A)\det(B) = \det(AB)}$$

- If we have B as the inverse of A

$$B = A^{-1} \Rightarrow \det(AA^{-1}) = \det(I)$$
$$= 1$$

- We then have this:

$$\det(A^{-1}) = 1/\det(A)$$

- *A* is a *n* by *n* invertible matrix!

Up for another mini challenge?

- Use a generic 2 by 2 (or 3X3) matrix as an example.

  - Mathematically, show in its entirety that the determinant of this generic matrix represents an area (or volume).

(submit with P.S 3, Optional)

Question 9

Consider the following matrices

$$\mathsf{A} = \begin{bmatrix} -8 & 0 \\ -1 & 2 \end{bmatrix} \quad \text{and} \quad \mathsf{B} = \begin{bmatrix} 4 & 1 & 4 \\ 1 & 4 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

Calculate the determinants and inverses of A and B.

Learning Outcomes?

Det. and Inverse: 2 by 2 vs. 3 by 3

Let's do the 2 by 2 one first!

$$A = \begin{bmatrix} -8 & 0 \\ -1 & 2 \end{bmatrix}$$

$$\det(A) = ad - bc$$

$$a = -8,\ b = 0,\ c = -1,\ d = 2$$

$$\det(A) = (-8 \cdot 2) - (0 \cdot -1)$$
$$\det(A) = -16 - 0$$
$$\det(A) = -16$$

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{1}{8} & 0 \\ -\frac{1}{16} & \frac{1}{2} \end{bmatrix}$$

You might want to take a snapshot of these slides!

Let’s now do the 3 by 3 (i): Det.

$$B = \begin{bmatrix} 4 & 1 & 4 \\ 1 & 4 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\det(B) = 4\left(\overset{-5}{\det \begin{bmatrix} 4 & 1 \\ 1 & -1 \end{bmatrix}}\right) - 1\left(\overset{-1}{\det \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}}\right)$$

$$+ 4\left(\overset{1}{\det \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}}\right)$$

$$\det(B) = -20 + 1 + 4$$
$$\det(B) = -15$$

$$B^{-1} = \frac{1}{\det(B)} \operatorname{adj}(B)$$

Let's now do the 3 by 3 (ii): Cofac.

$$B = \begin{bmatrix} 4 & 1 & 4 \\ 1 & 4 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$C_{11} = \det \begin{bmatrix} 4 & 1 \\ 1 & -1 \end{bmatrix} = -5$$

$$B = \begin{bmatrix} 4 & 1 & 4 \\ 1 & 4 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$C_{22} = \det \begin{bmatrix} 4 & 4 \\ 0 & -1 \end{bmatrix} = -4$$

$$\boxed{B^{-1} = \frac{1}{\det(B)} \operatorname{adj}(B)}$$

adjugate matrix

(transpose of the cofactor matrix)

Let's now do the 3 by 3 (ii) Cofac.

$$B = \begin{bmatrix} 4 & 1 & 4 \\ 1 & 4 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{\det(B)} \operatorname{adj}(B)$$

ROW 1

$$C_{11} = \det \begin{bmatrix} 4 & 1 \\ 1 & -1 \end{bmatrix} = -5$$

$$C_{12} = -\det \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = 1$$

$$C_{13} = \det \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} = 1$$

ROW 2

$$C_{21} = -\det \begin{bmatrix} 1 & 4 \\ 1 & -1 \end{bmatrix} = 5$$

$$C_{22} = \det \begin{bmatrix} 4 & 4 \\ 0 & -1 \end{bmatrix} = -4$$

$$C_{23} = -\det \begin{bmatrix} 4 & 1 \\ 0 & 1 \end{bmatrix} = -4$$

ROW 3

$$C_{31} = \det \begin{bmatrix} 1 & 4 \\ 4 & 1 \end{bmatrix} = -15$$

$$C_{32} = -\det \begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix} = 0$$

$$C_{33} = \det \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} = 15$$

$$\operatorname{Cof}(B) = \begin{bmatrix} -5 & 1 & 1 \\ 5 & -4 & -4 \\ -15 & 0 & 15 \end{bmatrix}$$

Let's now do the 3 by 3 (ii)

$$= \begin{bmatrix} -5 & 1 & 1 \\ 5 & -4 & -4 \\ -15 & 0 & 15 \end{bmatrix}$$

Transpose cofactor
to get *adj (B)*

$$= \begin{bmatrix} -5 & 1 & 1 \\ 5 & -4 & -4 \\ -15 & 0 & 15 \end{bmatrix}^T$$

Adjugate

$$B^{-1} = \frac{1}{15}\underbrace{\begin{pmatrix} 5 & -5 & 15 \\ -1 & 4 & 0 \\ -1 & 4 & -15 \end{pmatrix}}$$

$$B^{-1} = \frac{1}{\det(B)}\operatorname{adj}(B)$$

$$\det(B) = -15$$

adjugate matrix
(transpose of the cofactor matrix)

Some Comments!

(you decide which to use for the applied quiz later)

1. Formula *(more explicit/direct)*

2. Gauss Jordan *(less formula)* (Topic 9)

Thank You

Eng. Math

ENG 1005

(Workshop)Week 3: Matrices

(MEC) Senior Lecturer: K.B. Goh, Ph.D.

Tutor: (a) Ian Keen & (b) Jack

Pass Leader: (i) Zi Wei and (ii) Yvonne

Campus

kekboon.goh@monash.edu

Topics

Week	Topic
1	Vectors, Lines, and Planes
2	Systems of Linear Equations
3	Matrices
4	Eigenvalues & Eigenvectors
5	Multivariable Calculus 1
6	Multivariable Calculus 2
7	Integration techniques and hyperbolic functions
8	O.D.E 1
9	O.D.E 2
10	O.D.E 3
11	Series 1
12	Series 2

Admin. Stuff (1)

1. Feedback on Workshop Submission: On Thur/Fri

2. Pls join our MS TEAM group: (90%)_Email me

3. Submissions:

Summary

ASSESSMENT	DUE
Kick Starting Week 3: Lecture Quiz 3 (Total mark for all 12 weeks of lecture quizzes is 5%)	Tuesday, 6 August 2024, 11:55 PM **Due in 4 days**
Applied class quiz week 3 (Total mark for all 12 weeks of applied quizzes is 5%)	Wednesday, 7 August 2024, 11:55 PM **Due in 5 days**
Workshop 3 problem set (Total mark for all 12 weeks of workshop sets is 20%)	Sunday, 11 August 2024, 11:55 PM

Admin. Stuff (2)

4. Consultation/Feedback hour

- Wed: 10 am till 11 am
- Fri: 8 am till 9 am

Location: 5-4-68

- Sat: 1030 am till 1130 am

Library (entrance)

(every odd week, e.g. 3-Week, but msg me first at least a day before; appointment-based.)

Resources

1. PASS with Yvonne and Zi Wei

2. Math Centre

Pass Session: Monday or Wednesday

Monday: 12 pm – 2 pm (6302)

Wednesday: 10 am – 12 pm (LT6008)

Even if it is full in allocate+, feel free to attend!

MONASH University

School of Engineering

• MATHEMATICS & COMPUTING LEARNING
• CENTRE (MCLC)

- **What**
  - The Mathematics and computing Learning Centre is a drop-in centre for one-to-one help for students enrolled in any mathematic units (ENG1090, ENG1005, ENG2005, ENG 1013, ENG1014, and MEC3456)

When

The MCLC is open on Mondays to Fridays (except public holidays) ***including*** the mid-semester break and SwotVac

Monday - Friday -- 12:00 noon - 2:00 pm

Where

5-5-01

MATHEMATICS & COMPUTING LEARNING CENTRE (MCLC)

Semester 2, 2024 Tutors

Min Khant

Engineering Postgraduate Student

Python and MatLab

Nader Kamrani

Lecturer

Mathematics

Siow Kaei Te (Dantes)

Year 3 Mechanical Engineering Student

Python, Mathematics

Joshua Nah

PHD, Sessional Lecturer

Math, Python and MatLab

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Feedback: Workshop 2

- comment 1: writing/drawing & resolution.

Assessments breakdown

Task description	*Value*	*Due date*
Lecture quizzes	5%	Weekly 11:55pm the night before your applied class (except Monday classes) except in Week 1 when the due date is Friday at 11:55 pm (Malaysia time)
Applied class quizzes	5%	Weekly during your applied class
Workshop problems	20%	Weekly at 11:55pm Sunday, except for Weeks 7 (midsem)
Mid-semester exam	20%	During your workshop in Week 7

The Big Learning Outcomes for Week 3

After completing this week's task, you should be able to:

- Perform matrix algebra.
- Understand how matrices represent transformations of space.
- Understand what a matrix determinant represents and how to calculate it.
- Calculate matrix inverses.

Today's Activity

1. Workshop Problem Set

- **Subsection 1** Q1 to Q5

- **Subsection 2** Q6 to Q10

  Take Homework!

- **Subsection 3** Q11 to Q15

- **Subsection 1** Q1 to Q5

Missile Trajectory

Radars have long been used to track flying objects. However it is typically not possible to continuously track a object in real time. Instead, locations of the object are taken at different times. An interpolation process is then used to estimate the trajectory and make predictions about the future location of the object. In this workshop, we will study the methods of polynomial interpolation and least square approximation.

Suppose a missile is fired from sea level and travels directly east. Three radar stations are situated 1km, 2km and 3km due east of the launch site. As the missile passes directly overhead, each radar station measures the altitude of the missile. A short time after launch, the missile engine is switched off. For simplicity, ignore any aerodynamic forces, and assume that without any thrust from its engine, the only force acting on the missile is gravity. From Newtonian mechanics, we know that the missile will follow a parabolic trajectory, i.e., if we let x be the horizontal distance of the missile from the launch site, and y be its altitude, then we have

$$y(x) = Ax^2 + Bx + C$$

where A, B, C are constants.

From the three radar station measurements, we have the follow data about the location of the missile, with both x and y in kilometres.

x	1	2	3
y	6	7	5

We will try to find the constants A, B and C such that the three points $(1,6)$, $(2,7)$, and $(3,5)$ all lie on the parabola $y = Ax^2 + Bx + C$.

$$(1,6) \Rightarrow y(1) = A + B + C = 6$$

$$(2,7) \Rightarrow y(2) = 4A + 2B + C = 7$$

$$(3,5) \Rightarrow y(3) = 9A + 3B + C = 5$$

1. Using the given measurements from the table, write down a **matrix equation** for the unknown variables A, B, and C. **[1 marks]**

Systems of Eqs.

$$A + B + C = 6$$
$$4A + 2B + C = 7$$
$$9A + 3B + C = 5$$

Matrix Form

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 5 \end{bmatrix}$$

x	1	2	3
y	6	7	5

We will try to find the constants A, B and C such that the three points $(1,6)$, $(2,7)$, and $(3,5)$ all lie on the parabola $y = Ax^2 + Bx + C$.

2. Calculate the determinant of the matrix in your equation. It should be non-zero. What does this tell you about whether you can fit a unique quadratic curve through the three data points? **[2 marks]**

$$\det \begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix} = 1 \times (2 \times 1 - 3 \times 1) - 1 \times (4 \times 1 - 9) + 1 \times (4 \times 3 - 9 \times 2)$$
$$= -2$$

- The determinant has significant geometric interpretations, namely: **area**

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 5 \end{bmatrix}$$

- In other words, the system has a unique solution/invertible!

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\det(A) = a(ei - fh) - b(di - fg) + c(dh - eg)$$

3. Calculate the **inverse of the matrix**, and hence find the values of A, B and C.

❖Gauss-Jordan

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 4 & 2 & 1 & 0 & 1 & 0 \\ 9 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -1 & \frac{1}{2} \\ 0 & 1 & 0 & -\frac{5}{2} & 4 & -\frac{3}{2} \\ 0 & 0 & 1 & 3 & -3 & 1 \end{array}\right]$$

❖Row operations, the identity changes its position to the left of the vertical line!

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 5 \end{bmatrix}$$

3. Calculate the **inverse of the matrix**, and hence find the values of A, B and C.

❖Formulae

$$\boxed{A^{-1} = \frac{1}{\det(A)} \cdot \operatorname{adj}(A)}$$

$$\boxed{\begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 5 \end{bmatrix}}$$

Determinant (Q2)

$$\det\begin{bmatrix}1 & 1 & 1\\ 4 & 2 & 1\\ 9 & 3 & 1\end{bmatrix} = 1\times(2\times 1 - 3\times 1) - 1\times(4\times 1 - 9) + 1\times(4\times 3 - 9\times 2)$$
$$= -2$$

$$A = \begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}$$

$$\det(A) = a(ei - fh) - b(di - fg) + c(dh - eg)$$

Adjugate (a general form)

$$A^{-1} = \frac{1}{\det(A)} \cdot \operatorname{adj}(A)$$

- Can you calculate the adj. (10mins)

$$\operatorname{adj}(A) = \begin{bmatrix} ei - fh & -(bi - ch) & bf - ce \\ -(di - fg) & ai - cg & -(af - cd) \\ dh - eg & -(ah - bg) & ae - bd \end{bmatrix}$$

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix}$$

$$\operatorname{adj}(A) = \begin{bmatrix} -1 & 2 & -1 \\ 5 & -8 & 3 \\ -6 & 6 & -2 \end{bmatrix}$$

Inverse

$$A^{-1} = \frac{1}{\det(A)} \cdot \text{adj}(A)$$

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} ei - fh & -(bi - ch) & bf - ce \\ -(di - fg) & ai - cg & -(af - cd) \\ dh - eg & -(ah - bg) & ae - bd \end{bmatrix}$$

$$\det(A) = -2$$

$$\text{adj}(A) = \begin{bmatrix} -1 & 2 & -1 \\ 5 & -8 & 3 \\ -6 & 6 & -2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & -1 & \frac{1}{2} \\ -\frac{5}{2} & 4 & -\frac{3}{2} \\ 3 & -3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{2} & -1 & \frac{1}{2} \\ -\frac{5}{2} & 4 & -\frac{3}{2} \\ 3 & -3 & 1 \end{bmatrix}}_{\textbf{The Inverse}} \begin{bmatrix} 6 \\ 7 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -1 & \frac{1}{2} \\ -\frac{5}{2} & 4 & -\frac{3}{2} \\ 3 & -3 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 7 \\ 5 \end{bmatrix} = \begin{bmatrix} -\frac{3}{2} \\ \frac{11}{2} \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 5 \end{bmatrix}$$

4. Where do you predict the missile will land (i.e. hit sea level)?

$$y(x) = Ax^2 + Bx + C$$

$$y = -\frac{3}{2}x^2 + \frac{11}{2}x + 2.$$

- If it hits sea level, what do we have?

$$y = 0 \Rightarrow x = -\frac{1}{3}; 4\ ?$$

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} -\frac{3}{2} \\ \frac{11}{2} \\ 2 \end{bmatrix}$$

- Which one?

Usually having more data will give a more accurate prediction. Suppose a fourth radar station located 4km east also made a measurement, giving the following table:

x	1	2	3	4
y	6	7	5	4

5. Without doing any calculation, do you expect to be able to find a parabola through those four data points? Explain your answer. **[1 marks]**

$(1,6) \Rightarrow A + B + C = 6$

$(2,7) \Rightarrow 4A + 2B + C = 7$

$(3,5) \Rightarrow 9A + 3B + C = 5$

$(4,4) \Rightarrow 16A + 4B + C = 4$

- We have (A, B, C) 3 variables but only 4 equations;
  - Too much info;
- Overdetermined!

parabola $y = Ax^2 + Bx + C$.

- **Subsection 2** Q6 to Q10

Take Homework!

New System 2!

Try at Home! Following the questions above.

We will now try to find a cubic $\boxed{y = Ax^3 + Bx^2 + Cx + D}$ that goes through the four data points.

6. Using the given measurements from the new table, write down a matrix equation for the unknown variables A, B, C and D. **[1 marks]**

7. Using appropriate row and column operations and properties of determinants with those operations, find the determinant of the matrix in your equation. What does this tell you about whether there is a unique cubic through the four data points? **[4 marks]**

8. Using Matlab or CAS, find A, B, C and D. You might find the commands on the last page useful. **[2 marks]**

Help me check my derivation with your ans.

7. Using appropriate row and column operations and properties of determinants with those operations, find the determinant of the matrix in your equation. What does this tell you about whether there is a unique cubic through the four data points? **[4 marks]**

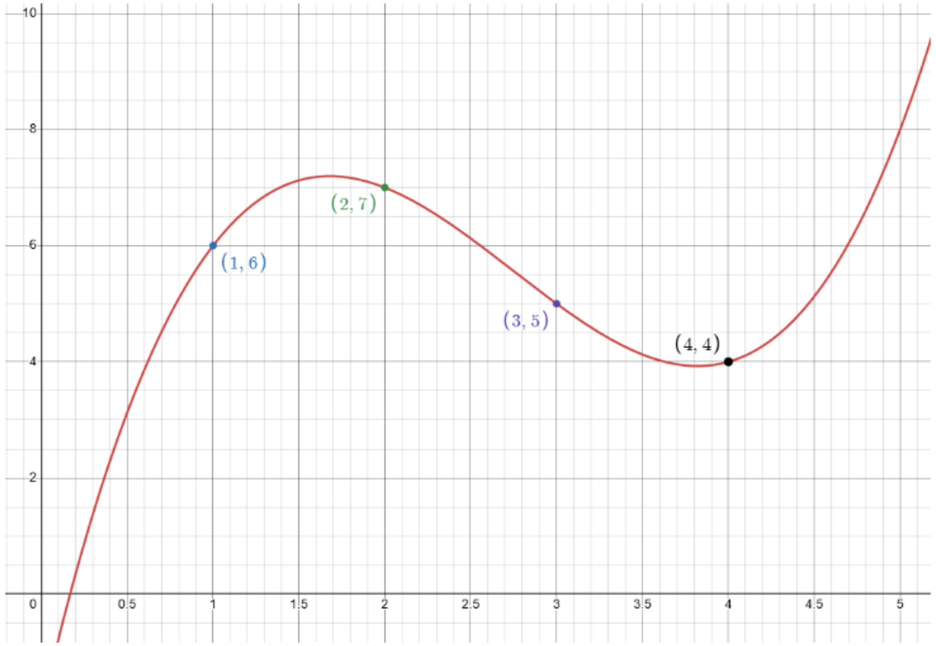
- See your higher dimension space topic 11 vid!

$$\begin{aligned}\det(B) = & b_{11}\left(b_{22}(b_{33}b_{44} - b_{34}b_{43}) - b_{23}(b_{32}b_{44} - b_{34}b_{42}) + b_{24}(b_{32}b_{43} - b_{33}b_{42})\right) \\ & - b_{12}\left(b_{21}(b_{33}b_{44} - b_{34}b_{43}) - b_{23}(b_{31}b_{44} - b_{34}b_{41}) + b_{24}(b_{31}b_{43} - b_{33}b_{41})\right) \\ & + b_{13}\left(b_{21}(b_{32}b_{44} - b_{34}b_{42}) - b_{22}(b_{31}b_{44} - b_{34}b_{41}) + b_{24}(b_{31}b_{42} - b_{32}b_{41})\right) \\ & - b_{14}\left(b_{21}(b_{32}b_{43} - b_{33}b_{42}) - b_{22}(b_{31}b_{43} - b_{33}b_{41}) + b_{23}(b_{31}b_{42} - b_{32}b_{41})\right)\end{aligned}$$

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix}$$

9. Plot your data points and the graph of the interpolating polynomial through them. Be sure to include axis labels.

[1 mark]



10. Where does the missile land? Does this plot seem a realistic trajectory for a missile?

x	1	2	3	4
y	6	7	5	4

➢ Off the space?

- **Subsection 3** Q11 to Q15

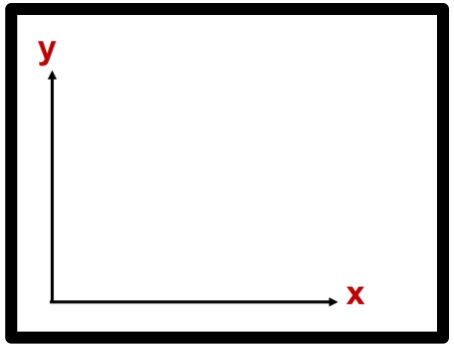
New System 3!

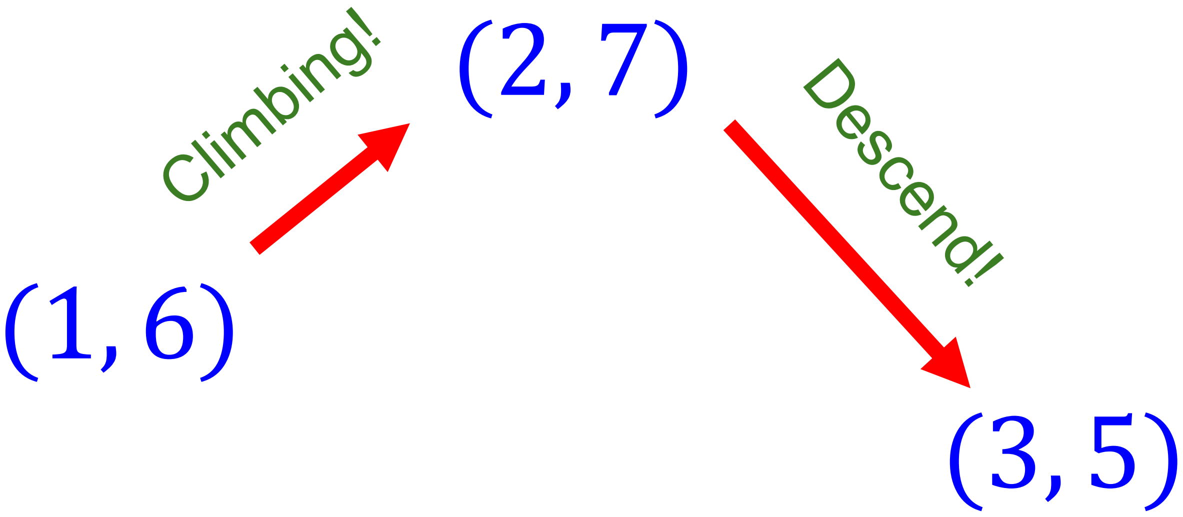
In practice, measurements are never exact and almost always contain some error. For example, you may find the data points don't fit a parabola precisely. In theses situations, the best we can do is to find the curve that best fits the data points. For simplicity, we will investigate the (straight) line of best fit, but similar ideas can be applied to find the parabola or any curve of best fit.

Suppose the missile did not turn of its engine, so that it flies in a straight line instead of having a parabolic trajectory. Let's return to the first set of measurements.

x	1	2	3
y	6	7	5

11. Show that the three observed points do not lie on a straight line. **[1 mark]**





So, the two lines are distinct, and the three points are not on the same line!

x	1	2	3
y	6	7	5

These points are to scale

Let $y(x) = Sx + T$ denote a straight line. We allow S and T to vary over all real numbers to cover all possible straight lines. For each pair (S, T), we let the triple $(y(1), y(2), y(3))$ be the *predicted value* of the radar station measurements. The actual *observed value* from the radar stations is $(6, 7, 5)$. In the method of least squares, the idea is to minimize the distance (as points in $\mathbf{R}^3$) between the predicted value and the observed value. The particular S and T that achieve this minimum will give us the line of best fit.

12. Express $(y(1), y(2), y(3))$ in terms of S and T. **[1 mark]**

$$(1, 6) \Rightarrow y(1) = S(1) + T$$

$$(2, 7) \Rightarrow y(2) = S(2) + T$$

$$(3, 5) \Rightarrow y(3) = S(3) + T$$

x	1	2	3
y	6	7	5

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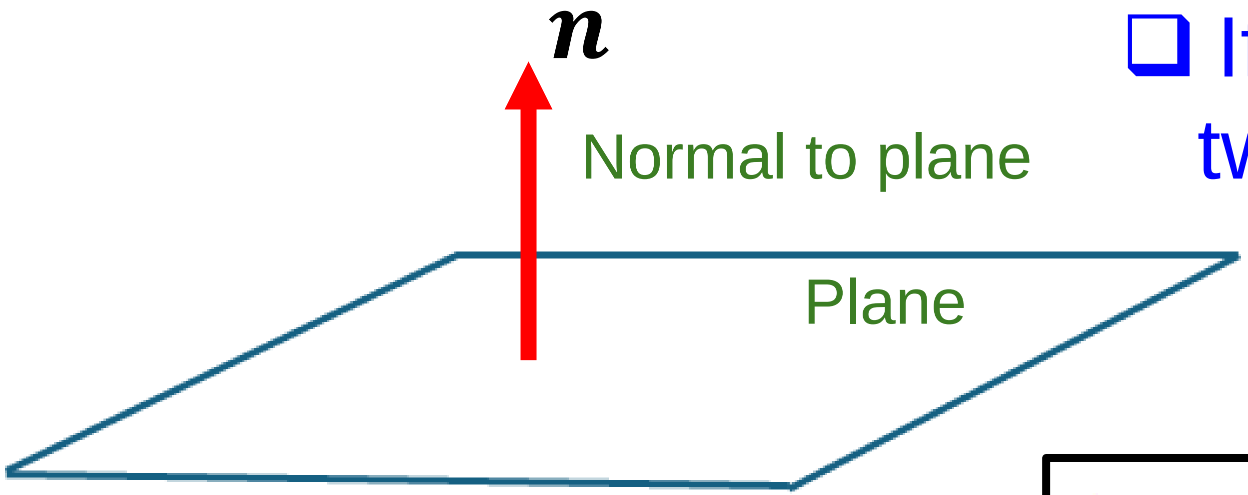
13. Explain that the set of all predicted values, as S and T vary, form a plane in $\mathbb{R}^3$, the space of all triples. **[1 marks]**

$$\begin{bmatrix} y(1) \\ y(2) \\ y(3) \end{bmatrix} = S \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + T \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} y(1) \\ y(2) \\ y(3) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{\text{Position Vector}} + \underbrace{S \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + T \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{\text{Direction Vectors}}$$

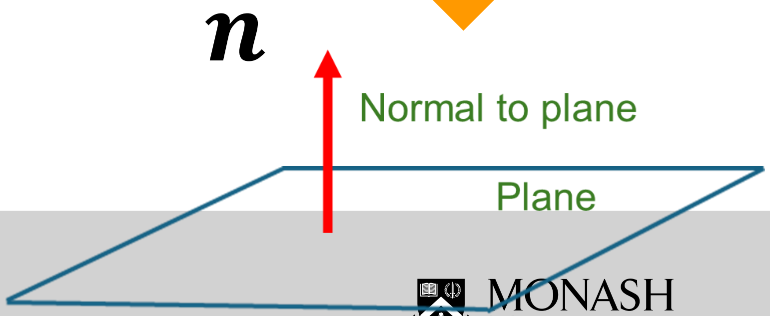
We observe that this is the parametric equation of a plane passing through (0, 0, 0), with vectors **(1, 2, 3)** and **(1, 1, 1)** on the plane.

14. Using ideas from week 1, find the minimum distance between the plane of all predicted values and $(6, 7, 5)$. **[3 marks]**



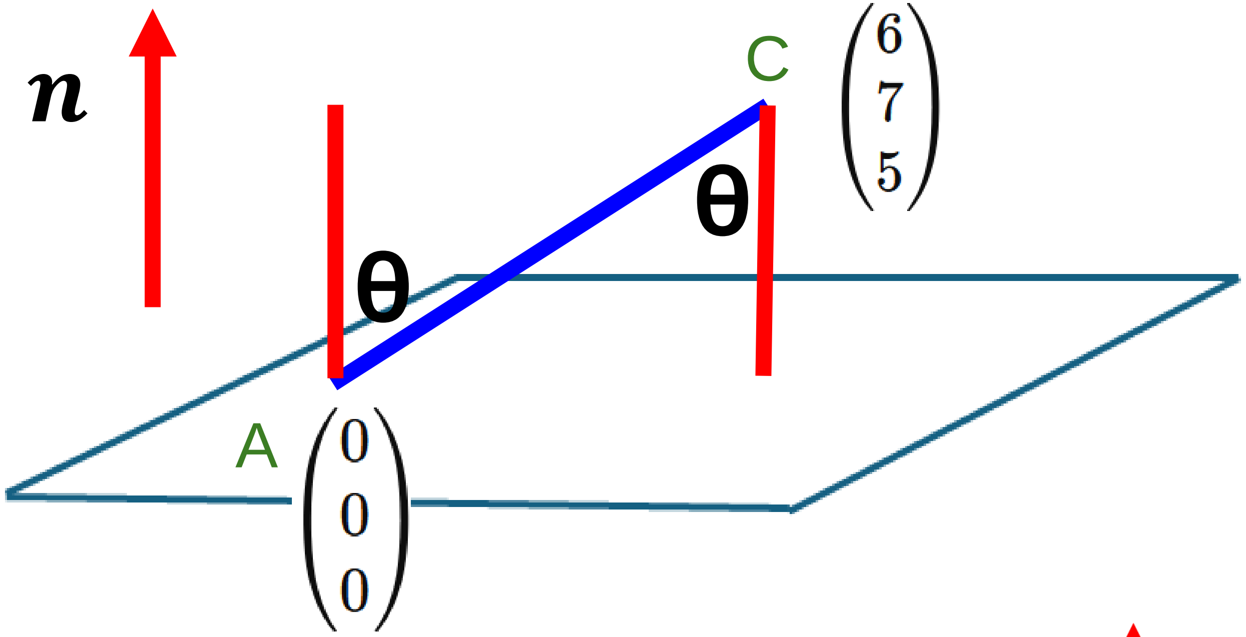
- If we do a cross-product between two vectors on the plane, we get
  - A normal to the plane!

$$\boxed{(1, 2, 3) \times (1, 1, 1) = (-1, 2, -1).}$$



$$\boxed{\begin{bmatrix} y(1) \\ y(2) \\ y(3) \end{bmatrix} = S \overbrace{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}^{\textbf{On the plane}} + T \overbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}^{\textbf{On the plane}}}$$

14. Using ideas from week 1, find the minimum distance between the plane of all predicted values and $(6,7,5)$. **[3 marks]**

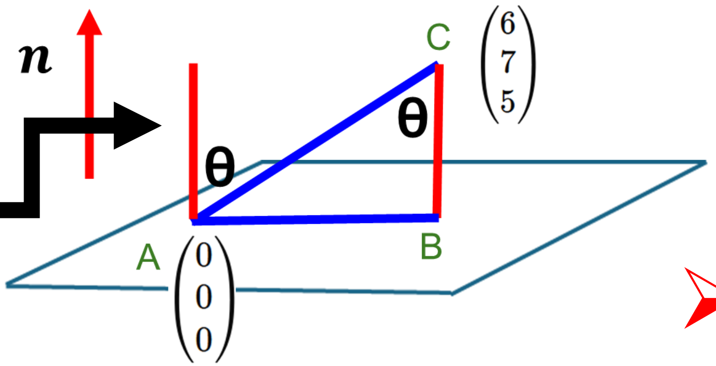


$$\overrightarrow{AC} \cdot n = |\overrightarrow{AC}|\,|n| \cos\theta$$

- Dot product

$$\begin{pmatrix}6\\7\\5\end{pmatrix} \cdot \begin{pmatrix}-1\\2\\-1\end{pmatrix} = \underbrace{\sqrt{1+4+1}}_{|n|}\,|\overrightarrow{AC}| \cos\theta$$

$$|\overrightarrow{AC}| \cos\theta = \frac{3}{\sqrt{6}}$$



- You can also use:

$$\begin{bmatrix} y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + S \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + T \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot \hat{\mathbf{n}}|$$

15. Find the line of best fit for the given data set.

If you're fitting a line of the form $y = Sx + T$ using the least squares method, you can use the following formulas to determine the coefficients S (slope) and T (intercept):

1. **Slope** (S):

$$S = \frac{N\sum(xy) - \sum x \sum y}{N\sum(x^2) - (\sum x)^2}$$

2. **Intercept** (T):

$$T = \frac{\sum y - S\sum x}{N}$$

where:

- N is the number of data points,
- $\sum xy$ is the sum of the product of x and y values,
- $\sum x$ is the sum of x values,
- $\sum y$ is the sum of y values,
- $\sum(x^2)$ is the sum of the squares of x values.

x	1	2	3
y	6	7	5

$$\boxed{y = -\frac{1}{2}x + 7}$$

The Big Learning Outcomes for Week 3

After completing this week’s task, you should be able to:

- Perform matrix algebra.
- Understand how matrices represent transformations of space.
- Understand what a matrix determinant represents and how to calculate it.
- Calculate matrix inverses.

Thank You