MONASH University

Eng. Math

ENG 1005

Week 9: O.D.E 2

(MEC) Senior Lecturer: K.B. Goh, Ph.D.

Tutor: (a) Ian Keen & (b) Jack

Pass Leader: (i) Zi Wei and (ii) Yvonne

kekboon.goh@monash.edu

Topics

Week	Topic
1	Vectors, Lines, and Planes
2	Systems of Linear Equations
3	Matrices
4	Eigenvalues & Eigenvectors
5	Multivariable Calculus 1
6	Multivariable Calculus 2
7	Integration techniques and hyperbolic functions
8	O.D.E 1
9	O.D.E 2
10	O.D.E 3
11	Series 1
12	Series 2

The Big Learning Outcomes for Week 8

After completing this week's task, you should be able to:

- Find general solutions of second-order, linear, constant-coefficient ODEs.
- Solve second-order, linear, constant-coefficient IVPs.

Attendance Codes (Week 9)

International students

Tutorial	Wednesday, 18 Sep	02	8:00AM	R8UV9
Tutorial	Wednesday, 18 Sep	01	2:00PM	WMZL5

Workshop	Thursday, 19 Sep	01	1:00PM	JQX69
Workshop	Friday, 20 Sep	02	10:00AM	7TB5A

Admin. Stuff (1)

1. Def. Mid Term: 26 Sept; 8am-10am (6-3-1)

2. Submissions:

Summary

ASSESSMENT	DUE
Kick Starting Week 3: Lecture Quiz 3 (Total mark for all 12 weeks of lecture quizzes is 5%)	Tuesday, 6 August 2024, 11:55 PM **Due in 4 days**
Applied class quiz week 3 (Total mark for all 12 weeks of applied quizzes is 5%)	Wednesday, 7 August 2024, 11:55 PM **Due in 5 days**
Workshop 3 problem set (Total mark for all 12 weeks of workshop sets is 20%)	Sunday, 11 August 2024, 11:55 PM

Admin. Stuff (2)

3. Consultation/Feedback hour

- Wed: 10 am till 11 am — Location: 9-4-01
- Fri: 8 am till 9 am — Location: 5-4-68
- Sat: 1030 am till 1130 am — NO (Vacation)

(every odd week, e.g. 3-Week, but msg me first at least a day before; appointment-based.)

Resources

1. Additional: Videos

2. Pass materials

Let us start! (from last week)

Calculating Homogeneous Solution

Let $\frac{dI(t)}{dt} + 5\,I(t) = 0$ $\rightarrow$ Assume $I_h(t) = Ae^{\lambda t}$

Substitute $I_h(t)$ expression

$$\frac{d(Ae^{\lambda t})}{dt} + 5\,(Ae^{\lambda t}) = 0$$

Expansion

$$\lambda \cancel{Ae^{\lambda t}} + 5\,(\cancel{Ae^{\lambda t}}) = 0$$

$$\lambda = -5$$

$$\frac{dI}{I} = -5\,dt$$

$$\ln I = -5t + C$$

$$I = \exp(-5t + C)$$

$$\boxed{I_h(t) = Ae^{-5t}}$$

A scaling up or down the solution; eigenvector?

Refresher!

Set $y(x) = y_h(x) + y_p(x)$

homogeneous solution

$$a\frac{d^2y_h}{dx^2} + b\frac{dy_h}{dx} + cy_h = 0$$

a particular solution
use an educated guess

Today's Activity

1. Applied Problem Set

2. Applied Quiz

- ✓ Q1
- ✓ Q4
- ✓ Q7
- ✓ Q8
- ✓ Q6

Question 1

1. Find the homogeneous solution of the ODE

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = r(x).$$

For each of the following functions $r(x)$, write down an appropriate guess for $y_p(x)$

(a) $r(x) = 2 + 3x + 6e^{-x}$

(b) $r(x) = x\sin(x)$

(c) $r(x) = 6xe^{3x}$. For this case, also find the particular solution. *This one is tricky - think carefully about your guess.*

Learning Outcomes?

2nd order, linear ODEs

Find the homogenous part 1st (1)

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$$

Step 1: Assume sol. y ∝ to exp (λx) & sub. into the ODE.

$$\frac{d^2}{dx^2}(\exp \lambda x) - 6\frac{d}{dx}(\exp \lambda x) + 9(\exp \lambda x) = 0$$

Step 2: Expand & Simplify

$$\lambda^2 \exp(\lambda x) - 6\lambda(\exp \lambda x) + 9(\exp \lambda x) = 0$$

Find the homogenous part 1st (2)

$$\lambda^2 \exp(\lambda x) - 6\lambda(\exp \lambda x) + 9(\exp \lambda x) = 0$$

Step 3: Simplified it to: and solve the quad. eq

$$\boxed{\exp(\lambda x) \neq 0}$$

$$\exp(\lambda x)(\lambda^2 - 6\lambda + 9) = 0$$

$$\boxed{\lambda = 3, 3}$$

$$\boxed{\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0}$$

Step 4: With two multiplicities, we write these

$$\boxed{y_h(x) = Ae^{3x} + Bxe^{3x}}$$

Particular 1

$$r(x) = \underbrace{2 + 3x} + \underbrace{6e^{-x}}$$

$$y_p(x) = \boxed{a + bx} + \boxed{ce^{-x}}$$

$$\boxed{\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = r(x).}$$

Particular 2

$$r(x) = x\sin(x)$$

$$\boxed{y_p(x) = (a + bx)\sin(x) + (c + dx)\cos(x)}$$

$$\boxed{\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = r(x).}$$

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Particular 3 $r(x) = 6xe^{3x}$

Step 1: Logically we can propose this:

$$yp(x) = (C + Dx)\exp(3x)$$

But, this is similar to the homogenous solution, so what is the point?

Step 2: Let us now propose a new one!

$$yp(x) = x.x(C + Dx)\exp(3x)$$

Multiple with x power 2, taking account two roots for the homogenous solution!

$$\boxed{y_h(x) = Ae^{3x} + Bxe^{3x}}$$

Question 4

4. The general solution of an ODE is

$$y(x) = Ae^{x} + Be^{2x} + x^2$$

What is the ODE that this solution satisfies, given that the coefficient for d^2y/dx^2 is 1?

Learni

Set $y(x) = y_h(x) + y_p(x)$

homogeneous solution

$$a\frac{d^2y_h}{dx^2} + b\frac{dy_h}{dx} + cy_h = 0$$

a particular solution

use an educated guess

$$\boxed{y(x) = \underbrace{A\exp(x) + B\exp(2x)}_{\text{Homogenous solutions!}} + x^2}$$

$$(\lambda - 1)(\lambda - 2) = (1\lambda^2 - 3\lambda + 2) = 0$$

Homogenous ODE!

$$(1)\frac{d^2yh}{dx^2} - (3)\frac{dyh}{dx} + (2)yh = 0$$

$$(1)\frac{d^2y}{dx^2} - (3)\frac{dy}{dx} + (2)y = r(x)$$

Original ODE

$\downarrow$ $\boxed{\text{sub. } yp(x) = x^2}$

$$(1)(2) - (3)(2x) + (2)x^2 = r(x)$$

$\downarrow$

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2 - 6x + 2x^2$$

$$\boxed{y(x) = A\exp(x) + B\exp(2x) + x^2}$$

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Question 7

The ODE

$$ax^2y'' + bxy' + cy = 0$$

where a, b and c are constants is known as the Cauchy–Euler equation. It can be solved in a similar way to constant-coefficient linear ODEs except the appropriate form to try for the homogeneous ODE is

$$y = Ax^n$$

where n is an exponent that we need to find. Using this information, find the general solution of the ODE

$$x^2y'' + 4xy' + 2y = 0.$$

Learning Outcomes?

Alternative assumption!

$$\boxed{x^2y'' + 4xy' + 2y = 0}$$

A new assumption!

$$\boxed{y = Ax^n}$$

$$\boxed{y' = Anx^{n-1}}$$

$$\boxed{y'' = An(n-1)^1x^{n-2}}$$

Sub. Into the ODE!

$$\boxed{A \;\&\; x^n \neq 0}$$

$$A(n^2 + 3n + 2)\, x^n = 0$$

$$\boxed{n = -1 \;\&\; -2}$$

$$\boxed{y = Ax^{-1} + Bx^{-2}}$$

Question 8

Sometimes, a constant-coefficient, linear, second order differential equation has solutions of the form $y(t) = e^{\lambda t}$ and $y(t) = te^{\lambda t}$. Can you construct a constant-cofficient, linear, **third-order** differential equation such that

$$y(t) = e^{2t}, \quad y(t) = te^{2t}, \quad \text{and} \quad y(t) = t^2e^{2t}$$

are all solutions?

Learning Outcomes?

Patterns!

$$\overbrace{(\lambda - 2)^3}^{\boxed{\lambda = 2,\ 2, 2}} = 0$$

Expanding!

$$\boxed{y''' - 6y'' + 12y' - 8y = 0}$$

Question 6 (Advanced/Challenging)

Consider a beam of length L with a load on top P. The vertical coordinate is z and the horizontal deflection of the beam from straight is w. The deflection of the beam w satisfies the ODE

$$Bw'' + Pw = 0$$

with boundary conditions $w(0) = 0$ and $w(L) = 0$. The constant B is the bending stiffness of the beam.

Let $B = 1$. Find the particular solution of the ODE above satisfying the boundary conditions given. Can you find more than the trivial solution $w = 0$ for some values of P? What does this te
when the column buckles under the load?

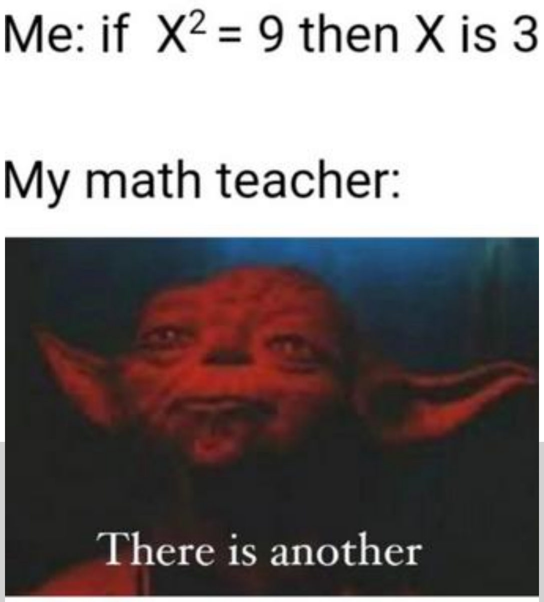
DIY!

- ✓ Any Questions on Complex Roots???

Today's Activity

1. Applied Problem Set

2. Applied Quiz

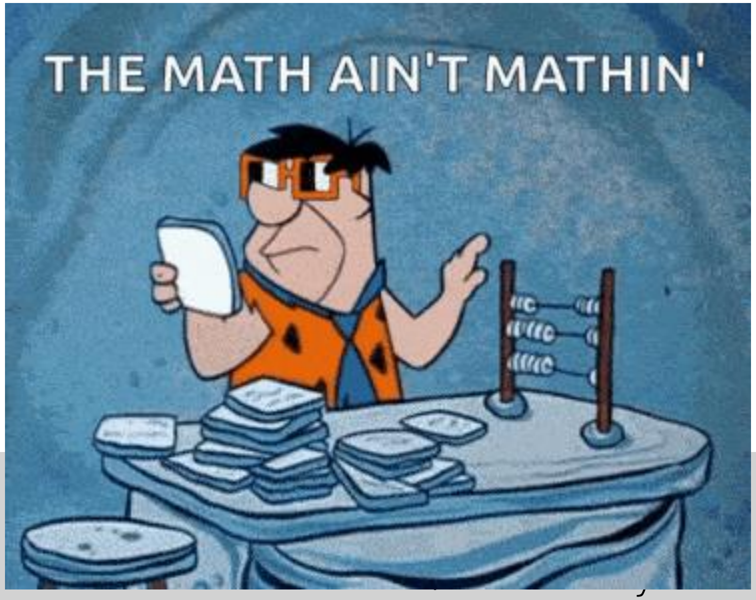


2. Applied Quiz

(*9.30 am till 9.50 am*)

Password: MidTermBreakIsComing!

Q2

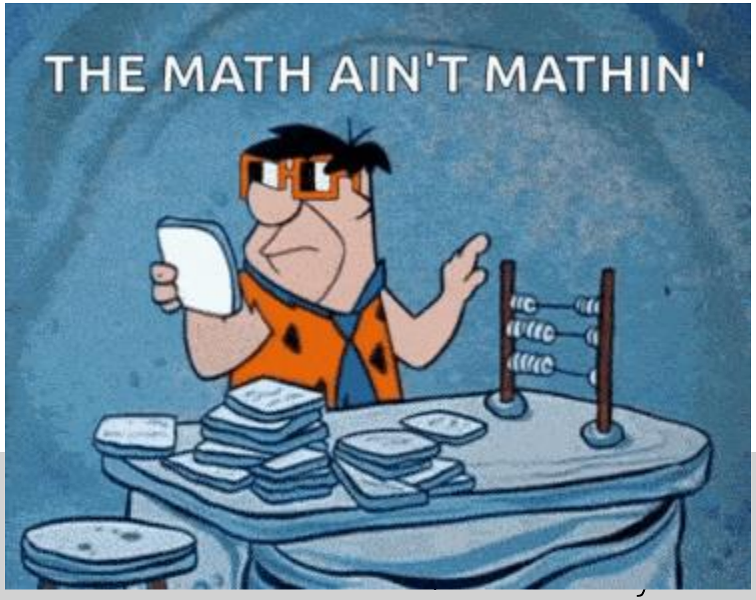


2. Applied Quiz

(3.20 pm till 3.50 pm)

Password: MidTermBreakIsComing!

Q2



Thank You

Ahmad Amiruddin

Eng. Math

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Assessments breakdown

Task description	*Value*	*Due date*
Lecture quizzes	5%	Weekly 11:55pm the night before your applied class (except Monday classes) except in Week 1 when the due date is Friday at 11:55 pm (Malaysia time)
Applied class quizzes	5%	Weekly during your applied class
Workshop problems	20%	Weekly at 11:55pm Sunday, except for Weeks 7 (midsem)
Mid-semester exam	20%	During your workshop in Week 7

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Let us start!

Particular 3 $r(x) = 6xe^{3x}$

Step 1: Logically we can propose this:

$$yp(x) = (C + Dx)\exp(3x)$$

(incorrect) $yp(x) = x.(C + Dx)\exp(3x)$

Step 2: Let us now propose a new one!

$$yp(x) = x.x(C + Dx)\exp(3x)$$

Multiple with x power 2, taking account two roots for the homogenous solution!

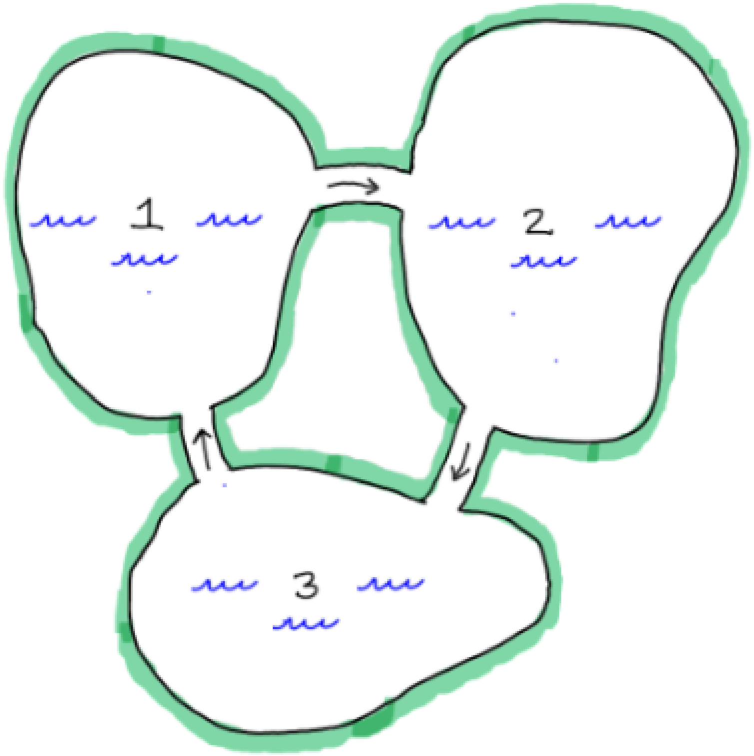
$$\boxed{y_h(x) = Ae^{3x} + Bxe^{3x}}$$

Today’s Activity

1. Workshop Problem Set

Pollution Modelling

In this workshop, we will use the systems of differential equations to model the spread of pollution in three connected lakes. Consider three lakes connected by rivers as in the diagram below.

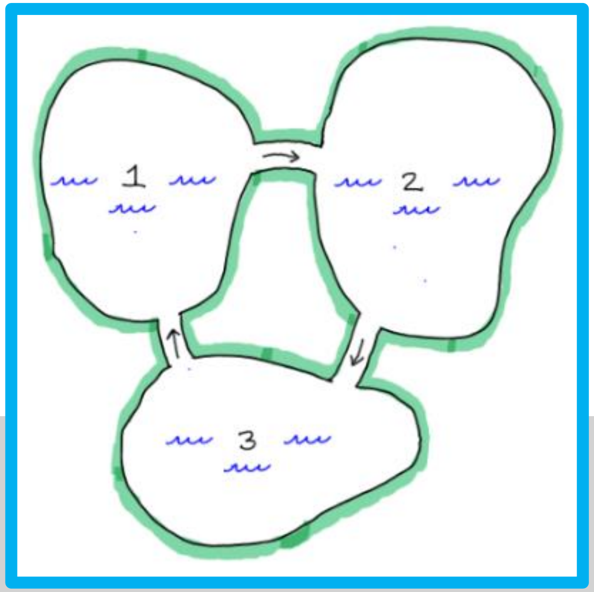


Suppose lake 1 holds 8000 megalitres of water, lake 2 holds 24000 megalitres and lake 3 holds 3000 megalitres. Three rivers connect the lakes, flowing in the directions indicated on the diagram. All three rivers have a constant flow rate of 24000 megalitre per day. Suppose at time 0, 10kg of pollutant is discharged into lake 1. You may assume that any amount of pollutant would instantaneously dissolve and spread evenly throughout each lake.

1. Show that the amount of water in each lake stays constant.

- All three rivers have a constant flow rate of 24000 megalitre per day!

- In = Out



2. Let x_i denote the amount of pollutant in lake i for $i = 1, 2, 3$. Calculate the rate of change $\dfrac{dx_1}{dt}$ of lake 1 in terms of the x_i's. **[2 marks]**

Suppose lake 1 holds 8000 megalitres of water, lake 2 holds 24000 megalitres and lake 3 holds 3000 megalitres. Three rivers connect the lakes, flowing in the directions indicated on the diagram. All three rivers have a constant flow rate of 24000 megalitre per day. Suppose at time 0, 10kg of pollutant is discharged into lake 1. You may assume that any amount of pollutant would instantaneously dissolve and spread evenly throughout each lake.

Tips: You sometime do not need to know the mass transport formula to solve this problem; unit checking!

$$\frac{dx}{dt} = A \cdot B$$

(i) Volume of lake;

(ii) Flow rate;

(iii) Total pollutant mass.

This is for CHE students!

Now let us talk about the unit! An inspired kindergarten chemistry case!

$$\underbrace{\frac{dc}{dt}}_{} = kc$$

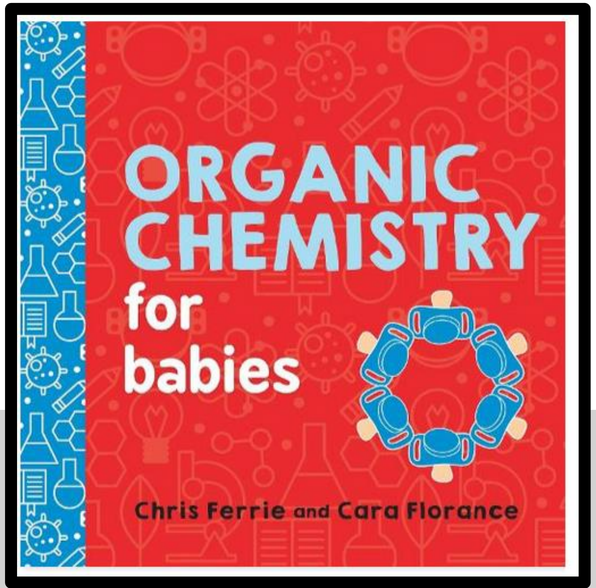
Change of concentration!

(Me) Computational Mechanics: number/volume.

Chem. Eng: mass/volume, mol/volume, volume/volume.

- The unit for concentration, c, is always taken as mol/volume.
  - But this narrative is incomplete.

https://en.wikipedia.org/wiki/Concentration#Volume_concentratio



- Lets see what info do we have!

$$\frac{dx}{dt} = A \cdot B$$

$$\underbrace{\frac{dx}{dt}}_{?/\text{time}} = \underbrace{24k}_{\text{vol/time}} \cdot \underbrace{B}_{?/\text{volume}}$$

$\boxed{?= \text{kg (mass)}}$

- x cant be volume!

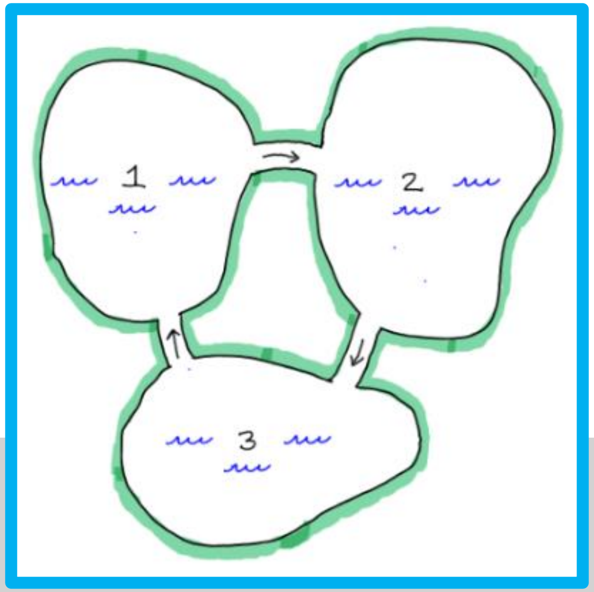
$$\frac{dx}{dt} = 24\cancel{k} \cdot \frac{x}{8\cancel{k}} \qquad \frac{?}{\text{time}} = \frac{\cancel{\text{vol}}}{\text{time}} \cdot \frac{?}{\cancel{\text{vol}}}$$

Suppose lake 1 holds 8000 megalitres of water, lake 2 holds 24000 megalitres and lake 3 holds 3000 megalitres. Three rivers connect the lakes, flowing in the directions indicated on the diagram. All three rivers have a constant flow rate of 24000 megalitre per day. Suppose at time 0, 10kg of pollutant is discharged into lake 1. You may assume that any amount of pollutant would instantaneously dissolve and spread evenly throughout each lake.

Suppose lake 1 holds 8000 megalitres of water, lake 2 holds 24000 megalitres and lake 3 holds 3000 megalitres. Three rivers connect the lakes, flowing in the directions indicated on the diagram. All three rivers have a constant flow rate of 24000 megalitre per day. Suppose at time 0, 10kg of pollutant is discharged into lake 1. You may assume that any amount of pollutant would instantaneously dissolve and spread evenly throughout each lake.

$$\frac{dx_1}{dt} = \underbrace{-24 \cdot \frac{x_1}{8}}_{-\text{ ve going out}} + \underbrace{24 \cdot \frac{x_3}{3}}_{+\text{ ve going in}}$$

$$\boxed{\frac{dx_1}{dt} = -3x_1 + 8x_3}$$



3. Repeat the calculation for lakes 2 and 3, and hence write down a system of first order differential equations in the form of $\frac{d\vec{x}}{dt} = A\vec{x}$ where $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. **[2 marks]**

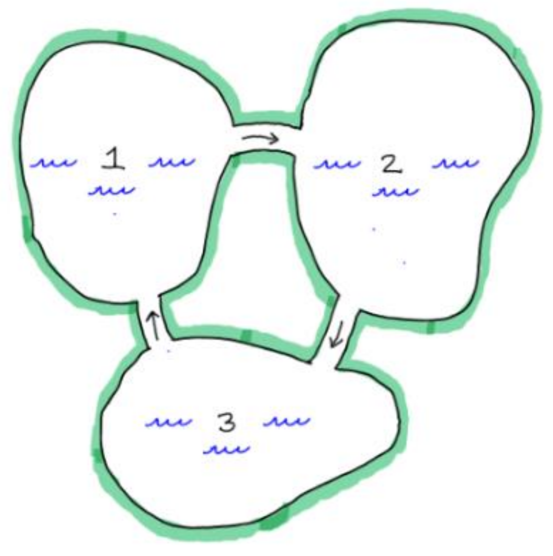
$$\frac{dx_1}{dt} = -3x_1 + 8x_3$$

Try: 5 mins!

$$\frac{dx_2}{dt} = 3x_1 - x_2$$

$$\frac{dx_3}{dt} = x_2 - 8x_3$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} -3 & 0 & 8 \\ 3 & -1 & 0 \\ 0 & 1 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



Suppose lake 1 holds 8000 megalitres of water, lake 2 holds 24000 megalitres and lake 3 holds 3000 megalitres. Three rivers connect the lakes, flowing in the directions indicated on the diagram. All three rivers have a constant flow rate of 24000 megalitre per day. Suppose at time 0, 10kg of pollutant is discharged into lake 1. You may assume that any amount of pollutant would instantaneously dissolve and spread evenly throughout each lake.

4. Find all the eigenvalues and corresponding eigenvectors of the matrix A.

Try: 5 mins!

Eigenvalue	Eigenvector
-7	$\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$
-5	$\begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix}$
0	$\begin{pmatrix} \frac{8}{3} \\ 8 \\ 1 \end{pmatrix}$

Calculating Homogeneous Solution

Let $\dfrac{dI(t)}{dt} + 5\,I(t) = 0$ → Assume $I_h(t) = Ae^{\lambda t}$

Substitute $I_h(t)$ expression

$$\frac{d(Ae^{\lambda t})}{dt} + 5\,(Ae^{\lambda t}) = 0$$

Expansion

$$\lambda \cancel{Ae^{\lambda t}} + 5\,(\cancel{Ae^{\lambda t}}) = 0$$

$$\lambda = -5$$

$$\frac{dI}{I} = -5\,dt$$

$$\ln I = -5t + C$$

$$I = \exp(-5t + C)$$

$$\boxed{I_h(t) = Ae^{-5t}}$$

A scaling up or down the solution; eigenvector?

Are You Camera ready?

5. Use the eigenvalue/eigenvector method to write down the general solution of the system of ODE's.

[2 marks]

Eigenvalue	Eigenvector
-7	$\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$
-5	$\begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix}$
0	$\begin{pmatrix} \frac{8}{3} \\ 8 \\ 1 \end{pmatrix}$

$$\frac{d\mathbf{x}(t)}{dt} = A\mathbf{x}(t)$$

$$\mathbf{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix}$$

Assume

$$\mathbf{x}(t) = \mathbf{v}e^{\lambda t}$$

$$\frac{d}{dt}\left(\mathbf{v}e^{\lambda t}\right) = A\mathbf{v}e^{\lambda t}$$

$$\lambda \mathbf{v}e^{\lambda t} = A\mathbf{v}e^{\lambda t}$$

$$\lambda \mathbf{v} = A\mathbf{v}$$

ASH
rsity

Example!

Consider the system:

$$\frac{d}{dt}\mathbf{x}(t) = A\mathbf{x}(t), \quad A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

The eigenvalues and eigenvectors of A are:

$$\lambda_1 = 3, \quad \mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\lambda_2 = 1, \quad \mathbf{v}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

(sense) *vectors*

Thus, the general solution is:

$$\mathbf{x}(t) = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{t}$$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = Ae^{-7t}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} + Be^{-5t}\begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix} + C\begin{bmatrix} 8 \\ 24 \\ 3 \end{bmatrix}}$$

Eigenvalue	Eigenvector
-7	$\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$
-5	$\begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix}$
0	$\begin{pmatrix} \frac{8}{3} \\ 8 \\ 1 \end{pmatrix}$

6. Hence solve the system of ODE's with initial condition.

- Solve these set of Eqs.

$$\boxed{\begin{bmatrix}10\\0\\0\end{bmatrix}} = \boxed{A\begin{bmatrix}-2\\1\\1\end{bmatrix} + B\begin{bmatrix}-4\\3\\1\end{bmatrix} + C\begin{bmatrix}8\\24\\3\end{bmatrix}}$$

Remember: At initial time, we have no pollutant in other lakes, apart from lake 1.

$\boxed{A = 15/7}$ $\boxed{B = -3}$ $\boxed{C = 2/7}$

7. What is the long term behaviour of the distribution of pollutant in the lakes? Can you offer a physical explanation? **[2 marks]**

$$\exp(-\infty) = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \cancel{Ae^{-7t}} \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} + \cancel{Be^{-5t}} \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix} + C \begin{bmatrix} 8 \\ 24 \\ 3 \end{bmatrix}$$

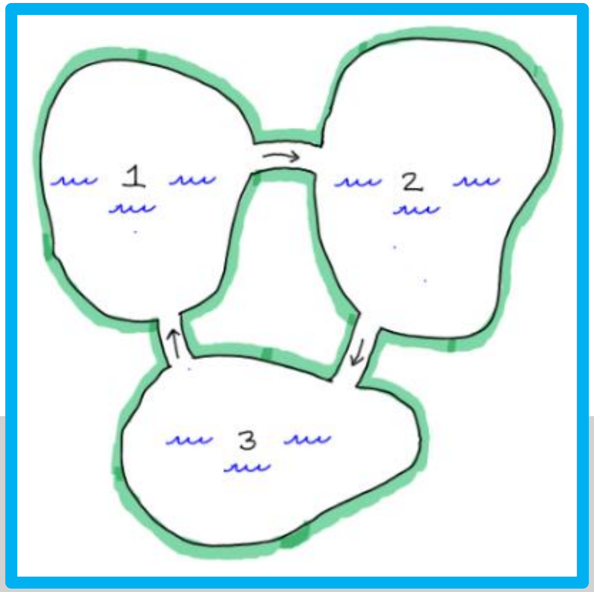
- Bigger-sized lake has the largest amount!

$\boxed{A = 15/7}$ $\boxed{B = -3}$ $\boxed{C = 2/7}$

8. A filtering plant is set up at the middle of the river between lake 3 and lake 1 to remove the pollutant from the water. As water passes through the plant, the concentration of pollutant is halved. Write down a system of first order differential equations in the form of $\frac{d\vec{x}}{dt} = B\vec{x}$ where $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ to model this situation. **[2 marks]**

$$\frac{dx_1}{dt} = \underbrace{-24 \cdot \frac{x_1}{8}} + \underbrace{24 \cdot \frac{x_3}{3}} \cdot \overbrace{\frac{1}{2}}^{\text{Filter}}$$

– ve going out + ve going in



$$\frac{d\vec{x}}{dt} = \begin{bmatrix} -3 & 0 & 4 \\ 3 & -1 & 0 \\ 0 & 1 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

9. Find the eigenvalues of B. **[2 marks]**

10. Without any further calculation, describe the long term behaviour of the solution $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. Does it match your expectation? **[2 marks]**

Q9: Follow the steps in Q4 (QR code)

$$\boxed{-4,\ -4+\sqrt{13},\ -4-\sqrt{13}.}$$

Q10: Demonstrate that the filtration plant should eventually remove almost all pollutant from the water

The Big Learning Outcomes for Week 8

After completing this week's task, you should be able to:

- Categorize ODEs as linear/nonlinear and identify their order.
- Solve first-order separable ODEs.
- Solve first-order linear ODEs.

Thank You