Practice 4

COMP9021, Term 3, 2019

1 A triangle of characters

Write a program characters_triangle.py that gets a strictly positive integer N as input and outputs a triangle of height N, following this kind of interaction:

```
$ python3 characters_triangle.py
Enter strictly positive number: 13
            A
           BCB
          DEFED
         GHIJIHG
        KLMNONMLK
       PQRSTUTSRQP
      VWXYZABAZYXWV
     CDEFGHIJIHGFEDC
    KLMNOPQRSRQPONMLK
   TUVWXYZABCBAZYXWVUT
  DEFGHIJKLMNMLKJIHGFED
 OPQRSTUVWXYZYXWVUTSRQPO
ABCDEFGHIJKLMLKJIHGFEDCBA
```

Two built-in functions are useful for this exercise:

- `ord()` returns the integer that encodes the character provided as argument;
- `chr()` returns the character encoded by the integer provided as argument.

For instance:

```
>>> ord('A')
65
>>> chr(65)
'A'
```

Consecutive uppercase letters are encoded by consecutive integers. For instance:

```
>>> ord('A'), ord('B'), ord('C')
(65, 66, 67)
```

2 Pascal triangle

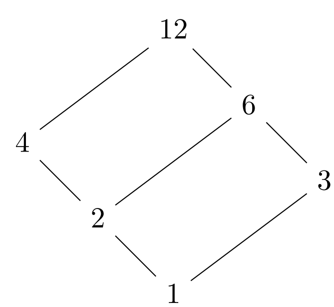
Write a program pascal_triangle.py that prompts the user for a number N and prints out the first $N + 1$ lines of Pascal triangle, making sure the numbers are nicely aligned, following this kind of interaction.

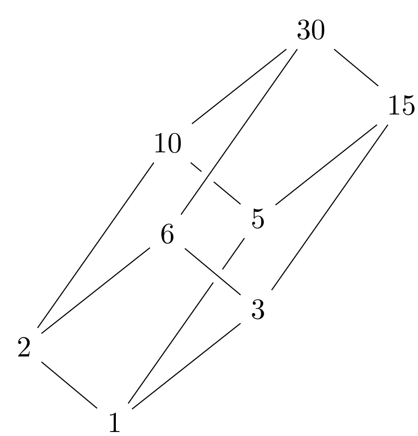
```
$ python3 pascal_triangle.py
Enter a nonnegative integer: 3
   1
  1 1
 1 2 1
1 3 3 1
$ python3 pascal_triangle.py
Enter a nonnegative integer: 7
               1
             1   1
           1   2   1
         1   3   3   1
       1   4   6   4   1
     1   5  10  10   5   1
   1   6  15  20  15   6   1
 1   7  21  35  35  21   7   1
$ python3 pascal_triangle.py
Enter a nonnegative integer: 11
                                   1
                                1     1
                             1     2     1
                          1     3     3     1
                       1     4     6     4     1
                    1     5    10    10     5     1
                 1     6    15    20    15     6     1
              1     7    21    35    35    21     7     1
           1     8    28    56    70    56    28     8     1
        1     9    36    84   126   126    84    36     9     1
     1    10    45   120   210   252   210   120    45    10     1
  1    11    55   165   330   462   462   330   165    55    11     1
```

3 Hasse diagrams

Let a strictly positive integer n be given. Let D be the set of divisors of n. Let k be the number of prime divisors of n (the number of prime numbers in D). The members of D can be arranged as the vertices of a solid in a k-dimensional space as illustrated below for $n = 12$ (in which case $D = \{1, 2, 3, 4, 6, 12\}$ and $k = 2$) and for $n = 30$ (in which case $D = \{1, 2, 3, 5, 6, 10, 15, 30\}$ and $k = 3$).

- Each of the solids' vertices is associated with two collections of nodes: those "directly below" it, and those "directly above" it. In particular, the prime divisors of n are "directly above" 1, and no vertex is below 1; n has exactly k vertices "directly below" it, and no vertex is above n. This suggests considering a dictionary whose keys are the members of D (inserted from smallest to largest), and as value for a given key d, the pair of ordered lists of members of D "directly below" d and "directly above" d, respectively.
- The solids exhibit k distinct "edge directions", one for each prime divisor of n, defining a partition of the solids' edges. One can represent this partition as a dictionary whose keys are the prime divisors of n (inserted from smallest to largest), and as value for a given key p, the ordered list of ordered pairs of members of D that make up the endpoints of the edges whose "direction" is associated with p.





Write a Python program `hasse_diagram.py` that defines a function `make_hasse_diagram()` that returns a named tuple `HasseDiagram` with three attributes:

- `factors`, for a dictionary whose keys are the members of D, and as value for a given key d—1 excepted—, a string that represents the prime decomposition of d, using `x` for multiplication and `^` for exponentiation, displaying only exponents greater than 1;
- `vertices`, for the first dictionary previously defined;
- `edges`, for the second dictionary previously defined.

Using the `doctest` module to test `make_hasse_diagram()` and the `pprint()` function from the `pprint` module, the following behaviour would then be observed:

```
>>> HD = make_hasse_diagram(12)
>>> HD # doctest: +ELLIPSIS
HasseDiagram(factors=..., edges=..., vertices=...)
>>> HD.factors
{1: '1', 2: '2', 3: '3', 4: '2^2', 6: '2x3', 12: '2^2x3'}
>>> pprint(HD.vertices)
{1: ([], [2, 3]),
 2: ([1], [4, 6]),
 3: ([1], [6]),
 4: ([2], [12]),
 6: ([2, 3], [12]),
 12: ([4, 6], [])}
>>> HD.edges
{2: [(1, 2), (2, 4), (3, 6), (6, 12)], 3: [(1, 3), (2, 6), (4, 12)]}
>>> HD = make_hasse_diagram(30)
>>> HD # doctest: +ELLIPSIS
HasseDiagram(factors=..., edges=..., vertices=...)
>>> HD.factors
{1: '1', 2: '2', 3: '3', 5: '5', 6: '2x3', 10: '2x5', 15: '3x5', 30: '2x3x5'}
>>> pprint(HD.vertices)
{1: ([], [2, 3, 5]),
 2: ([1], [6, 10]),
 3: ([1], [6, 15]),
 5: ([1], [10, 15]),
 6: ([2, 3], [30]),
 10: ([2, 5], [30]),
 15: ([3, 5], [30]),
 30: ([6, 10, 15], [])}
>>> pprint(HD.edges)
{2: [(1, 2), (3, 6), (5, 10), (15, 30)],
 3: [(1, 3), (2, 6), (5, 15), (10, 30)],
 5: [(1, 5), (2, 10), (3, 15), (6, 30)]}
```

4 Encoding pairs of integers as natural numbers (optional)

Write a program `plane_encoding.py` that implements a function `encode(a, b)` and a function `decode(n)` for the one-to-one mapping from the set of pairs of integers onto the set of natural numbers, that can be graphically described as follows:

16	15	14	13	12
17	4	3	2	11
18	5	0	1	10
19	6	7	8	9
20	21	...		

That is, starting from the point $(0, 0)$ of the plane, we move to $(1, 0)$ and then spiral counterclockwise:

- `encode(0,0)` returns `0` and `decode(0)` returns `(0,0)`
- `encode(1,0)` returns `1` and `decode(1)` returns `(1,0)`
- `encode(1,1)` returns `2` and `decode(2)` returns `(1,1)`
- `encode(0,1)` returns `3` and `decode(3)` returns `(0,1)`
- `encode(-1,1)` returns `4` and `decode(4)` returns `(-1,1)`
- `encode(-1,0)` returns `5` and `decode(5)` returns `(-1,0)`
- `encode(-1,-1)` returns `6` and `decode(6)` returns `(-1,-1)`
- `encode(0,-1)` returns `7` and `decode(7)` returns `(0,-1)`
- `encode(1,-1)` returns `8` and `decode(8)` returns `(1,-1)`
- `encode(2,-1)` returns `9` and `decode(9)` returns `(2,-1)`
- ...

5 Map of CO_2 emissions (optional, needs a module not installed on CSE computers)

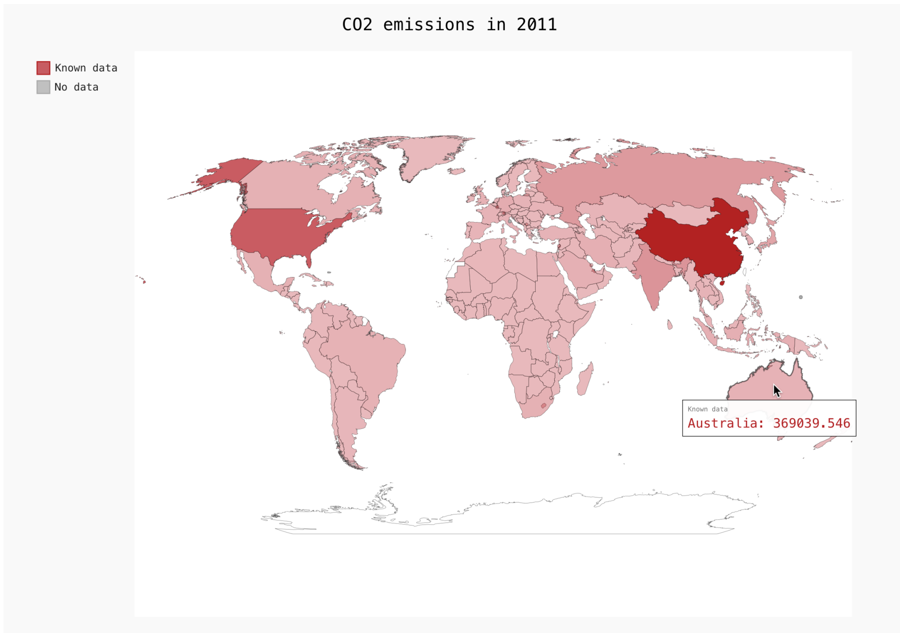
Write a program that extracts from the file `API_EN.ATM.CO2E.KT_DS2_en_csv_v2.csv`, stored in the subdirectory `API_EN` of the working directory, the country CO_2 emissions for the year 2011. Some data in this file are for entities different to countries, or for countries which are not values of the `COUNTRIES` dictionary of the `pygal.maps.world` module. The program will produce an output of the form

```
Leaving out Aruba
Leaving out Arab World
Leaving out American Samoa
Leaving out Antigua and Barbuda
Leaving out Bahamas, The
...
Leaving out Latin America & Caribbean (all income levels)
Leaving out Least developed countries: UN classification
Leaving out Low income
Leaving out Lower middle income
Leaving out Low & middle income
...
Leaving out Virgin Islands (U.S.)
Leaving out Vanuatu
Leaving out West Bank and Gaza
Leaving out World
Leaving out Samoa
```

to let the user know of all those entities and countries, which will be ignored. Some countries are described differently in the dictionary and in the file; these countries will not be ignored. The data will be shown interactively on a map, created as an object of class `World` of the `pygal.maps.world` module, that can be displayed in a browser by opening a file named `CO2_emissions.svg`—check out `render_to_file()`. To create the `World` object from a dictionary having as keys the keys of `COUNTRIES`, check out `add()`. The map should have—check out the `Style` class from the `pygal.style` module:

- as title for the map, `CO2 emissions in 2011`;
- one group of data with `Known data` as legend and with `#B22222` as colour, another group of data with `No data` as legend and with `#A9A9A9` as colour, both with a font size of 10pt;
- tooltips providing standard display for the first group, but with the amount of CO_2 emissions replaced by `?` for the second group, both with a font size of 8pt.

Here is the map with the cursor hovering over Australia, for which the CO_2 emissions are known.



Here is the map with the cursor hovering over Puerto Rico, for which the CO_2 emissions are not known.