A5 Eryk Halicki

1. a) $Q: \mathbb{R}^n \to \mathbb{R}$

$$Q(x) = x^T A x$$
$$Q(cx) = (cx^T) A (cx)$$
$$Q(cx) = c^2 (x^T A x)$$
$$= c^2 Q(x) \blacksquare$$

b) $Q(x) = 2x_1^2 - 6x_2^2 + 6x_1 x_2$

i) $x = Py$. P comes from PDP^{-1}

$$Q(x) = x^T A x$$

$$A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix} \quad \det(A - \lambda I) = \det\left(\begin{bmatrix} 2-\lambda & 3 \\ 3 & -6-\lambda \end{bmatrix}\right)$$
$$= (2-\lambda)(-6-\lambda) - 9$$
$$= -12 - 2\lambda + 6\lambda + \lambda^2 - 9$$
$$= \lambda^2 + 4\lambda - 21$$
$$= (\lambda - 3)(\lambda + 7)$$
$$\lambda = 3, -7$$

$(A - 3I)x = 0$

$$\begin{bmatrix} -1 & 3 \\ 3 & -9 \end{bmatrix} x = 0$$

$-x_1 + 3x_2 = 0 \quad x_1 = 3x_2$

$3x_1 - 9x_2 = 0$

$$v_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix}$$

$(A + 7I)x = 0$

$$\begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix} x = 0$$

$x_2 = -3x_1$

$$v_2 = \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{10} \\ -3/\sqrt{10} \end{bmatrix}$$

$$P = \begin{bmatrix} 3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & -3/\sqrt{10} \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 0 \\ 0 & -7 \end{bmatrix} \quad Q(y) = y^T D y$$
$$= 3y_1^2 - 7y_2^2$$

ii) $x = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$ $\quad P^{-1} = P^T$ since P is orthogonal

$P^T = \begin{bmatrix} 3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & -3/\sqrt{10} \end{bmatrix} = P^{-1}$ $\quad A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$

$y = P^{-1}x$
$= \begin{bmatrix} 3.4785 \\ 2.2135 \end{bmatrix}$

$Q(x) = x^T A x = 2x_1^2 - 6x_2^2 + 6x_1x_2$
$= \begin{bmatrix} 4 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix}$
$= 2$

$Q(y) = y^T D y$
$= 3y_1^2 - 7y_2^2$
$= 2$

c) i) False quadratic function

$x^2 + x$ is q function,

but not q form, since x $(x^2 + x)$ is not ~~An~~ of degree 2

ii) True in $x^T A x$, A is always symmetric, and as such always orthogonally diagonalizable. So, there always exists an orthogonal P matrix, which by definition has orthonormal columns, which are also the principal axes of the quadratic forms.

iii) False P can't be any orthogonal matrix eg. $P = I$ will result in $x = y$, and $Q(x)$ remains unchanged, ~~possibly~~ retaining any cross product terms it had before

iv) False eg. $x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $Q(x) = 2$

$2 \geq 0$, $\therefore$ cannot be negative definite.

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d) $A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$ $Q(x) = x^T A x$ $\boxed{\text{Proof by contradiction}}$

A is symetric since $A^T = A$, so we know it has all real eigenvalues.

~~If~~ $\det(A) > 0$, $\lambda_1 \lambda_2 > 0$, so either λ_1 and λ_2 are both positive or both negative.

2 cases:

both $+$: $\lambda_1 + \lambda_2 > 0$

both $-$: $\lambda_1 + \lambda_2 < 0$

Take the case where λ_1 and λ_2 are both negative. we know $\lambda_1 + \lambda_2 = a + d$, so $a+d < 0$ as well. It then follows that $d < -a$, and since $a > 0$, $-a < 0$, so, $d < -a < 0$. Combined with the fact $ad - bc = \det(A)$ for a 2×2 matrix, and $b = c$ in this case, we know $ad - b^2 > 0$.

This can be rearranged to $ad > b^2$.

since $d < 0$ and $a > 0$, ad is negative.

But, $b^2 \geq 0$ for any real b,

So we get a contradiction:

$$ad > b^2 \geq 0$$

but, $ad < 0$.

As such, it is impossible that both eigenvalues of A are negative, imply they are both positive. This means A is positive definite, since all its eigenvalues are positive.

2. a) i) if A is positive definite, all its eigenvalues are positive.

Furthermore, $Av_i = \lambda_i v_i$ for eigenvector/value pair $v_i \lambda_i$. When taking powers of A, we get:

$A^2 v_i = (AA)v_i = A(Av_i) = A(\lambda_i v_i) = \lambda_i (Av_i) = \lambda_i^2 v_i$

so for λ_i in A, there is a corresponding λ_i^2 in A^2.

Since all eigenvalues of A are real and positive, (symmetric and positive definite), all eigenvalues of A^2 are also real and positive, since $\lambda_i > 0$, $\lambda_i^2 > 0$, for all λ_i in A.

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ii) Matrix A is positive def iff its quadratic form $Q_A(x) = x^T A x$ is also positive definite.
So, $A+B$ is positive def iff its quadratic form $Q_{A+B}(x) = x^T(A+B)x$ is positive definite.

$x^T(A+B)x = x^T(Ax + Bx) = \underline{x^T Ax + x^T Bx}$

so $Q_A(x) + Q_B(x) = Q_{A+B}(x)$

and since we know both $Q_A(x)$ and $Q_B(x)$ are positive definite, $Q_{A+B}(x)$ is also positive definite, since,

$Q_A(x) > 0$ for all x

$Q_B(x) > 0$ for all x

so $Q_A(x) + Q_B(x) > 0$ for all x

and $Q_{A+B}(x) = Q_A(x) + Q_B(x)$.

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b) i) all eigenvalues of A satisfy

$Av_i = \lambda_i v_i$, and if A^{-1} exists,

$$\cancel{A^{-1}A} v_i = A^{-1} \lambda_i v_i$$

$$v_i = \lambda_i A^{-1} v_i$$

$$\frac{1}{\lambda_i} v_i = A^{-1} v_i$$

So for eigenvector v_i corresponding to λ_i of A, there is an eigenvalue λ_i^{-1} for v_i in A^{-1}! Since all eigenvalues of $A > 0$ (since A is positive definite) it follows that all eigenvalues of A^{-1}, $\frac{1}{\lambda_i} > 0$, so A^{-1} is also positive definite. True

ii) from 2.a.i, we know for λ_i and v_i in A there is corresponding λ_i^2 for v_i in A^2.

if A^2 is positive definite, all λ_i^2 are >0, however, it is possible for λ_i to be <0 and λ_i^2 to be >0, so A may contain negative λ_i, breaking the positive eigenvalue assumption required for it to be positive definite.

False

iii) iff $Q(x) = x^T A^T A x$ is positive semi definite, $A^T A$ is positive definite.

Furthermore, $x^T A^T A x = (Ax)^T Ax = \|Ax\|^2$

So $A^T A$ is only positive definite if $\|Ax\|^2$ is always >0.

But, $\|Ax\|^2$ can equal 0 when $Ax = 0$ has a non-trivial solution, which occurs when A does not have full rank.

False

c)

i) $\begin{bmatrix} 1 & -2 & 2 \\ -2 & 5 & -5 \\ 2 & -5 & 6 \end{bmatrix}$

$A_1 = [1] \quad \det(A_1) = 1 > 0$ ✓

$A_2 = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} \quad \det(A_2) = 1 \cdot 5 - (-2 \cdot -2)$

$= 5 - 4$

$= 1$

$1 > 0 \therefore \det(A_2) > 0$ ✓

$A_3 = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 5 & -5 \\ 2 & -5 & 6 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$\det(A_3) = aei + bfg + cdh - gec - hfa - idb$

$= 1, \quad 1 > 0 \therefore \det(A_3) > 0$ ✓

since A_1, A_2, and A_3 all have det > 0,

A is positive definite

ii) $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 10 & -4 \\ -1 & -4 & 11 \end{bmatrix}$

$A_1 = [1] \quad \det(A_1) = 1 > 0$ ✓

$A_2 = \begin{bmatrix} 1 & 1 \\ 1 & 10 \end{bmatrix} \quad \det(A_2) = ad - bc$

$= 10 - 1$

$= 9 > 0$ ✓

$A_3 = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 10 & -4 \\ -1 & -4 & 11 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$\det(A_3) = aei + bfg + cdh - gec - hfa - idb$

$= 81 > 0$ ✓

since all determinants are $>$, A has an LU factorization.

3. a)

$$A = \begin{bmatrix} 1 & 4 & 1 \\ 4 & 25 & 7 \\ 1 & 7 & 6 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 1 \\ 8 \end{bmatrix}$$

$R_2 - 4R_1 \to R_2$
$R_3 - R_1 \to R_3$

$$\begin{bmatrix} 1 & 4 & 1 \\ 0 & 9 & 3 \\ 0 & 3 & 5 \end{bmatrix} \xrightarrow{R_3 - \frac{1}{3}R_2 \to R_3} \begin{bmatrix} 1 & 4 & 1 \\ 0 & 9 & 3 \\ 0 & 0 & 4 \end{bmatrix} = U_1$$

$$U = \begin{bmatrix} 1 & 4 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$U^T U \cancel{x} = b \to$

$U^T y = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 3 & 0 \\ 1 & 1 & 2 \end{bmatrix} y = \begin{bmatrix} 1 \\ 1 \\ 8 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$$

$Ux = y$

$$\begin{bmatrix} 1 & 4 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix} x = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$$

$$x = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

b) i) $A^TA\hat{x} = A^Tb$

$A = U^TU$

$$A^T(U^TU)\hat{x} = A^Tb$$

$$\cancel{(A^T)^{-1}A^T}(U^TU)\hat{x} = \cancel{(A^T)^{-1}A^T}b$$

$$U^TU\hat{x} = b$$

to be positive definite, A must have all positive λ_i, so $\det(A) > 0$ by thm 0.1

Since $\det(A) \neq 0$, A is invertible.

This also implies A has full rank, so the system $A\hat{x} = b$ has only one unique solution (its not really a LS problem)

ii) $A = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 5 & -4 \\ 2 & -4 & 6 \end{bmatrix}$ $b = \begin{bmatrix} -1 \\ 3 \\ -5 \end{bmatrix}$

$R_2 + R_1 \to R_2$

$R_3 - 2R_1 \to R_3$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 4 & -2 \\ 0 & -2 & 2 \end{bmatrix} \xrightarrow{R_3 + \frac{1}{2}R_2 \to R_3} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 4 & -2 \\ 0 & 0 & 1 \end{bmatrix} = U_1$$

$$U = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$U^T U x = b \quad Ux = y$

$U^T y = b$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ 2 & -1 & 1 \end{bmatrix} y = \begin{bmatrix} -1 \\ 3 \\ -5 \end{bmatrix} \quad y = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} \quad x = \begin{bmatrix} 5/2 \\ -1/2 \\ -2 \end{bmatrix}$$

c) False

$A = U^T U$ makes sense,

$A^T = (U^T U)^T$ since A is symmetric

$A^T = U^T U^{TT}$ $A = A^T$

$A^T = U^T U$

4. a) $Q(x) = 6x_1^2 - 5x_2^2 + 9x_3^2$

$\|x\| = 1$

$$Q(x) \leq 9x_1^2 + 9x_2^2 + 9x_3^2$$
$$\leq 9(x_1^2 + x_2^2 + x_3^2) \leftarrow \|x\| = 1$$
$$\leq 9$$

$x = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $Q(x) = 9$ $\therefore$ 9 is max

$$Q(x) \geq -5x_1^2 - 5x_2^2 - 5x_3^2$$
$$\geq -5(\ldots)$$
$$\geq -5$$

$x = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $Q(x) = -5$ $\therefore$ -5 is min

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b) i) $Q(x) = x_2^2 + 2x_1x_2 + 4x_1x_3 + 2x_2x_3$

$Q(x) = x^TAx$ $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix}$

need to find λ_1 and λ_3

$\det(A - \lambda I) = \det\left(\begin{bmatrix} -\lambda & 1 & 2 \\ 1 & 1-\lambda & 1 \\ 2 & 1 & -\lambda \end{bmatrix}\right)$

$\det(A) = aei + bfg + cdh - ceg - bdi - afh$

$$\det\left(\begin{bmatrix} -\lambda & 1 & 2 \\ 1 & 1-\lambda & 1 \\ 2 & 1 & -\lambda \end{bmatrix}\right) = \underbrace{(-\lambda)(1-\lambda)(-\lambda)}_{-\lambda^3+\lambda^2}$$

$$\left.\begin{aligned} &+ 1\cdot 1\cdot 2 \\ &+ 2\cdot 1\cdot 1 \end{aligned}\right\} +4$$

$$- \underbrace{(2)(1-\lambda)(2)}_{(4-4\lambda)} -4+4\lambda$$

$$\left.\begin{aligned} &- \underbrace{(1)(1)(-\lambda)}_{(-\lambda)} \\ &- \underbrace{(-\lambda)(1)(1)}_{(-\lambda)} \end{aligned}\right\} +2\lambda$$

$$= -\lambda^3 + \lambda^2 + 6\lambda$$

$$= -\lambda(\lambda^2 - \lambda - 6)$$

$$= \lambda(\lambda - 3)(\lambda + 2)$$

$\lambda = -2, 0, 3$

$\max(Q(x)) = 3$

$\min(Q(x)) = -2$

$(A + 2I)x = 0$

$$\begin{bmatrix} 2 & 1 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 1 & 2 \\ 0 & 5/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 - R_1 \frac{1}{2} \to R_2$

$R_3 - R_1 \to R_3$

$2x_1 + x_2 + 2x_3 = 0$

$\frac{5}{2}x_2 = 0$

$x_2 = 0$

$2x_1 + 0 + 2x_3 = 0$

$x_1 = -x_3$

$u_3' = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$u_3 = \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} = \frac{u_3'}{\|u_3'\|}$

$(A - 3I)x = 0$

$$\begin{bmatrix} -3 & 1 & 2 \\ 1 & -2 & 1 \\ 2 & 1 & -3 \end{bmatrix} \xrightarrow[R_3 + 2/3 R_1 \to R_3]{R_2 + \frac{1}{3} R_1 \to R_2} \begin{bmatrix} -3 & 1 & 2 \\ 0 & -5/3 & 5/3 \\ 0 & 5/3 & -5/3 \end{bmatrix}$$

$R_3 + R_2 \to R_3$

$$\begin{bmatrix} -3 & 1 & 2 \\ 0 & -5/3 & 5/3 \\ 0 & 0 & 0 \end{bmatrix} x = 0$$

$u_1' = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $u_1 = \frac{u_1'}{\|u_1'\|} = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$

$-3x_1 + x_2 + 2x_3 = 0$

$x_2 = x_3$

$-3x_1 + 3x_3 = 0$ $x_1 = x_3$

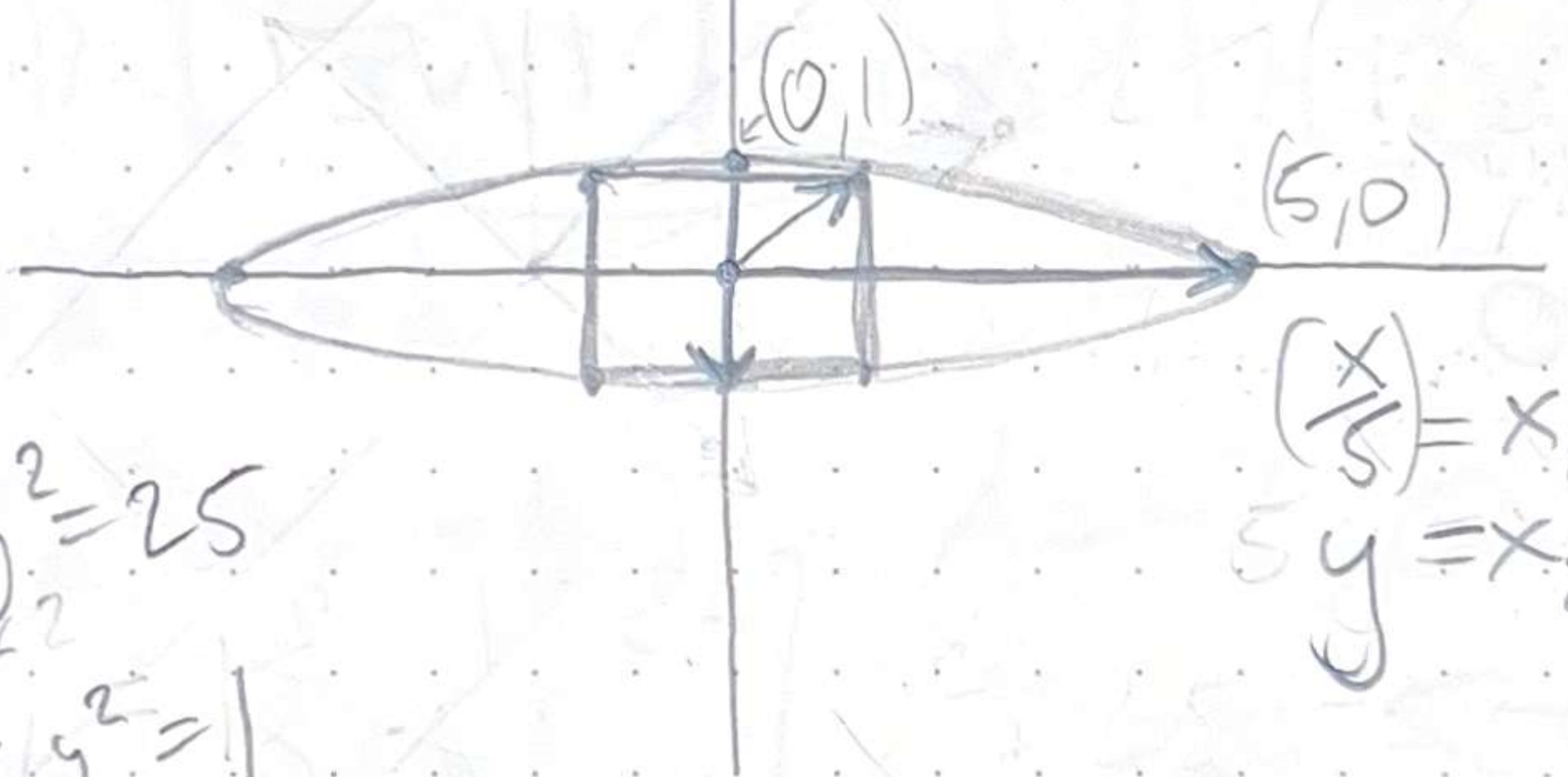
max of $Q(x) = 3$, achieved when $x = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$

min of $Q(x) = -2$, achieved when

$$x = \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

ii) Python

iii)



$\left(\frac{x}{5}\right) = x_1$

$y = x_2$

$x^2 + 25y^2 = 25$

$\frac{x^2}{25} + \frac{25}{25}y^2 = 1$

$\left(\frac{x}{5}\right)^2 + y^2 = 1$

@ $x = 0$
$25y^2 = 25$
$y^2 = 1$
$y = \pm 1$

@ $y = 0$
$x^2 = 25$
$x = \pm 5$

area:

$l \times w = 2x_1 \cdot 2x_2$
$= 4x_1x_2$

$Q(x) = 4x_1x_2$

so we want

$\max(Q(x) \mid \|x\| = 1)$ since

$\|x\| = \sqrt{x_1^2 + x_2^2}$
$= \sqrt{\left(\frac{x}{5}\right)^2 + y^2}$
$= \sqrt{1} = 1$

where $\vec{x} = \begin{bmatrix} \frac{x}{5} \\ y \end{bmatrix}$

$Q(x) = x^T A x \qquad A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$

$= 4x_1x_2$

$\det(A - \lambda I) = \det\left(\begin{bmatrix} -\lambda & 2 \\ 2 & -\lambda \end{bmatrix}\right) = 0$

$\lambda^2 - 4 = 0$

$\lambda^2 = 4$

$\lambda = \pm 2$

$(A - 2I)x = 0$

$\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} x = 0 \qquad -2x_1 + 2x_2 = 0$

$2x_2 = 2x_1$

$x_2 = x_1$

$(A + 2I)x = 0$

$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} x = 0 \qquad x_1 = -x_2$

$\lambda_1 = 2 \quad v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad u_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

$\lambda_2 = -2 \quad v_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

max achieved at u_1,

$x/5 = x_1 = 1/\sqrt{2} \qquad x = 5/\sqrt{2}$

$y = x_2 = 1/\sqrt{2} \qquad y = 1/\sqrt{2}$

c) By spectral decomp, we know

$$A = \lambda_1 u_1 u_2^T + \lambda_2 u_2 u_2^T \cdots \lambda_n u_n u_n^T$$

so,

$$x^T A x = \sum_{i=1}^{n} \lambda_i x^T u_i u_i^T x$$

and since the normalized eigenvectors of A form an orthonormal basis for $\mathbb{R}^n$, $x = \sum_{i=1}^{n} c_i u_i$

, x is a linear combination of the eigenvectors of A. So,

$$x^T A x = \sum_{i=1}^{n} \lambda_i (c_1 u_1 + c_2 u_2 \cdots c_n u_n)^T u_i \ldots$$

$$= \sum_{i=1}^{n} \lambda_i \left(\sum_{j=1}^{n} c_j u_j \cdot u_i \right)^2 \text{ where } c_j \text{ is } x \cdot u_j$$

but for every $i \neq j$, $u_i \cdot u_j = 0$, and when $i = j$, $u_i \cdot u_j = 1$, so

$$\boxed{x^T A x = \sum_{i=1}^{n} \lambda_i (x \cdot u_i)^2}$$

Since $\|x\|=1$, $\|x\|^2=1$, so

$$\sum_{i=1}^{n}(x \cdot u_i)^2 = \|x\|^2 = 1$$

and since $\lambda_1 \geq \lambda_2 \geq \lambda_3 \dots \geq \lambda_n$

we know we obtain the max of $Q(x) \mid \|x\|=1$

when $x \cdot u_1 = 1$, so when $x = u_1$.

However, we also have the constraint

$x \cdot u_1 = 0$, so x must be in the subspace

spanned by $[u_2, u_3 \dots u_n]$.

This changes the starting index of our sum to 2,

so $x^T A x = \sum_{i=2}^{n} \lambda_i (x \cdot u_i)^2$, and similarly,

we obtain max when $x \cdot u_2 = 1$, since

$$\lambda_2 \geq \lambda_3 \dots \geq \lambda_n$$

and min when $x \cdot u_n = 1$.

$\therefore$, we obtain the max $(Q(x) \mid \|x\|=1, x \cdot u_1 = 0)$

when $x = u_2$ and min when $x = u_n$.

$\blacksquare$

ii) $Q(x) = x_2^2 + 2x_1x_2 + 4x_1x_3 + 2x_2x_3$

from question 4,b,i) $A = \begin{matrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{matrix}$

we found $\lambda = -2, 0, 3$

and $\min(Q(x) \mid \|x\| = 1, x \cdot u_1 = 0) = -2$

Using thm 0.6, we know

$\max(Q(x) \mid \|x\| = 1, x \cdot u_1 = 0) = 0$,

since $\max(Q(x) \mid \|x\| = 1) = 3$ when $x = u_1$,

so we take $\max = \lambda_2$.

```python
import numpy as np

#Q(x) = -6x1^2 - 10x2^2 - 13x3^2 - 13x4^2 - 4x1x2 - 4x1x3 - 4x1x4 + 6

A = [[-6,-2 ,-2 , -2],
     [-2 ,-10, 0, 0],
     [-2, 0, -13,3 ],
     [-2, 0, 3, 13]]

A = np.array(A)

eigvalues,eigvectors=np.linalg.eig(A)

print(f"max(Q(x) | ||x|| = 1) = {np.max(eigvalues)}")
print(f"min(Q(x) | ||x|| = 1) = {np.min(eigvalues)}")
```

.py

```
erykhalicki@mac-2 A5 % python Q4.py
max(Q(x) | ||x|| = 1) = 13.59537962670595
min(Q(x) | ||x|| = 1) = -13.81265322440764
erykhalicki@mac-2 A5 %
```