MAT216 Fall 2020

Assignment 2 on Week 4–6

Due: 23/11/2020 (23:59) **Total Marks:** $8 \times 12.5 = 100$

Problem 1. ($A = LU$)

$$A = \begin{bmatrix} 2 & -2 & -2 \\ 0 & -2 & 2 \\ -1 & 5 & 2 \end{bmatrix}$$

i) Find an LU-decomposition of A.

ii) If $\mathbf{b} = \begin{bmatrix} -4 \\ -2 \\ 6 \end{bmatrix}$, solve the system $A\mathbf{x} = \mathbf{b}$ by using the LU obtained in i).

Problem 2. (*Gauss–Jordan Elimination Method*)

Consider the following system.

$$-x_1 + 3x_2 - 2x_3 + 4x_4 = 0$$

$$2x_1 - 6x_2 + x_3 - 2x_4 = -3$$

$$x_1 - 3x_2 + 4x_3 - 8x_4 = 2$$

i) Write the augmented matrix for the given system.

ii) Reduce the system into the reduced row echelon form.

iii) Identify the leading and free variables.

iv) Solve and represent the solution of the system by using column vectors.

Problem 3. (*Inverse Matrix*)

$$A = \begin{bmatrix} 0 & -1 & 1 & 0 \\ 2 & 1 & 0 & 2 \\ 1 & -1 & 3 & 0 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

i) Find A^{-1}, if it exists.

ii) Solve the system $A\mathbf{x} = \mathbf{b}$ by using A^{-1} from i) with $\mathbf{b} = \begin{bmatrix} 2 \\ 2 \\ 2 \\ -3 \end{bmatrix}$.

iii) Verify the solution.

Problem 4. (*Span*)

Consider the following subset S of $\mathbb{R}^3$.

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix} \right\}$$

i) Determine if $\text{span}(S) = \mathbb{R}^3$.

ii) Write $\mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$ as a linear combination of vectors in S, if possible.

iii) Find at least one vector $\mathbf{v} \in \mathbb{R}^3$, if possible, such that it is not a linear combination of the vectors in S.

Problem 5. (*Basis and Dimension*)

$$S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\}$$

i) Determine if S is a basis for $M_{2,2}$.

ii) Write $A = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$ as a linear combination of vectors in S, if possible.

iii) Find at least one vector $A \in M_{2,2}$, if possible, such that it is not a linear combination of the vectors in S.

Problem 6. (*Basis and Dimension of Subspace*)

$$W = \left\{ \begin{bmatrix} s + 4t \\ t \\ s \\ 2s - t \end{bmatrix} : s, t \in \mathbb{R} \right\}$$

Find

i) a basis for W.

ii) the dimension of the subspace W of $\mathbb{R}^4$.

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Problem 7. (*Row Space, Column Space, and Rank of a Matrix*)

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 2 & 5 & 1 & 1 & 0 \\ 3 & 7 & 2 & 2 & -2 \\ 4 & 9 & 3 & -1 & 4 \end{bmatrix}$$

i) Find the basis for row space of A.

ii) Find the basis for column space of A.

iii) Find the rank(A).

Problem 8. (*Coordinates and Change of Basis*)

Let B and B' are two ordered bases for $\mathbb{R}^3$.

$$B = \left\{ \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix} \right\}, \qquad B' = \left\{ \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

i) Find the coordinate matrices of $\mathbf{v} = \begin{bmatrix} 9 \\ 2 \\ 4 \end{bmatrix}$ relative to the bases B and B'.

ii) Find the transition matrix $P_{B \to B'}$.

iii) Verify that $[\mathbf{x}(\mathbf{v})]_{B'} = P_{B \to B'} [\mathbf{x}(\mathbf{v})]_B$.