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Principle of Physics II

PHY 112

Lecture Notes

Chapter 1

Fundamentals of Electrostatics

In this chapter, we introduce the notion of electric charges. We describe the different kinds of charges and the quantization and conservation of charge. We consider charges which are static and discuss the law, known as Coulomb's law, that describes the interaction of one charge with another. We discuss the generalization to many charges using the superposition principle. The chapter is concluded with a comparison between Coulomb's law and Newton's law of gravitation.

1.1 Introduction

The aim of these set of lecture notes is to introduce the subject of electricity and magnetism at the introductory undergraduate level. Electricity and Magnetism are deeply interlinked and they were unified into one set of equations by the famous Scottish physicist James Clerk Maxwell in the 19th century. The combined subject is known as *classical electromagnetism* or *electromagnetism* for short and it provides the foundation of modern physics as well much of today's engineering (especially electrical engineering). For example, Albert Einstein's inspiration for his famous discovery of the special theory of relativity was based on a deep understanding of electromagnetism. A thorough understanding of electromagnetism is necessary to excel in the physical sciences.

1.2 Electrostatics

To begin, let us keep things simple. A good place to start is to consider electric charges which are static.[1] The study of the physics of static electric charges is known as *electrostatics.*

[1]When charges move they give rise to magnetism which we shall study in the second half of this course.

1.3 Charge

All matter is made out of charged particles known as protons and electrons. There are also charge-neutral particles inside the nucleus of an atom known as *neutrons* but if we look closely we shall see that even they are made out of smaller charged particles. Experimentally it has been established that there are two *types* of electric charges, called *positive* and *negative* charges. By convention, the electron is assigned negative charge $-e$. In this course we shall be using the MKS system as our unit of measurement and in this system the unit of charge is a *coulomb* denoted by the symbol C. In these units the value of the constant e is given by

$$e = 1.6 \times 10^{-19} \mathrm{C}. \tag{1.1}$$

The assignment of the relative sign of charges comes from the fact that two positive charges exert repulsive forces on each other. Similarly, two negative charges also exert repulsive forces on each other. On the other hand, a positive change and a negative charge exert attractive forces on each other. We summarize this situation by saying: *like charges repel each other and unlike charges attract each other.* The precise manner in which this force, known as the *electrostatic force*, works is encapsulated in *Coulomb's law* which we shall discuss below shortly.

Apart from this experimental fact about the sign of charges there are two other fundamental experimental facts which are

1. Charge is conserved.
2. Charge is quantized.

Let us discuss these important topics in turn.

1.4 Conservation of Charge

The total charge on an *isolated* system is always conserved. By isolated we mean that no charged particles may leave or enter the system. For example, we can consider a neutral atom which has the structure of a positively charged nuclear core surrounded by a cloud of electrons whose total charge is exactly equal to that of the nucleus but with a negative sign. It may happen that such a neutral atom can suffer a collision with another particle or photons (which are particles of light) and one of the outer electrons may get knocked off the atom. As a result we shall have one negatively charged electron and a positively charged *ion* – an atom or a molecule with an electron missing or an extra electron attached. This is an example of a process by which we have a pair creation of a positively and negatively charged particles from matter. Under the right circumstances the opposite process can occur and a positively charged ion may merge with an electron and become a neutral atom.

It may also happen that a photon may collide with matter and the energy that it carries may be converted into an electron and its *antiparticle* known as a *positron.* Recall that according to Einstein's theory of relativity the mass of a particle can be converted into radiant energy according to the formula $E = mc^2$, where c is the speed of light. Of course, for a photon to be converted to an electron and a positron the photon must have energy that is at least $2m_ec^2$ where m_e is the mass of the electron or the positron.

We can summarize this discussion by the following statement about charge conservation:

The total algebraic sum of positive and negative charges in an isolated system never changes.

1.5 Quantization of Charge

An important aspect of charge is that the particles that we can directly observe in nature come with charges whose values are integer multiple of the constant e. This fact is known as *charge quantization.*

The charge of the proton is e and the charge of the electron is $-e$. It is an experimental fact that, as far as we can measure, the charge of the electron is *exactly* equal to minus the charge of the proton. This has been experimentally verified in 1 part in 10^{20}. There is no deeply understood and experimentally verified theoretical reason for this coincidence. But this fact has the effect that, since most ordinary matter that we come across every day has an equal number of positive and negative charges, we are largely unaware of electrostatic forces.

Although particles we can directly measure can only have charge which come in integer multiple of e, there is a vast amount of experimental evidence that the *internal* structure of strongly interacting particles – known as *hadrons* – consists of particles known as *quarks* whose electric charges are fractional and their charges come in multiples of $\frac{1}{3}e$. For example, the proton, which is an example of a hadron, consists of two quarks with charge $\frac{2}{3}e$ and one quark with charge $-\frac{1}{3}e$. Similarly, the neutron contains two quarks of charge $-\frac{1}{3}e$ and one quark of charge $\frac{2}{3}e$. What is remarkable about these quarks is that they are never observed outside the hadron and they are said to be *confined* inside the hadron. So, as far as particles that we can freely observe in nature it is safe to say that their charge is quantized in units of e. In this course, we shall not discuss quarks any further but it's important that you know of their existence.

Experimentally the electron appears to be a point particle whereas the dimension of the proton is about 10^{-15} meters. In this course we shall be concerned with length scales which are typically much larger than the proton length scale. Thus when we talk about smooth charge distributions later in the course we shall assume that that we are averaging over a large number of very small charges.

1.6 Coulomb's Law

Let us start by considering the force between point-like charges. Consider two point-like objects, labelled 1 and 2 with charges q_1 and q_2 respectively. Then we denote the force on object 2 due to object 1 by F_{12}. Then Coulomb's law states that this force is given by

$$\mathbf{F}_{12} = k\frac{q_1 q_2 \hat{\mathbf{r}}_{12}}{r_{12}^2}. \tag{1.2}$$

Here $\hat{\mathbf{r}}_{12}$ denotes the *unit vector* pointing from object 1 in the direction of object 2, and r_{12} denotes the distance between the two objects (see the figure 1.1). Observe several important aspects of this law:

1. The force described by this law is *central*, that is, the force is along the line joining the two objects. The word central here describes the fact that the force on object 2 is towards the *centre*

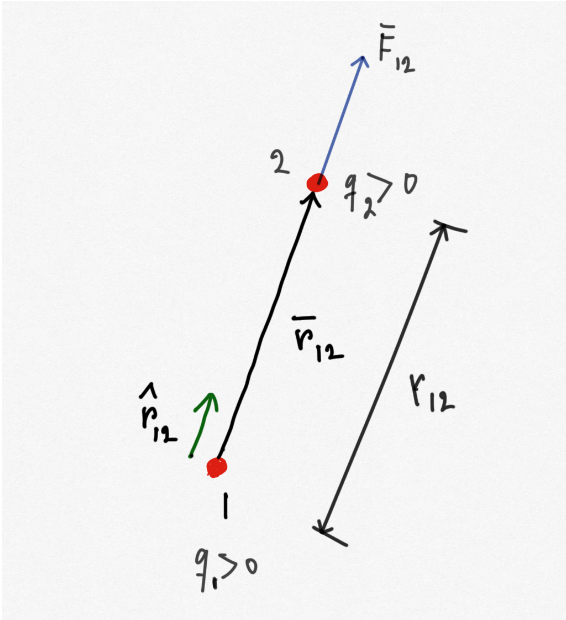

Figure 1.1: Two charges 1 and 2 are joined by a position vector

of the source of that force which is object 1. The direction of the force is given by the unit vector $\hat{\mathbf{r}}_{12}$ which points from charge 1 to charge 2. If we are given the vector $\mathbf{r}_{12}$ which joins charge 1 to charge 2, then this unit vector is given by

$$\hat{\mathbf{r}}_{12} = \frac{\mathbf{r}_{12}}{|\mathbf{r}_{12}|} = \frac{\mathbf{r}_{12}}{r_{12}}. \tag{1.3}$$

2. The force is an *inverse square law*. This refers to the fact that the *magnitude*, $|\mathbf{F}_{12}|$, of the force is proportional to the $1/(r_{12})^2$, that is inverse-square of the distance, r_{12}, between the two point charges. This means that if double of the distance between the two charges the magnitude of the force changes by a factor of $\frac{1}{4}$, and if we reduce the distance between the two charges by a factor of two the magnitude of the force changes by a factor of 4.

   The exponent in the denominator of Coulomb's law (1.2) appears to be exactly equal to 2 so far as experiments can determine. In principle, the exponent in the denominator could be be $2 + \delta$ where δ is a small number. However, modern experiments have determined δ to be smaller than 10^{-15}.

   Coulomb's law is not the only fundamental law of nature which displays an inverse square law. Newton's law of universal gravitation is also an inverse square law. Indeed there are important differences and similarities between these two laws which we shall discuss below.

3. Coulomb's law encodes both the cases when the forces between the charges are attractive as well as repulsive. This is encoded in the sign of the factor $q_1 q_2$ in the numerator of equation (1.2). Note that $q_1 q_2 > 0$ when both charges are positive, $\{q_1 > 0$ and $q_2 > 0\}$, as well as when both charges are negative, $\{q_1 < 0$ and $q_2 < 0\}$. In this case, the direction as the force $\mathbf{F}_{12}$ is in the same direction as the unit vector $\hat{\mathbf{r}}_{12}$ which results in a repulsive force.

   On the other hand, note that $q_1 q_2 < 0$ when the two charges have dissimilar signs, that is, either $\{q_1 > 0$ and $q_2 < 0\}$, or $\{q_1 < 0$ and $q_2 > 0\}$. Under these circumstances the force $\mathbf{F}_{12}$ points in the opposite direction to the unit vector $\hat{\mathbf{r}}_{12}$ and so we see that the force in these situations is attractive.

4. The constant k in Coulomb's law is a universal constant (i.e., its value is independent of the value of the charges and their distance). Its precise value is dependent on the choice of units. For the MKS system we have chosen here k is given by

$$k = 9 \times 10^9 \;\; N \cdot m^2 \cdot C^{-2}. \tag{1.4}$$

   The dimensions of k is determined by dimensional analysis of equation (1.2).

5. Lastly, Coulomb's law encapsulates in it Newton's third law of mechanics which states that for every action by one object on another there is an equal and opposite action by the second body on the first. To see this let us consider the force on charge 1 due to charge 2. This done easily by just switching the two labels $1 \leftrightarrow 2$ in equation (1.2). In this way we obtain

$$\begin{aligned} \mathbf{F}_{21} &= k \frac{q_2 q_1 \hat{\mathbf{r}}_{21}}{r_{21}^2} \\ &= k \frac{q_1 q_2 \hat{\mathbf{r}}_{21}}{r_{21}^2}. \end{aligned} \tag{1.5}$$

In this equation the unit vector $\hat{\mathbf{r}}_{21}$ points from charge 2 towards charge 1. This is in the opposite direction of $\hat{\mathbf{r}}_{12}$. Thus we see

$$\hat{\mathbf{r}}_{21} = -\hat{\mathbf{r}}_{12}. \tag{1.6}$$

Using this in equation (1.5) and comparing the resulting equation with equation (1.2) we obtain

$$\mathbf{F}_{12} = -\mathbf{F}_{21}. \tag{1.7}$$

1.7 Multiple Charges: Superposition Principle

If we only had two charges then we could not measure them since Coulomb's law depends on the product $q_1 q_2$ of the two charges. So we need to consider the case of more than two charges.

We can also ask what happens when we bring more than two charges near each other. Suppose we have three charges q_1, q_2, and q_3, then we may ask what is the force on the charge q_3 due to the other two charges. We know from Coulomb's law that the forces on q_3 due to q_1 and q_2 would be respectively give by

$$\mathbf{F}_{13} = k\frac{q_1 q_3 \hat{\mathbf{r}}_{13}}{r_{13}^2} \tag{1.8}$$

and

$$\mathbf{F}_{23} = k\frac{q_2 q_3 \hat{\mathbf{r}}_{23}}{r_{23}^2}. \tag{1.9}$$

But the question is what is the *total* force on charge 3? It is an experimental fact that the total force on charge 3 is given by the vector sum of the two individual forces. If we denote the net force on charge 3 by $\mathbf{F}_3$ then it is given by

$$\mathbf{F}_3 = k\frac{q_1 q_3 \hat{\mathbf{r}}_{13}}{r_{13}^2} + k\frac{q_2 q_3 \hat{\mathbf{r}}_{23}}{r_{23}^2}. \tag{1.10}$$

This is considered to be a new law of nature *in addition* to Coulomb's law and it is known as the *superposition principle.* This principle is non-trivial since we know that in the quantum domain this principle fails to hold for electromagnetism. Fortunately, in this course

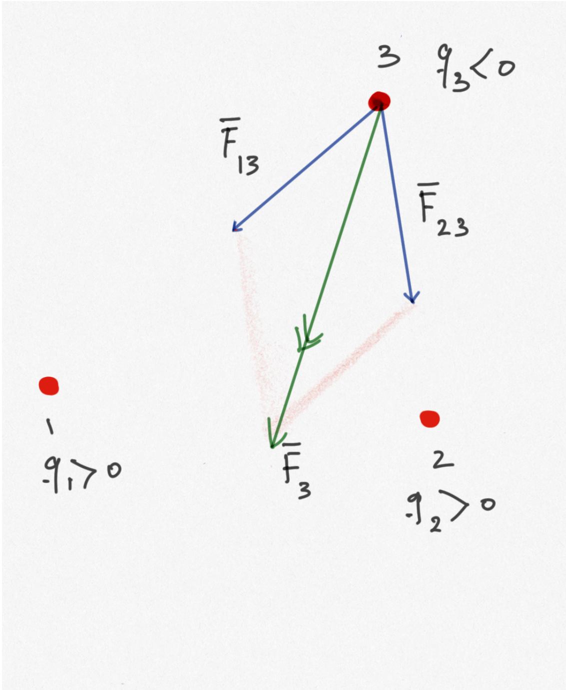

Figure 1.2: The net force $\mathbf{F}_3$ on charge 3 is given by resultant on the sum of the two forces $\mathbf{F}_{12}$ and $\mathbf{F}_{13}$.

we shall not consider the quantum domain but will stick to *classical* electromagnetism.

The superposition principle generalizes to an arbitrary number of charges. Thus if we have N charges $q_1, q_2, q_3, \ldots, q_N$ and then ask what is the force on the k-th charge then the superposition principle states that $\mathbf{F}_k$ is given by

$$\mathbf{F}_k = {\sum_{i=1}^{N}}' \mathbf{F}_{ik}, \tag{1.11}$$

where $\mathbf{F}_{ik}$ are given by Coulomb's law and the primed sum $\sum'$ denotes that we are *excluding* the term $i = k$ in the summation. See figure 1.2.

1.8 Comparison Between Coulomb's Law and Newton's Law of Gravitation

In your previous physics course you learned about Newton's universal law of gravitation which is also an inverse square law. According to this law if we have two point-like masses m_1 and m_2 then the force between them is attractive. Then the force on mass 2 exerted by mass 1 is given by

$$\mathbf{F}^g_{12} = -G_N \frac{m_1 m_2 \hat{\mathbf{r}}_{12}}{r^2_{12}}. \tag{1.12}$$

Here we put a superscript g on the force to make it clear that this is the force due to gravity and not any charge that the two masses may have. Let us compare this equation with Coulomb's law (1.2). Because mass and charge have different units, the universal gravitational constant G_N (known as *Newton's constant*) is of course different from the constant k in Coulomb's law. However another important difference between the two laws is the *minus* sign in Newton's law (1.12). Because mass is always a positive quantity and $G_N > 0$ this means that gravitational force between two point massive particles is always attractive.

Because gravitational force is always attractive it has a cumulative effect that an object with equal amount positive and negative charges does not have. This is even though the gravitational force is an extremely weak force. To see how weak the gravitational force is let us compare the gravitational force between two electrons which are 1 centimeters apart with the electrostatic repulsion between the same electrons. To do this computation we need to know the value of G_N which is given by

$$G_N = 6.674 \times 10^{-11} m^3 kg^{-1} s^{-2}. \tag{1.13}$$

Using this fact and the fact that 1 newton (unit of force) is equal to 1 $\mathrm{kg} \cdot \mathrm{m\ s}^{-2}$ we can calculate the ratio of magnitudes F^e (the electrostsatic force) with F^g of the two forces to be

$$\frac{F^e}{F^g} \sim 10^{42}. \tag{1.14}$$

Thus we see that the gravitational force amazingly weak compared to the electrostatic force. Yet because the gravitational force is always attractive we are always aware of its existence. The forces due to the positive and negative charges from a neutral object almost always cancel out. And so, even though the electrostatic force is many orders of magnitude larger than the gravitational force we are not always aware of the effect of electrical forces.