5.2 The Indefinite Integral
Solutions to the Selected Problems

Formula

$$\int cf(x)\,dx = c\int f(x)\,dx$$

$$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

15–34. Evaluate the integral and check your answer by differentiating.

15.

$$\int x(1+x^3)\,dx$$

Solution

$$\int x(1+x^3)\,dx = \int (x+x^4)\,dx = \int x\,dx + \int x^4\,dx = \frac{x^2}{2} + \frac{x^5}{5} + C$$

$$\boxed{\int x(1+x^3)\,dx = \frac{x^2}{2} + \frac{x^5}{5} + C}.$$

✓Check

$$\frac{d}{dx}\left(\frac{x^2}{2} + \frac{x^5}{5} + C\right) = \frac{2x}{2} + \frac{5x^4}{5} + 0 = x + x^4$$

17.

$$\int x^{\frac{1}{3}}(2-x)^2\,dx$$

Solution

$$\int x^{\frac{1}{3}}(2-x)^2\,dx = \int x^{\frac{1}{3}}(4-4x+x^2)\,dx$$

$$= \int \left(4x^{\frac{1}{3}} - 4x^{\frac{4}{3}} + x^{\frac{7}{3}}\right)dx$$

$$= 4\int x^{\frac{1}{3}}\,dx - 4\int x^{\frac{4}{3}}\,dx + \int x^{\frac{7}{3}}\,dx$$

$$= 4\frac{x^{\frac{4}{3}}}{\frac{4}{3}} - 4\frac{x^{\frac{7}{3}}}{\frac{7}{3}} + \frac{x^{\frac{10}{3}}}{\frac{10}{3}} + C$$

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$$= 3x^{\frac{4}{3}} - \frac{12}{7}x^{\frac{7}{3}} + \frac{3}{10}x^{\frac{10}{3}} + C$$

$$\boxed{\int x^{\frac{1}{3}}(2-x)^2\,dx = \left(3x - \frac{12}{7}x^2 + \frac{3}{10}x^3\right)\sqrt[3]{x} + C}.$$

✓Check

$$\frac{d}{dx}\left(3x^{\frac{4}{3}} - \frac{12}{7}x^{\frac{7}{3}} + \frac{3}{10}x^{\frac{10}{3}} + C\right) = 3 \times \frac{4}{3}x^{\frac{1}{3}} - \frac{12}{7} \times \frac{7}{3}x^{\frac{4}{3}} + \frac{3}{10} \times \frac{10}{3}x^{\frac{7}{3}} + 0$$

$$= 4x^{\frac{1}{3}} - 4x^{\frac{4}{3}} + x^{\frac{7}{3}} = x^{\frac{1}{3}}(4 - 4x + x^2) = x^{\frac{1}{3}}(2-x)^2$$

19.

$$\int \frac{x^5 + 2x^2 - 1}{x^4}\,dx$$

Solution

$$\int \frac{x^5 + 2x^2 - 1}{x^4}\,dx = \int \left(\frac{x^5}{x^4} + \frac{2x^2}{x^4} - \frac{1}{x^4}\right)dx = \int \left(x + \frac{2}{x^2} - \frac{1}{x^4}\right)dx$$

$$= \int x\,dx + 2\int \frac{1}{x^2}\,dx - \int \frac{1}{x^4}\,dx = \frac{x^2}{2} + \left(2\frac{x^{-1}}{-1}\right) - \left(\frac{x^{-3}}{-3}\right) + C$$

$$\boxed{\int \frac{x^5 + 2x^2 - 1}{x^4}\,dx = \frac{x^2}{2} - \frac{2}{x} + \frac{1}{3x^3} + C}.$$

✓Check

$$\frac{d}{dx}\left(\frac{x^2}{2} - \frac{2}{x} + \frac{1}{3x^3} + C\right) = \frac{2x}{2} - 2\frac{-1}{x^2} + \frac{1}{3}(-3x^{-4}) + 0 = x + \frac{2}{x^2} - \frac{1}{x^4} = \frac{x^5 + 2x^2 - 1}{x^4}$$

23.

$$\int (3\sin x - 2\sec^2 x)\,dx$$

Solution

$$\int (3\sin x - 2\sec^2 x)\,dx = 3\int \sin x\,dx - 2\int \sec^2 x\,dx = -3\cos x - 2\tan x + C$$

$$\boxed{\int (3\sin x - 2\sec^2 x)\,dx = -3\cos x - 2\tan x + C}.$$

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✓**Check**

Try.

25.

$$\int \sec x\,(\sec x + \tan x)\,dx$$

Solution

$$\int \sec x\,(\sec x + \tan x)\,dx = \int (\sec^2 x + \sec x \tan x)\,dx = \int \sec^2 x\,dx + \int \sec x \tan x\,dx$$

$$= \tan x + \sec x + C$$

$$\boxed{\int \sec x\,(\sec x + \tan x)\,dx = \tan x + \sec x + C}.$$

✓**Check**

Try.

29.

$$\int \frac{\sin x}{\cos^2 x}\,dx$$

Solution

$$\int \frac{\sin x}{\cos^2 x}\,dx = \int \left(\frac{1}{\cos x} \times \frac{\sin x}{\cos x}\right) dx = \int \sec x \tan x\,dx = \sec x + C$$

$$\boxed{\int \frac{\sin x}{\cos^2 x}\,dx = \sec x + C}.$$

✓**Check**

Try.

33.

$$\int \left(\frac{1}{2\sqrt{1-x^2}} - \frac{3}{1+x^2}\right) dx$$

Solution

$$\int \left(\frac{1}{2\sqrt{1-x^2}} - \frac{3}{1+x^2}\right) dx = \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx - 3\int \frac{1}{1+x^2}\,dx = \frac{1}{2}\arcsin x - 3\arctan x + C$$

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$$\boxed{\int \left(\frac{1}{2\sqrt{1-x^2}} - \frac{3}{1+x^2}\right) dx = \frac{1}{2}\arcsin x - 3\arctan x + C}.$$

✓**Check**

Try.

35. Evaluate the integral

$$\int \frac{1}{1+\sin x}\, dx$$

by multiplying the numerator and denominator by an appropriate expression.

Solution

$$\int \frac{1}{1+\sin x}\, dx = \int \frac{(1-\sin x)}{(1+\sin x)(1-\sin x)}\, dx = \int \frac{(1-\sin x)}{(1-\sin^2 x)}\, dx = \int \frac{(1-\sin x)}{\cos^2 x}\, dx$$

$$= \int \frac{1}{\cos^2 x}\, dx - \int \frac{\sin x}{\cos^2 x}\, dx = \int \sec^2 x\, dx - \int \sec x \tan x\, dx = \tan x - \sec x + C.$$

$$\boxed{\int \frac{1}{1+\sin x}\, dx = \tan x - \sec x + C}$$

36. Use the double-angle formula $\cos 2x = 2\cos^2 x - 1$ to evaluate the integral

$$\int \frac{1}{1+\cos 2x}\, dx.$$

Solution

$$\int \frac{1}{1+\cos 2x}\, dx = \int \frac{1}{2\cos^2 x}\, dx = \frac{1}{2}\int \frac{1}{\cos^2 x}\, dx = \frac{1}{2}\int \sec^2 x\, dx = \frac{1}{2}\tan x + C$$

$$\boxed{\int \frac{1}{1+\cos 2x}\, dx = \frac{1}{2}\tan x + C}$$

Alternative

$$\int \frac{1}{1+\cos 2x}\, dx = \int \frac{(1-\cos 2x)}{(1+\cos 2x)(1-\cos 2x)}\, dx = \int \frac{(1-\cos 2x)}{(1-\cos^2 2x)}\, dx$$

$$= \int \frac{1}{\sin^2 2x}\, dx - \int \frac{\cos 2x}{\sin^2 2x}\, dx = \int \csc^2 2x\, dx - \int \csc 2x \cot 2x\, dx$$

$$= -\frac{1}{2}\cot 2x + \frac{1}{2}\csc 2x + C = \frac{1}{2}\left(\frac{1-\cos 2x}{\sin 2x}\right) + C = \frac{1}{2}\left(\frac{2\sin^2 x}{2\sin x \cos x}\right) + C$$

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$$\boxed{\int \frac{1}{1+\cos 2x}\,dx = -\frac{1}{2}\cot 2x + \frac{1}{2}\csc 2x + C}$$

The results are not different. In fact, these can be proved to be equivalent in the following way:

$$-\frac{1}{2}\cot 2x + \frac{1}{2}\csc 2x = \frac{1}{2}\left(\frac{1-\cos 2x}{\sin 2x}\right) = \frac{1}{2}\left(\frac{2\sin^2 x}{2\sin x\cos x}\right) = \frac{1}{2}\left(\frac{\sin x}{\cos x}\right) = \frac{1}{2}\tan x$$

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