A linear polynomial passing through

$(0,1)$ and $(0.6, 1.8221)$

$\therefore \quad x_0 = 0$ and $x_1 = 0.6$

Now. $P(x_0) = a_0 + a_1 x_0$

$P(x_1) = a_0 + a_1 x_1$

$\therefore$ Vandermonde matrix $= \begin{pmatrix} 1 & 0 \\ 1 & 0.6 \end{pmatrix}$

Augmented matrix $= \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 1 & 0.6 & 0 & 1 \end{array}\right]$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 0.6 & -1 & 1 \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & -1.667 & 1.667 \end{array}\right]$$

= Inverse of the Vandermonde matrix

Again,
$$\begin{pmatrix} 1 & 0 \\ 1 & 0.6 \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1.8221 \end{pmatrix}$$

Or,
$$\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 1 & 0.6 & 1.8221 \end{array}\right) = \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 0.6 & 0.8221 \end{array}\right)$$

$$= \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1.3701 \end{array}\right)$$

$\therefore\ a_0 = 1$ and $a_1 = 1.3701$

$$\therefore\ P(0.75) = 1 + (1.3702 \times 0.75)$$
$$= 2.02765$$
$$\approx 2.0277$$

We know,

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$$

Given, a quadratic polynomial passes through $(0, 1)$, $(0.6, 1.8221)$, $(1.2, 3.3261)$

$$\therefore\ L_0(x) = \frac{(x-0.6)(x-1.2)}{(0-0.6)(0-1.2)} = \frac{1}{0.72}(x-0.6)(x-1.2)$$

$$= 1.388\cancel{8}9\,(x-0.6)(x-1.2)$$

$$\therefore\ L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-0)(x-1.2)}{(0.6-0)(0.6-1.2)}$$

$$= -2.77\cancel{7}8\,(x)(x-1.2)$$

$$\therefore\ L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-0)(x-0.6)}{(1.2-0)(1.2-0.6)}$$

$$= 1.3889x\,(x-0.6)$$

Using Lagrange method

$$p(x) = l_0 f(x_0) + l_1 f(x_1) + l_2 f(x_2)$$

$$\therefore p(0.75) = 1.3889(0.75-0.6)(0.75-1.2)(1)$$

$$+ (-2.7778)(0.75)(0.75-1.2)(1.8221)$$

$$+ 1.3889(0.75)(0.75-0.6)(3.3201)$$

$$= -0.0938 + 1.7082 + 0.5188$$

$$= \cancel{2.1332}\ 2.1332$$

Now

maximum true error

$$= |F(x) - P(x)| = |e^{0.75} - 2.1332|$$

$$= \cancel{0.0162}\ 0.0162$$

(Ans.)