ASSIGNMENT - 05

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Section : 09

Set : J

Ans. to Q No - 1

Given, $|z-1| = 9$ is a circle.

Now, we can parameterise the contour by $z = 9e^{i\theta}$ ~~where~~ where $0 \le \theta \le 2\pi$.

$\therefore dz = 9ie^{i\theta}\,d\theta$

$$\therefore \oint \frac{1}{z-2}\,dz$$

$$= \int_{0}^{2\pi} \frac{1}{9e^{i\theta}-2} \cdot 9ie^{i\theta}\,d\theta$$

$$= \Big[\ln|9e^{i\theta}-2|\Big]_{0}^{2\pi}$$

$$= 2\pi i$$

(Ans.)

Ans to Q No - 2

Given, $\oint_C (\bar{z})^2 dz$

We can parameterise the contour by

$z = e^{i\theta}$ where $0 \leq \theta \leq 2\pi$

$\therefore dz = ie^{i\theta} d\theta$ and $(\bar{z})^2 = e^{-2i\theta}$

$\therefore \oint_c (\bar{z})^2 dz$

$\Rightarrow \oint_0^{2\pi} e^{-2i\theta} \cdot ie^{i\theta} d\theta \Rightarrow i\int_0^{2\pi} e^{-i\theta} d\theta$

$\Rightarrow i^2 [\cdot e^{-i\theta}]_0^{2\pi} \Rightarrow -(e^{-2\pi i} - e^0)$

$\Rightarrow -(1-1) = 0$ (Ans.)

Ans. to Q No - 3

Given, $\int_c (x^2 - iy^2)\,dz$

We know, $z = x + iy$ $\therefore$ $dz = dx + i\,dy$

Now, $(x^2 - iy^2)\,dz$

$= (x^2 - iy^2)(dx + i\,dy)$

$= x^2 dx + ix^2 dy - iy^2 dx + y^2 dy$

Given, C is a straight line from (1,1) to (2,8)

$\therefore$ slope of the straight line $= \dfrac{8-1}{2-1} = 7$

$\therefore (y-1) = 7(x-1)$

$\Rightarrow y - 1 = 7x - 7$

$\Rightarrow y = 7x - 6$

$\therefore dy = 0$

$$\therefore \int_C (x^2 - iy^2)\, dz$$

$$= \int_C x^2 dx + ix^2 dy - iy^2 dx + y^2 dy$$

$$= \int_C x^2 dx + 0 + 0 - i(7x-6)^2 dx$$ [putting value of y and dy]

$$= \int_C x^2 dx - i(49x^2 - 84x + 36)\, dx$$

$$= \int_C x^2 dx - i49x^2 dx + i84x\, dx - i36\, dx$$

$$= \left[\frac{x^3}{3} - i49\frac{x^3}{3} + i84\frac{x^2}{2} - i36x\right]_1^2$$

[$\because$ x varies from 1 to 2]

$$= \frac{1}{3}\left[x^3\right]_1^2 - \frac{49}{3}i\left[x^3\right]_1^2 + 42i\left[x^2\right]_1^2 - 36i\left[x\right]_1^2$$

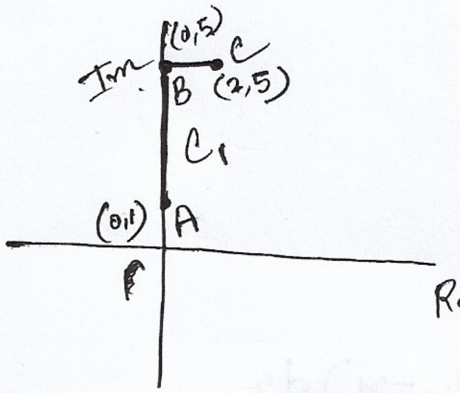
$$= \frac{1}{3}(8-1) - \frac{49}{3}i(8-1) + 42i(4-1) - 36i(2-1)$$

$$= \frac{7}{3} - \frac{343}{3}i + 126i - 36i$$

(Ans.)

Ans to Q No - 4

The figure looks like this:-



Here,

$C_1 = AB + BC$

Now, $\int_{AB} (3x+y)dx + (2y-x)\,dy$

Along AB:

$x = 0$; $dx = 0$; $y_{low} = 1$; $y_{up} = 5$

Thus, $\int_1^5 2y\,dy = 2\left[\frac{y^2}{2}\right]_1^5$

$= 25 - 1 = 24$

Along BC:

$y = 0$; $dy = 0$; $x_{low} = 0$; $x_{up} = 2$

Thus,

$$\int_0^2 3x\,dx = 3\left[\frac{x^2}{2}\right]_0^2 = \frac{3}{2}(4-0)$$

$$= 6$$

Therefore,

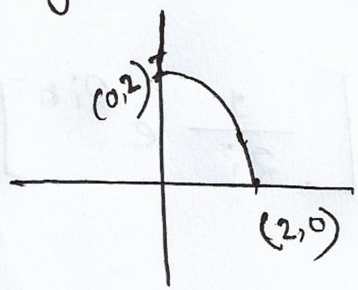
$$\int_{C_1} (3x+y)\,dx + (2y-x)\,dy$$

$$= \int_{AB} (3x+y)\,dx + (2y-x)\,dy + \int_{BC} (3x+y)\,dx + (2y-x)\,dy$$

$$= 24 + 6 = 30 \quad \text{(Ans)}$$

Ans. to Q No-5

The figure looks like :



We can parameterise the contour by

$z = 2e^{i\theta}$ where $\theta = [0, \frac{\pi}{2}]$

$\therefore dz = 2ie^{i\theta} d\theta$

$z^2 = 4e^{2i\theta}$ and $3z = 6e^{i\theta}$

and, $\theta_{min} = 0$; $\theta_{max} = \frac{\pi}{2}$

Therefore,

$$\int_C (z^2+3z)\,dz$$

$$= \int_0^{\pi/2} \left(4e^{2i\theta} + 6e^{i\theta}\right) 2ie^{i\theta}\,d\theta$$

$$= \int_0^{\pi/2} \left(8ie^{3i\theta} + 12ie^{2i\theta}\right) d\theta$$

$$= \int_0^{\pi/2} 8ie^{i3\theta}\,d\theta + \int_0^{\pi/2} 12ie^{2i\theta}\,d\theta$$

$$= 8i\left[\frac{1}{3i}e^{3i\theta}\right]_0^{\pi/2} + 12i\left[\frac{1}{2i}e^{2i\theta}\right]_0^{\pi/2}$$

$$= \frac{8}{3}(-i-1) + 6(-1-1)$$

$$= \frac{-8i-8}{3} - 12$$

$$= \frac{-32i-32-144}{12}$$

$$= -\frac{32i+176}{12}$$

$$= -\frac{44+8i}{3} \quad \text{(Ans)}$$