BRAC UNIVERSITY

Inspiring Excellence

MAT 215

Complex Variable and Laplace Transformations

The Principal Argument

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1 Preliminaries

Since a complete rotation around the origin leaves a complex number unchanged, there are many choices which could be made for ϕ (here we denoted the polar angle by ϕ instead of the traditional θ) by circling the origin any number of times, look at fig:(1)

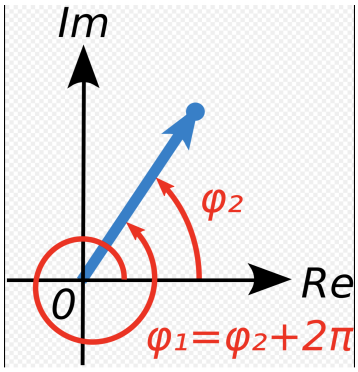

Figure 1: Two choice for argument ϕ

Which is why we devised what is known as the principal value, denoted by $Arg(z)$. The values that ϕ take when calculating the principal value range from $(-\pi, \pi]$. Remember that ϕ is positive when calculated anti-clockwise from the positive x-axis, and negative when calculated clockwise from the positive x-axis.

2 Calculating Principal Arguments

Calculating principal arguments are not as straightforward as calculating the argument, but it isn't complicated either. A simple rule of thumb is to calculate the $arg(z)$, after which you intuitively add or subtract π such that ϕ is a value between $(-\pi, \pi]$. Or, use the computational cheat sheet below for calculating the principal argument for different values of x and y.

$$\text{Arg}(z) = \text{Arg}(\text{x} + \text{iy}) = \begin{cases} \tan^{-1}\left(\frac{y}{x}\right) & \text{if } x > 0 \\ \tan^{-1}\left(\frac{y}{x}\right) + \pi & \text{if } x < 0 \text{ and } y \geq 0 \\ \tan^{-1}\left(\frac{y}{x}\right) - \pi & \text{if } x < 0 \text{ and } y < 0 \\ +\frac{\pi}{2} & \text{if } x = 0 \text{ and } y > 0 \\ -\frac{\pi}{2} & \text{if } x = 0 \text{ and } y < 0 \\ \text{undefined} & \text{if } x = 0 \text{ and } y = 0 \end{cases}$$