Summer 2020

MAT 216

<u>Problem Sheet - 2:</u>

Solve the following Problems:

1. Find the domain and codomain of the transformation defined by the equations, and determine whether the transformation is linear:

(i) $$\begin{aligned} w_1 &= 3x_1 - 2x_2 + 4x_3 \\ w_2 &= 5x_1 - 8x_2 + x_3 \end{aligned}$$

(ii) $$\begin{aligned} w_1 &= 2x_1x_2 - x_2 \\ w_2 &= x_1 + 3x_1x_2 \\ w_3 &= x_1 + x_2 \end{aligned}.$$

2. Find the standard matrix for the linear transformation $T : \Re^3 \to \Re^3$ given by:

$$\begin{aligned} w_1 &= 3x_1 + 5x_2 - x_3 \\ w_2 &= 4x_1 - x_2 + x_3 \\ w_3 &= 3x_1 + 2x_2 - x_3 \end{aligned}$$

and then calculate $T(-1, 2, 4)$ by matrix multiplication.

3. Let $T_A : \Re^3 \to \Re^3$ be multiplication by

$$A = \begin{bmatrix} -1 & 3 & 0 \\ 2 & 1 & 2 \\ 4 & 5 & -3 \end{bmatrix}$$

and let e_1, e_2 and e_3 be the standard basis vectors for $\Re^3$. Find the following vectors:

(i) $T_A(e_1), T_A(e_2)$ and $T_A(e_3)$ (ii) $T_A(e_1 + e_2 + e_3)$ (iii) $T_A(7e_3)$.

4. Let $T : \Re^3 \to \Re^2$ be defined by $T(x, y, z) = (x - y, x - z)$. Show that T is a linear transformation.

5. Suppose the mapping $T:\Re^3 \to \Re^2$ is defined by $T(u)=(3x-2y+z, x-3y-2z)$, where $u=(x,y,z)$ in $\Re^3$. Show that T is a linear transformation.

6. Determine whether $T:\Re^3 \to \Re^2$ is a linear transformation:

   (i) $T(x,y,z)=(1,1)$ (ii) $T(x,y,z)=(3x-4y, 2x-5z)$.

7. Find a system of linear equations corresponding to the augmented matrix:

   (i) $\begin{bmatrix} 3 & 0 & -2 & 5 \\ 7 & 1 & 4 & -3 \\ 0 & -2 & 1 & 7 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 0 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$.

8. (a) Find a linear equation in the variables x and y that has the general solution:

   $$x=5+2t,\ y=t.$$

   (b) Show that $x=t,\ y=\frac{1}{2}t-\frac{5}{2}$ is also the general solution of the equation in part (a).

9. For which value(s) of the constant k does the system

$$\begin{aligned} x - y &= 3 \\ 2x-2y &= k \end{aligned}$$

have no solutions? Exactly one solution? Infinitely many solutions? Explain your reasoning.