BRAC UNIVERSITY

MAT215

MATHEMATICS III: COMPLEX VARIABLES & LAPLACE TRANSFORMATIONS

Assignment 04

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Section: 09

Assignment Set: N

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Ans To The Question No. (1)

Given, $e^{2z-1} = 1$

R.H.S in Polar form:

$$r = \sqrt{1^2 + 0^2} = 1$$

$$\theta = tan^{-1}\left(\frac{0}{1}\right) = 0 + 2n\pi \;\; [where\ n = 0, \pm1, \pm2, ...]$$

Therefore,

$$e^{2z-1} = 1 \times e^{i(0+2\pi n)}$$

$$\Rightarrow e^{(2x+i2y-1)} = e^{(i2\pi n)}$$

$$\Rightarrow e^{(2x-1)} \times e^{(i2y)} = e^{(i2\pi n)} \;\; [where\ n = 0, \pm1, \pm2, ...]$$

Now,

$$e^{2x-1} = 1$$

$$\Rightarrow 2x - 1 = ln(1)$$

$$\Rightarrow 2x = 0 + 1$$

$$\Rightarrow x = \frac{1}{2}$$

Again,

$$e^{i2y} = e^{i2\pi n}$$

$$\Rightarrow i2y = i2\pi n$$

$$\Rightarrow y = n\pi \;\; [where\ n = 0, \pm1, \pm2, ...]$$

Therefore, $z = \frac{1}{2} + in\pi [where\ n = 0, \pm1, \pm2, ...]$ (Answer)

Ans To The Question No. (2)

Given, $ln(\sqrt{3}-i)$
Here,

$$r=\sqrt{(\sqrt{3})^2+(-1)^2}=\sqrt{3+1}=2$$
$$and, \theta=tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)=-\frac{\pi}{6}$$

We know, $log(z)=ln(r)+i(\theta+2\pi n), \ \ where\ n=0,\pm1,\pm2....$
Therefore,

$$ln(\sqrt{3}-i)$$
$$=ln(2)+i(-\frac{\pi}{6}+2\pi n), \ \ where\ n=0,\pm1,\pm2....$$

Ans To The Question No. (3)

Given, $ln(1-i)$
Here,

$$r=\sqrt{(1)^2+(-1)^2}=\sqrt{1+1}=\sqrt{2}$$
$$and, \theta=tan^{-1}\left(\frac{-1}{1}\right)=-\frac{\pi}{4}$$

We know, $log(z)=ln(r)+i(\theta+2\pi n), \ \ where\ n=0,\pm1,\pm2....$
Therefore,

$$ln(1-i)$$
$$=ln(\sqrt{2})+i(-\frac{\pi}{4}+2\pi n), \ \ where\ n=0,\pm1,\pm2....$$

Ans To The Question No. (4)

Given that, $Exp(z + \pi i) = -e^z$

Now, L.H.S =

$$Exp(z + \pi i)$$
$$= e^z \cdot e^{\pi i}$$
$$= e^z \cdot (\cos \pi + i \sin \pi)$$
$$= e^z \cdot (-1 + i \cdot 0)$$
$$= -e^z$$

Therefore, L.H.S = R.H.S [Showed]

Ans To The Question No. [5(a)]

Given,

$$w^2 = z$$
$$\Rightarrow w = z^{\frac{1}{2}}$$
$$= \left(r \cdot e^{i(\theta + 2\pi n)}\right)^{\frac{1}{2}}$$
$$= r^{\frac{1}{2}} \cdot e^{\frac{i\theta}{2}} \cdot e^{i\pi n}$$

Now, if we start at z = 1 in the z-plane and make one complete circuit counter clockwise around the origin,

$$w_1 = \sqrt{r} \cdot e^{\frac{i\theta}{2}} \cdot e^{i\pi}$$
$$= \sqrt{r} \cdot e^{\frac{i\theta}{2}} \cdot (-1)$$
$$= -\sqrt{r} \cdot e^{\frac{i\theta}{2}} \quad (Ans)$$

Ans To The Question No. [5(b)]

From the previous question, we found that $w = r^{\frac{1}{2}} \cdot e^{\frac{i\theta}{2}} \cdot e^{i\pi n}$

Now,

The value of w after returning to $z = 1$ after 2 complete circuits about the origin is

$$\begin{aligned} w_2 &= \sqrt{r} \cdot e^{\frac{i\theta}{2}} \cdot e^{2i\pi} \\ &= \sqrt{r} \cdot e^{\frac{i\theta}{2}} \end{aligned}$$

The value of w after returning to $z = 1$ after 3 complete circuits about the origin is

$$\begin{aligned} w_3 &= \sqrt{r} \cdot e^{\frac{i\theta}{2}} \cdot e^{3i\pi} \\ &= -\sqrt{r} \cdot e^{\frac{i\theta}{2}} \end{aligned}$$

The value of w after returning to $z = 1$ after 4 complete circuits about the origin is

$$\begin{aligned} w_4 &= \sqrt{r} \cdot e^{\frac{i\theta}{2}} \cdot e^{4i\pi} \\ &= \sqrt{r} \cdot e^{\frac{i\theta}{2}} \end{aligned}$$

So, we can see a pattern that one time the result is positive and the next time it is negative and vice-versa. So, it can be said that as it is right now, i.e circuiting about the origin, this function is a multi-valued function.