1(a)

$$f(x) = x^3 - 6x^2 + 11x - 6$$

$$\therefore f'(x) = 3x^2 - 12x + 11$$

$$\therefore g(x) = x - \frac{f(x)}{f'(x)}$$

$$= x - \frac{x^3 - 6x^2 + 11x - 6}{3x^2 - 12x + 11}$$

$$= \frac{3x^3 - 12x^2 + 11x - x^3 + 6x^2 - 11x + 6}{3x^2 - 12x + 11}$$

$$= \frac{2x^3 - 6x^2 + 6}{3x^2 - 12x + 11}$$

$$= \frac{2(x^3 - 3x^2 + 3)}{3x^2 - 12x + 11}$$

(Ans)

1 (b)

$$g'(x) = \frac{f(x)\, f''(x)}{(f'(x))^2}$$

Now, $f''(x) = 6x - 12$

or, $f''(1) = -6 \quad [\because x_* = 1]$

Again, $f'(1) = 2 \quad [\because x_* = 1]$

and $f(1) = 0 \quad [x_* = 1]$

Now, $\lambda \equiv g'(x_*) = \dfrac{f(x)\, f''(x)}{(f'(x))^2}$

$$= g'(1) = \frac{0 \times (-6)}{(2)^2}$$

$$= 0$$

Since, $\lambda = 0$, the convergence rate for $x_* = 1$ is superlinear

Similarly,

$f(2) = 0$ | So, $g'(2) = 0$

and $f(3) = 0$ | and $g'(3) = 0$

Hence we can verify that $g(x)$ is superlinear.

1(c)

We know, $f'(x) = 3x^2 - 12x + 11$

The turning points are:

$3x^2 - 12x + 11 = 0$

$\Rightarrow x = 2 \pm \frac{1}{\sqrt{3}}$ [using quadratic eqn]

At these points, the iteration methods fails.

2(a)

Iteration formula: $x_{k+1} = x_k - \dfrac{f(x_k)}{f'(x_k)}$

$$= x_k - \frac{\cos(2x) - 4x\cos(x) + 2x^2 + 1}{-2\{\sin(2x) - 2x(\sin(x)+1) + 2\cos(x)\}}$$

k	x_k	$f(x_k)$	is $f(x_k) < 10^{-5}$
0	0.250000	1.0033670	No
1	0.538163	0.205426	No
2	0.644106	0.048380	No
3	0.692695	0.011807	No
4	0.716140	0.002919	No
5	0.727671	0.000726	No
6	0.733392	0.000181	No
7	0.736241	0.000045	No
8	0.737656	0.000011	No
9	0.738344	0.000003	YES

$\therefore f(x_9) \approx 0$.

2(b)

The new Aitken acceleration formula for iteration point:

$$\hat{x}_{k+2} = x_k - \frac{(x_{k+1} - x_k)^2}{x_{k+2} - (2x_{k+1}) + x_k}$$

k	x_k	$f(x_k)$	is $f(x_k) < 10^{-5}$
0	0.250000	1.033670	No
1	0.538163	0.205426	No
2	0.644106	0.048380	No
$\hat{2}$	0.705701	0.006151	No
3	0.722521	0.001526	No
4	0.730835	0.000380	No
$\hat{4}$	0.738961	8.632×10^{-8}	Yes

$\therefore f(x_{\hat{4}}) \approx 0$