

$$\begin{array}{l}
 32 \rightarrow 5 \rightarrow 2^5 \\
 16 \rightarrow 4 \rightarrow 2^4 \\
 8 \rightarrow 3 \rightarrow 2^3 \\
 4 \rightarrow 2 \rightarrow 2^2 \\
 2 \rightarrow 1 \rightarrow 2^1
 \end{array}$$

$O(\log_2 N)$

$$13 \rightarrow \log_2 13 = \cancel{\text{Boo}} = \underline{\underline{4}}$$

$$\begin{aligned}
 N &\rightarrow \text{steps} \\
 N &= 2^{\text{steps}} \\
 \log_2 N &= \log_2 2^{\text{steps}}
 \end{aligned}$$

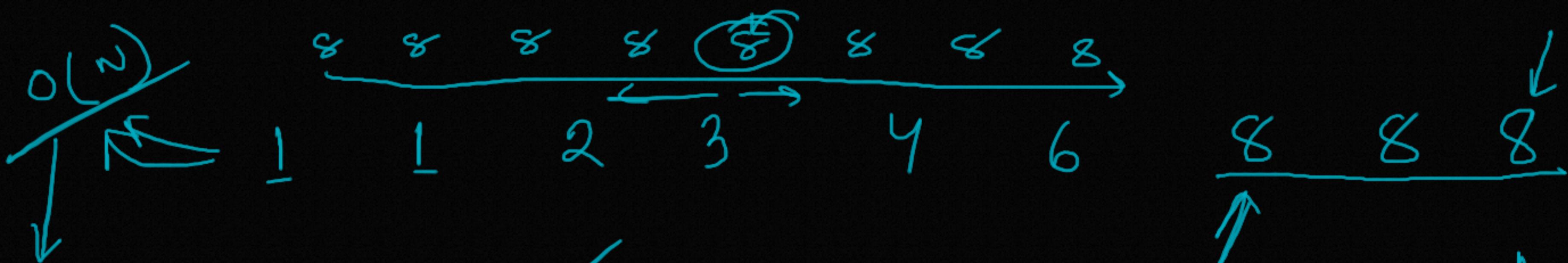
$$\text{steps} : \frac{\log_2 N}{\cancel{\cancel{2}}}$$

Value: 3 / 6

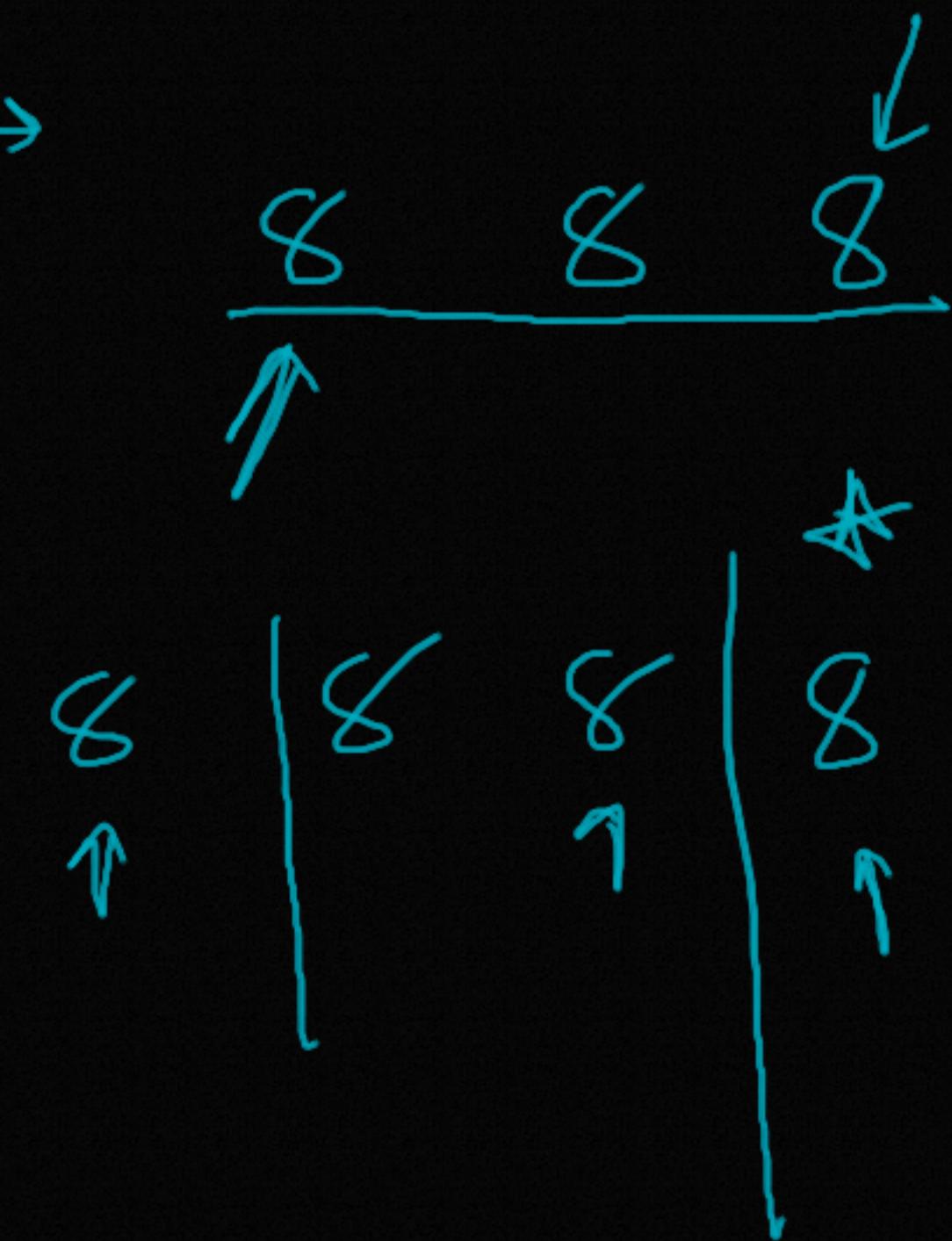
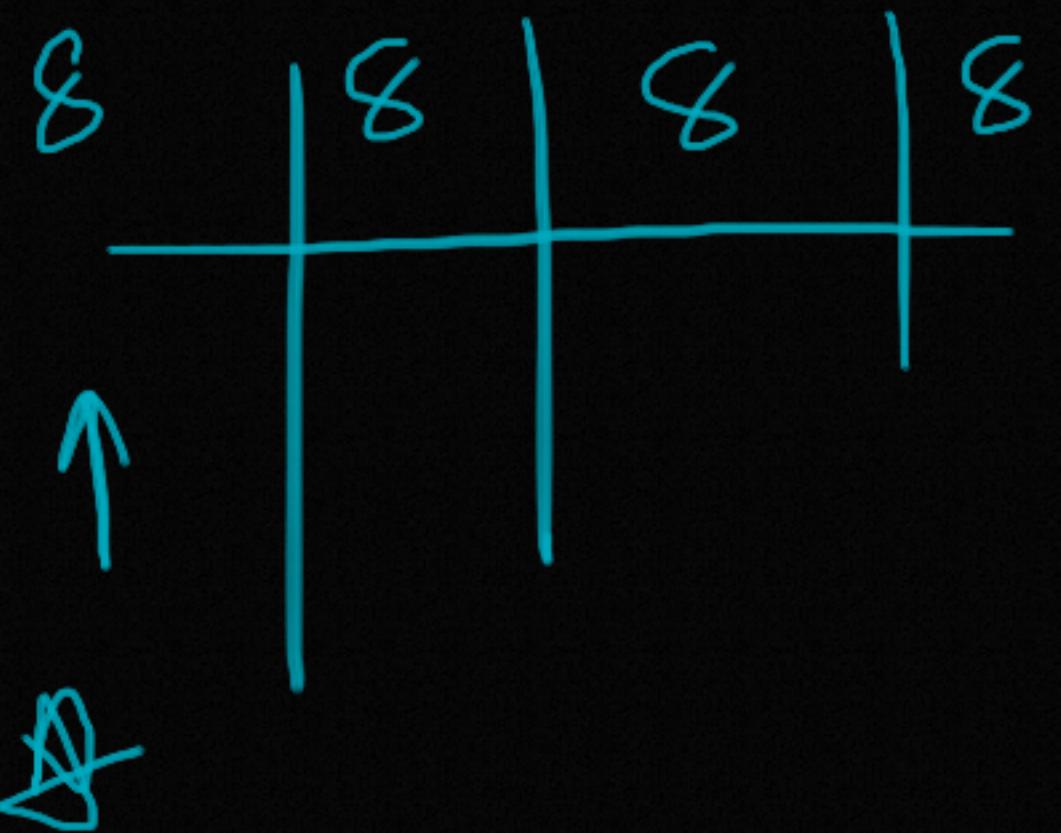
⇒ | 5 3 4 | | 2 8 7 |

↳ " 1 1 2 3 " ↓ ↓  
| 4 | 5 7 8  
X X ↓

8 →  $\log_2 8$   
63



Value: 8



$$1 - 8 + 1 = 8$$



## Topics for Today

### .Binary Search (variations)

- Lower Bound
- Upper Bound
- Binary Search on answer
- Bisection Method (to solve algebraic equations)

### .Ternary Search

- Basic Concept
- finding a key value from a sorted sequence
- Complexity - Solving equations

## Lower and Upper Bound

	0	1	2	3	4	5	6	7	8
A:	10	10	10	20	20	20	30	40	

→ Key: ~~20~~ 40

Lower Bound: <sup>position of</sup>  $\nwarrow$  Smallest (and leftmost) value  $\geqslant$  Key

Upper Bound: <sup>position of</sup>  $\nwarrow$  Smallest value  $>$  Key

## Binary Search on answer

Q: Given a sequence of numbers, we have to divide it into k parts such that summation of all numbers in every part is  $\leq N$ . Find the smallest value of N.

Say:  $10 \ 20 | 30 | 10 \ 15 ; 10$   
k: 3

$N = ?$  [  $10 \ 15 \ 20 \ 25 \ 30 \ 35 \ 40$   
-----  $80$  ]  
 $\rightarrow 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1$

$\Rightarrow 1 \ \downarrow \ \boxed{15} \ 10 \ 10^6$   
Satisfiability () {

## BS to solve polynomials [Bisection method]

$$f(x) = x^3 - x - 1 = 0, \quad x = ?$$

$$O(\log_2 n)$$

-11000

$$\frac{l}{2} = 1100$$

$$\boxed{50, 100, 200}$$
$$f(x) = 0.0000195$$

## Ternary Search

puzzle of identical balls



"=" → 2

"<" → 2

10 — 27 28

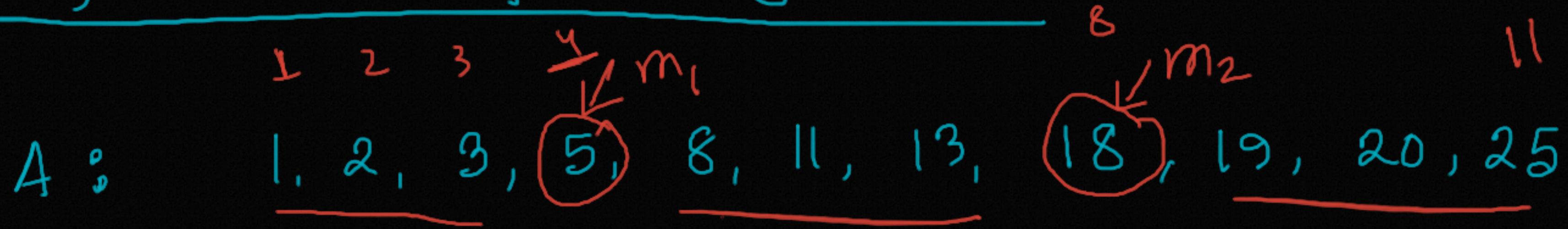
## Use of the Scale

7, 4, 3, 2  
✓ ✓ ↴ BS ?

2<sup>7</sup>  
↓  
9 ✓  
↓  
3 ✓  
↓  
1 ←

0 0 ✓  
—  
"="  
"<"

## Ternary Search to find key value



key = 19  $\geq$

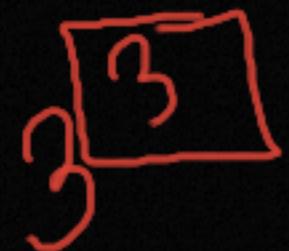
if ( $A[m_2] < \text{key}$ )  $\rightarrow$   $(m_2, \text{end}]$

if ( $A[m_1] > \text{key}$ )  $\rightarrow$   $[s_t, m_1)$

if ( $A[m_1] < \text{key} < A[m_2]$ )  $\rightarrow$   $(m_1, m_2)$

# Ternary Search (Complexity & Constraints)

$$N \rightarrow 27 \rightarrow 3$$



$$\downarrow$$
$$O(\log_3 N)$$

$$3^x = N$$

$$\log_3 3^x = \log_3 N$$

$$9 \rightarrow 2$$



# Solving Polynomials again!

$$y = x^2 - x - 1, \quad x = ?$$

