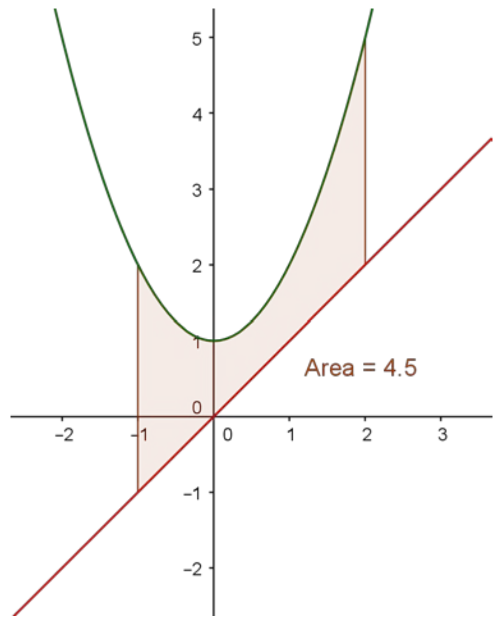
6.1 Area Between Two Curves
Solutions to the Selected Problems

Formula

$$\text{Area} = \int_a^b [f(x) - g(x)]\, dx$$

1–4. Find the area of the shaded region.

1. Solution



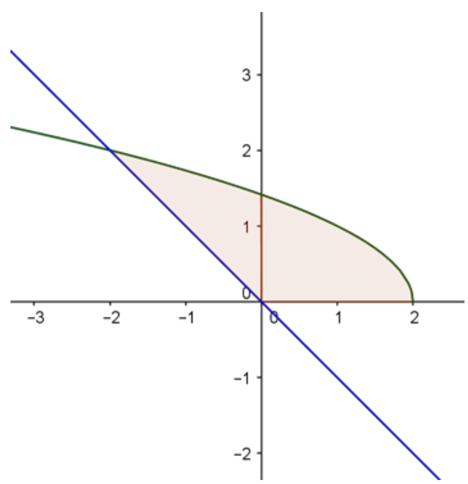
$$\text{Area} = \int_{-1}^{2} (x^2 + 1 - x)\, dx$$

$$= \left(\frac{x^3}{3} + x - \frac{x^2}{2}\right)\Bigg|_{-1}^{2}$$

$$= \left(\frac{8}{3} + 2 - \frac{4}{2}\right) - \left(-\frac{1}{3} - 1 - \frac{1}{2}\right)$$

$$= \frac{9}{2}$$

4. Solution



$$\text{Area} = \int_{-2}^{0} \left(\sqrt{2 - x} + x\right) dx + \int_{0}^{2} \left(\sqrt{2 - x}\right) dx$$

$$= \left(-\frac{2}{3}(2 - x)^{\frac{3}{2}} + \frac{x^2}{2}\right)\Bigg|_{-2}^{0} + \left(-\frac{2}{3}(2 - x)^{\frac{3}{2}}\right)\Bigg|_{0}^{2}$$

$$= \left(-\frac{2}{3}(2)^{\frac{3}{2}}\right) - \left(-\frac{2}{3}(4)^{\frac{3}{2}} + 2\right) + \left(0 + \frac{2}{3}(2)^{\frac{3}{2}}\right)$$

$$= -\frac{2}{3}(2)^{\frac{3}{2}} + \frac{2}{3}(4)^{\frac{3}{2}} - 2 + \frac{2}{3}(2)^{\frac{3}{2}}$$

$$= \frac{10}{3}$$

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Alternative

$$\text{Area} = \int_0^2 [(2 - y^2) - (-y)]\, dy$$

$$= \int_0^2 (2 - y^2 + y)\, dy$$

$$= \left(2y - \frac{y^3}{3} + \frac{y^2}{2}\right)\Bigg|_0^2$$

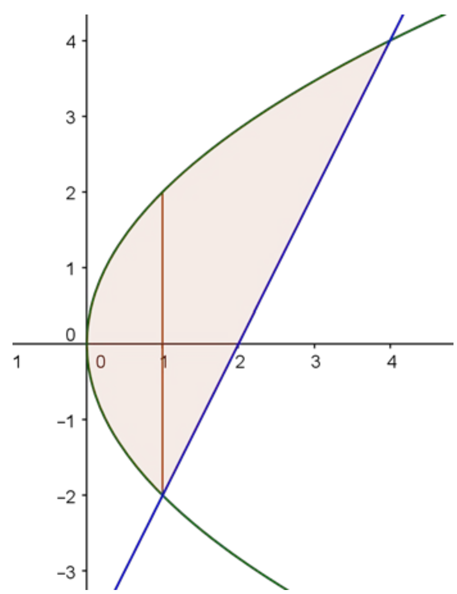
$$= \left(4 - \frac{8}{3} + \frac{4}{2}\right) - (0 - 0 + 0)$$

$$= \frac{10}{3}$$

5–6. Find the area of the shaded region by (a) integrating with respect to x and (b) integrating with respect to y.

6. Solution

(a)



$$\text{Area} = \int_0^1 \left[\sqrt{4x} - \left(-\sqrt{4x}\right)\right] dx + \int_1^4 \left[\sqrt{4x} - (2x - 4)\right] dx$$

$$= 9$$

(b)

$$\text{Area} = \int_{-2}^4 \left[\frac{1}{2}(y + 4) - \frac{y^2}{4}\right] dy$$

$$= 9$$

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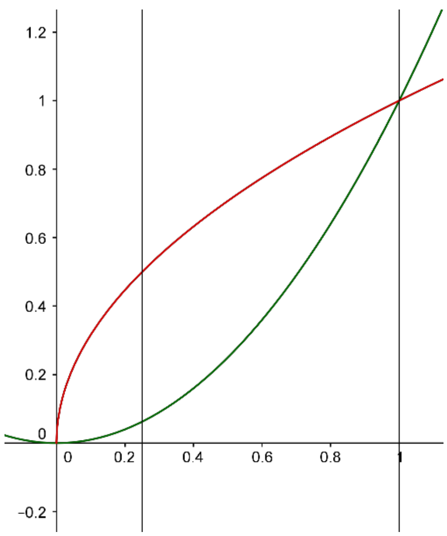
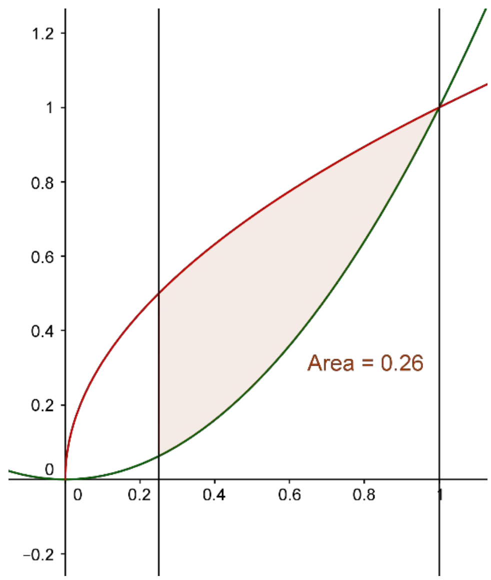
7–18. Sketch the region enclosed by the curves and find its area.

7. $y = x^2, \qquad y = \sqrt{x}, \qquad x = \frac{1}{4}, \qquad x = 1.$

Solution

Sketch





$$\int_{x=a}^{x=b} [f(x) - g(x)]\, dx = \int_{\frac{1}{4}}^{1} \left(\sqrt{x} - x^2\right) dx = \left(\frac{2}{3}x^{\frac{3}{2}} - \frac{x^3}{3}\right)\Bigg|_{\frac{1}{4}}^{1}$$

$$\boxed{\text{Area} = \frac{49}{192}}$$

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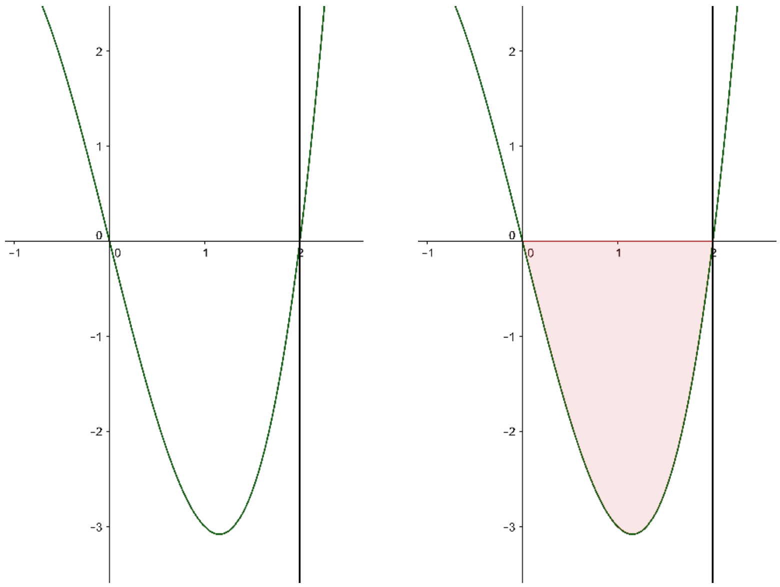
6.1 Area Between Two Curves
Solutions to the Selected Problems

8. $y = x^3 - 4x, \qquad y = 0, \qquad x = 0, \qquad x = 2.$

Solution

Sketch



$$\int_{x=a}^{x=b} [f(x) - g(x)]\, dx = \int_0^2 [0 - (x^3 - 4x)]\, dx =$$

$$\boxed{\text{Area} = 4}$$

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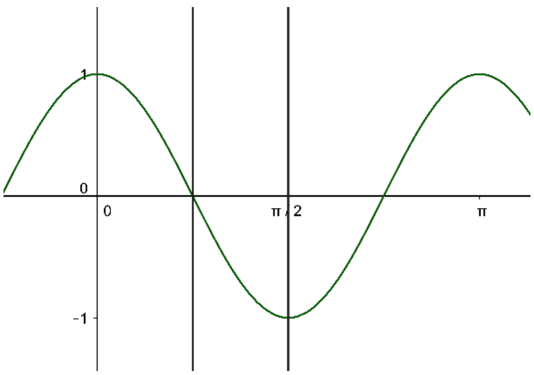
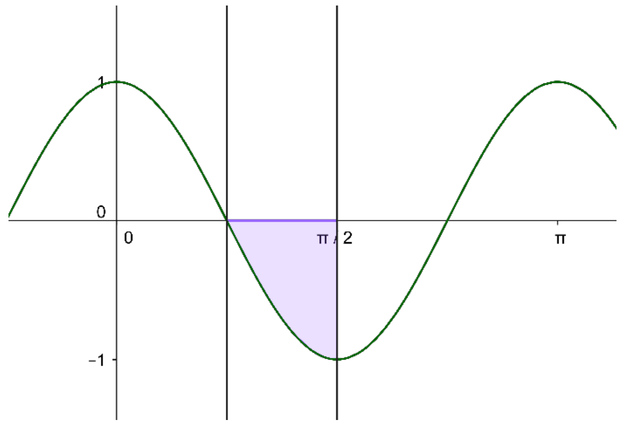
6.1 Area Between Two Curves
Solutions to the Selected Problems

9. $y = \cos 2x\,,\qquad y = 0,\qquad x = \frac{\pi}{4},\qquad x = \frac{\pi}{2}.$

Solution

Sketch





$$\int_{x=a}^{x=b} [f(x) - g(x)]\,dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} [0 - \cos 2x]\,dx$$

$$\boxed{\text{Area} = \frac{1}{2}}$$

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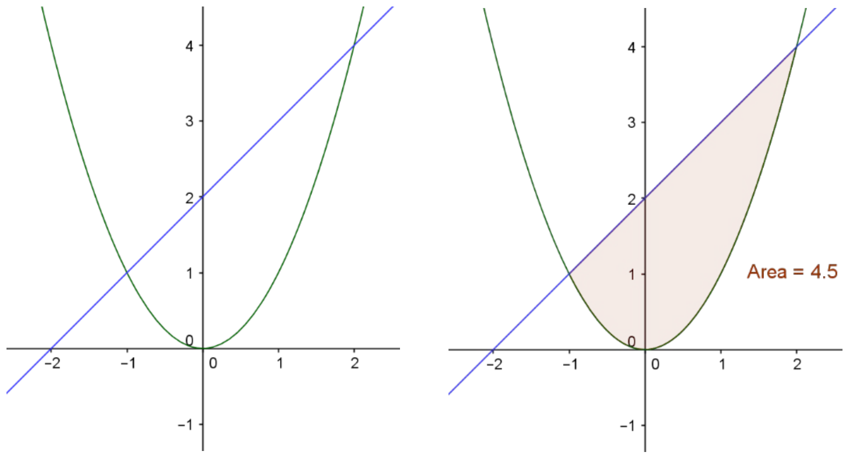
6.1 Area Between Two Curves
Solutions to the Selected Problems

12. $x^2 = y, \qquad x = y - 2.$

Solution

Sketch



$$\int_{x=a}^{x=b} [f(x) - g(x)]\, dx = \int_{-1}^{2} [(x + 2) - x^2]\, dx =$$

$$\boxed{\text{Area} = \frac{9}{2}}$$

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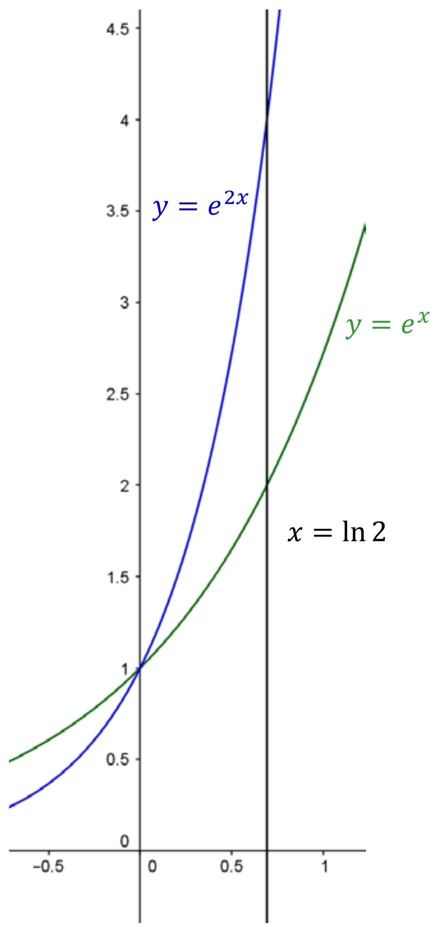
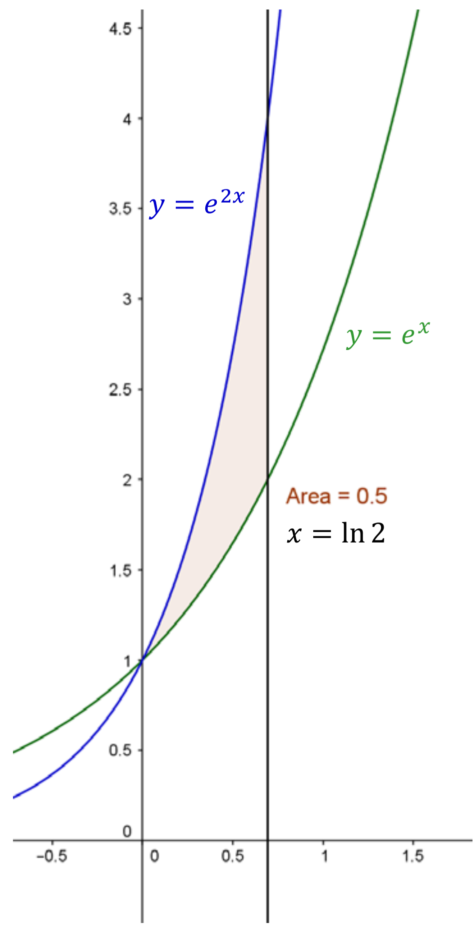
6.1 Area Between Two Curves
Solutions to the Selected Problems

13. $y = e^{x}$, $y = e^{2x}$, $x = 0$, $x = \ln 2$.

Solution

Sketch





$$\int_{x=a}^{x=b} [f(x) - g(x)]\, dx = \int_{0}^{\ln 2} [e^{2x} - e^{x}]\, dx =$$

$$\boxed{\text{Area} = \frac{1}{2}}$$

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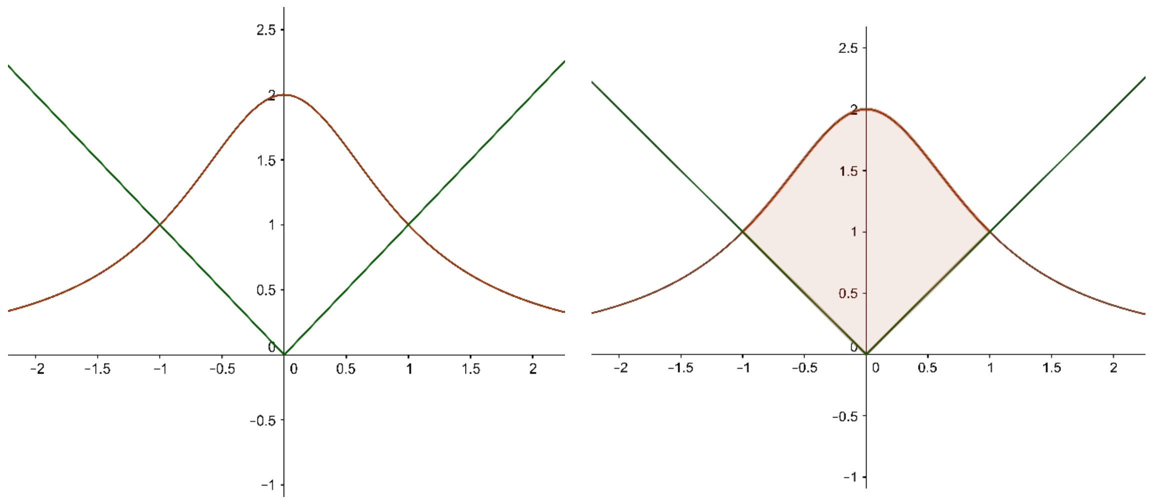
6.1 Area Between Two Curves
Solutions to the Selected Problems

15. $y = \dfrac{2}{1+x^2}, \qquad y = |x|.$

Solution

Sketch



$$\int_{-1}^{1}\left[\frac{2}{1+x^2}-|x|\right]dx = \int_{-1}^{0}\left[\frac{2}{1+x^2}-|x|\right]dx + \int_{0}^{1}\left[\frac{2}{1+x^2}-|x|\right]dx$$

$$\boxed{\text{Area} = \pi - 1}$$

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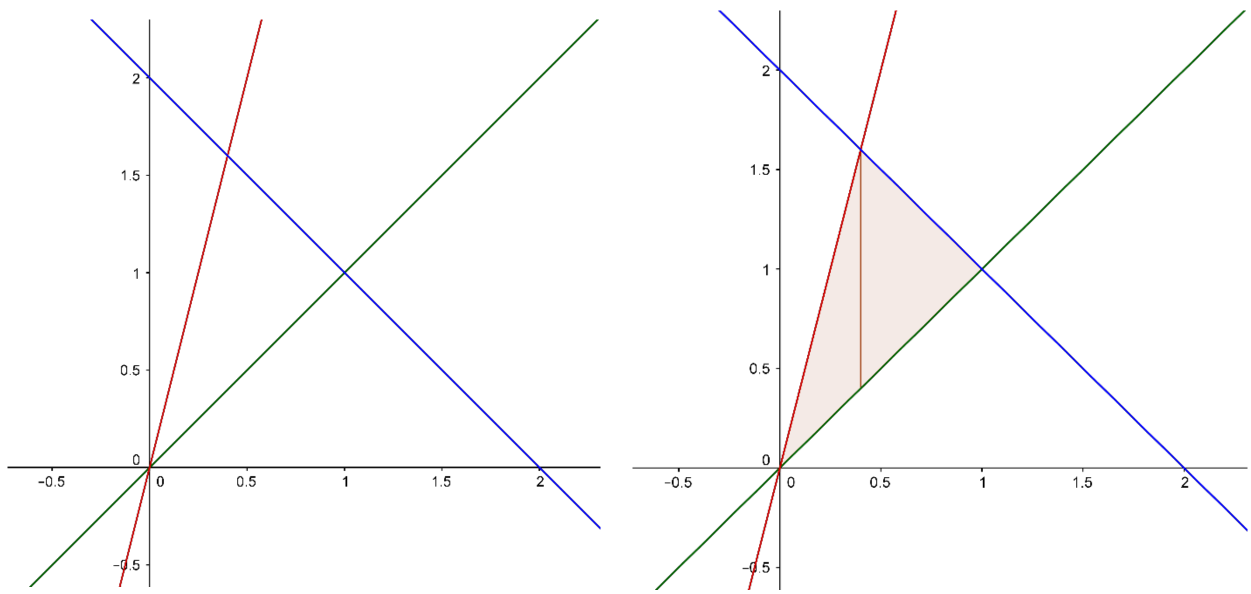
6.1 Area Between Two Curves

Solutions to the Selected Problems

18. $y = x, \qquad y = 4x, \qquad y = -x + 2.$

Solution

Sketch



$$\text{Area} = \int_0^{\frac{2}{5}} [4x - x]\,dx + \int_{\frac{2}{5}}^{1} [(-x+2) - x]\,dx =$$

$$\boxed{\text{Area} = \frac{3}{5}}$$

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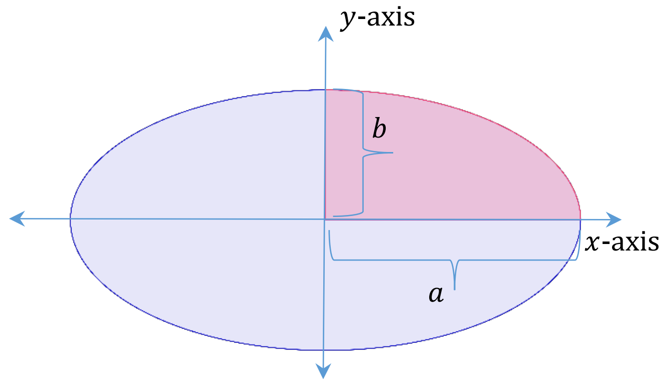
6.1 Area Between Two Curves
Solutions to the Selected Problems

50. Show that the area of the ellipse in the accompanying figure is πab.

Solution



Area of the quarter of the ellipse is given by

$$\frac{b}{a}\int_0^a \sqrt{a^2 - x^2}\,dx$$

The integral

$$\int_0^a \sqrt{a^2 - x^2}\,dx$$

represents the area of the quarter of a circle of radius a. Therefore,

$$\int_0^a \sqrt{a^2 - x^2}\,dx = \frac{1}{4}\pi a^2$$

Total area is given by

$$A = 4 \times \frac{b}{a} \times \frac{1}{4}\pi a^2 = \pi ab.$$

$$\boxed{A = \pi ab}.$$

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