Assessment - 5

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Ans to Q. No – 1

We know,

$$g(x_k) = x_k - \frac{f(x_k)}{f'(x_k)}$$

$x_k \to x_*$,

$f(x_k)\big|_{x_k \to x_*} \to 0$

So, now, $x_{k+1} = g(x_k) = x_k = x_*$

$\therefore$ The rate of convergence is

$$g'(x) = \frac{d}{dx}\left(x - \frac{f(x)}{f'(x)}\right)$$

$$= 1 - \frac{f'(x)f'(x) - f(x)f''(x)}{(f'(x))^2}$$

$$= \frac{f(x)\,f''(x)}{(f'(x))^2}$$

Now, $\lambda \equiv g'(x_*) = \left.\frac{f(x)\,f''(x)}{(f'(x))^2}\right|_{x=x_*} = \frac{f(x_*)f''(x_*)}{(f'(x_*))^2}$

$$= 0$$

As, $\lambda = 0$, the convergence is superlinear.

[Proved]

Ans. to Q.No - 2

a

Given, $f(x) = x^3 - 2x + 2$

$\therefore f'(x) = 3x^2 - 2$

We know, $x_{k+1} = x_k - \dfrac{f(x_k)}{f'(x_k)}$

k	x	f(x)	f'(x)
0	1	+1.00000	1.00000
1	0	+2.00000	-2.00000
2	1	+1.00000	1.00000
3	0	+2.00000	-2.00000

So, we can clearly see that ~~from~~ starting from initial point $x_0 = 1$, the Newton's method is stuck in an infinite loop.

b

Since Newton's Method requires $f'(x_k) \neq 0$, we have to be careful while choosing initial point x_0. We need to make sure that the turning point of the function $f(x)$ doesn't fall between successive iterated points because then $f'(x_k)$ and $f'(x_{k+1})$ will have opposite signs. As a result, instedd of converging to a fixed point or root, the iteration will fluctuate around turning point indefinitely.

In the above function, turning point is at

$\Rightarrow f'(x) = 3x^2 - 2 = 0$, or, $x = \pm\sqrt{\frac{2}{3}}$.

So, we can clearly see that the turning point $x = \pm 0.81649$ lies between $x_0 = 1.00000$ and $x_1 = 0$. Thus, resulting infinite loop.

Ans. to Q No-3

For converting a matrix to upper diagonal matrix, we need to do the following steps:

- Defining row multipliers.

$$m_{ik} = \frac{a_{ik}^{(k)}}{a_{kk}^{(k)}} \quad \text{where } i = k+1, k+2, \ldots\ldots n$$

here, the ~~subscript~~ superscript (k) is the k-th row operation and the subscripts ik and kk are the matrix element indices.

- Then we use these multipliers to eliminate the elements in entire k-th column below

- After performing all row operations, final upper diagonal matrix is achieved.

Now,

- for i-th multiplier, we need $(n-(k+1)+1)$ $= (n-k)$ number of divisions.

and for (ij)-th element, we need $(n-k)^2$ number of subtractions and $(n-k)^2$ number of multiplications,

So, total number of operations required:

$$N = \sum_{k=1}^{n-1} \left[2(n-k)^2 + (n-k) \right]$$

$$= n(2n+1) \sum_{k=1}^{n-1} 1 - (4n+1) \sum_{k=1}^{n-1} k + 2 \sum_{k=1}^{n-1} k^2$$

$$= \left\{ n(2n+1)(n-1) \right\} - \left\{ (4n+1) \frac{1}{2} (n-1) n \right\}$$
$$+ \left\{ 2 \left(\frac{1}{6}\right) n \left(n+1(2n+1)\right) \right\}$$

$$= \frac{2}{3} n^3 - \frac{1}{2} n^2 - \frac{1}{6} n$$

[Showed]

Ans. to Q No - 4

For finding Aitken's acceleration, we need to have previous two values of x i.e x_{k-1} and x_k. So, we need to complete minimum 2 iterations. Again, we can notice that $|\hat{x}_k - x_*|$ is significantly lower than $|x_k - x_*|$. So, there's no point in waiting for another iteration and that's why we don't do it in every 4th iteration.