Measure of Dispersion or Variation
Variance or Standard Deviation and Coefficient of variance:

The standard deviation may be defined as the root of the mean of squares of the deviation of individual items from the AM.

Calculation of population variance:
The formula for computing population variance of a set of data is given below:

Case 1 *(For ungroup data)***:**
If $X_1, X_2,...,X_N$ are N values of a population of size N, then the population variance commonly designated as σ^2, is defined as

$$\sigma^2 = \frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}, \text{ Where } \mu = Mean \text{ of the distribution}$$

Problem 1 *(Population ungroup data)***:**
Let a **population** of 10 students got the marks in the examination as given in the table below. Find the variance of the given data.

| 13 | 15 | 14 | 16 | 2 | 8 | 9 | 23 | 28 | 12 |

Solution 1 *(Population ungroup data***:**

X_i	$(X_i - \mu)$	$(X_i - \mu)^2$
13		
15		
14		
16		
2		
8		
9		
23		
28		
12		
		$\sum (x_i - \mu)^2 =?$

For the required solution please complete the following steps and table:
Step 1: First find the AM of the given value. Population AM, $\mu =?$?

Step 2: Then complete the following table:
Step 3: Here N= total number of observations= 10.

$$\textbf{Step 4: } \text{compute } \sigma^2 = \frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N} =??$$

Answer:
AM = 14
Variance = 49.2

Case 2 *(For group data)***:**

For grouped data if the values $X_1, X_2, ..., X_k$ occur with frequencies $f_1, f_2, ..., f_k$ respectively then the variance of the distribution will be

$$\sigma^2 = \frac{\sum_{i=1}^{k} f_i(X_i - \mu)^2}{\sum_{i=1}^{k} f_i} = \frac{\sum_{i=1}^{k} f_i(X_i - \mu)^2}{N}$$

Problem *(Population group data***:**

Let a **population** of 40 students got the marks in the examination as given in the table below. Find the variance of the given data.

X_i	15	20	25	30	35
f_i	6	8	15	7	4

Solution 2 *(Population group data***:**

X_i	f_i	$(X_i - \mu)$	$f_i(X_i - \mu)^2$
15	6		
20	8		
25	15		
30	7		
35	4		
	$\sum_{i=1}^{k=5} f_i =$		$\sum_{i=1}^{k} f_i(X_i - \mu)^2 = ?$

For the required solution please complete the following steps and table:

    Step 1: First find the AM of the given value. Population AM, $\mu = ? ?$

    Step 2: Then complete the attached table:

    Step 3: Here N = total number of observations= 40.

    Step 4: compute $\sigma^2 = \dfrac{\sum_{i=1}^{k} f_i(X_i - \mu)^2}{N} = ?$

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Answer:
AM = 24.37
Variance = 33.98 or 34

Calculation of sample variance:

The formula for computing variance of a set of sample observations is given below:

Case 3 *(Sample ungroup data***:**

If $X_1, X_2,...,X_n$ are n values of a sample of size n, then the sample variance commonly designated as s^2, is defined as

$$s^2 = \frac{\sum_{i=1}^{N}(X_i - \bar{x})^2}{n-1}$$, Where $\bar{x}$ = Sample mean of the distribution

Problem 3 *(Sample ungroup data***:**

Let a **sample** of 10 students got the marks in the examination as given in the table below. Find the variance of the given data.

| 13 | 15 | 14 | 16 | 2 | 8 | 9 | 23 | 28 | 12 |

Solution 3 *(Sample ungroup data***:**

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
13		
15		
14		
16		
2		
8		
9		
23		
28		
12		
		$\sum(x_i - \bar{x})^2 =?$

For the required solution please complete the following steps and table:

Step 1: First find the AM of the given value. Sample Mean AM, $\bar{x} = ?$?

Step 2: Then complete the following table:

Step 3: Here N= total number of observations= 10.

Step 4: compute $s^2 = \frac{\sum_{i=1}^{N}(X_i - \bar{x})^2}{n-1} = ???$

Answer:
AM = 14
Variance = 54.6

Case 4 *(Sample group data)***:**

For grouped data if the values $X_1, X_2, ..., X_k$ occur with frequencies $f_1, f_2, ..., f_k$ respectively then the variance of the distribution will be

$$s^2 = \frac{\sum\limits_{i=1}^{k} f_i(X_i - \bar{x})^2}{\sum\limits_{i=1}^{k} f_i - 1} = \frac{\sum\limits_{i=1}^{k} f_i(X_i - \bar{x})^2}{n-1}$$

Problem 4 *(Sample group data***:**

Let a *sample* of 40 students got the marks in the examination as given in the table below. Find the variance of the given data.

X_i	15	20	25	30	35
f_i	6	8	15	7	4

Solution 4 *(Sample ungroup data***:**

X_i	f_i	$(X_i - \bar{x})$	$f_i(X_i - \bar{x})^2$
15	6		
20	8		
25	15		
30	7		
35	4		
$\sum\limits_{i=1}^{k=5} f_i = ?$			$\sum\limits_{i=1}^{k=?} f_i(X_i - \bar{x})^2$

For the required solution please complete the following steps and table:

Step 1: First find the AM of the given value. Population AM, $\bar{x} = ?$?

Step 2: Then complete the attached table:

Step 3: Here N = total number of observations= 40.

Step 4: compute $\sigma^2 = \dfrac{\sum\limits_{i=1}^{k} f_i(X_i - \bar{x})^2}{n-1} = ?$

Answer:
AM = Find your self
Variance = Find yourself

Standard deviation:

The standard deviation of a given data is obtained by taking the square root of the corresponding variance value.

That is standard deviation of the variable X, $SD(X) = \sqrt{VAR(X)}$

The coefficient of dispersion corresponding to variance is known as coefficient of variation (CV) and is obtained by dividing standard deviation by the AM.

$$\text{That is coefficient of variation}(CV) = \frac{SD(X)}{AM(X)}$$

NOTE: For computational convenience we will use the following formulae

	Ungroup data	Group data	
Population	$\sigma^2 = \frac{1}{N}\left[\sum_{i=1}^{N} x_i^2 - \frac{\left(\sum_{i=1}^{N} x_i\right)^2}{N}\right]$	$\sigma^2 = \frac{1}{N}\left[\sum_{i=1}^{k} f_i x_i^2 - \frac{\left(\sum_{i=1}^{k} f_1 x_1\right)^2}{N}\right]$	**Population**
Sample	$s^2 = \frac{1}{n-1}\left[\sum_{i=1}^{N} x_i^2 - \frac{\left(\sum_{i=1} x_i\right)^2}{n}\right]$	$s^2 = \frac{1}{n-1}\left[\sum_{i=1}^{k} f_i x_i^2 - \frac{\left(\sum_{i=1}^{k} f_1 x_1\right)^2}{n}\right]$	**Sample**
	Ungroup data	Group data	

Test your self
Variance and Standard Deviation

1. An Advertising company is looking for a group of extras to shoot a sequence for a movie. The ages of the first 20 candidates to be interviewed are

50	56	44	49	52	57	56	57	56	59
54	55	61	60	51	59	62	52	54	49

The director of the movie wants men whose ages are tightly grouped around 55 years. Being a statistics buff of sorts, the director suggests that a standard deviation of 3 years would be acceptable. Does this group of extras qualify?

Hints: Calculate AM. Variance and take the Square root to get the SD. Compare the standard deviation with 3 as given in the problem. Hence make the decision

Calculation of combined Standard deviation:

The combined standard deviation of two sets of data containing n_1 and n_2 observaitons with means $\bar{x}_1$ and $\bar{x}_2$ and standard deviations σ_1 and σ_2 respectively is given by

$$\sigma_{12} = \sqrt{\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}}$$

Where,

σ_{12} = combined standard deviation
$$d_1 = \bar{x}_{12} - \bar{x}_1$$
$$d_2 = \bar{x}_{12} - \bar{x}_2$$

And $\bar{x}_{12} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

This formula combined standard deviation of two sets of data can be extended to compute the standard deviation of more than two sets of data on the same lines

Test your self
Computation of Combined Standard deviation

1. From the analysis of monthly wages paid to employees in two service organizations X and Y, the following results were obtained:

	Organization X	Organization Y
Number of wage-earners	550	650
Average monthly wages	5000	4500
Variance of the distribution of wages	900	1600

   a. Which organization pays a larger amount as monthly wages?
   b. Determine the combined variance of all the employees taken together?

2. For a group of 50 male workers, the mean and standard deviation of their monthly wages are tk. 6300 and tk. 600 respectively. For a group of 40 female workers, these are tk. 5400 and tk. 600 respectively. Find the standard deviation of monthly wages for the combined group of workers.

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