6.2 Volume by Slicing; Disks and Washers
Solutions to the Selected Problems

Formula

11–18. Find the volume of the solid that results when the region enclosed by the given curves is revolved about the x-axis.

11. $y = \sqrt{25 - x^2},\ y = 3$

Solution

$$V = \pi \int_{-4}^{4} \left[\left(\sqrt{25 - x^2} \right)^2 - 3^2 \right] dx$$

$$= \pi \int_{-4}^{4} (16 - x^2)\, dx$$

$$= \pi \left(16x - \frac{x^3}{3} \right) \Bigg|_{-4}^{4} = \pi \left[\left(64 - \frac{64}{3} \right) - \left(-64 + \frac{64}{3} \right) \right] = \frac{256}{3} \pi$$

12. $y = 9 - x^2,\ y = 0$

Solution

$$V = \pi \int_{-3}^{3} (9 - x^2)^2\, dx$$

$$= \pi \int_{-3}^{3} (81 - 18x^2 + x^4)\, dx$$

$$= \pi \left(81x - 6x^3 + \frac{x^5}{5} \right) \Bigg|_{-3}^{3} = \frac{1296}{5} \pi$$

13. $x = \sqrt{y},\ x = y/4$

Solution

$$V = \pi \int_{0}^{4} [(4x)^2 - (x^2)^2]\, dx$$

$$= \pi \int_{0}^{4} (16x^2 - x^4)\, dx$$

$$= \pi \left(\frac{16}{3} x^3 - \frac{1}{5} x^5 \right) \Bigg|_{0}^{4} = \frac{2048}{15} \pi$$

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14. $y = \sin x\,,\ \ y = \cos x\,, x = 0, x = \pi/4$

Solution

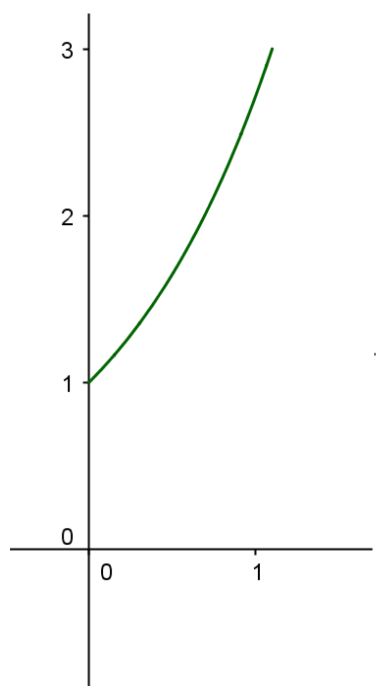
$$V = \pi \int_0^{\frac{\pi}{4}} [(\cos x)^2 - (\sin x)^2]\, dx$$

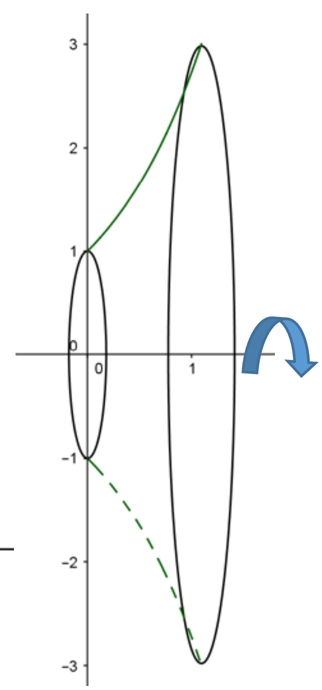
$$= \frac{\pi}{2}$$

15. $y = e^x,\ \ y = 0, x = 0, x = \ln 3$

Solution

$$V = \pi \int_0^{\ln 3} (e^x)^2\, dx$$





$$= 4\pi$$

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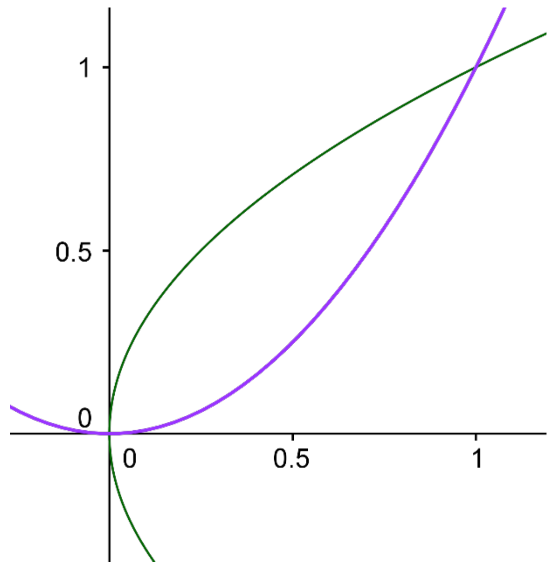
21–26. Find the volume of the solid that results when the region enclosed by the given curves is revolved about the y-axis.

22. $y = x^2,\ x = y^2$

Solution

$$V = \pi \int_0^1 \left[\left(\sqrt{y}\right)^2 - (y^2)^2\right] dy$$

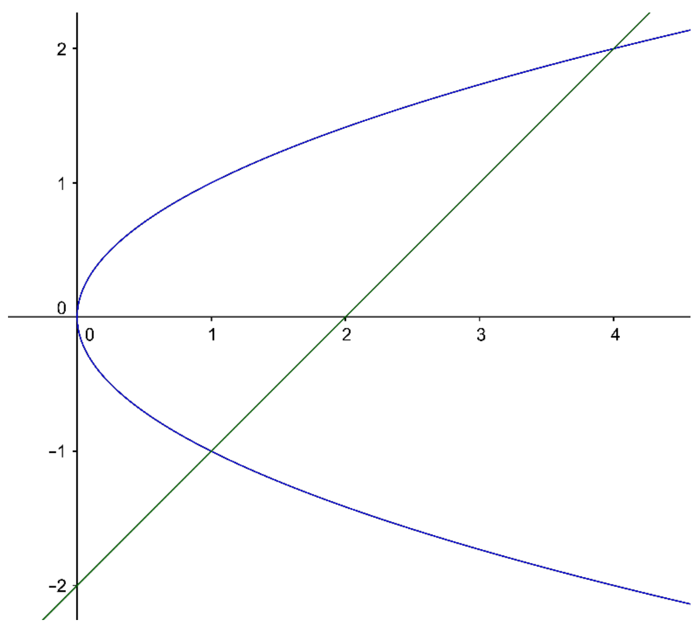


$$= \frac{3}{10}\pi$$

23. $x = y^2,\ x = y + 2$

Solution

$$V = \pi \int_{-1}^{2} [(y+2)^2 - (y^2)^2]\, dy$$



$$= \frac{92}{15}\pi$$

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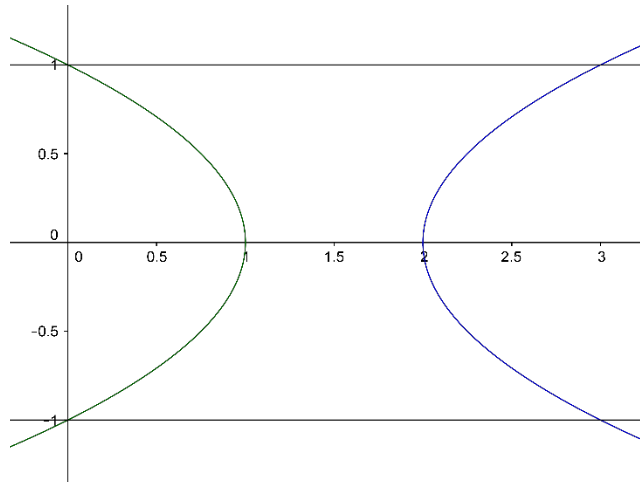
24. $x = 1 - y^2,\ x = 2 + y^2, y = -1, y = 1$

Solution

$$V = \pi \int_{-1}^{1} [(2 + y^2)^2 - (1 - y^2)^2]\, dy$$



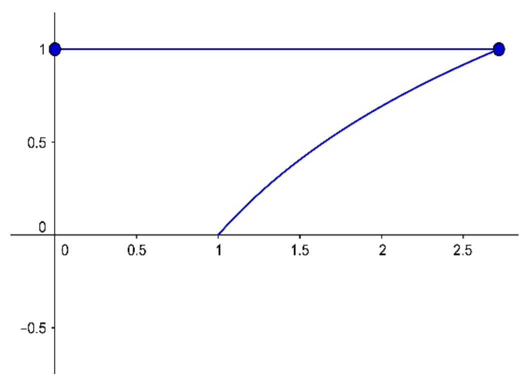
$= 10\pi$

25. $y = \ln x\,, x = 0, y = 0, y = 1$

Solution

$$V = \pi \int_{0}^{1} (e^y)^2\, dy$$

$$= \frac{\pi}{2}(e^2 - 1)$$



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31. Find the volume of the solid that results when the region above the x-axis and below the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad (a>0, b>0)$$

is revolved around the x-axis.

Solution

$$V=\pi\int_{-a}^{a}\left[b\sqrt{1-\frac{x^2}{a^2}}\right]^2 dx$$

$$=b^2\pi\int_{-a}^{a}\left(1-\frac{x^2}{a^2}\right)dx$$

$$=b^2\pi\times\left(x-\frac{x^3}{3a^2}\right)\Bigg|_{-a}^{a}$$

$$=b^2\pi\times\left[\left(a-\frac{a^3}{3a^2}\right)-\left(-a+\frac{a^3}{3a^2}\right)\right]$$

$$=b^2\pi\times\frac{4a}{3}$$

$$\boxed{V=\frac{4}{3}\pi ab^2}$$

33. Find the volume of the solid generated when the region enclosed by

$$y=\sqrt{x+1}, y=\sqrt{2x}, \text{and } y=0$$

is revolved about the x-axis.

Solution

$$V_1=\pi\int_{-1}^{0}\left[\sqrt{x+1}\right]^2 dx=\pi\int_{-1}^{0}(x+1)\ dx=\frac{\pi}{2}$$

$$V_2=\pi\int_{0}^{1}\left[\left(\sqrt{x+1}\right)^2-\left(\sqrt{2x}\right)^2\right]dx=\pi\int_{0}^{1}(1-x)\,dx=\frac{\pi}{2}$$

$$\boxed{V=V_1+V_2=\pi}$$

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Alternative

$$V = \pi \int_{-1}^{1} \left(\sqrt{x+1}\right)^2 dx - \pi \int_{0}^{1} \left(\sqrt{2x}\right)^2 dx = \pi \int_{-1}^{1} (x+1)\, dx - \pi \int_{0}^{1} 2x\, dx = \pi$$

34. Find the volume of the solid generated when the region enclosed by

$$y = \sqrt{x}, y = 6 - x, \text{and } y = 0$$

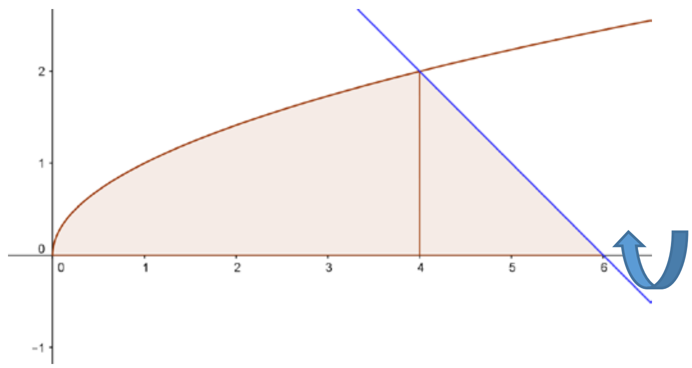
is revolved about the x-axis.

Solution

$$V_1 = \pi \int_{0}^{4} \left(\sqrt{x}\right)^2 dx = 8\pi$$

$$V_2 = \pi \int_{4}^{6} (6-x)^2\, dx$$

$$= \frac{8\pi}{3}$$



$$\boxed{V = 8\pi + \frac{8\pi}{3} = \frac{32\pi}{3}}$$

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