Geometric mean:

The geometric mean of $x_1, x_2, \ldots, x_n$ - n positive quantities is the n^{th} root of their product. i.e. for $x_1, x_2, \ldots, x_n - n$ nonzero positive observation the geometric mean GM will be

$$GM = \sqrt[n]{(x_1 * x_2 * \ldots * x_n)} = (x_1 * x_2 * \ldots * x_n)^{\frac{1}{n}}$$

Or

$$GM = Antilog\frac{\sum logx_i}{n}$$

For group data,

If $x_1, x_2, \ldots, x_k$ - k non zero positive quantities with corresponding frequencies $f_1, f_2, \ldots, f_k (where\ \sum_{i=1}^{k} f_i = n)$, then the geometric mean GM will be will be calculated as follows

$$GM = Antilog\frac{\sum_{i=1}^{k} f_i logx_i}{n}$$

When to use Geometric mean

Geometric mean is usually used for dealing with data related to growth rates (like population growth etc.) or interest rates, index number etc.

Test yourself
Geometric Mean

1. Find the Geometric mean of 45, 60, 48, 100, 65

Answer: 60.95

2. Find the geometric mean of the following data

x_i	50	63	65	130	135
f_i	5	10	5	15	15

Answer: 96.43

Hints:

Problem 1.

x_i	$logX$
45	1.65
60	
48	
100	
65	1.81
Total	8.92

$$GM = Antilog\frac{\sum logx_i}{n} = 60.95$$

Problem 2.

x_i	f_i	$logX$	$f_i * logx_i$
50	5	1.69	8.49
63	10		
65	5		
130	15		
135	15	2.13	31.95
Total	50		99.21

$$GM = Antilog\frac{\sum_{i=1}^{k} f_i logx_i}{n} = 96.43$$

Harmonic mean:

If $x_1, x_2, \ldots, x_n$ - n values of a series of data then the harmonic mean of the given data will be the reciprocal[1] of the arithmetic mean of the reciprocal of the given value.

i.e. for $x_1, x_2, \ldots, x_n\ -n$ nonzero observations the harmonic mean HM will be

$$HM = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots \ldots + \frac{1}{x_n}}$$

For group data,

If $x_1, x_2, \ldots, x_k$ - k non zero quantities with corresponding frequencies $f_1, f_2, \ldots, f_k (where\ \sum_{i=1}^{k} f_i = n)$, then the harmonic mean HM will be will be calculated as follows

$$HM = \frac{n}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \cdots \ldots + \frac{f_k}{x_k}}$$

When to use Harmonic mean

Harmonic means are often **used** in averaging things like rates (e.g., the **average** travel speed given a duration of several trips). The weighted **harmonic mean** is **used** in finance to **average** multiples like the price-earnings ratio because it gives equal weight to each data point.

Test yourself
Harmonic Mean

1. Find the Harmonic mean of 45, 60, 48, 100, 65

Answer: 58.74

2. Find the Harmonic mean of the following data

x_i	50	63	65	130	135
f_i	5	10	5	15	15

Answer: 89.28

Hints:

Problem 1.

x_i	$\frac{1}{X}$
45	0.022
60	0.017
48	0.021
100	0.010
65	0.015
Total	8.92

$$HM = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots \ldots + \frac{1}{x_n}} = \frac{5}{0.085} = 58.74$$

Problem 2.

x_i	f_i	$\frac{1}{X}$	f*(1/X)
50	5	0.02	0.10
63	10	0.015873	0.16
65	5	0.015385	0.08
130	15	0.007692	0.12
135	15	0.007407	0.11
Total	50		0.46

$$HM = \frac{n}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \cdots \ldots + \frac{f_k}{x_k}} = \frac{50}{0.56} = 89.28$$

[1] The reciprocal of any value X is $\frac{1}{X}$.

The Weighted Mean:
The weighted mean is a special case of the arithmetic mean. It occurs when there are several observations of the same value.

A **weighted mean** is a kind of **average**. Instead of each data point contributing equally to the final **mean**, some data points contribute more "weight" than others. If all the weights are equal, then the **weighted mean** equals the arithmetic **mean**.

To explain: Suppose the ***Shumi's Hot Cake*** offers three different kinds of burger packages small, medium and large for Tk. 100, Tk. 125 and Tk. 150. Of the last 10 burgers sold 3 were small, 4 were medium and 3 were large. To find the mean price of the last 10 burger packages sold we can calculate using the usual formula of the arithmetic mean as follows –

$$\bar{X} = \frac{Tk.(100+100+100+125+125+125+125+150+150+150)}{10} = \frac{Tk.1250}{10} = Tk.125$$

The mean selling price of the last 10 burger packages sold is Tk. 125.

An easier way to find the mean selling price is to determine the weighted mean. In this method we multiply each observation by the number of times it happens as described below –

$$\bar{X}_w = \frac{(3*100)+(4*125)+(3*150)}{3+4+3} = \frac{1250}{10} = 125$$

In this case the weights are ***frequency counts***. However, any measure of importance could be used as a weight. In general, the weighted mean of a set of numbers designated $X_1, X_2, \ldots, X_n$ with the corresponding weights $W_1, W_2, \ldots, W_n$ is computed by:

$$\bar{X}_w = \frac{\sum(WX)}{\sum W} = \frac{W_1X_1 + W_2X_2 + \ldots + W_nX_n}{W_1 + W_2 + \ldots + W_n}$$

Example:
Madina Construction Company pays its part time employees hourly basis. For different level of employee, the hourly rate are Tk. 50, Tk. 75 and Tk. 90. There are 260 hourly employees, 140 of which are paid at Tk. 50 rate, 100 at Tk. 75 and 20 at the Tk. 90 rate. What is the mean hourly rate paid to the employees?

Answer:
To find the mean hourly rate, we multiply each of the hourly rates by the number of employees earning that rate as follows -

$$\bar{X}_w = \frac{\sum(WX)}{\sum W} = \frac{140*50+100*75+20*90}{140+100+20} = \frac{16300}{260} = Tk.\ 62.69\,.$$

The weighted mean hourly wage is Tk. 62.69 or Tk. 63.00 (approximately).

For more detail about weighted mean watch the following tutorial

1. **https://www.youtube.com/watch?v=LdrBNhWw9AM**

Test yourself
Weighted mean

1. The postal service handles seven basic types of letters and cards: 3rd class, 2nd class, 1st class, airmail, special delivery, registered and certified. The mail volume during a given year is given in the following table

Types of mailing	gm delivered (in millions)	Price per gm
1st class	77600	0.13
AIR mail	19000	0.17
Special delivery	1300	0.35
Registered mail	750	0.40
Certified mail	800	0.45

What was the average revenue per gm for these services during the year?

Answer: $0.14 per gram

2. WESTECS sold 95 Executive Men's Suits for the regular price of TK. 4,900. For the summer sale the suits were reduced to Tk. 3,500 and 126 were sold. At the final year end clearance, the price was further reduced to Tk. 2,500 and the remaining 79 suits were sold.
   i. What was the weighted mean price of a WESTECS suit?
   ii. WESTECS paid Tk. 2000 a suit for the 300 suits. Comment on the store's profit per suit if a salesperson received a Tk. 150 for each one sold.

Answer: tk. 3680 and tk.1530

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Work out
Weighted Mean

Problem 1:

Types of mailing	gm delivered **(in millions)** **W**	Price **($ per gm)** **X**	**W*X**
1st class	77600	0.13	10088
AIR mail	19000	0.17	
Special delivery	1300	0.35	
Registered mail	750	0.4	
Certified mail	800	0.45	
Total	99450		14433

Weighted Mean= 0.145128 $ per gm

Problem 2:

Types suits	Item sold (in millions)	Price per Suit	
	W	X	W*X
Item 1	95	4900	465500
Item 2	126	3500	
Item 3	79	2500	
Total	300		1104000

Weighted Mean= 3680 tk per suit

Per suit expenditure=tk (2000+150) = tk 2150

Profit per suit =Tk (3680 – 2150) = tk 1530

Some other measures of Central tendency
Quartiles, Deciles and Percentiles

Quartile:
If the items in a series are arranged in ascending order of their magnitudes then those values of the variable that ***divide the total frequency in to four equal parts*** are called ***quartiles***.

There are three quartiles denoted by Q_1, Q_2 and Q_3. The second quartile (Q_2) coincides with the median. The lower quartile (Q_1) is the point such that one fourth of the total frequency is less than Q_1 and three forth is greater than Q_1.

Problem:
For the following data compute the three quartiles.

99	75	84	33	45	66	97	69	55	61
72	91	74	93	54	76	62	91	77	68

Answer:
Arrange the data

33	45	54	55	61	62	66	68	69	72
74	75	76	77	84	91	91	93	97	99

Hints:

First find the median (Q_2)

Median (Q_2) = AM of the values of $(\frac{n}{2})^{th}$ and $(\frac{n}{2}+1)^{th}$

= AM of the values of 10^{th} and 11^{th} observation

$= \frac{72+74}{2} = 73$

1^{st} quartile Q_1 = median of the 1^{st} half of observations =? ? ?

3^{rd} quartile Q_3 = median of the 2^{nd} half of observations =? ? ?

Thus,

- ***Median*** is the quantity of the variable that ***divides the into 2 equal halves.***
- ***Quartiles (Denoted by Q_1, Q_2 and Q_3)*** are the quantities of the variable that ***divides the total frequencies in to 4 equal parts.***

Similarly

- ***Deciles (Denoted by $D_1, D_2, \ldots, \ldots, \ldots, D_9$)*** are the quantities of the variable that ***divides the total frequencies into 10 equal parts.***

And

- ***Percentiles (Denoted by $P_1, P_2, \ldots, \ldots, \ldots, P_{99}$)*** are the quantities of the variable that ***divides the total frequencies in to 100 equal parts.***

Note that,

Median = $Q_2 = D_5 = P_{50}$ >> All these quantities divide the total frequencies in to two equal halves.

- **Merits and demerits of different measure of central tendency -**

	Merits	Demerits	
Arithmetic mean	1. Rigidly defined. 2. Easy to understand and calculate. 3. Based upon all observation. 4. Most amenable to algebraic treatment. 5. Not based on position in the series.	1. Cannot be defined graphically. 2. Cannot be used in case of qualitative data. 3. Affected very much by extreme values. 4. May not occur in the series. 5. Difficult to calculate in the case of the data with open-end class.	Arithmetic mean
Median	1. Rigidly defined. 2. Easy to understand and calculate. 3. Not affected very much by extreme values. 4. Can be calculated in the case of the data with open-end class. 5. Can be defined graphically.	1. In case of even number of observations, it is not defined exactly. 2. Not based on all observations. 3. Not easy for algebraic treatment. 4. For calculating median, it is necessary to arrange the data either ascending or descending order.	Median
Mode	1. Most typical and representative value of a distribution. 2. Not at all affected by extreme values. 3. Can be calculated in the case of the data with open-end class. 4. Easy to understand and calculate. 5. Can be defined graphically.	1. Not clearly defined in case of bimodal or multi modal distribution. 2. Not based on all observation. 3. Not suitable for further algebraic treatment. 4. Affected by sampling fluctuations.	Mode
Geometric Mean	1. It is rigidly defined. 2. It is based on all the observations of the series. 3. It is suitable for measuring the relative changes. 4. It gives more weights to the small values and less weights to the large values. 5. It is used in averaging the ratios, percentages and in determining the rate gradual increase and decrease. 6. It is capable of further algebraic treatment.	1. It is not easy to understand by a man of ordinary prudence as it involves logarithmic operations. As such it is not popular like that of arithmetic average. 2. It is difficult to calculate as it involves finding out of the root of the products of certain values either directly, or through logarithmic operations. 3. It cannot be calculated, if the number of negative values is odd. 4. It cannot be calculated, if any value of a series is zero.	Geometric Mean
Harmonic mean	1. It is rigidly defined. 2. It is defined on all observations. 3. It is amenable to further algebraic treatment. 4. It is the most suitable average when it is desired to give greater weight to smaller observations and less weight to the larger ones	1. It is not easily understood. 2. It is difficult to compute. 3. It is only a summary figure and may not be the actual item in the series. 4. It gives greater importance to small items and is therefore, useful only when small items have to be given greater weightage. 5. It is rarely used in grouped data	Harmonic mean

Test yourself

1. The middle value of an ordered series is called
   A. 2nd quartile
   B. 5th decile
   C. 50th percentile
   D. all the above

   Ans: All the above

2. For a set of values, the model value can be
   A. Uni-model
   B. Bimodal
   C. Tri-model
   D. All of these

   Ans: All the above

3. Mode is suitable for qualitative data. ***True / False***

   Ans: True

4. Decile divides the group in to ten equal parts. ***True / False***

   Ans: True

5. Mean is affected by extreme values. ***True / False***

   Ans: True

6. Geometric mean can be calculated for negative values. ***True / False***

   Ans: False

7. Define mean and median
8. For what type of data mode can be calculated.
9. Explain how to calculate the arithmetic mean for raw and grouped data.
10. Explain how to calculate median and mode for grouped data.

For any queries related to this presentation please contact
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