Problem-1

The minimum dimension of the vector space is 4.

The basis elements are :

$\{1, x, x^2, x^3\}$

Problem-2

We have to use Taylor's theoream when only one node and it's function is given to us.

And we have to use Cauchy's theoream when multiple nodes and their functions are given.

Problem-3

$$P_1(x) = \frac{x_1 f(x_0) - x_0 f(x_1)}{x_1 - x_0} + \frac{f(x_1) - f(x_0)}{x_1 - x_0} x$$

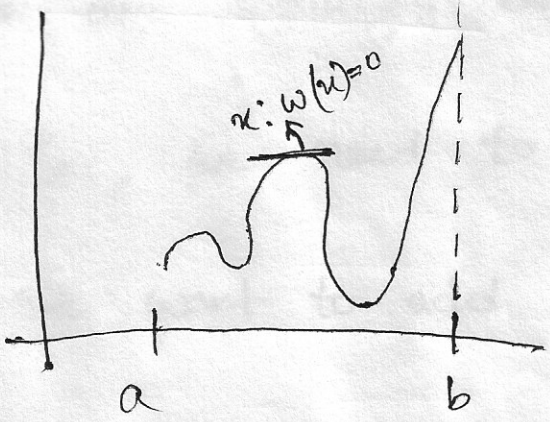
$$= \frac{x_1}{x_1 - x_0} f(x_0) - \frac{x_0}{x_1 - x_0} f(x_1) + \frac{x}{x_1 - x_0} f(x_1) - \frac{x}{x_1 - x_0} f(x_0)$$

$$= f(x_0)\left(\frac{x_1 - x}{x_1 - x_0}\right) + f(x_1)\left(\frac{x - x_0}{x_1 - x_0}\right)$$

$$= f(x_0)\underbrace{\left(\frac{x - x_1}{x_0 - x_1}\right)}_{l_0(x)} + f(x_1)\underbrace{\left(\frac{x - x_0}{x_1 - x_0}\right)}_{l_1(x)}$$

$$= f(x_0) \cdot l_0(x) + f(x_1) \cdot l_1(x)$$

Problem-4



To calculate the maxima of a function, only taking derivative of the function equal to 0 is not enough. Because, the function can be maximum at the boundary points but may not come as a root, only by taking derivative of function = 0. That's why we also have to manually check the function at boundary points.

Problem-5

If we use lagrange basis, to calculate $l_0, l_1 \dots l_n$, we need to all the nodes. So, if we want to add 1 more node, we have to add 1 more factors at each polynomial, resulting in running the whole code again, which is not efficient.

But in Newton-divided method, if we want to add a new node, all the previous calculations come handy. We just have to do a few more calculations to get the desired update answer.

That's why Newton-divided method is beneficial when increasing nodes.

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