Ans. to Q No - 1

Given, $\beta = 2$, $m = 5$, $e \in [-3, 4]$

$\therefore$ The convention will be

$$= \pm (0.1 d_1 d_2 d_3 d_4 d_5)_2 \times 2^e$$

a

The maximum number will be

$$= (0.111111)_2 \times 2^4$$

$$= 0 \times 2^0 . (1 \times 2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} + 2^{-5} + 2^{-6}) \times 16$$

$$= 0.984375 \times 16$$

$$= (15.75)_{10}$$

(Ans.)

b) The minimum number will be the negative of the maximum number which can be stored in the system.

$\therefore$ Minimum number = -15.75

(Ans).

c) The 5 mantissa bits can have $2^5 = 32$ possible combinations of 0, 1.

For 1 exponent of e, there are 32 number.

$\therefore$ " 8 " " " e, " " (32×8) "

= 256 number.

(Ans.)

e) The minimum number will be

$= (0.100000)_2 \times 2^{-3}$

$= 0.5 \times 0.125$

$= 0.0625$

(Ans.)

Ans. to Q No - 2

Given $\beta = 2$, $m = 3$, $e_{min} = -2$, $e_{max} = 1$

$\therefore$ The convention will be

$= \pm (0.1d_1d_2d_3)_2 \times 2^e$

a) $\therefore$ Maximum number

$= (0.1111)_2 \times 2^1$

$= (0.9375)_{10} \times 2$

$= (1.875)_{10}$

(Ans.)

b) Minimum number = -1.875.

c) The decimal numbers will be

$n_1 \Rightarrow (0.1000) \times 2^{-2} = (0.125)_{10}$

$n_2 \Rightarrow (0.1001) \times 2^{-2} = 0.140625$

$n_3 \Rightarrow (0.1010) \times 2^{-2} = 0.15625$

Difference $= 0.15625 - 0.140625 = \frac{1}{64}$

$\therefore n_4 = 0.15625 + \frac{1}{64} = 0.171875$

$n_5 = 0.171875 + \frac{1}{64} = 0.1875$

$n_6 = 0.1875 + \frac{1}{64} = 0.203125$

$n_7 = 0.21875$

$n_8 = 0.234375$

$n_9 = 0.25 = (0.1000) \times 2^{-1}$

$n_{10} = (0.1001) \times 2^{-1} = 0.28125$

Difference, $n_{10} - n_9 = 0.03125$

$\therefore n_{11} = 0.28125 + 0.03125 = 0.3125$

$n_{12} = 0.34375$

$n_{13} = 0.375$

$n_{14} = 0.40625$

$n_{15} = 0.4375$

$n_{16} = 0.46875$

$n_{17} = 0.5 = (0.1000)_2 \times 2^0$

$n_{18} = (0.1001) \times 2^0 = 0.5625$

Difference $= n_{18} - n_{17} = 0.0625$

$\therefore n_{19} = 0.5625 + 0.0625 = 0.625$

$n_{20} = 0.6875$, $n_{21} = 0.75$

$n_{22} = 0.8125$, $n_{23} = 0.875$

$n_{24} = 0.9375$ $n_{25} = 1$

$n_{26} = (0.1001)_2 \times 2^1 = 1.125$

$\therefore$ Difference, $n_{26} - n_{25} = 0.125$

$\therefore n_{27} = 1.125 + 0.125 = 1.25$

$n_{28} = 1.375$, $n_{29} = 1.5$

$n_{30} = 1.625$ $n_{31} = 1.75$

$n_{32} = 1.875$

d) machine epsilon

$= 0.5 \times 2^{-2}$

$= \cancel{0.0625}\ 0.125$

(Ans.)

Hermite Interpolation

Given, $f(x) = \cos(x)$

$x_0 = 0, \quad x_1 = \pi/2$

$f(x_0) = 1, \quad f(x_1) = 0$

$f'(x) = -\sin(x)$

$f'(x_0) = 0$

$f'(x_1) = -1$

As, $n=1$, ~~number of~~ degree of the polynomial will be $(2\times 1) + 1 = 3$.

$$\therefore P_3(x) = f(x_0)\, h_0(x) + f'(x_0)\, \hat{h}_0(x) + f(x_1)\, h_1(x) + f'(x_1)\, \hat{h}_1(x)$$

$$= h_0(x) - \hat{h}_1(x)$$

Now, we know

$$h_0(x) = \{1 - 2(x - x_0)\, l_0'(x_0)\} \times l_0^2(x)$$

Again, $l_0(x) = \dfrac{x - x_1}{x_0 - x_1} = \dfrac{x - \pi/2}{-\pi/2}$

$$= \frac{x}{-\pi/2} + 1 = 1 - \frac{2x}{\pi}$$

$$\therefore l_0'(x) = -\frac{2}{\pi}$$

$$\therefore h_0(x) = \left\{1 - 2(x - 0)\cdot\left(-\frac{2}{\pi}\right)\right\} \times \left(1 - \frac{2x}{\pi}\right)^2$$

$$= \left\{1 - (2x)\left(-\frac{2}{\pi}\right)\right\} \times \left(1 - \frac{4x}{\pi} + \frac{4x^2}{\pi^2}\right)$$

$$= \left\{1 + \frac{4x}{\pi}\right\} \times \left(1 - \frac{4x}{\pi} + \frac{4x^2}{\pi^2}\right)$$

$$= 1 - \frac{4x}{\pi} + \frac{4x^2}{\pi^2} + \frac{4x}{\pi} - \frac{16x^2}{\pi^2} + \frac{16x^3}{\pi^3}$$

$$= 1 - \frac{12x^2}{\pi^2} + \frac{16x^3}{\pi^3}$$

Then, $\hat{h}_1(x) = (x - x_1)\, l_1^2(x)$

$$l_1(x) = \frac{x - x_0}{x_1 - x_0} = \frac{x}{\pi/2} = \frac{2x}{\pi}$$

$$\therefore \hat{h}_1(x) = \left(x - \frac{\pi}{2}\right)\cdot\left(\frac{4x^2}{\pi^2}\right)$$

$$= \frac{4x^3}{\pi^2} - \frac{2x^2}{\pi}$$

$$\therefore P_3(x) = h_0(x) - \hat{h}_1(x)$$

$$= 1 - \frac{12x^2}{\pi^2} + \frac{16x^3}{\pi^3} - \frac{4x^3}{\pi^2} + \frac{2x^2}{\pi}$$

(Ans.)