BRAC UNIVERSITY

MAT215

MATHEMATICS III: COMPLEX VARIABLES & LAPLACE TRANSFORMATIONS

Assignment 02

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Assignment Set: E

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Ans To The Question No. (1)

Let, $z = x + iy$
Given,

$$f(z) = z^2 - z + 2$$
$$= (x + iy)^2 - (x + iy) + 2$$
$$= x^2 + i2xy + i^2y^2 - x - iy + 2$$
$$= x^2 + i2xy - y^2 - x - iy + 2$$
$$= (x^2 - x - y^2 + 2) + i(2xy - y)$$

Therfore, in Cartesian form,
$u(x, y) = x^2 - x - y^2 + 2$
$v(x, y) = 2xy - y$

Again,

$$f(z) = z^2 - z + 2$$
$$= (r(cos\theta + isin\theta))^2 - r(cos\theta + isin\theta) + 2$$
$$= r^2(cos2\theta + isin2\theta) - r(cos\theta + isin\theta) + 2$$
$$= r^2cos2\theta + ir^2sin2\theta - rcos\theta + irsin\theta + 2$$
$$= (r^2cos2\theta - rcos\theta + 2) + i(r^2sin2\theta + rsin\theta)$$

Therfore, in Polar form,
$u(x, y) = r^2cos2\theta - rcos\theta + 2$
$v(x, y) = r^2sin2\theta + rsin\theta$
(Ans)

Ans To The Question No. (2)

Given,

$$
\begin{aligned}
f(z) &= \frac{1}{1+z} \\
\Rightarrow f'(z) &= \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z} \\
&= \lim_{\Delta z \to 0} \frac{\frac{1}{1+z+\Delta z} - \frac{1}{1+z}}{\Delta z} \\
&= \lim_{\Delta z \to 0} \frac{\frac{(1+z)-(1+z+\Delta z)}{(1+z+\Delta z)(1+z)}}{\Delta z} \\
&= \lim_{\Delta z \to 0} \frac{1+z-1-z-\Delta z}{\Delta z(1+z+z+z^2+\Delta z+\Delta z z)} \\
&= \lim_{\Delta z \to 0} \frac{-1}{1+2z+z^2+\Delta z+\Delta z z} \\
&= -\frac{1}{1+2z+z^2+0+0} \\
&= -\frac{1}{(1+z)^2} (Ans)
\end{aligned}
$$

Ans To The Question No. (3)

Given,

$$\lim_{z\to 0}\frac{Re(z)}{|z|}$$

$$= \lim_{z\to 0}\frac{Re(x+iy)^2}{\sqrt{x^2+y^2}}$$

$$= \lim_{z\to 0}\frac{Re(x^2+i2xy-y^2)}{\sqrt{x^2+y^2}}$$

$$= \lim_{z\to 0}\frac{x^2-y^2}{\sqrt{x^2+y^2}}$$

$$= \frac{0^2-0^2}{\sqrt{0^2+0^2}}$$

$$= \infty(Ans)$$

Ans To The Question No. (4)

Given,

$$\lim_{z\to 1+i}\frac{z^2-z+1-i}{z^2-2z+2}$$

$$= \frac{(1+i)^2-(1+i)+1-i}{(1+i)^2-2(1+i)+2}$$

$$= \frac{1+2i-1-1-i+1-i}{1+2i-1-2-2i+2}$$

$$= \frac{0}{0}$$

$$= \infty(Ans)$$

Ans To The Question No. (5)

We know,
$\lim_{z\to z_0} f(z) = \infty$ if and only if $\lim_{z\to z_0} \frac{1}{f(z)} = 0$

Here,

$$\lim_{z\to z_0} \frac{1}{f(z)}$$
$$= \lim_{z\to 1} \frac{(z-1)^3}{1}$$
$$= (1-1)^3$$
$$= 0$$

Therefore, it is showed that,

$\lim_{z\to 1} \frac{1}{(z-1)^3} = \infty$