Principles of Physics I (PHY111)

Lab

Experiment no: 1

Name of the experiment: Determination of the modulus of rigidity of the element of wire by the method of oscillation

Theory

Let's consider a cylinder of mass m and radius R which is suspended vertically by a thin circular wire from a point A, called the suspension point as shown in figure 1. The thin wire has negligible mass compare to the cylinder and has radius r and the modulus of rigidity η (if you are not familiar with this term please see appendix A given in the soft copy of this script which is available in the server). The wire is also aligned along the z axis.

If the cylinder is rotated through a small angle about z axis and then released, due to the rigidity modulus of the wire, the system (cylinder and the wire) will also oscillate rotationally. This is because of the Hooke's law. Let T be the time period of the circular oscillation. There is a relationship among η, T and the geometrical parameters of the wire. If l is the length of the wire, this relation is given by the following equation:

$$\eta = \frac{8\pi I l}{T^2 r^4} \qquad (1)$$

Figure 1: The arrangement of the experiment. A cylinder is suspended vertically by means of a wire.

Where, I is the moment of inertia of the cylinder about z axis as shown in the diagram. Those who are interested to know how to derive this formula please see appendix B (given in the soft copy of this script which is available in the server).

If the mass of the cylinder is m and its radius is R, then its moment of inertia about its own axis is given by,

$$I = \frac{1}{2} mR^2 \qquad (2)$$

In this experiment you are going to measure the modulus of rigidity of the element of the wire by using equations (1) and (2).

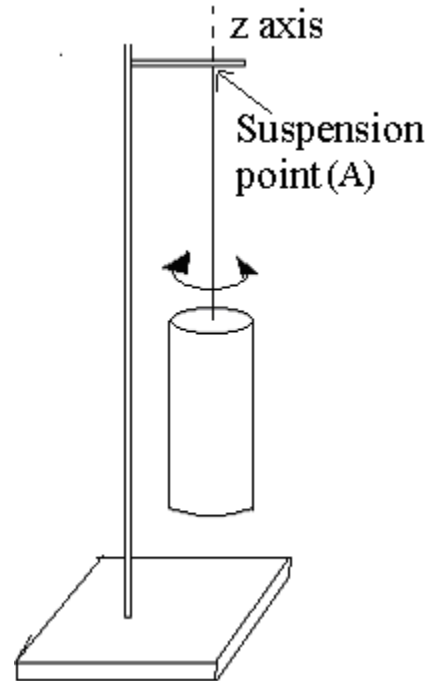**Apparatus:** Meter scale, slide calipers, screw gauge, stop watch, mass meter.

Procedure:

1. Measure the length, *l* of the wire by using meter scale and record it in section A of the data sheet.

2. Measure the diameter of the wire three times at three different positions by using screw gauge. Compute their average. Find out the radius, *r* of the cross section of the wire. Use table 1- provided in the section B of data sheet, to record data of this step. To know how to use a screw gauge please see appendix C (given in the soft copy of this script that is available in the server).

3. Measure the diameter of the cylinder for three times at three different positions at the top, middle and bottom, by using slide calipers. Write down the data in table 2 of section B of the data sheet. Find out the radius, *R* of the cylinder. To know how to use a slide calipers please see appendix D (given in the soft copy of this script which is available in the server).

4. Record the mass of the cylinder in section D of the data sheet. You may possibly be given the value of the mass, *m* of the cylinder, if not, then measure the mass of the cylinder using the mass meter.

5. Rotate the cylinder through a small angle about its axis and release. It will oscillate rotationally. Record the time required to complete 20 oscillations by using stop watch and write it down in table 3 of section E. Repeat it for two more times and find their average. Then work out the period of oscillation, *T* by dividing total time by number of oscillations.

6. In the calculation part in the data sheet, compute the moment of inertia, *I* of the cylinder using equation (2) and using the data collected in step 3 and 5.

7. Putting the values found in steps 1, 2, 4 and 6 in equation (1) compute the modulus of rigidity of the wire

Read carefully and follow the following instructions:

- **Please READ the theory carefully, TAKE printout of the 'Questions on Theory' and ANSWER the questions in the specified space BEFORE you go to the lab class.**

- **To get full marks for the 'Questions on Theory' portion, you must answer ALL of these questions CORRECTLY and with PROPER UNDERSTANDING, BEFORE you go to the lab class. However, to ATTEND the lab class you are REQUIRED to answer AT LEAST the questions with asterisk mark.**

- **Write down your NAME, ID, THEORY SECTION, GROUP, DATE, EXPERIMENT NO AND NAME OF THE EXPERIMENT on the top of the first paper.**

- **If you face difficulties to understand the theory, please meet us BEFORE the lab class. However, you must read the theory first.**

- **DO NOT PLAGIARIZE. Plagiarism will bring ZERO marks in this WHOLE EXPERIMENT. Be sure that you have understood the questions and the answers what you have written, and all of these are your own works. You WILL BE asked questions on these tasks in the class. If you plagiarize for more than once, WHOLE lab marks will be ZERO.**

- **After entering the class, please submit this portion before you start the experiment.**

Name: _____ **ID:** _____ **Sec:** ___ **Group:** __ **Date:** _____

Experiment no: ___

Name of the Experiment: _____

_____

Questions on Theory

*1) What is elasticity? [0.5]

Ans:

*2) Define 'modulus of rigidity'. [1]

Ans:

*3) Draw the arrangement of the experiment. [0.5]

Ans:

*4) Write down the working formula for this experiment. Mention what every variable represents. [1]

Ans:

5) Suppose, we have two wires consisting of two <u>different materials</u>. Both of them are having same length and radius of cross section. Two identical cylinders are suspended vertically by means of them. The first one shows period of oscillation of 2s and the second one shows the period of oscillation of 4s. Which wire consists of the material of higher modulus of rigidity- the wire attached to the first cylinder or the wire attached to second cylinder? Why? [1]

Ans:

6) Suppose, we have two wires consisting of <u>same material</u>. Both of them are having same radius of cross section. Two identical cylinders are suspended vertically by means of them. The length of the first wire is 1cm and that of the second wire is 2cm. Will the modulus of rigidity for different wires be different or same? If they are different then which one has higher modulus of rigidity? If they are same then why are they same? [1]

Ans:

Data:

A) The length of the wire, $l =$ _____ cm

B) Measurement of the radius of wire, r:

The pitch of the screw gauge = _____ mm

Total number of divisions of the circular scale = _____

Least count = pitch/number of divisions of the circular scale = _____ mm

Table 1: Screw gauge readings for measuring r

Position	Linear scale reading L. S. R (mm)	Circular scale reading C. S. R (mm)	Mechanical Error M.E. (mm)	Diameter of wire = LSR+CSR ± M.E (mm)	Average value of diameter of wire, d (mm)	Radius, $r = d/2$ (mm)	r (cm)
Top							
Middle							
Bottom							

C) Measurement of the radius of cylinder, R:

Table 2: Slide calipers readings for measuring R

Position	Linear scale reading L. S. R (cm)	Vernier scale reading V. S. R (cm)	Mechanical Error M.E. (cm)	Diameter of wire = LSR+VSR ± M.E (cm)	Average value of diameter of wire, D (cm)	Radius $R = D/2$ (cm)
Top						
Middle						
Bottom						

D) Mass of the cylinder, $m =$ _____ gm

E) Measurement of period of oscillation, *T:*

Table 3: Stop watch readings for measuring *T*

Number of oscillation, n	Total time t (sec)	Average of total time t (s)	Period, T = t/n

Calculations

Result:

Name: _____ **ID:** _____

Questions for Discussions

1) In this experiment you kept the length of the wire fixed. To reduce the error in measurement if you performed the experiment for different lengths of the wire (keeping r fixed) how the period of oscillations would change with length? [1]

Ans:

2) If the cylinder is replaced by a cubic metal whose mass is different but the moment of inertia about the Z axis is as same as that of the cylinder, will it change your result? Explain your answer. [0.5]

Ans:

3) Does the period of oscillation depend on the amplitude of oscillation of the cylinder? [0.5]

Ans:

APPENDIX A: Definition of modulus of rigidity

If a force is applied on a solid object, its shape and size may change. There are some solid objects which regains its initial shape and size when the applied force is removed. They are called elastic objects. Elasticity is the property of material- the object consists of, due to which it regains its initial shape and size.

Modulus of rigidity is the measure of how much rigid a material is to oppose the force which attempts to deform the shape of the object. Before, define it mathematically, we need to know about two other concepts- shear stress and shear strain.
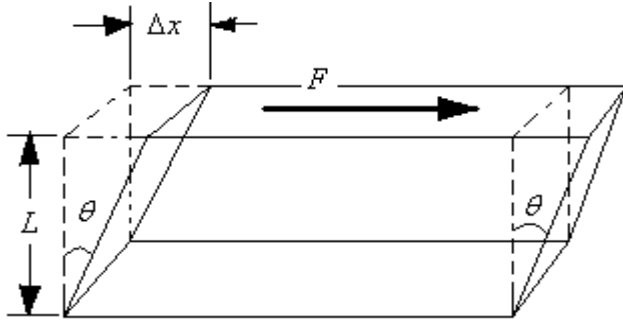
Figure A1: A force *F* is applied tangentially on the top surface of a solid object which is a rectangular parallelepiped.

Let's consider a solid rectangular parallelepiped. The area of top and bottom surface is *A*. The distance between the top and bottom surface is *L*. A force *F* is applied tangentially on the top surface. So the top surface is displaced and has moved rightward for distance Δx from its initial position.

Now, shear stress is defined by,

$$\text{Shear stress} = \frac{F}{A}$$

On the other hand shear strain is defined to be

$$\text{Shear strain} = \frac{\Delta x}{L} = \tan \theta \text{ [From the figure]}$$

If the angle θ is very small and measured in radian then we can write, $\tan \theta \approx \theta$

So, shear strain = θ, which gives us a measure of distortion of the shape of the body, i.e., more the value of θ is more the distortion of the shape is.

Now, modulus of rigidity of the material of the object is given by,

$$\text{Modulus of rigidity, } \eta = \text{shear stress / shear strain} = \frac{F/A}{\theta} = \frac{F}{A\theta}$$

APPENDIX B: Derivation of $\eta = \dfrac{8\pi I l}{T^2 r^4}$ (For the students who are interested)
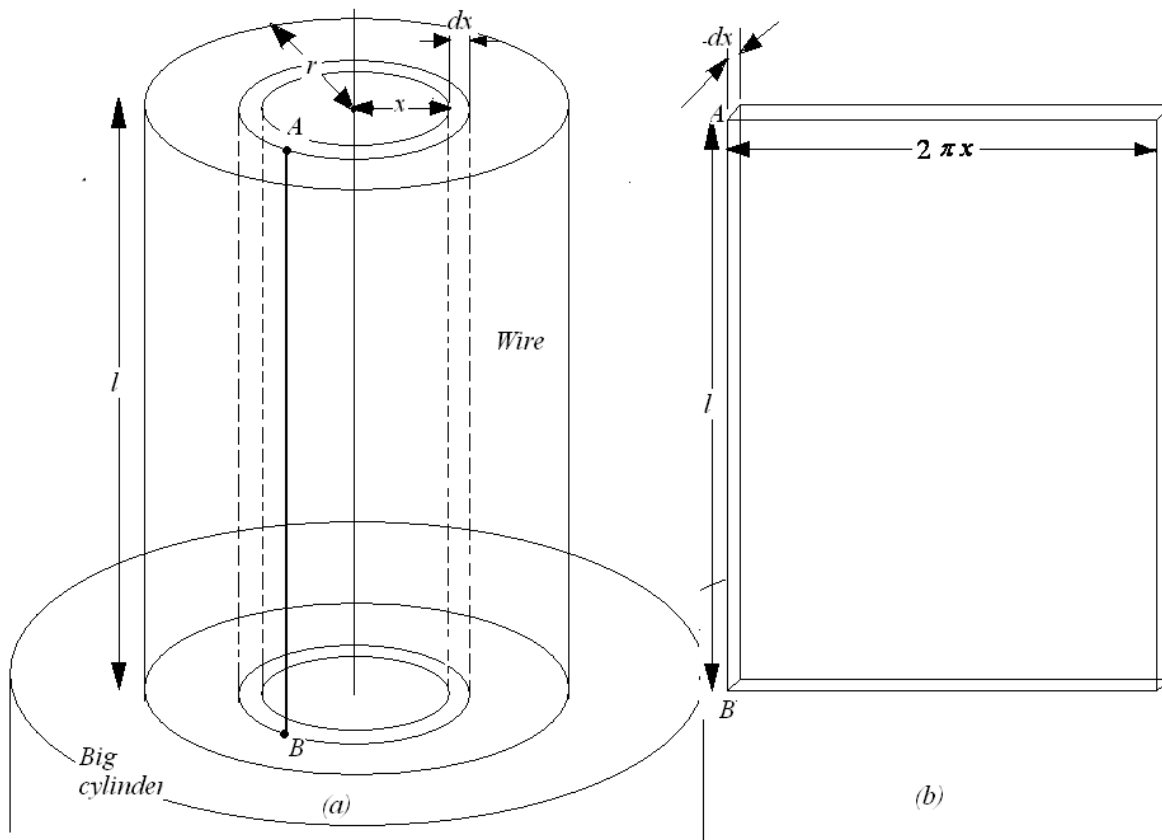

Figure B1: a) A cylindrical wire of radius *r* and length *l* with a big cylindrical body at its bottom end. In our experiment the wire is much thinner than the cylindrical body joined with it. A thick wire is shown to view it clearly. b) A rectangular parallelepiped which is found by cutting the cylindrical shell of radius *x* (shown in figure B1 a) along line AB and unrolling the shell.

Let's consider a wire of length *l* and having radius of cross section, *r* (figure B1 a). A cylindrical shell is joined at the bottom end of the wire. Consider a cylindrical shell of radius x and thickness dx (figure B1 a). The wire is nothing but a combination of lots of such cylindrical shell of different radii ranging from *x= 0* to x = *r* . If we cut that cylindrical shell along line AB and unroll it we get a rectangular parallelepiped shown in figure B1 b.

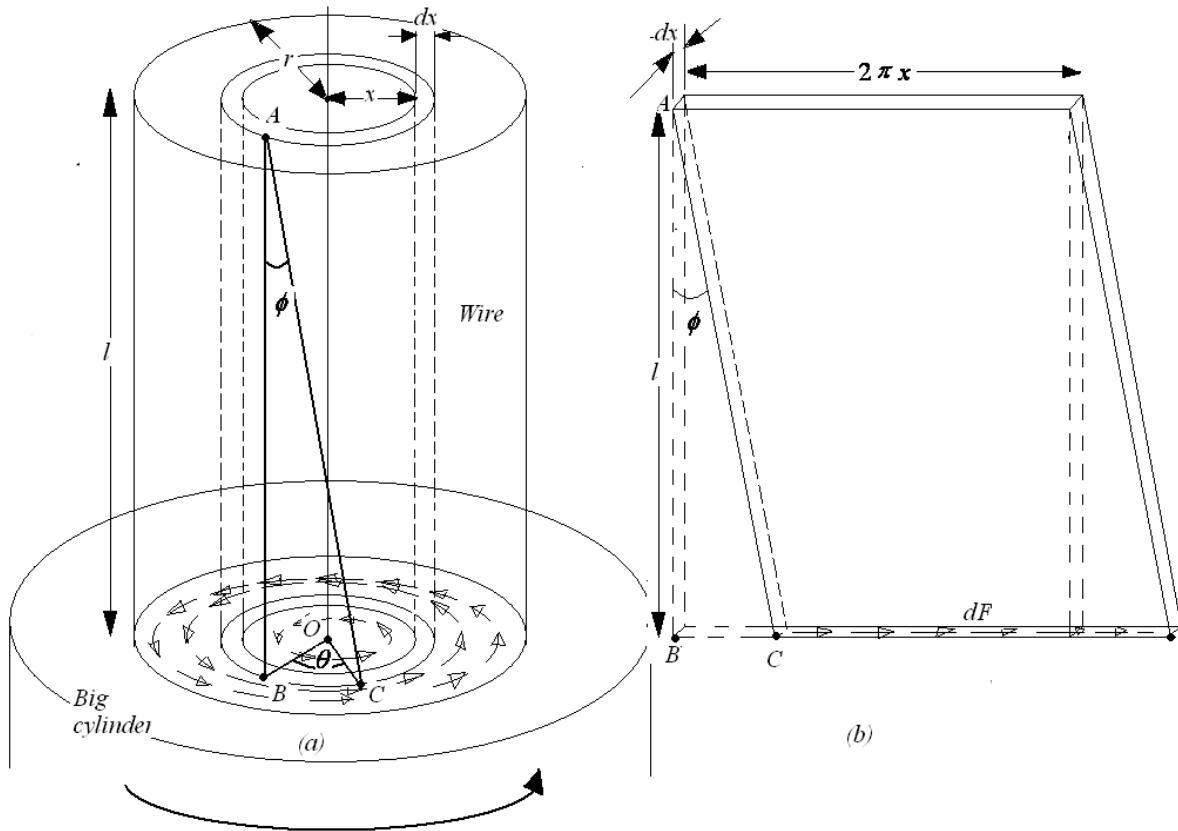The parallelepiped has length, *l*; width, $2\pi x$ (= perimeter of the shell) and thickness *dx*.

Figure B2: a) The cylindrical big body is rotated through an angle of θ about the axis. So the wire-particle of point B is displaced and C is the new position of that particle. b) If now we cut the cylinder along AC and unroll it we find the parallelepiped shown here.

Now we rotate the big cylindrical object at the bottom of the wire through an angle of θ about the axis. S a total force of F acts tangentially along some circular paths at the bottom surface of the wire, as shown in figure B2 a, which twisted the wire. Now the wire-particle which initially lied at point B is displaced and its new position is point C.

$\angle BOC = \theta$ and

$\angle BAC = \phi$

Now $BC = x\theta$, according to the definition of radian angle.

Now if we cut the shell along AC and unroll it we find a parallelepiped shown in figure B2 b.

We can find out the shear strain for this parallelepiped, in other word for that shell of the wire.

$$\text{Shear strain} = \frac{BC}{AC} = \frac{x\theta}{l}$$

I said that total F amount of force is acting tangentially on the whole bottom surface of the wire. For that cylindrical shell of radius x, dF amount of force is acting tangentially on its bottom surface. From figure 3b we see the area of the bottom surface of the parallelepiped, i.e., of the shell is $2\pi x dx$

So, the shear stress $= \dfrac{dF}{2\pi x dx}$

Now, the modulus of rigidity of the material of the wire,

$\eta =$ shear stress/ shear strain

$\Rightarrow \eta = \dfrac{\dfrac{dF}{2\pi x dx}}{\dfrac{x\theta}{l}}$

Therefore, $dF = \dfrac{2\pi\eta\theta\ x^2}{l}dx$

We know that torque is the property of force which causes a rotational tendency on a body about an axis when the force is applied on it. If $\vec{r}$ is the position vector about the axis of rotation of the point of action of a force, $\vec{F}$, then mathematically torque is given by,

$\vec{\tau} = \vec{r} \times \vec{F}$

Magnitude of torque, $\tau = rF\sin(\alpha)$, here α is the angle between $\vec{r}$ and $\vec{F}$

Now the torque acting on the bottom surface of the cylindrical shell is,

$$d\tau = xdF\sin 90^0 = xdF = \dfrac{2\pi\eta\theta x^3}{l}dx$$

We can find the total torque acting on the whole bottom surface of the wire by just integrating the above mentioned equation from limit $x = 0$ to $x = r$. So the total torque on the bottom surface of the wire by the big cylindrical object,

$$\tau = \dfrac{2\pi\eta\theta}{l}\int_0^r x^3 dx = \dfrac{\pi\eta r^4}{2l}\theta$$

According to the third law of Newton, the bottom surface of the wire also exerts a torque on the big cylinder having equal magnitude but opposing direction.

So, the torque on the big cylinder,

$$\tau' = -\tau = -\frac{\pi\eta r^4}{2l}\theta \qquad\qquad\qquad\qquad \textbf{B1}$$

This torque will cause an angular acceleration of the cylinder, which is $\dfrac{d^2\theta}{dt^2}$ [here *t* stands for time].

Now if I is the moment of inertia of the cylinder about it's own axis then the torque applied on it is given by:

$$\tau' = I\frac{d^2\theta}{dt^2} \qquad\qquad\qquad\qquad \textbf{B2}$$

By equating B1 and B2 we get

$$I\frac{d^2\theta}{dt^2} = -\frac{\pi\eta r^4}{2l}\theta$$

$$=> \frac{d^2\theta}{dt^2} + \frac{\pi\eta r^4}{2lI}\theta = 0$$

This is exactly the equation of simple harmonic oscillation-

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0, \text{ where } \omega \text{ is the angular frequency}$$

We can easily identify that,

$$\omega^2 = \frac{\pi\eta r^4}{2lI}$$

Again we know, $\omega = \dfrac{2\pi}{T}$, where T is the period of oscillation.

Hence $\left(\dfrac{2\pi}{T}\right)^2 = \dfrac{\pi\eta r^4}{2lI}$

$$\Rightarrow \eta = \frac{8\pi Il}{T^2 r^4}$$

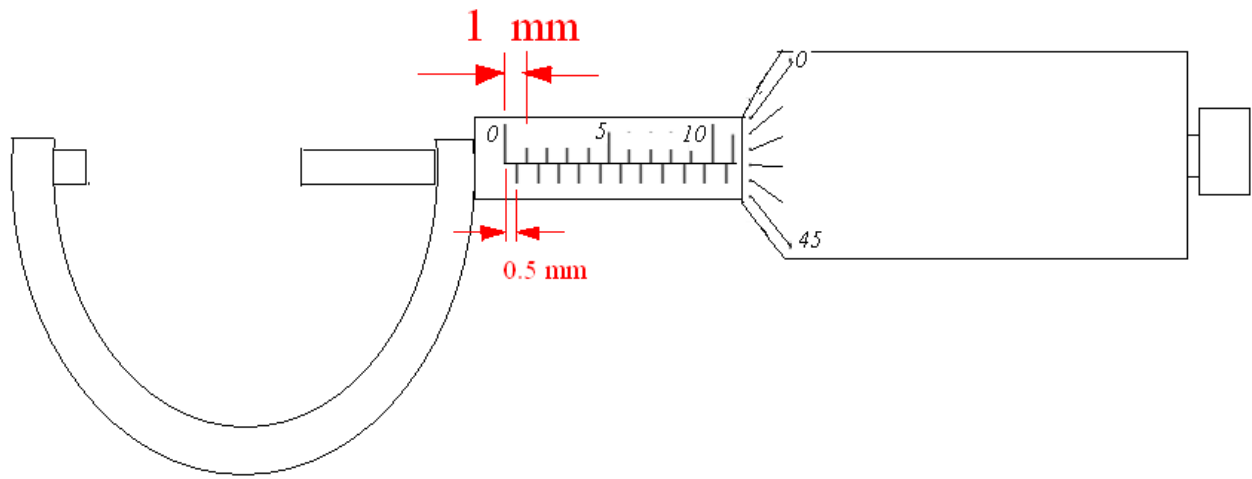**APPENDIX C: How to measure length with a screw gauge**

Figure 1: A screw gauge

Figure 1 shows the magnified view of the screw gauge what you are using in the lab.

Step 1: First notice the length of the smallest division of the linear scale.

You can see the separation between the two consecutive lines of the upper portion of the linear scale is 1 mm. Each of the line of the lower portion of linear scale has divided this 1mm segment equally. So the gap between a line of the upper scale and the next line of the lower scale is 0.5 mm.
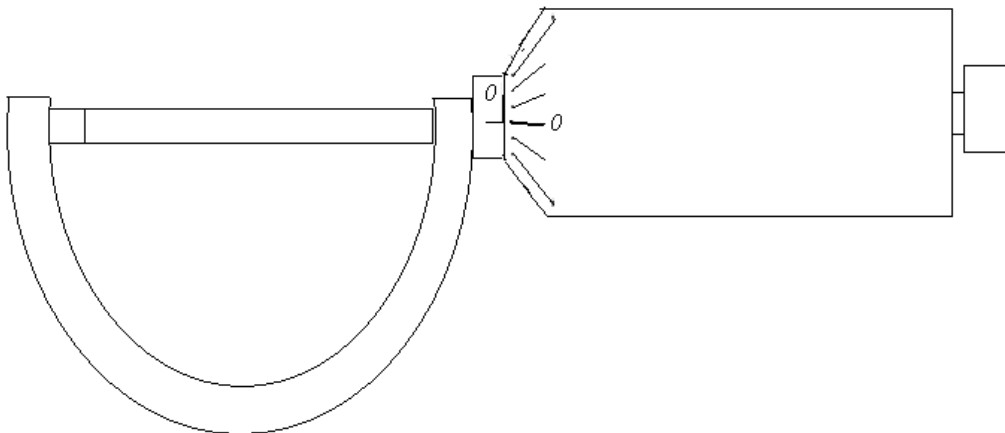

Figure 2: If the screw gauge is free from mechanical error, when the screw touches the left end of the frame of screw gauge, then 0 of circular scale coincides with the linear scale
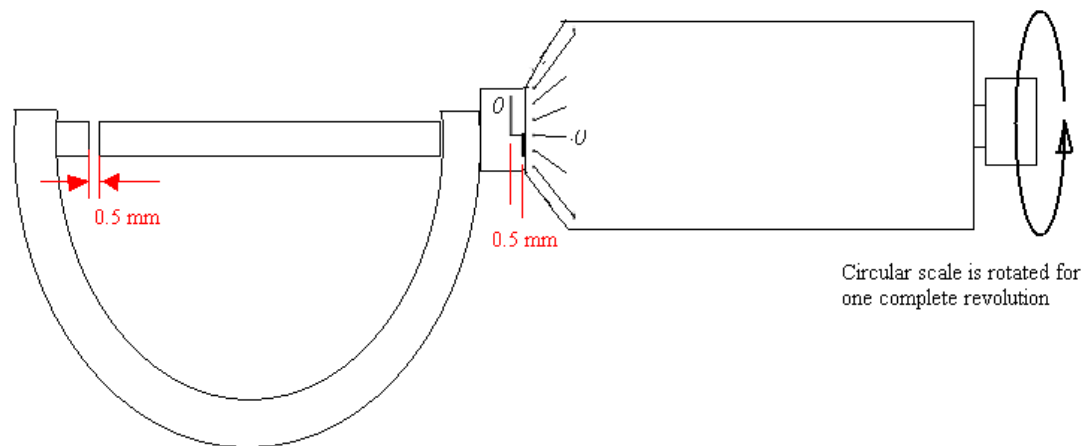
Figure 3: The position of the circular scale on the linear scale when it is rotated for one complete revolution

Step 2: Rotate the circular scale for one complete revolution and notice the displacement of the circular scale along the linear scale. This is the pitch of the screw gauge

To do so suppose initially 0 mark of circular scale coincided with the linear scale (figure 2) and then the position of the circular scale on linear scale was at 0. Now the circular scale is rotated for one complete revolution and 0 of circular scale has been brought back to coincide with the linear scale (figure 3). Here we see circular scale is displaced for 0.5 mm along the linear scale.

So the Pitch = 0.5 mm

Step 3: Now see how many number of divisions are there in the circular scale.

Here, it is 50.

Step 4: Find the Least Count.

Least Count = Pitch/ number of division of circular scale

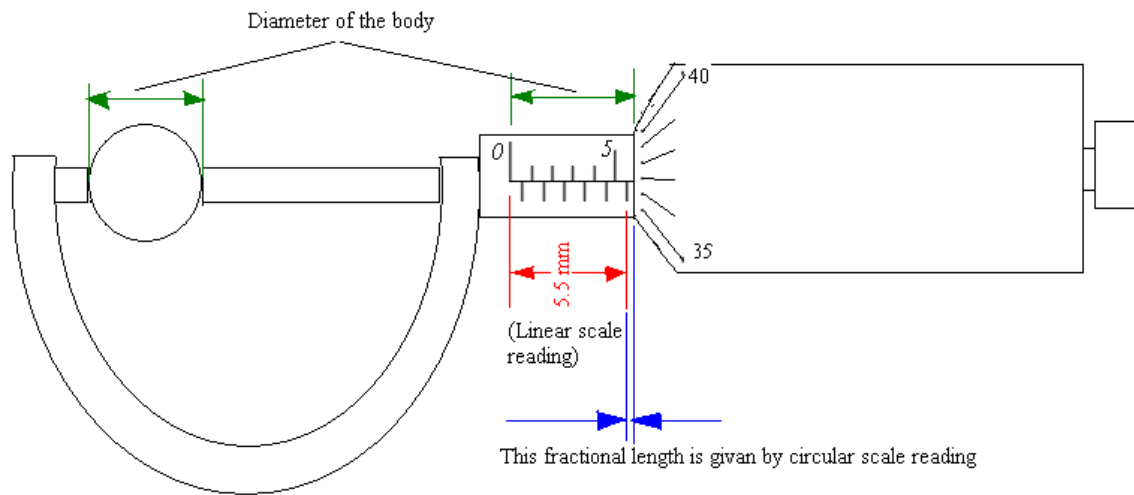Least Count = (0.5/50) mm = 0.01 mm

Figure 4: Measuring diameter of a ball.

Step 5: Now keep the body whose diameter or length is supposed to be measured between the left end point and the screw of the screw gauge. Rotate the circular scale until left end point and the screw both touch the body (figure 4)

Step 6: See which division of the linear scale the circular scale just crosses. That will be the linear scale reading.

In this case we see it is 5.5 mm (= linear scale reading)

Step 7: See which line of the circular scale coincides with the linear scale.

In this case it is 37

Step 8: By multiplying it by least count find the circular scale reading

37×0.01 mm = 0.37 mm (= Circular scale reading)

Step 9: Find the total reading of the diameter of the body by adding linear scale reading and the circular scale reading.

Total reading = (5.5 + 0.37) mm = 5.87 mm

Mechanical Error

When the screw touches the left end point of the screw gauge, then if the 0 of circular scale does not coincide with the linear scale then there is mechanical error.
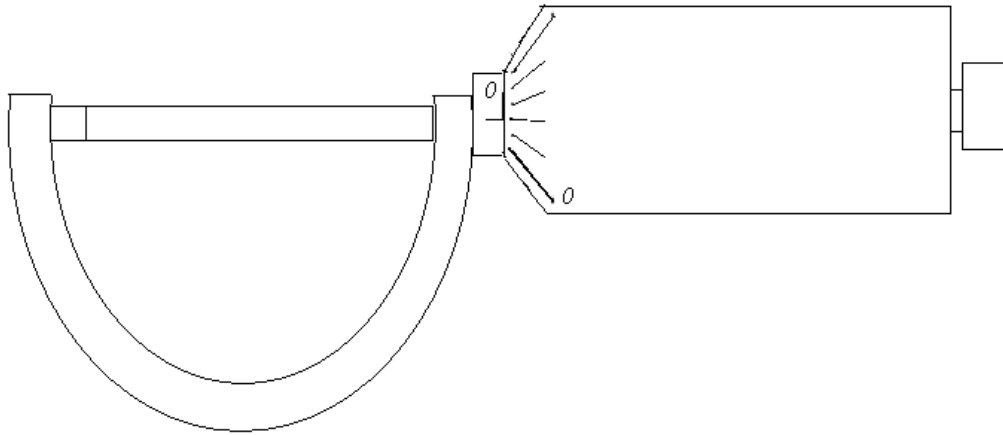

Figure 5: Screw gauge with mechanical error where 0 of circular scale lies below the linear scale when the screw touches the left end point.

Case 1: Where 0 of circular scale lies below the linear scale when the screw touches the left end point.

- See how many number of divisions 0 of circular scale lies below linear scale.

  Here it is 2 (figure 5)

- Multiply it by least count. 2×0.01 mm $= 0.02$ mm

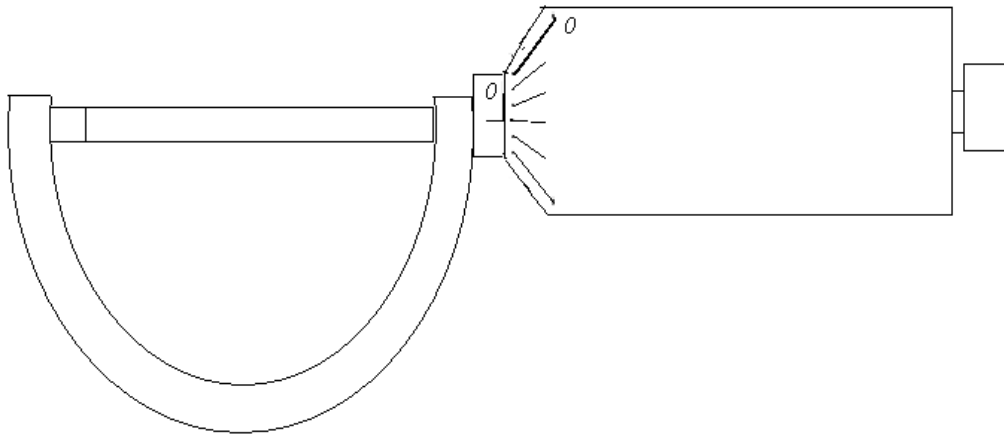- Subtract it from the total reading to correct the measurement.

Figure 6: Screw gauge with mechanical error where 0 of circular scale lies above the linear scale when the screw touches the left end point.

Case 2: Where 0 of circular scale lies above the linear scale when the screw touches the left end point.

- See how many number of divisions 0 of circular scale lies above the linear scale.

   Here it is 3 (figure 6)

- Multiply it by least count. 3×0.01 mm $= 0.03$ mm

- Add it with the total reading to correct the measurement.

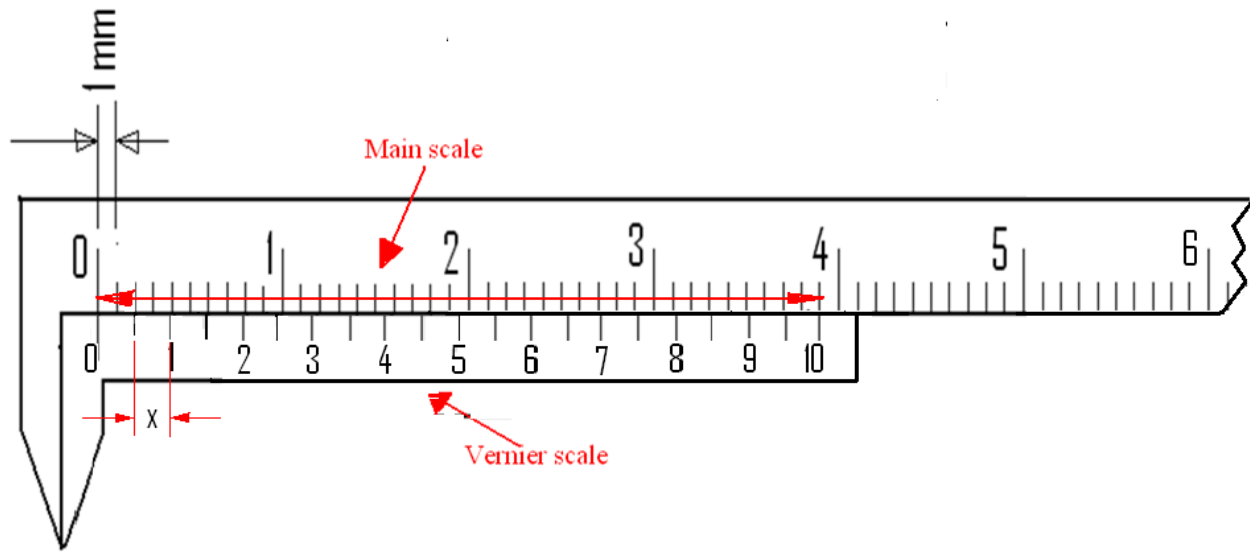**APPENDIX D: How to measure length with a slide calipers**

Figure 1: A slide calipers when its two jaws touch each other.

Figure 1 shows you the magnified view of the slide calipers what you are using in the laboratory. When the two jaws touch each other the 0 mark of main scale coincides the 0 mark of the Vernier scale, if the slide calipers is free from mechanical error.

Step 1: Notice what the length of smallest division of the main scale is.

It is 1 mm, for the slide calipers shown above

Step 2: Count the number of the smallest divisions of the Vernier scale.

Here, it is 20.

Step 3: Measure the length of the all 20 divisions of the Vernier scale by using the main scale (If your vernier scale had 40 divisions instead of 20, then you would measure the length of all 40 divisions in this step).

Here, it is 39 mm.

Step 4: Work out the length of the smallest division of the Vernier scale.

If x is the length of the smallest division of the Vernier scale then we can write,

$20 \, x = 39$ mm
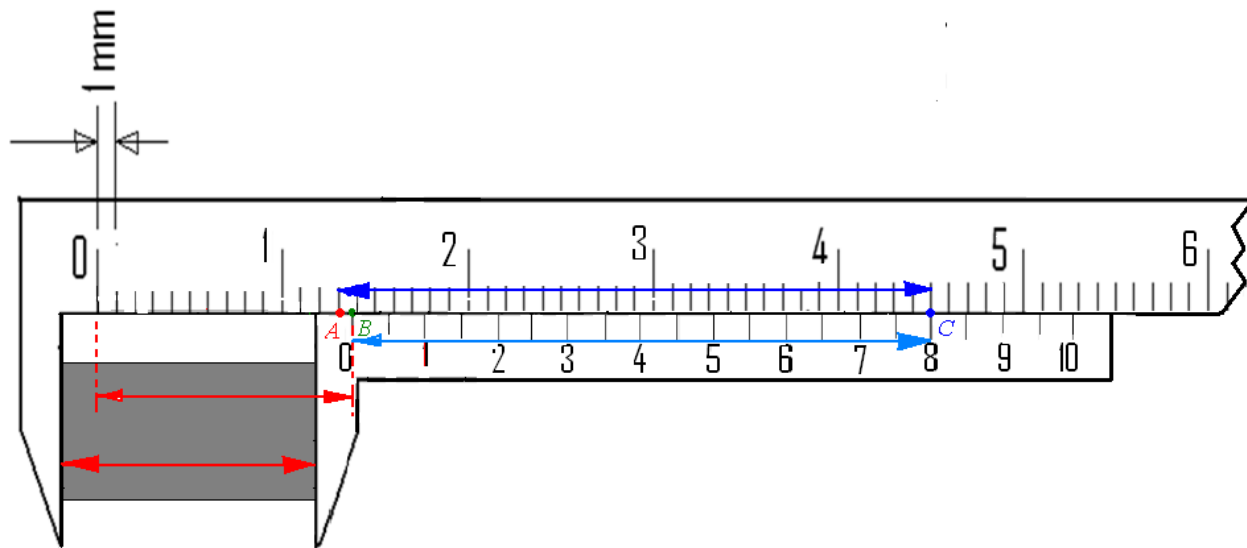So, $x = (39/ 20)$ mm $= 1.95$ mm

Figure 2: Same slide calipers containing a body between its two jaws.

Step 5: Now keep the body between the jaws of the slide calipers. So the 0 mark of the Vernier scale is displaced. Separation between the 0 mark of the main scale and the 0 mark of the Verinier scale is equal to the length of the body. Which mark of the main scale does the 0 of Vernier scale (B) just cross?

We can see, for this body it the mark of 13 mm. Let's denote it by A (figure 2)and the 0 of Vernier by B.

So, the length of the body = 13 mm + AB

Main scale reading = 13 mm

Step 6: Now, see which mark of the Vernier scale coincides with any mark of the main scale.

We can see it is the mark of 16th division of the Vernier scale. In the Vernier scale this mark is denoted by 8. Let's call it point C.

Step 7: Measure AC by using main scale.

AC = 32 mm

Step 8: Measure BC by using Vernier scale.

BC = 16 x = 16×1.95 mm = 31.2 mm

Step 9: Find AB.

AB = AC – BC = (32-31.2) mm = 0.8 mm
So Vernier scale reading = 0.8 mm

Step 10: Now find the total length of the body

Length of the body = Main scale reading + Vernier scale reading

= (13 + 0.8) mm

Vernier constant:

Go back at step 9. Look how we find AB (Excess by Vernier scale)

AB = (32 – 31.2) = 16 (2-1.95) mm = 16 × 0.05 mm

Here 16 was the mark-number of Vernier which coincided with a mark of main scale. This is called "vernier coincident". By multiplying it by 0.05 mm we get the Vernier scale reading.

We say 0.05 mm is the Vernier constant of the slide calipers. You must have noticed that 0.05 mm is written at the top left corner of the Vernier scale of the slide calipers what you are using.

So, to find the Vernier scale reading, you can just notice the Vernier scale division which coincides with a mark of main scale and then multiply it by Vernier constant.

Mechanical Error:

If 0 of Vernier scale does not coincide with the 0 of main scale when the two jaws touch each other, then there is mechanical error.
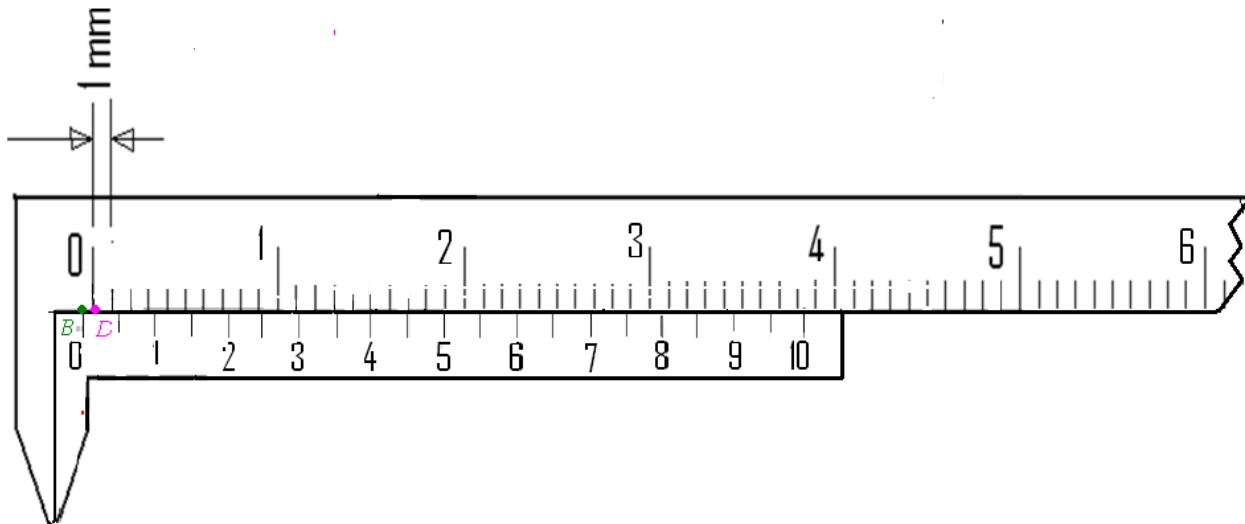

Figure 3: A slide calipers with mechanical error, where 0 of Vernier scale lies at left of the 0 or main scale.
In figure 3 we see the 0 of Vernier scale (B) lies at left of the 0 of main scale (D). We have to add this distance BD with the measured length of any body to correct it.

We can see the mark of 8th division (denoted by 4 in the Vernier scale) coincides with a line of the main scale.

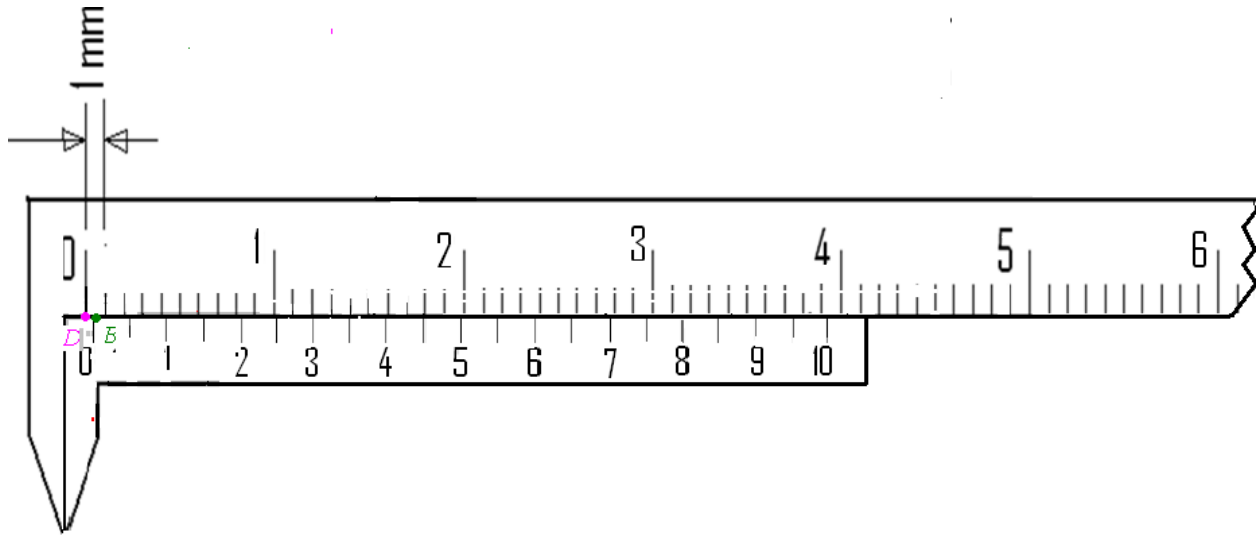So BD $= 8 \times 0.05$ mm $= 0.40$ mm

Figure 4: A slide calipers with mechanical error, where 0 of Vernier scale lies at right of the 0 or main scale.

In figure 4 we see the 0 of Vernier scale (B) lies at left of the 0 of main scale (D). We have to subtract this distance BD from the measured length of any body to correct it.

We can see the mark of 6th division (denoted by 3) in the Vernier scale coincides with a line of the main scale.

So BD $= 6 \times 0.05$ mm $= 0.30$ mm