6.3 Volume by Cylindrical Shells
Solutions to the Selected Problems

5–12. Use cylindrical shells to find the volume of the solid generated when the region enclosed by the given curves is revolved about the y-axis.

5. $y = x^3, x = 1, y = 0.$

Solution

$$V = \int_0^1 2\pi x \times x^3 \, dx$$

$$= 2\pi \int_0^1 x^4 \, dx$$

$$= \frac{2\pi}{5}$$

6. $y = \sqrt{x}, x = 4, x = 9, y = 0.$

Solution

$$V = \int_4^9 2\pi x\sqrt{x} \, dx$$

$$= 2\pi \int_4^9 x^{\frac{3}{2}} dx$$

$$= 2\pi \frac{2}{5}\left(9^{\frac{5}{2}} - 4^{\frac{5}{2}}\right)$$

$$= \frac{844\pi}{5}$$

7. $y = 1/x\,, y = 0, x = 1, x = 3.$

Solution

$$V = \int_1^3 2\pi x \times \frac{1}{x} dx$$

$$= 2\pi \int_1^3 1 \, dx$$

$$= 4\pi$$

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8. $y = \cos(x^2)\,, x = 0, x = \sqrt{\pi}/2\,, y = 0.$

Solution

$$V = \int_0^{\sqrt{\pi}/2} 2\pi x \cos(x^2)\, dx$$

$$= \frac{\pi\sqrt{2}}{2}$$

9. $y = 2x - 1, y = -2x + 3, x = 2.$

Solution

$$V = \int_1^2 2\pi x[(2x-1) - (-2x+3)]\, dx$$

$$= \frac{20}{3}\pi$$

10. $y = 2x - x^2, y = 0.$

Solution

$$V = \int_0^2 2\pi x(2x - x^2)\, dx$$

$$= \frac{8\pi}{3}$$

11. $y = \dfrac{1}{1+x^2}, x = 0, x = 1, y = 0.$

Solution

$$V = \int_0^1 2\pi x \left(\frac{1}{1+x^2}\right) dx$$

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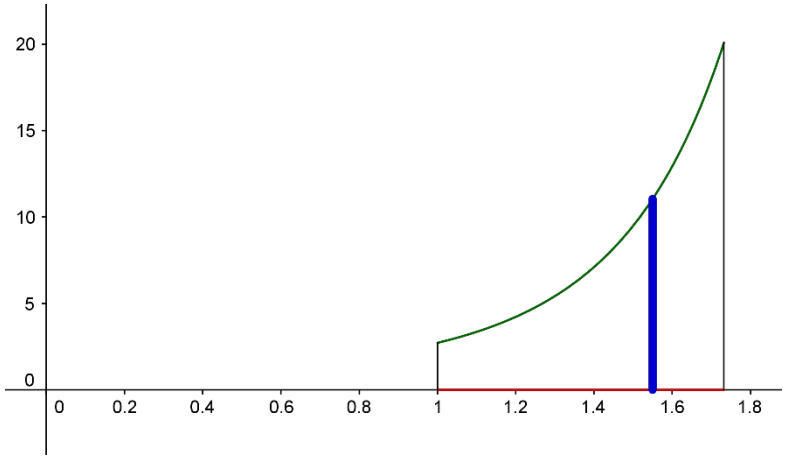
$$= 2\pi \int_0^1 \frac{x}{1+x^2}\,dx$$

$$= \pi \ln 2$$

12. $y = e^{x^2}, x = 1, x = \sqrt{3}, y = 0.$

Solution

$$V = \int_1^{\sqrt{3}} 2\pi x e^{x^2}\,dx$$



$$= \pi(e^3 - e)$$

13–16. Use cylindrical shells to find the volume of the solid generated when the region enclosed by the given curves is revolved about the x-axis.

13. $y^2 = x, y = 1, x = 0.$

Solution

$$V = \int_0^1 2\pi y \times y^2\,dy$$

$$= \frac{\pi}{2}$$

14. $x = 2y, y = 2, y = 3, x = 0.$

Solution

$$V = \int_0^1 2\pi y \times 2y\,dy$$

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$$= \frac{4\pi}{5}$$

15. $y = x^2, x = 1, y = 0.$

Solution

$$V = \int_0^1 2\pi y \times (1 - x)\, dy$$

$$= \int_0^1 2\pi y \times \left(1 - \sqrt{y}\right) dy$$

$$= \frac{\pi}{5}$$

16. $xy = 4, x + y = 5.$

Solution

$$V = \int_1^4 2\pi y \times \left(5 - y - \frac{4}{y}\right) dy = 9\pi$$

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