7.2 Integration by Parts
Solutions to the Selected Problems

Formulas

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Or,

$$\boxed{\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx}$$

1–38. Evaluate the integrals.

1. $\int xe^{-2x}\,dx$

Solution

Identifying

$$f(x) = x, \qquad g'(x) = e^{-2x}$$

$$f'(x) = 1, \qquad g(x) = -\frac{1}{2}e^{-2x}$$

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

$$\int xe^{-2x}\,dx = -\frac{1}{2}xe^{-2x} - \int \left(-\frac{1}{2}e^{-2x}\right)dx$$

$$= -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$$

$$\boxed{\int xe^{-2x}\,dx = -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C}$$

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3. $\int x^2 e^x \, dx$

Solution (Tabular Method of Integration)

f	g'
x^2	e^x
$2x$	e^x
2	e^x
0	e^x

$$\boxed{\int x^2 e^x \, dx = x^2 e^x - 2xe^x + 2e^x + C}$$

6. $\int x \cos 2x \, dx$

Solution

f	g'
x	$\cos 2x$
1	$\frac{1}{2}\sin 2x$
0	$-\frac{1}{4}\cos 2x$

$$\boxed{\int x \cos 2x \, dx = \frac{1}{2} x \sin 2x + \frac{1}{4}\cos 2x + C}$$

8. $\int x^2 \sin x \, dx$

Solution

Sign	f	g'
$+$	x^2	$\sin x$
$-$	$2x$	$-\cos x$
$+$	2	$-\sin x$
$-$	0	$\cos x$

$$\int x^2 \sin x \, dx = (x^2)(-\cos x) - (2x)(-\sin x) + (2)(\cos x)$$

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$$\boxed{\int x^2 \sin x\, dx = -x^2 \cos x + 2x \sin x + 2\cos x + C}$$

15. $\int \sin^{-1} x\, dx$

Solution

f	g'
$\sin^{-1} x$	1
$\dfrac{1}{\sqrt{1-x^2}}$	x

$$\begin{aligned}\int \sin^{-1} x\, dx &= x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}}\, dx \\ &= x\sin^{-1} x + \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\, dx \\ &= x\sin^{-1} x + \sqrt{1-x^2}\end{aligned}$$

$$\boxed{\int x\sin^{-1} x\, dx = x\sin^{-1} x + \sqrt{1-x^2} + C}$$

20. $\int e^{3x} \cos 2x\, dx$

Solution

Sign	f	g'
$+$	e^{3x}	$\cos 2x$
$-$	$3e^{3x}$	$\frac{1}{2}\sin 2x$
$+$	$9e^{3x}$	$-\frac{1}{4}\cos 2x$

$$\int e^{3x}\cos 2x\, dx = (e^{3x})\left(\frac{1}{2}\sin 2x\right) - (3e^{3x})\left(-\frac{1}{4}\cos 2x\right) + \int (9e^{3x})\left(-\frac{1}{4}\cos 2x\right) dx$$

$$\int e^{3x}\cos 2x\, dx = \frac{1}{2}e^{3x}\sin 2x + \frac{3}{4}e^{3x}\cos 2x - \frac{9}{4}\int e^{3x}\cos 2x\, dx$$

$$\left(1+\frac{9}{4}\right)\int e^{3x}\cos 2x\, dx = \frac{1}{2}e^{3x}\sin 2x + \frac{3}{4}e^{3x}\cos 2x$$

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$$\int e^{3x}\cos 2x\,dx = \frac{2}{13}e^{3x}\sin 2x + \frac{3}{13}e^{3x}\cos 2x$$

$$\boxed{\int e^{3x}\cos 2x\,dx = \frac{2}{13}e^{3x}\sin 2x + \frac{3}{13}e^{3x}\cos 2x + C}$$

28. $\int_0^1 xe^{-5x}\,dx$

Solution

$$\int xe^{-5x}\,dx$$

f	g'
x	e^{-5x}
1	$-\frac{1}{5}e^{-5x}$
0	$\frac{1}{25}e^{-5x}$

$$\int xe^{-5x}\,dx = -\frac{1}{5}xe^{-5x} - \frac{1}{25}e^{-5x}$$

$$\int_0^1 xe^{-5x}\,dx = \left(-\frac{1}{5}e^{-5} - \frac{1}{25}e^{-5}\right) - \left(0 - \frac{1}{25}\right)$$

$$= \frac{1}{25} - \frac{6}{25}e^{-5}$$

$$\boxed{\int_0^1 xe^{-5x}\,dx = \frac{1}{25}(1 - 6e^{-5})}$$

29. $\int_1^e x^2 \ln x\,dx$

Solution

$$\int x^2 \ln x\,dx$$

f	g'
$\ln x$	x^2

$\frac{1}{x}$	$\frac{x^3}{3}$
$\vdots$	$\vdots$

$$\int x^2 \ln x \, dx = \frac{1}{3} x^3 \ln x - \frac{1}{3} \int x^2 \, dx$$

$$= \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3$$

$$\int_1^e x^2 \ln x \, dx = \left(\frac{1}{3} e^3 \ln e - \frac{1}{9} e^3\right) - \left(\frac{1}{3} \ln 1 - \frac{1}{9}\right)$$

$$= \frac{2}{9} e^3 + \frac{1}{9}$$

$$\boxed{\int_1^e x^2 \ln x \, dx = \frac{1}{9}(2e^3 + 1)}$$

47–52. Evaluate the integral using tabular integration by parts.

47. $\int (3x^2 - x + 2) e^{-x} \, dx$

Solution

$$\int (3x^2 - x + 2) e^{-x} \, dx$$

Sign	f	g'
$+$	$3x^2 - x + 2$	e^{-x}
$-$	$6x - 1$	$-e^{-x}$
$+$	6	e^{-x}
$-$	0	$-e^{-x}$

$$\int (3x^2 - x + 2) e^{-x} \, dx = -(3x^2 - x + 2) e^{-x} - (6x - 1) e^{-x} - 6e^{-x}$$

$$\boxed{\int (3x^2 - x + 2) e^{-x} \, dx = (-3x^2 - 5x - 7) e^{-x} + C}$$

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49. $\int 4x^4 \sin 2x \, dx$

Solution

$$\int x^4 \sin 2x \, dx$$

Sign	f	g'
$+$	x^4	$\sin 2x$
$-$	$4x^3$	$-\frac{1}{2}\cos 2x$
$+$	$12x^2$	$-\frac{1}{4}\sin 2x$
$-$	$24x$	$\frac{1}{8}\cos 2x$
$+$	24	$\frac{1}{16}\sin 2x$
$-$	0	$-\frac{1}{32}\cos 2x$

$$\int x^4 \sin 2x \, dx = -\frac{1}{2}x^4 \cos 2x + x^3 \sin 2x + \frac{3}{2}x^2 \cos 2x - \frac{3}{2}x \sin 2x - \frac{3}{4}\cos 2x$$

$$\int x^4 \sin 2x \, dx = \left(-\frac{1}{2}x^4 + \frac{3}{2}x^2 - \frac{3}{4}\right)\cos 2x + \left(x^3 - \frac{3}{2}x\right)\sin 2x$$

$$\boxed{\int 4x^4 \sin 2x \, dx = (-2x^4 + 6x^2 - 3)\cos 2x + (4x^3 - 6x)\sin 2x + C}$$

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