Assignment No- 03

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Section: 15

Ans to the Q No - 1

$$\int_0^1 \left(1-\frac{1}{x}\right)^{1/3} dx$$

$$= \int_0^1 \left(\frac{x-1}{x}\right)^{1/3} dx$$

$$= \int_0^1 (x-1)^{1/3} \cdot (x^{-1})^{1/3} dx$$

$$= -\int_0^1 x^{-1/3} (1-x)^{1/3} dx$$

Now, $m-1 = -1/3 \Rightarrow m = 2/3$ | $n-1 = 1/3 \Rightarrow n = 4/3$

So, $\beta(m,n) = \beta(2/3, 4/3)$

$$= \frac{\Gamma(\frac{2}{3}) \cdot \Gamma(\frac{4}{3})}{\Gamma(\frac{2}{3}+\frac{4}{3})}$$

$$= \frac{\Gamma\frac{2}{3} \cdot \Gamma\frac{4}{3}}{\Gamma 2} = \frac{\Gamma\frac{2}{3} \cdot \Gamma\frac{4}{3}}{1!} = \Gamma\frac{2}{3} \cdot \Gamma\frac{4}{3}$$

(Ans.)

Ans. to the Q. No - 2

$$\int_0^a y^7 \sqrt{a^4 - y^4}\, dy$$

Let, $au = y$

$\Rightarrow \frac{d}{dy}(au) = \frac{d}{dy}(y)$

$\Rightarrow a\frac{du}{dy} = 1$

$\Rightarrow dy = a\,du$

$$= \int_0^1 (au)^7 \sqrt{(a)^4 - (au)^4}\, a\, du$$

$$= \int_0^1 a^8 u^7 \sqrt{a^4(1-u^4)}\, du$$

$$= \int_0^1 a^8 u^7 . a^2 \sqrt{1-u^4}\, du$$

$$= a^{10} \int_0^1 u^7 \sqrt{1-u^4}\, du$$

$$= a^{10} \int_0^1 u^4 (1-u^4)^{1/2} . u^3\, du$$

$$= \frac{1}{4} a^{10} \int_0^1 n\,(1-n)^{1/2}\, dn$$

$$= \frac{a^{10}}{4} \int_0^1 (n)^{2-1} . (1-n)^{3/2-1}\, dn$$

Comparing with beta function,

we get $m = 2$, $n = 3/2$

$\therefore$ Ans. $\frac{1}{4} a^{10} B(2, 3/2)$

limits

y	a	0
u	1	0

Let,

$u^4 = n$

$\Rightarrow 4u^3 = \frac{dn}{du}$

$\Rightarrow u^3 du = \frac{1}{4} dn$

limits

u	1	0
n	1	0