Assessment - 4

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Sec: 06

Ans. to Q No - 1

Given, $\delta = \varepsilon_M = 1.1 \times 10^{-11}$

$a_0 = 5, \quad b_0 = 27$

$\therefore$ Number of iterations required,

$$n \geq \frac{\log(|b_0 - a_0|) - \log(\delta)}{\log(2)} - 1$$

or, $$n \geq \frac{\log(27-5) - \log(1.1 \times 10^{-11})}{\log(2)} - 1$$

or, $n \geq 40$ iterations.

(Ans.)

Ans to Q No - 2

a

Given, $f(x) = x^4 + 2x^2 - x - 3$

Now, $x^4 + 2x^2 - x - 3 = 0$

or, $2x^2 = x + 3 - x^4$

or, $x^2 = \dfrac{x+3-x^4}{2}$

or, $x = \left(\dfrac{x+3-x^4}{2}\right)^{1/2}$

Again, $x^4 + 2x^2 - x - 3 = 0$

or, $x^2(x^2+2) - x - 3 = 0$

or, $x^2(x^2+2) = x + 3$

or, $x^2 = \dfrac{x+3}{x^2+2}$

or, $x = \left(\dfrac{x+3}{x^2+2}\right)^{\frac{1}{2}}$

And, $x^4 + 2x^2 - x - 3 = 0$

or, $(4x^4 - 3x^4) + (4x^2 - 2x^2) + x - 3 = 0$

or, $3x^4 + 2x^2 + 3 = 4x^4 + 4x^2 - x$

or, $3x^4 + 2x^2 + 3 = x(4x^3 + 4x - 1)$

or, $$x = \frac{3x^4 + 2x^2 + 3}{4x^3 + 4x - 1}$$

[Showed]

b

For $g_1(x)$,

$g_1(1) = 1.225 = x_1$ $g_1(1.225) = 0.993 = x_2$ $g_1(0.993) = 1.229 = x_3$ $g_1(1.229) = 0.987 = x_4$	Value of $g(x)$ doesn't converge to any root and so no fixed point.

For $g_2(x)$,

$g_2(1) = 1.155 = x_1$

$g_2(1.155) = 1.116 = x_2$

$g_3(1.116) = 1.126 = x_3$

$g_4(1.126) = 1.124 \approx 1.12 = x_4$

Here, x_4 is the fixed point as the function $g(x)$ is converging to a single value. And, x_4 is also a root.

For $g_3(x)$,

$g_3(1) = 1.143 = x_1$

$g_3(1.143) = 1.124 = x_2$

$g_3(1.124) = 1.124 = x_3$

$g_3(1.124) = 1.124 = x_4$

Here, we can see that the function rapidly converges to the root.

c

For $g_1(x) = \left(\frac{x+3-x^4}{2}\right)^{\frac{1}{2}}$

$\Rightarrow g'(1.12) = 1.04 > 1$ [non-linear]

For $g_2(x) = \left(\frac{x+3}{x^2+2}\right)^{\frac{1}{2}}$

$\Rightarrow g'(1.12) = 0.25 < 1$ [~~non~~ Linear]

For $g_3(x) = \frac{3x^4+2x^2+3}{4x^3+4x-1}$

$\Rightarrow g'(1.12) = 0$ [Super linear]

So, we can see that $g_3(x)$ gives the best approximation as it is a super linear convergence.