Principles of Physics I (PHY111)

Lab

Experiment no: 4

Name of the experiment: Determination of the spring constant and effective mass of a given spiral spring

YOU HAVE TO BRING TWO GRAPH PAPERS (cm scale) TO DO THIS EXPERIMENT.

Theory

In this experiment a spring is suspended vertically from a clamp attached to a rigid frame work of heavy metal rods. At the bottom end (which is the free end) of the spring a load of mass, m_0 is suspended. So the force acting on the spring is the weight m_0 g of the load which acts vertically downward and the spring gets extended. Due to the elastic property of the spring, it tries to regain its initial length, hence applies a counter force on the load, which is called the restoring force of the spring. According to Hooke's law, within the elastic limit the magnitude of this restoring force is directly proportional to the extension of the spring and the direction of this restoring force is always towards the equilibrium position. If k is the spring constant of the spring and l is the extension of the spring, then restoring force = - $k\ l$

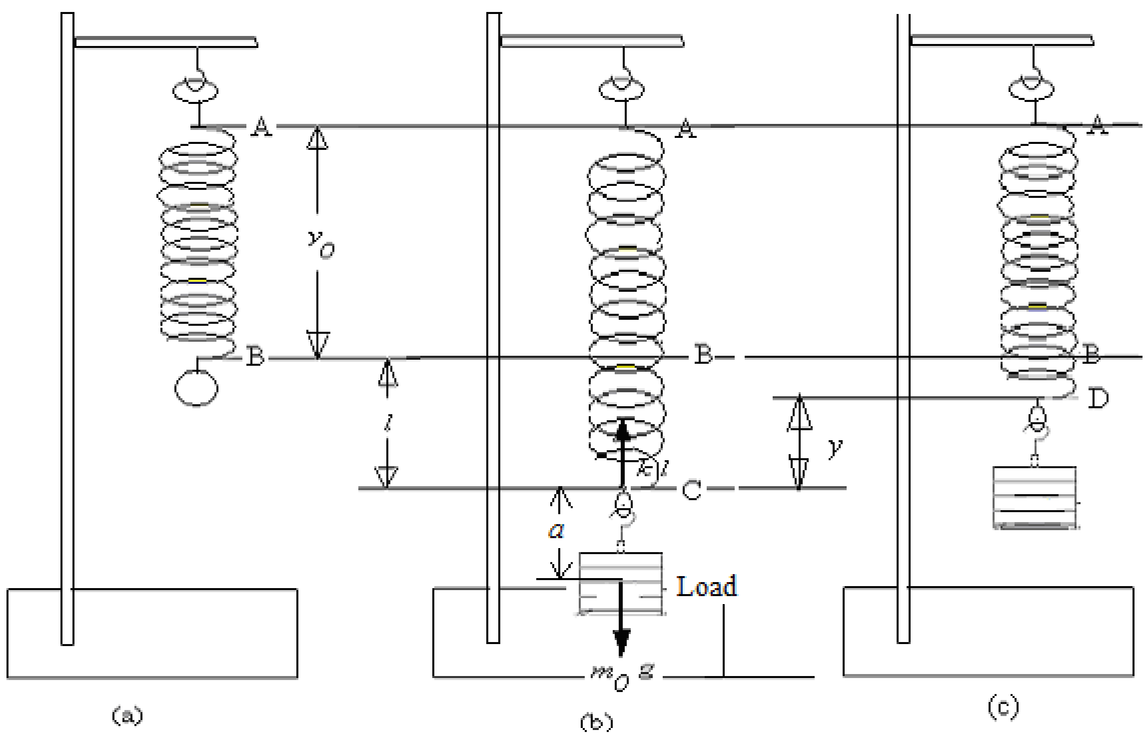

Figure 1: a) A vertically suspended spring, b) A load of mass m_0 is attached at the bottom end of the spring and c) The spring is oscillating, the load is y distance above the equilibrium position.

Let the spring is in equilibrium with mass m_0 attached as in figure 1 (a), and so we can write

$$m_0 g = kl$$

$$=> \; l = \frac{g}{k} m_0 \qquad (1)$$

Here k is the spring constant and g is the acceleration due to gravity.

Equation (1) is an equation of a straight line. The slope of this line is given by-

$$\text{Slope} = \frac{g}{k} \quad => k = \frac{g}{slope} \qquad (2)$$

We can plot l vs. m_0 graph and determine its slope and hence, the value of k.

If the load is slightly pulled down and released, the spring will oscillate simple harmonically. Suppose, at time t the velocity of the load is v and the spring is compressed by a distance y above the point C.

As you know from your earlier schools, if the mass of the spring were negligible then the period of oscillation would be given by $T = 2\pi\sqrt{\frac{m_0}{k}}$

Due to the mass, m of the spring an extra term m' will be added with the mass of the load m_0 in the above mentioned equation. So, the period of oscillation is,

$$T = 2\pi\sqrt{\frac{m_0 + m'}{k}} \qquad (3)$$

m' is called to be the effective mass of the spring. It can be shown that m' is related with the mass of the spring by the following equation:

$$m' = \frac{m}{3} \qquad (4)$$

Please see appendix A (provided in the soft copy of this script in the server) to learn how to derive equations (4) and (3).

From equation (3) we get,

$$T^2 = \frac{4\pi^2}{k} m_0 + \frac{4\pi^2}{k} m'$$

For different mass, m_0 of the load we find different periods of oscillation, T. If we draw a graph by plotting m_0 along X axis and corresponding T^2 along Y axis, it will be a straight line. The point where the line intersects the X axis, y-coordinate is 0, i.e., T^2 =0. We can find the X coordinate of the point, (i.e. the value of m_0 at that point) by putting T^2=0 in the above mentioned equation.

$$0 = \frac{4\pi^2}{k} m_0 + \frac{4\pi^2}{k} m'$$

=> $m_0 = -m'$

That means x coordinate of the point is equal to the negative value of the effective mass.

So, if we draw a T^2 vs. m_o graph, it will be a straight line and its x-interception gives us the effective mass of the spring.

Apparatus

A spiral spring, convenient masses with hanging arrangement, a hook attached to a rigid framework of heavy metal rods, weighing balance, stop watch and a scale

Procedure

1. Suspend the spring by a hook attached to the rigid framework of heavy metals.

2. Measure the length y_0 =AB (figure 1a) of the spring with a meter scale and write it down in section A of data sheet.

3. Attach a load of mass m_0=700 gm at the bottom end of the spring. So the spring gets stretched. Measure the length AC by using meter scale. Find the extension, l=BC (=AB-AC) and record m_0 and l in the Table 1 of section B shown in the data-sheet.

4. Pull down the load slightly and release it to oscillate. By using the stopwatch measure how much time, t the load takes to complete 20 oscillations and record it in the same table.

5. Repeat the steps 2, 3 and 4 for load of mass m_0 = 600, 500, 400, 300, 200 gm.

6. Draw an l vs. m_0 graphs which should be a straight line passing through the origin. Work out the slope of the straight line. Find out the spring constant of the spring by using equation 2.

7. Measure the mass of the spring, m by using the weight meter. Find the effective mass, $m' = \frac{m}{3}$ from this value.

8. Draw a T^2 vs. m_0 graph which should be a straight line which should intersect the negative X-axis. X-interception of the line is the effective mass, m' of the spring.

9. Compare the value of m' what you deduced in step 8 with the value of m' what you deduced in step 7.

Read carefully and follow the following instructions:

- **Please READ the theory carefully, TAKE printout of the 'Questions on Theory' and ANSWER the questions in the specified space BEFORE you go to the lab class.**
- **To get full marks for the 'Questions on Theory' portion, you must answer ALL of these questions CORRECTLY and with PROPER UNDERSTANDING, BEFORE you go to the lab class. However, to ATTEND the lab class you are REQUIRED to answer AT LEAST the questions with asterisk mark.**
- **Write down your NAME, ID, THEORY SECTION, GROUP, DATE, EXPERIMENT NO AND NAME OF THE EXPERIMENT on the top of the first paper.**
- **If you face difficulties to understand the theory, please meet us BEFORE the lab class. However, you must read the theory first.**
- **DO NOT PLAGIARIZE. Plagiarism will bring ZERO marks in this WHOLE EXPERIMENT. Be sure that you have understood the questions and the answers what you have written, and all of these are your own works. You WILL BE asked questions on these tasks in the class. If you plagiarize for more than once, WHOLE lab marks will be ZERO.**
- **After entering the class, please submit this portion before you start the experiment.**

Name: ______________________ ID: _______________ Sec: ___ Group: __ Date: ___________

Experiment no: ___

Name of the Experiment: ____

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Questions on Theory

*1) Draw the arrangement of this experiment. [0.5]

Ans:

*2) States Hooke's law for an elastic spring. [0.5]

Ans:

*3) Suppose, an external force is applied on a spring to stretch it. Extension of the spring is l. If the spring constant is k then what is the restoring force of the spring due to its elasticity? [0.5]

Ans:

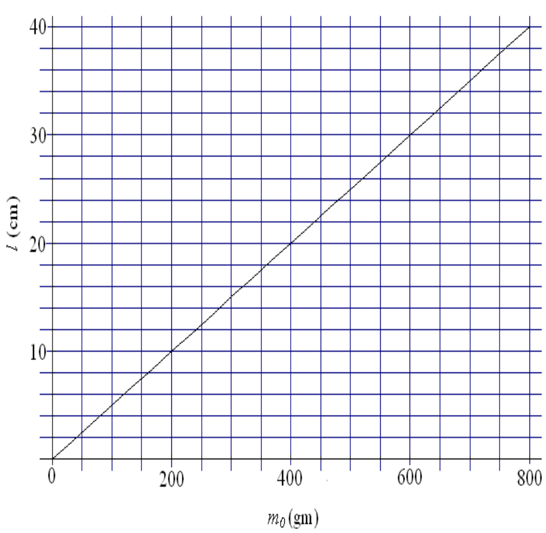
*4) If a load of m_0 is connected with the bottom end of a vertically suspended spring, the extension of the spring is *l*. The spring constant is *k*. At the equilibrium condition what is the relationship among m_0, *k*, *l* and g? [0.5]

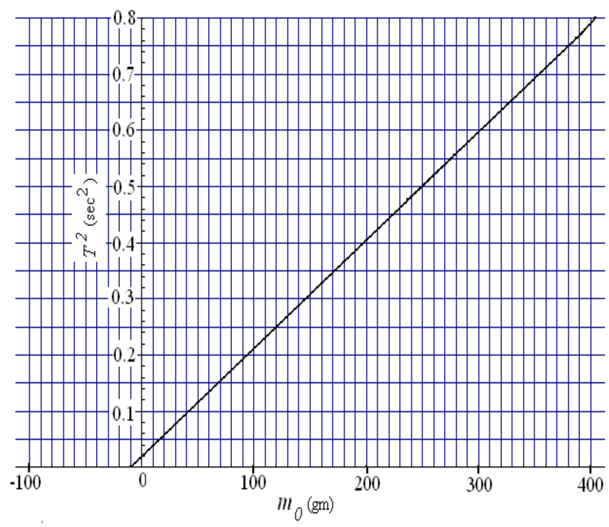
Ans:

5) What is the effective mass of the spring? (See Appendix A/textbook/websites) [1]

Ans:

6) An *l* vs. m_0 and a T^2 vs. m_0 graphs are shown below. Work out the spring constant and effective mass. [2]





[You may use additional page(s) to answer this question]

Ans:

- **Draw the data table(s) and write down the variables to be measured shown below (in the 'Data' section), using pencil and ruler BEFORE you go to the lab class.**

- **Write down your NAME and ID on the top of the page.**
- **This part should be separated from your Answers of "Questions on Theory" part.**
- **Keep it with yourself after coming to the lab.**
- **DO NOT forget to bring two GRAPH PAPERS.**

Data Data

A) Initial length of the spring, y_0 = AB = ____________________ cm

B) Table 1: Data of m_0, l and T

Mass of load m_0 (gm)	Extension of the spring, l (cm)	Time required to complete 20 oscillations, t (s)	Period of oscillation, T (s)	T^2 (s^2)

C) Mass of the spring by using weight-meter, m = _____________________ gm

D) Effective mass of the spring by using the value of its mass- measured by weight-meter, m' =__________gm

Please attach two graphs here.

- **READ the PROCEDURE carefully and perform the experiment by YOURSELVES. If you need help to understand any specific point draw attention of the instructors.**
- **DO NOT PLAGIARIZE data from other group and/or DO NOT hand in your data to other group. It will bring ZERO mark in this experiment. Repetition of such activities will bring zero mark for the whole lab.**
- **Perform calculations by following the PROCEDURE . Show every step in the Calculations section.**
- **Write down the final result(s).**

Calculations

Result:

- **TAKE printout of the ‘Questions for Discussions’ BEFORE you go to the lab class. Keep this printout with you during the experiment. ANSWER the questions in the specified space AFTER you have performed the experiment.**
- **Attach Data, Graphs, Calculations, Results and the Answers of ‘Questions for Discussions’ parts to your previously submitted Answers of ‘Questions on Theory’ part to make the whole lab report.**
- **Finally, submit the lab report before you leave the lab.**

Name: ___________________________ ID: __________________

Questions for Discussions

i) Was the value of the effective mass what you found from the graph consistent with the value of effective mass what you calculated by using the mass directly measured from the mass meter? [1]

Ans:

ii) Did you face any difficulty while measuring T for load of small mass, e.g., m_0 = 100 gm. Please mention. [0.5]

Ans:

iii) What will happen if you take load of a heavy mass? [0.5]

Ans:

Appendix A (Derivation of formula $T = 2\pi\sqrt{\frac{m_0 + m'}{k}}$)

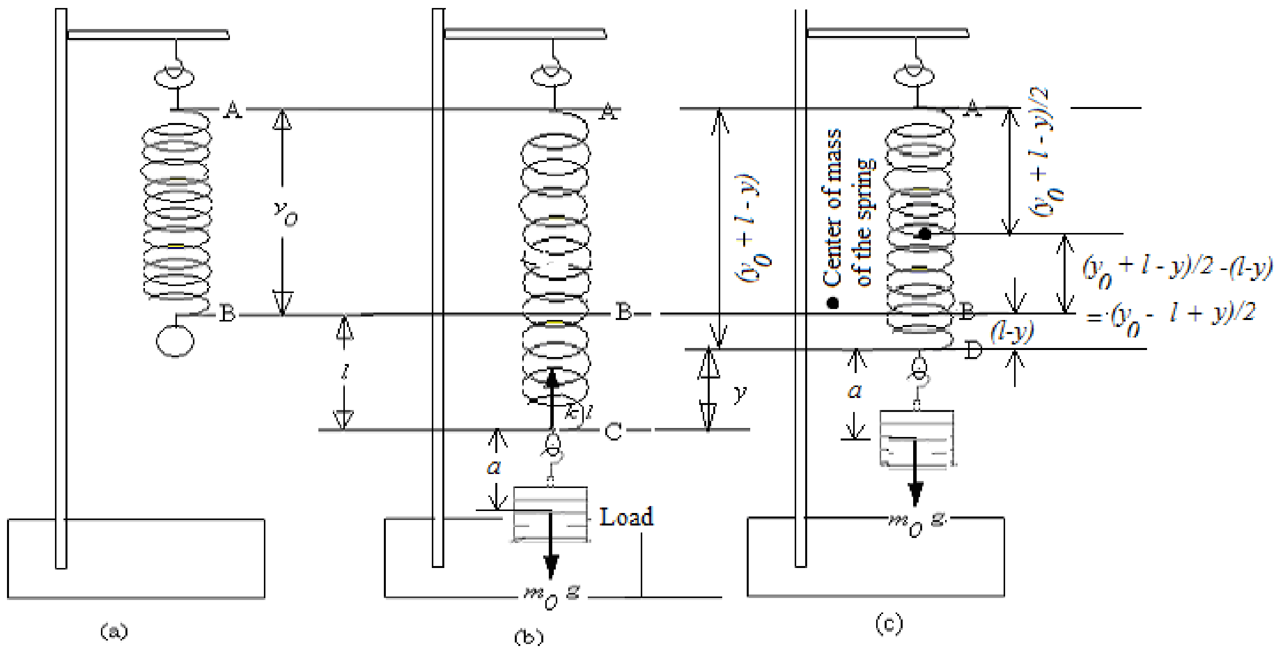


Figure 1: a) A vertically suspended spring, b) A load of mass m_0 is attached at the bottom end of the spring and c) The spring is oscillating, the load is y distance above the equilibrium position.

If the load is slightly pulled down and released, the spring will oscillate simple harmonically. Suppose, at time t the velocity of the load is v and the spring is compressed by a distance y above the point C. Now,

Total energy of the spring-load system, E = kinetic energy of the load + gravitational potential energy of the load + elastic potential energy of the spring + gravitational potential energy of the spring + kinetic energy of the spring

Now,

Kinetic energy of the load = $\frac{1}{2} m_0 v^2$

Gravitational potential energy of the load about point B = $-m_0 g(l - y + a)$ (Here, a is the separation between the free end of the spring and the center of mass of the load)

Elastic potential energy of the load about point B = $\frac{1}{2} k(l - y)^2$

Now, from figure 1 c the length AD = $y_0 + l - y$ (y_0 is the initial length of the spring)

Centre of mass of the spring lies at the midpoint of AD. The distance of centre of mass from point B =

$$y_0 - (y_0 + l - y)/2 = \frac{y_0 + y - l}{2}$$

Gravitational potential energy of the spring about point B = $mg\left(\frac{y_0 + y - l}{2}\right)$, where *m* is the mass of the spring.

Now, let's find out an expression of the kinetic energy of the spring. At a certain time different portions of the spring have different velocities. The top most point of the spring always remains stationary. The bottom most point of the spring moves at same velocity as the load.

Say, at time *t* the length of the wire is AD = *L*. Linear mass density of the wire, $\lambda = m / L$

It is observed that the velocity, v' of a particular point of the spring is directly proportion to its distance from the point of suspension (A), y. That means

$v' \propto y$

=> $v' = cy$, here c is an arbitrary constant (4)

When $y = L$ then $v' = v$

So, from (4) $c = v / L$

Therefore, from (4):

$$v' = \frac{v}{L} y \qquad (5)$$

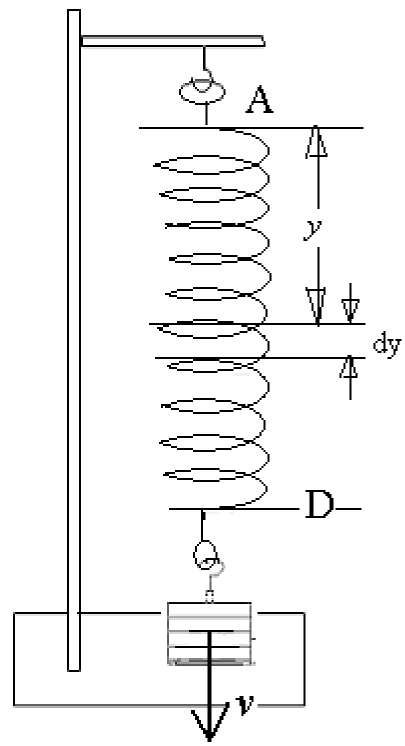


Figure 2: Calculation of effective mass

Now, let's consider a tiny portion of the spring at distance y below the point A, having length dy.

At time t the velocity of this portion is v'. The mass of this tiny portion = $dm = \lambda dy$

Therefore, the kinetic energy of this portion, $dK = \frac{1}{2}(v')^2 dm = \frac{1}{2}\left(\frac{v}{L} y\right)^2 \lambda dy = \frac{v^2}{2L^2} y^2 \frac{m}{L} dy = \frac{mv^2}{2L^3} y^2 dy$

The total kinetic energy of the spring, $K = \frac{mv^2}{2L^3}\int_0^L y^2 dy = \frac{mv^2}{2L^3}\left[\frac{y^3}{3}\right]_0^L = \frac{mv^2}{2L^3}\left(\frac{L^3}{3} - \frac{0^3}{3}\right) = \frac{1}{2}\frac{m}{3}v^2$

Here, $\frac{m}{3}$ is called to be the effective mass, m' of the spring.

$\therefore$ Kinetic energy of the spring = $\frac{1}{2}m'v^2$

Now, we can write the expression of the total energy, E of the spring and load system:

$$E = \frac{1}{2} m_0 v^2 - m_0 g(l - y) + \frac{1}{2} k(l - y)^2 + mg\left(\frac{y_0 + y - l}{2}\right) + \frac{1}{2} m' v^2 \qquad (6)$$

The total energy of the system remains conserved, i.e., it does not change with time. Therefore, $\frac{dE}{dt} = 0$

By differentiating equation (6) with respect to time, t we find,

$$\frac{1}{2} m_0 2v \frac{dv}{dt} - m_0 g\left(0 - \frac{dy}{dt}\right) + \frac{1}{2} k 2(l - y)\left(-\frac{dy}{dt}\right) + \frac{mg}{2}\frac{dy}{dt} + \frac{1}{2} m' 2v \frac{dv}{dt} = 0$$

=> $m_0 v \frac{dv}{dt} + m_0 gv - k(l - y)v + \frac{mg}{2} v + m' v \frac{dv}{dt} = 0$ [since, $\frac{dy}{dt} = v$]

=> $(m_0 + m')\frac{dv}{dt} + m_0 g - kl + ky + \frac{mg}{2} = 0$

=> $(m_0 + m')\frac{dv}{dt} + ky + \frac{mg}{2} = 0$, since $m_0 g - kl = 0$

=> $(m_0 + m')\frac{d^2 y}{dt^2} + ky + \frac{mg}{2} = 0$, since $\frac{dv}{dt} = \frac{d^2 y}{dt^2}$

=> $\frac{d^2 y}{dt^2} + \frac{k}{(m_0 + m')} y + \frac{mg}{2(m_0 + m')} = 0$

=> $\frac{d^2 y}{dt^2} + \frac{k}{(m_0 + m')}\left\{y + \frac{mg}{2k}\right\} = 0$ (7)

Let, $Y = y + \frac{mg}{2K}$

So,

$$\frac{dY}{dt} = \frac{dy}{dt} + 0$$

=> $\frac{d^2 Y}{dt^2} = \frac{d^2 y}{dt^2}$

Equation (7) becomes,

$$\frac{d^2 Y}{dt^2} + \frac{k}{(m_0 + m')} Y = 0$$

This is an equation of simple harmonic motion where the angular frequency ω is given by

$$\omega = \sqrt{\frac{k}{m_0 + m'}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{k}{m_0 + m'}}$$, Here T is the period of oscillation

$$\Rightarrow T = 2\pi\sqrt{\frac{m_0 + m'}{k}}$$