BRAC UNIVERSITY

MAT215

MATHEMATICS III: COMPLEX VARIABLES & LAPLACE TRANSFORMATIONS

Assignment 01

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Assignment Set: N

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Ans To The Question No. (1)

We know, $i^{123} = i^3$ and $i^{14} = -1$

Now,

$$\left|\frac{z_1 + z_2 + i^{123}}{z_1 - z_2 - i^{14}}\right|$$

$$= \left|\frac{-3 - 5i - 5 - 7i - i}{-3 + 5i + 5 + 7i + 1}\right|$$

$$= \left|\frac{-8 - 13i}{3 + 2i}\right|$$

$$= \left|\frac{(-8 - 13i)(3 - 2i)}{3^2 + 2^2}\right|$$

$$= \left|\frac{-24 + 16i - 39i + 26i^2}{9 + 4}\right|$$

$$= \left|\frac{-50 - 23i}{13}\right|$$

$$= \left|-\frac{50}{13} - i\frac{23}{13}\right| = \sqrt{\frac{233}{13}}(Ans)$$

Ans To The Question No. (2)

Given, $z = -2\sqrt{3} + 2i$

Comparing with the general form of complex number, we get
$x = -2\sqrt{3}$ and $y = 2$

Therefore, Modulus of $z = \sqrt{x^2 + y^2}$

$$= (-2\sqrt{3})^2 + 2^2$$

$$= 12 + 4$$

$$= 16$$

And, Argument of $z = \tan^{-1}(\frac{y}{x})$

$$= \tan^{-1}(\frac{2}{-2\sqrt{3}})$$

$$= 30^{\circ}(Ans)$$

Ans To The Question No. (3)

Given,

$$z^4 - \sqrt{2} + \sqrt{6}i = 0$$
$$or,\ z^4 = \sqrt{2} - \sqrt{6}i$$

Now, $|z| = 2\sqrt{2}$ and $\theta = \frac{5\pi}{3}$

$$So,\ (r(cos\theta + isin\theta))^4 = 2\sqrt{2}(cos\frac{5\pi}{3} + isin\frac{5\pi}{3})$$
$$or,\ r^4(cos4\theta + isin4\theta) = 2\sqrt{2}(cos\frac{5\pi}{3} + isin\frac{5\pi}{3})$$

Here,
$r^4 = 2\sqrt{2}$ therefore, $r = 1.3$
and, $4\theta = \frac{5\pi}{3} + n2\pi$ therefore, $\theta = \frac{5\pi}{12} + \frac{n2\pi}{4}$

When $n = 0$,

$$z = 1.3(cos\frac{5\pi}{12} + isin\frac{5\pi}{12})$$
$$= 0.34 + 1.26i$$

When $n = 1$,

$$z = 1.3(cos\frac{11\pi}{12} + isin\frac{11\pi}{12})$$
$$= -1.26 + 0.34i$$

When $n = 2$,

$$z = 1.3(cos\frac{17\pi}{12} + isin\frac{17\pi}{12})$$
$$= -0.34 - 1.26i$$

When $n = 3$,

$$z = 1.3(cos\frac{23\pi}{12} + isin\frac{23\pi}{12})$$
$$= 1.26 - 0.34i$$

Therefore, these four complex numbers are the roots of the given equation

Ans To The Question No. (4)

Given, $\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$

Now, $|z| = 1$ and $\theta = \frac{\pi}{6}$

Therefore,

$$\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^{12}$$
$$= \left(1(cos\frac{\pi}{6} + isin\frac{\pi}{6})\right)^{12}$$
$$= \left(cos\frac{12\pi}{6} + isin\frac{12\pi}{6}\right)$$
$$= 1 + 0i\ (Ans)$$

Ans To The Question No. (5)

Given,

$$z = \frac{1}{(1-2i)(1+3i)}$$

$$= \left[\frac{1}{1+3i-2i-6i^2}\right]$$

$$= \left[\frac{1}{1+3i-2i+6}\right]$$

$$= \left[\frac{1}{7+i}\right]$$

$$= \left[\frac{1(7-i)}{(7+i)(7-i)}\right]$$

$$= \left[\frac{7-i}{7^2+1^2}\right]$$

$$= \left[\frac{7-i}{50}\right]$$

$$= \left[\frac{7}{50} - \frac{1}{50}i\right]$$

$$= 0.14 - 0.02i$$

Therefore,

$Re(z) = 0.14$
$Im(z) = -0.02$ (Ans)