Assignment No- 03

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Course: MAT120

Section: 15

Semester: Spring 2020
Course ID: MAT 120
Course Title: Mathematics I
Section:15
Quiz: 03

Name: SHADAB IQBAL
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Date: 30.03.2020
Time: 15 Minutes
Marks obtained:

Answer the following questions: (Total Marks: 15)

Q1. Evaluate the following integral [8]

$$\int_0^{\frac{\pi}{6}} \sin^2 6x \ \cos^4 3x \ dx.$$

Solution:

$$= \int_0^{\pi/6} [\sin^2(2\cdot 3x) \cos^4 3x] \, dx$$

$$= \int_0^{\pi/6} (2\sin 3x \cos 3x)^2 \cos^4 x \, dx \quad [\because \sin 2x = 2\sin x \cos x]$$

$$= \int_0^{\pi/6} 4\sin^2 3x \cos^6 3x \, dx$$

Let, $3x = u$

$\Rightarrow dx = \frac{du}{3}$

Limits

x	0	$\pi/6$
u	0	$\pi/2$

$$= \frac{4}{3}\int_0^{\pi/2} \sin^2 u \cdot \cos^2 u \, du$$

$$= 4/3 \cdot \frac{\Gamma\left(\frac{2+1}{2}\right) \Gamma\left(\frac{6+1}{2}\right)}{2 \cdot \Gamma\left(\frac{2+6+2}{2}\right)}$$

$$\left[\because \int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \frac{\Gamma\frac{p+1}{2} \cdot \Gamma\frac{q+1}{2}}{2\,\Gamma\frac{p+q+2}{2}}\right]$$

$$= \frac{4}{3} \cdot \frac{\Gamma\frac{3}{2} \cdot \Gamma\frac{7}{2}}{2\,\Gamma 5}$$

(Ans)

Q2. Evaluate the following improper integral [7]

$$\int_0^{\frac{\pi}{2}} \tan x \ dx.$$

Solution:

Answer is on the next page

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$$= \lim_{k \to \pi/2^-} \int_0^k \tan x \, dx$$

$$= \lim_{k \to \pi/2^-} \Big[-\ln|\cos x| \Big]_0^k$$

$$= \lim_{k \to \pi/2^-} \left(-\ln(\cos k) + \ln(\cos 0) \right)$$

$$= -\ln(\cos \pi/2) + \ln(\cos 0)$$

$$= \infty + 0$$

$$= \infty$$

$\therefore$ Divergent.

(Ans)