Ans: to Q No - (1)

Advantage of central difference over forward difference is such :-

I Central difference method gives a better value of the slope we are looking for as they remain almost parallel to the original tangent line.

II In central difference, error $\propto h^2$ and in forward difference error $\propto h$. So central difference method gives lesser error as $0<h<1$.

Again, when we are unable to use ~~the~~ a node before the original node but are allowed to use a next node, only in that scenario forward difference method will have an advantage over central difference.

Ans. to Q No – 2

Given, $f(x) = \ln(x)$; $x = 3$; $h = 0.1$

Using forward difference method:

$$f'(x) = \frac{f(x+h) - f(x)}{x} = \frac{f(3.1) - f(3)}{0.1}$$

$$= \frac{\ln(3.1) - \ln(3)}{0.1} = 0.3278982282$$

and, $\text{error} = \frac{d}{dx}(\ln(x)) - 0.3278982282$

$$= \frac{1}{3} - 0.3278982282$$

$$= 5.435105103 \times 10^{-3}$$

Using central difference method:

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} = \frac{f(3.1) - f(2.9)}{0.2}$$

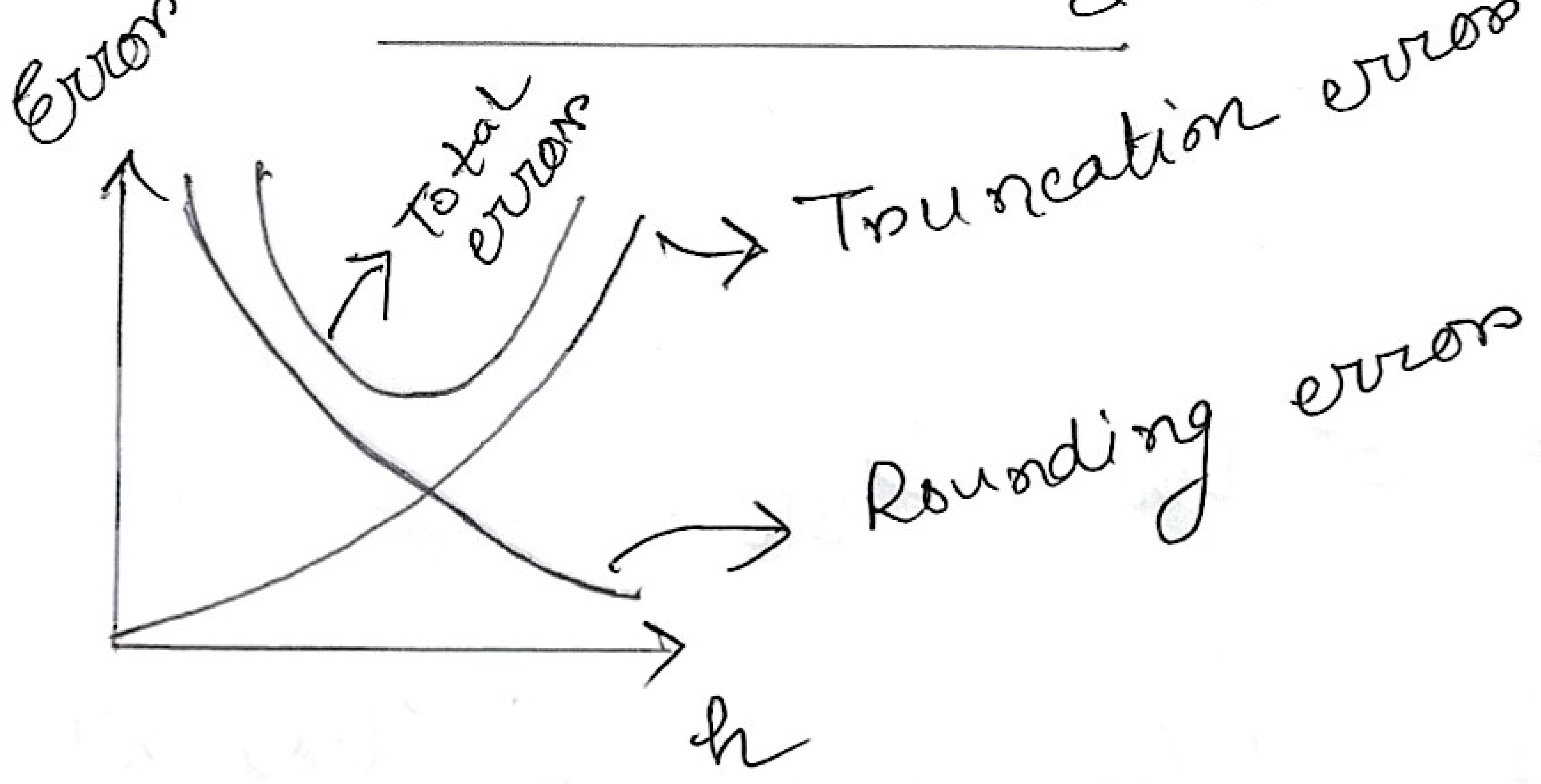
$$= \frac{\ln(3.1) - \ln(2.9)}{0.2} = 0.3334568725$$

and error $= \frac{1}{3} - 0.333456 8725$

$= -1.2353916 \times 10^{-4}$.

(Ans.)

Ans. to Q No - 3



So, from the graph we can see that, when our value of h is small, our rounding error is big despite our truncation error being small and vice-versa. This happens in computer because truncation error $\propto h^2$, but rounding error $\propto \frac{1}{h}$. ~~To~~ The total error is the sum of the aforementioned two errors. So, we can see that there is an optimal value of h where error is minimum.

Ans. to Q No - 4

$$D_{3h} = f^{(1)}(x) + \frac{f^{(3)}(x)}{6}(3h)^2 + \frac{f^{(5)}(x)}{120}(3h)^4 + O(h^6)$$

$$= f^{(1)}(x) + \frac{3}{2} f^{(3)}(x) h^2 + \frac{27}{40} f^{(5)}(x) h^4 + O(h^6)$$

Now, $3^2 D_h - D_{3h}$

$$= (3^2-1) f'(x) + \frac{9}{5!} f^{(5)}(x) h^4 - \frac{f^{(5)}(x)}{40} \times 27 h^4 + O(h^6)$$

$$\Rightarrow f'(x) + \frac{f^{(5)}(x)}{1} h^4 \left(\frac{9}{5!} - \frac{27}{40} \right) + O(h^6)$$

$$= \frac{3^2 D_h - D_{3h}}{3^2 - 1}$$

$$\therefore D_h^{(1)} = \frac{9 D_h - D_{3h}}{8}$$ (Ans)