Principles of Physics I (PHY111)

Lab

Experiment no: 2

Name of the experiment: Determination of the acceleration due to gravity, g by means of a compound pendulum

YOU HAVE TO BRING A GRAPH PAPER (cm scale) TO DO THIS EXPERIMENT.

Theory

A rigid body of any shape which is free to oscillate without any friction on a vertical plane is called compound pendulum. It swings harmonically back and forth about a vertical z-axis. The harmonic motion is due to gravitational force, which is directed downward, acting on the pendulum.

In this experiment you are going to measure the magnitude of acceleration due to gravity, g by observing the motion of a compound pendulum.

Let us consider a compound pendulum shown in figure 1. The pendulum can oscillate about a horizontal axis (perpendicular to the plane of this paper) passing through the point of suspension, O. We call it the axis of oscillation. At a certain time t the pendulum makes an angle θ with the vertical line from the point, O. Total mass of the pendulum is M. G is the centre of gravity of the pendulum. The gravitational attraction force of the earth on the pendulum (in other word, the weight of the pendulum), $F = M g$ acts at point G vertically downward. This force causes the pendulum to oscillate, i.e. a rotational motion, about the axis of oscillation.

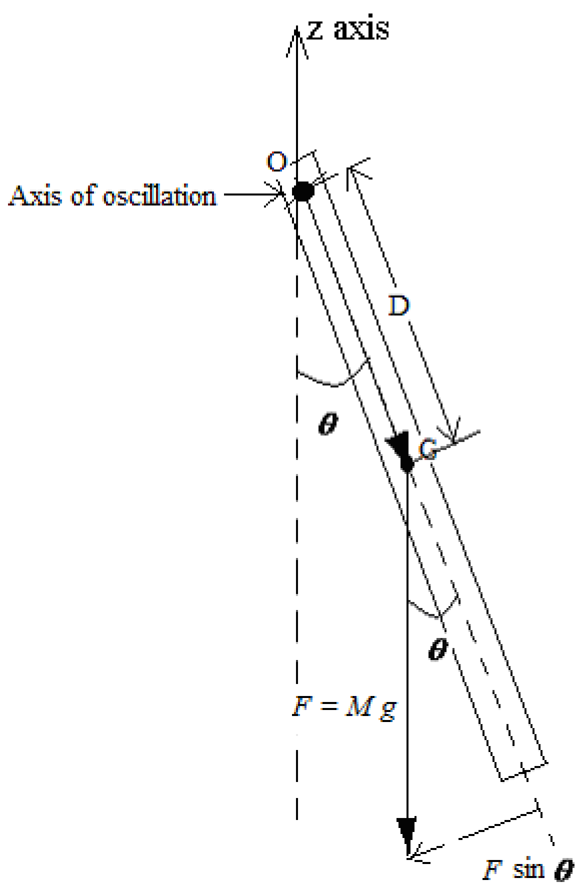


Figure 1: A compound pendulum

From our experience we notice that, when a force is applied on a body like a compound pendulum which can rotate around a fixed axis, then the angular acceleration of the body depends on two factors:

1) **The normal distance between the axis of rotation and the point at which the force is applied**: Suppose, if the force F (Figure 1) worked on the pendulum at point O, then it would not cause any rotation, i.e. angular acceleration would be zero.

On the other hand, if F worked at the very bottom edge of the pendulum then it would cause more angular acceleration (rotational tendency) than the angular acceleration when F acts at point G. Hence, for a certain force F angular acceleration of the pendulum is proportional to the normal distance between the axis of rotation and the point of application of the force *F*, i.e., OG (=*D*).

2) **The component of the force (*F*) which is perpendicular to the line passing through the axis of rotation and the point of at which the force is acting (OG):** Suppose, if the force *F* (figure: 1) were acting along the line OG ($\theta = 0$) then it will not cause any rotation of the pendulum. On the other hand, had the force *F* acted perpendicular to the line OG then it would cause maximum angular acceleration. In figure 1, the component of the force F perpendicular to the line OG is $F\sin\theta$. For a certain length of OG, angular acceleration of the pendulum is proportional to $F\sin\theta$.

From this discussion we can find the magnitude of the physical quantity called 'torque' (τ) which causes the angular acceleration of the pendulum, in the following way-

$$\tau = -DF\sin\theta = -MgD\sin\theta \qquad (1)$$

Now, the question is why the negative sign appears in this equation (1). Look at figure 1. We are measuring the angular displacement θ in an anticlockwise direction. On the other hand the twisting effect of the force *F*, i.e. the torque τ, tries to rotate the pendulum in a clockwise direction. Hence, we put a negative sign to indicate that the torque is acting opposite to the angular displacement.

Now, we will introduce a physical quantity called moment of inertia of a body. We know that mass of a body is a measure of its inertia in the case of linear motion. If a same amount of force is applied on bodies of different masses then their accelerations will be different depending on their masses. More massive a body is less is its acceleration due to a certain force. Similarly, the moment of inertia, *I* of a body about an axis of rotation, is a quantity which measures its inertia in the case of rotational motion. Due to the action of a same amount of torque in the bodies of different moments of inertia, their angular accelerations will also be different. More the moment of inertia is of a body, less is its angular acceleration due to the action of a certain torque.

Moment of inertia depends on the position and orientation of the axis of rotation and how the mass of the body is distributed around the axis of rotation. The farther away the mass of the body is distributed from the axis of rotation, more is the moment of inertia of the body. If I is the moment of inertia of the pendulum and α is the angular acceleration about point O, then the torque τ acting on the pendulum can be expressed in the following way:

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2} \qquad (2)$$

Here, we use the relation $\alpha = \frac{d^2\theta}{dt^2}$

Recall the equation *F* = *ma* and compare it with equation (2). In equation (2) the roles of *F*, *m* and *a* are played by τ, *I* and α.

From equations (1) and (2) we can write,

$$I\frac{d^2\theta}{dt^2} = -MgD\sin\theta$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{MgD}{I}\sin\theta = 0 \qquad (3)$$

Now if the angle θ is very small and measured in radian we can write $\sin\theta \approx \theta$

So, equation (3) simplifies to,

$$\frac{d^2\theta}{dt^2} + \frac{MgD}{I}\theta = 0 \Rightarrow \frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \qquad (4)$$

which is the equation of a simple harmonic motion, where ω is the angular frequency of this simple harmonic motion and given by,

$$\omega = \sqrt{\frac{MgD}{I}}$$

$$\Rightarrow \frac{2\pi}{T} = \sqrt{\frac{MgD}{I}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{I}{MgD}} \qquad (5)$$

Here T is the period of oscillation.

We said that, the moment of inertia of a body depends on the position of the axis of rotation. Now, think about an axis which passes through the center of mass of the pendulum, G and parallel to the actual axis of rotation.

Suppose, The moment of inertia of the pendulum about this axis is I_G. Using parallel axis theorem we can compute I from the following equation,

$$I = MD^2 + I_G \qquad (6)$$

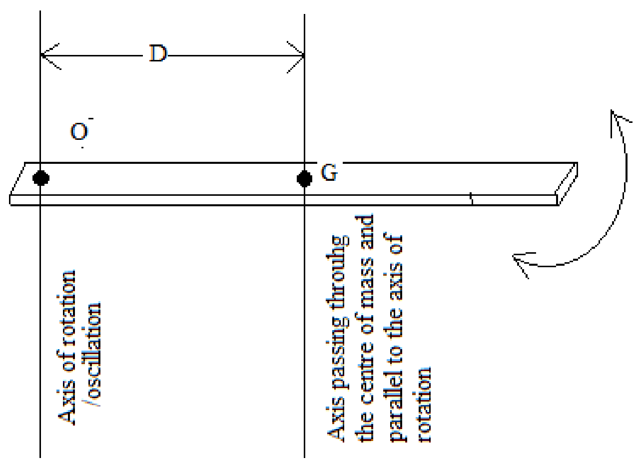


Figure 2: Axis of rotation and the axis passing through G and parallel to the axis of rotation

This theorem will be discusses in your theory class.

I_G can be represented in the following way:

$$I_G = MK^2 \qquad (7)$$

Here, K is the radius of gyration of the pendulum about that axis which we have imagined to be passing through the center of gravity. What radius of gyration is, will be discusses in your theory class.

Using equations (5), (6) and (7) we get,

$$T = 2\pi\sqrt{\frac{(D^2 + K^2)/D}{g}} \qquad (8)$$

Recall the equation of the time period of a simple pendulum which you studied in your school:

$T = 2\pi\sqrt{\frac{L}{g}}$, here L was the length between the center of mass of the bob and point at which the thread is connected with the stand. We call it the length of the simple pendulum.

Notice that, in equation (8) the term $\frac{D^2 + K^2}{D}$ plays the role of L.

Let us imagine a simple pendulum whose length L is equal to $\frac{D^2 + K^2}{D}$. Then the time period of this simple pendulum is equal to the time period of our compound pendulum. We can say that this simple pendulum is equivalent to our compound pendulum. For such an equivalent simple pendulum we can write,

$$L = \frac{D^2 + K^2}{D} \tag{9}$$

By making D as subject we can solve this equation.

$$L = \frac{D^2 + K^2}{D}$$
$$\Rightarrow D^2 - LD + K^2 = 0$$

This is a quadratic equation and its solutions are-

$$D_1 = \frac{L + \sqrt{L^2 - 4K^2}}{2} \text{ and } D_2 = \frac{L - \sqrt{L^2 - 4K^2}}{2}$$

Hence, we can say that, for each half of the compound pendulum, there are two specific points of suspensions. They are D_1 and D_2 distances away from the centre of gravity (G). If we hinge the compound pendulum at these two points, then we will find a same equivalent simple pendulum having length L. Moreover, if L is same for D_1 and D_2, then the period of oscillation, T is also same for them.

We can summarize that, if we hinge the compound pendulum at any point and measure the corresponding period of oscillation, T then there is another possible point of suspension at the same half of the pendulum such that if we hinge the pendulum there then we will find the same period of oscillation, T. In both cases we will have a same equivalent simple pendulum of length L. D_1 and D_2 are the distances of those two points of suspension from the center of gravity (G) separately. Now, we can write,

$$D_1 + D_2 = \frac{L + \sqrt{L^2 - 4K^2} + L - \sqrt{L^2 - 4K^2}}{2} = \frac{2L}{2} = L \tag{10}$$

If we add up the distances of the points of suspension from G, for which a same period of oscillation is observed, then that gives us the corresponding length L of equivalent simple pendulum.

If we plot a period, T vs. distance between the point of suspension, O and the centre of mass, D graph for a single half of the compound pendulum we get a curve similar to the Figure 3.

A line PAB parallel to X axis is drawn which cuts the Y axis at P and the curves at points A and B. For D_1=PB and D_2 =PA the period of oscillation, T is same.

Now according to equation (10)

$L = D_1 + D_2 = PA + PB$

From the graph we measure PA and PB to compute L using above mentioned equation. Then we measure T, i.e. y coordinate of point P.

From equation (8) we can write,

$$g = 4\pi^2 \frac{L}{T^2} \tag{11}$$

Putting the values of L and T in equation (11) we can easily calculate g.

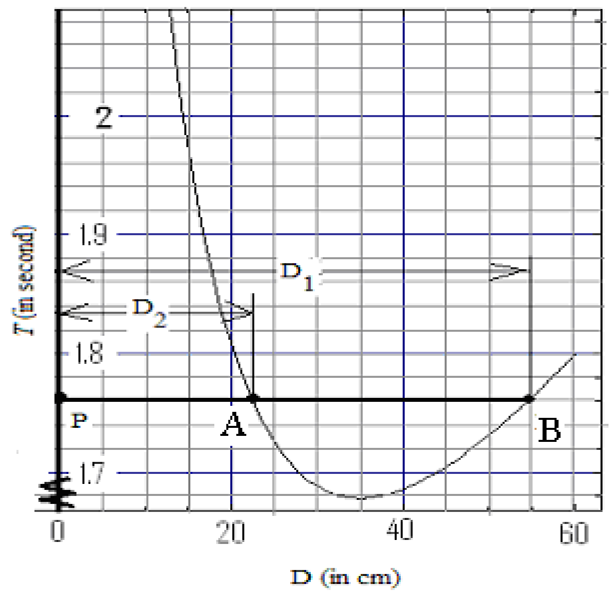


Figure 3: T vs. D graph for each half of the pendulum.

APPARATUS

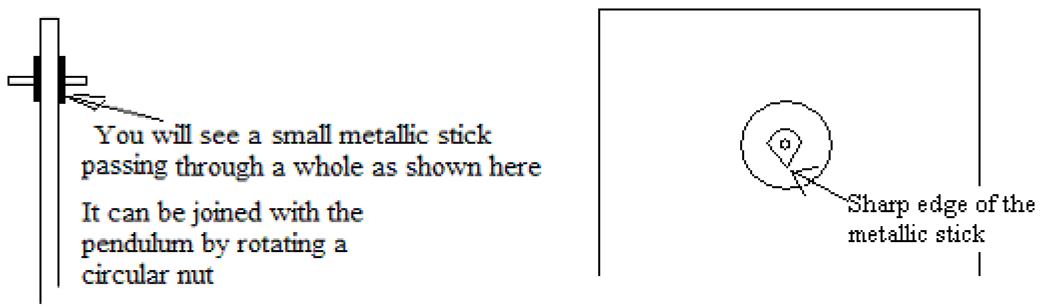
A compound pendulum, a stop watch and a meter scale.

PROCEDURE

1. First count how many number of holes are there in the compound pendulum.

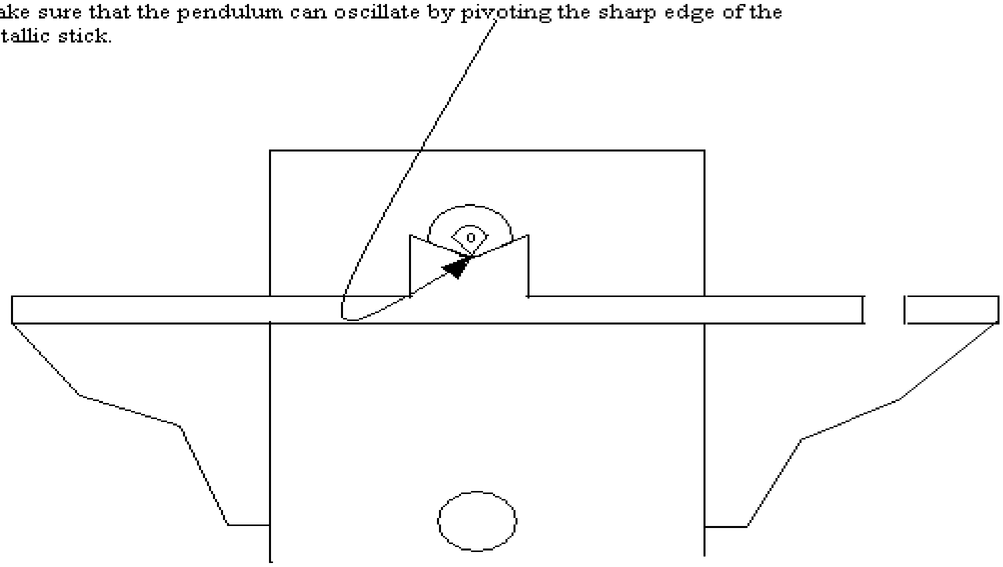
2. Mass of the pendulum is uniformly distributed. So, the centre of mass lies at the centre of the middle hole. Mark this hole, so that you can easily identify it.

3. By using a small scale measure the separation between the centers of two consecutive holes.



4. Join the metallic stick with the pendulum, by passing it through the first hole at one end of the pendulum. Make the joint tight by rotating the nut. Make sure that the sharp edge of the stick is pointed downward.

5. Now place the pendulum on the holder as it is shown in the following figure. Make sure that the pendulum can oscillate by pivoting the sharp edge of the metallic stick.



6. Write down the distance between the centre of mass and the point, O (here, the midpoint of the first hole), *D* in the data-table 1.

7. Pull the pendulum slightly and release it to oscillate. Make sure that the pendulum oscillates in a single plane.

8. Record the time, *t* taken by the pendulum to complete 10 oscillations. Write down it in the data table-1. Next work out the period of oscillation, *T* (time required to complete a single oscillation), by dividing *t* by 10

9. Now repeat the whole process for second hole, third hole, fourth hole, and so on, up to the nearest hole of the centre of gravity, G of the pendulum.

10. Draw a *T* vs. *D* curve, as shown in Figure (3).

12. Now, deduce the value of g as discussed in the theory.

Read carefully and follow the following instructions:

- **Please READ the theory carefully, TAKE printout of the 'Questions on Theory' and ANSWER the questions in the specified space BEFORE you go to the lab class.**
- **To get full marks for the 'Questions on Theory' portion, you must answer ALL of these questions CORRECTLY and with PROPER UNDERSTANDING, BEFORE you go to the lab class. However, to ATTEND the lab class you are REQUIRED to answer AT LEAST the questions with asterisk mark.**
- **Write down your NAME, ID, THEORY SECTION, GROUP, DATE, EXPERIMENT NO AND NAME OF THE EXPERIMENT on the top of the first paper.**
- **If you face difficulties to understand the theory, please meet us BEFORE the lab class. However, you must read the theory first.**
- **DO NOT PLAGIARIZE. Plagiarism will bring ZERO marks in this WHOLE EXPERIMENT. Be sure that you have understood the questions and the answers what you have written, and all of these are your own works. You WILL BE asked questions on these tasks in the class. If you plagiarize for more than once, WHOLE lab marks will be ZERO.**
- **After entering the class, please submit this portion before you start the experiment.**

Name: ________________________ ID: _________________ Sec: ___ Group: __ Date: _____________

Experiment no: ___

Name of the Experiment: ____

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Questions on Theory

*1) What is a compound pendulum? [0.5]

Ans:

*2) Draw a figure of a compound pendulum of mass *M*. Clearly show the point at which the pendulum is hinged (point O), center of mass of the pendulum (G), distance between O and G (D), the force(s) acting on the pendulum and its angular displacement about the vertical axis passing through O. [0.5]

Ans:

*3) What is the torque acting on the compound pendulum in terms of *M*, g, *D* and θ? [0.5]

Ans:

*4) If the moment of inertia of the compound pendulum about its axis of rotation is *I* and at a certain moment its angular acceleration is α, then what is the torque acting on the pendulum? [0.5]

Ans:

*5) Equate the both expressions of the torque acting on the compound pendulum and find out an expression of period of oscillation. [0.5]

Ans:

*6) $I_G = MK^2$ is the moment of inertia of the compound pendulum about an axis passing through the center of mass and parallel to the real axis of rotation. Here, K is the radius of gyration about this axis and M is the mass of the pendulum. Applying the parallel axis theorem what is the moment of inertia of the compound pendulum, I about the axis of rotation? Distance between the center of mass the axis of rotation is D. [0.5]

Ans:

*7) What is the length, L of a simple pendulum equivalent to our compound pendulum, in terms of K and D? [0.5]

Ans:

*8) Solve the equation of L by making D as subject. [0.5]

Ans:

*9) Show that the addition of two roots of the equation is equal to the length of equivalent simple pendulum, L. [0.5]

Ans:

10) See Figure 3. Find out the value of g approximately from this figure. [0.5]

Ans:

- **Draw the data table(s) and write down the variables to be measured shown below (in the 'Data' section), using pencil and ruler BEFORE you go to the lab class.**

- **Write down your NAME and ID on the top of the page.**
- **This part should be separated from your Answers of "Questions on Theory" part.**
- **Keep it with yourself after coming to the lab.**
- **DO NOT forget to bring a GRAPH PAPER.**

Data

Please attach the *T* vs. *D* graph.

Table 1: Observation for the time period *T* and the distance of the point, O from the centre of mass, *D* (for the first half)

Hole no	Distance of the hole from the centre of mass, *D*(cm)	Time for 10 oscillations, t (sec)	Period,*T* (sec)
1			
2			
3			
4			
5			
6			
7			
8			
9			

- **READ the PROCEDURE carefully and perform the experiment by YOURSELVES. If you need help to understand any specific point draw attention of the instructors.**
- **DO NOT PLAGIARIZE data from other group and/or DO NOT hand in your data to other group. It will bring ZERO mark in this experiment. Repetition of such activities will bring zero mark for the whole lab.**
- **Perform calculations by following the PROCEDURE . Show every step in the Calculations section.**
- **Write down the final result(s).**

Calculations

Result:

- **TAKE printout of the 'Questions for Discussions' BEFORE you go to the lab class. Keep this printout with you during the experiment. ANSWER the questions in the specified space AFTER you have performed the experiment.**
- **Attach Data, Graph, Calculations, Results and the Answers of 'Questions for Discussions' parts to your previously submitted Answers of 'Questions on Theory' part to make the whole lab report.**
- **Finally, submit the lab report before you leave the lab.**

Name: ___________________________ ID: ___________________

Questions for Discussions

i) What is the percentage of error occurs in measuring the value of g? [0.5]

Ans:

ii) What is the advantage of using a compound pendulum? [1]

Ans:

iii) What are the difficulties of using a compound pendulum to measure g? [You may use the textbook “Practical Physics for Degree Students” written by Dr. Gias Uddin Ahmed]_ [0.5]

Ans: