MAT 216

Assignment 1

Solutions

1. The vector equation:

$$c\,v + d\,w = b$$

$$\therefore\ c\begin{pmatrix}2\\-1\end{pmatrix} + d\begin{pmatrix}-1\\2\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

The required equations for c & d just come from two components separately,

$$2c - d = 1 \ \ \&\ \ -c + 2d = 0\,.$$

2. (i) Using P as the initial point, we move 2 units in the positive x-direction and -1 units in the positive y-direction to arrive at the terminal point $P' = (5,1)$ as drawn in Figure…

The magnitude of $\vec{v}$ is determined directly from the component form:

$$\|\vec{v}\| = \sqrt{2^2 + (-1)^2} = \sqrt{5}\,.$$

(ii) The component form of the vector w:

$$\begin{aligned} RS &= (x_2 - x_1, y_2 - y_1) \\ &= (-1-(-3), 2-(-2)) \\ &= (2,4). \end{aligned}$$

(iii) Using Q as the initial point, we move 2 units in the positive x-direction, -1 unit in the positive y-direction, and 3 units in the positive z-direction to arrive at the terminal point $Q = (3,0,4)$, illustrated in Figure…

The magnitude of $\vec{u}$ is:

$$\|\vec{u}\| = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{14}\,.$$

3. (i) $v_3 = -v_1 - v_2$, (ii) $v_4 = 5v_1 + 3v_3$, (iii) $v_4 = 2v_1 - 3v_2$.

4. (i) $5u - 2v = 5\begin{pmatrix} 5 \\ 3 \\ -4 \end{pmatrix} - 2\begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 27 \\ 5 \\ -24 \end{pmatrix}$.

   (ii) $-2u + 4v - 3w = \begin{pmatrix} -23 \\ 17 \\ 22 \end{pmatrix}$.

5. $v_4 = -6v_1 + 3v_2 + 2v_3$.

6. Find the dot product of each pair of vectors:

   $u.v = 15 - 16 + 1 = 0, \quad v.w = 3 + 8 + 3 = 14, \quad u.w = 5 - 8 + 3 = 0$.

   So (i) u and v are perpendicular, (ii) u and w are perpendicular, (iii) v and w are not .

7. (i) $u = \overrightarrow{PQ} = Q - P = (6-1, 1-(-2), -5-4) = (5, 3, -9)$,

   (ii) $u = \overrightarrow{PQ} = Q - P = (7-2, 1-3, 4+6, -8-5) = (5, -2, 10, -13)$.