1) Here, $x_0 = 0$, $x_1 = 1$, $x_2 = -1$

$f(x_0) = 5$ $\quad f(x_1) = 20$, $\quad f(x_2) = 10$

$$P_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$= f[x_0] + f[x_0, x_1](x - x_0)$$

$$+ f[x_0, x_1, x_2](x - x_0)(x - x_1)$$

Now

$x_0 = 0$ $\quad f[x_0] = 5$

$f[x_0, x_1] = 15$

$x_1 = 1$ $\quad f[x_1] = 20$ $\qquad f[x_0, x_1, x_2] = 10$

$f[x_1, x_2] = 5$

$x_2 = -1$ $\quad f[x_2] = 10$

$\therefore\ b_0 = f[x_0] = 5$

$b_1 = f[x_0, x_1] = 15$

$b_2 = f[x_0, x_1, x_2] = 10$

2) Therefore,

$$P_2(x) = 5 + 15(x - x_0) + 10(x - x_0)(x - x_1)$$

$$= 5 + 15(x) + 10(x)(x-1)$$

Now,

$$P_2(0.6) = 5 + (15 \times 0.6) + (10 \times 0.6)(-0.4)$$

$$= 11.6$$

and, $P_2(-0.7) = 5 + (15 \times -0.7) + (10 \times -0.7 \times -1.7)$

$$= 6.4$$

3) $f(x) = \sin^2\left(\frac{x}{2}\right)$

$x_0 = -\pi/2;\ x_1 = 0;\ x_2 = \frac{\pi}{2}$

Now,

$$|f(x) - P_2(x)| = \frac{f^3(\xi)}{3!}\left(x+\frac{\pi}{2}\right)\left(x-\frac{\pi}{2}\right)x$$

$$= \frac{-\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}{6}\left(x^2 - \frac{\pi^2}{4}\right)x$$

$$= \frac{\sin(x)}{12}\left(x^3 - \frac{\pi^2}{4}x\right) \quad \left[\because 2\cos(a)\sin(a) = \sin(2a)\right]$$

Now,

$\max \sin(\xi)$ where $\xi \in [-1, 1]$

$= \sin(1)$

Again, $w(x) = x^3 - \frac{\pi^2}{4}x$

$\therefore w'(x) = 3x^2 - \frac{\pi^2}{4} = 0$

$\Rightarrow 3x^2 = \frac{\pi^2}{4}$

$\Rightarrow x = \pm\frac{\pi}{2\sqrt{3}}$

$w(x)$ at $x = \pm \frac{\pi}{2\sqrt{3}}, \pm 1$

x	$w(x)$
$-\frac{\pi}{2\sqrt{3}}$	1.492
$\frac{\pi}{2\sqrt{3}}$	-1.492
-1	1.467
1	-1.467

$\therefore$ max error $= \frac{\sin(1)}{12} \times 1.492$

$= 0.1046$

(Ans.)