1. Aufgabe

a.

$$\forall Z \exists Y \forall X (f(X,Y) <=> (f(X,Z) \& \sim f(X,X)))$$

Simplify
$\forall Z \exists Y \forall X (f(X,Y) => (f(X,Z) \& \sim f(X,X))) \& ((f(X,Z) \& \sim f(X,X)) => f(X,Y))$
$\forall Z \exists Y \forall X (\sim f(X,Y) | (f(X,Z) \& \sim f(X,X))) \& (\sim (f(X,Z) \& \sim f(X,X)) | f(X,Y))$

Move negations in
$\forall Z \exists Y \forall X (\sim f(X,Y) | (f(X,Z) \& \sim f(X,X))) \& (\sim f(X,Z) | f(X,X) | f(X,Y))$

Skolemize
$\exists Y \forall X (\sim f(X,Y) | (f(X,Z) \& \sim f(X,X))) \& (\sim f(X,Z) | f(X,X) | f(X,Y))$
$\gamma = \{Z\}$
$\forall X (\sim f(X, skY(Z)) | (f(X,Z) \& \sim f(X,X))) \& (\sim f(X,Z) | f(X,X) | f(X, skY(Z)))$
$\gamma = \{Z\}$
$(\sim f(X, skY(Z)) | (f(X,Z) \& \sim f(X,X))) \& (\sim f(X,Z) | f(X,X) | f(X, skY(Z)))$
$\gamma = \{Z, X\}$

Distribute disjunctions
$(\sim f(X, skY(Z)) | f(X,Z)) \& (\sim f(X, skY(Z)) | \sim f(X,X)) \& (\sim f(X,Z) | f(X,X) | f(X, skY(Z)))$

Convert to CNF
$\{\sim f(X, skY(Z)) | f(X,Z), \sim f(X, skY(Z)) | \sim f(X,X), \sim f(X,Z) | f(X,X) | f(X, skY(Z))\}$

b.

$\forall X \forall Y(q(X,Y) <=> \forall Z(f(Z,X) <=> f(Z,Y)))$

Simplify

$\forall X \forall Y(\sim q(X,Y) | \forall Z((\sim f(Z,X) | f(Z,Y)) \& (\sim f(Z,Y) | f(Z,X))))$
$\quad \& (\sim \forall Z((\sim f(Z,X) | f(Z,Y)) \& (\sim f(Z,Y) | f(Z,X))) | q(X,Y))$

Move negations in

$\forall X \forall Y(\sim q(X,Y) | \forall Z((\sim f(Z,X) | f(Z,Y)) \& (\sim f(Z,Y) | f(Z,X))))$
$\quad \& (\exists Z((f(Z,X) \& \sim f(Z,Y)) | (f(Z,Y) \& \sim f(Z,X))) | q(X,Y))$

Rename variables

$\forall X \forall Y(\sim q(X,Y) | \forall Z((\sim f(Z,X) | f(Z,Y)) \& (\sim f(Z,Y) | f(Z,X))))$
$\quad \& (\exists A((f(A,X) \& \sim f(A,Y)) | (f(A,Y) \& \sim f(A,X))) | q(X,Y))$

Skolemize

$\quad (\sim q(X,Y) | ((\sim f(Z,X) | f(Z,Y)) \& (\sim f(Z,Y) | f(Z,X))))$
$\quad \& (((f(skA(X,Y),X) \& \sim f(skA(X,Y),Y)) | (f(skA(X,Y),Y) \& \sim f(skA(X,Y),X))) | q(X,Y))$

Distribute disjunctions

$\quad (\sim q(X,Y) | \sim f(Z,X) | f(Z,Y))$
$\quad \& (\sim q(X,Y) | \sim f(Z,Y) | f(Z,X))$
$\quad \& (f(skA(X,Y),X) | f(skA(X,Y),Y) | q(X,Y))$
$\quad \& (f(skA(X,Y),X) | \sim f(skA(X,Y),X) | q(X,Y))$
$\quad \& (\sim f(skA(X,Y),Y) | f(skA(X,Y),Y) | q(X,Y))$
$\quad \& (\sim f(skA(X,Y),Y) | \sim f(skA(X,Y),X) | q(X,Y))$

Convert to CNF

$\quad \{(\sim q(X,Y) | \sim f(Z,X) | f(Z,Y))$
$\quad ,(\sim q(X,Y) | \sim f(Z,Y) | f(Z,X))$
$\quad ,(f(skA(X,Y),X) | f(skA(X,Y),Y) | q(X,Y))$
$\quad ,(f(skA(X,Y),X) | \sim f(skA(X,Y),X) | q(X,Y))$
$\quad ,(\sim f(skA(X,Y),Y) | f(skA(X,Y),Y) | q(X,Y))$
$\quad ,(\sim f(skA(X,Y),Y) | \sim f(skA(X,Y),X) | q(X,Y))\}$