1. Aufgabe

a.

$$\forall Z \exists Y \forall X (f(X,Y) <=> (f(X,Z) \& \sim f(X,X)))$$

Simplify
$\forall Z \exists Y \forall X (f(X,Y) => (f(X,Z) \& \sim f(X,X))) \& ((f(X,Z) \& \sim f(X,X)) => f(X,Y))$
$\forall Z \exists Y \forall X (\sim f(X,Y) | (f(X,Z) \& \sim f(X,X))) \& (\sim (f(X,Z) \& \sim f(X,X)) | f(X,Y))$

Move negations in
$\forall Z \exists Y \forall X (\sim f(X,Y) | (f(X,Z) \& \sim f(X,X))) \& (\sim f(X,Z) | f(X,X) | f(X,Y))$

Skolemize
$\exists Y \forall X (\sim f(X,Y) | (f(X,Z) \& \sim f(X,X))) \& (\sim f(X,Z) | f(X,X) | f(X,Y))$
$\gamma = \{Z\}$
$\forall X (\sim f(X, skY(Z)) | (f(X,Z) \& \sim f(X,X))) \& (\sim f(X,Z) | f(X,X) | f(X, skY(Z)))$
$\gamma = \{Z\}$
$(\sim f(X, skY(Z)) | (f(X,Z) \& \sim f(X,X))) \& (\sim f(X,Z) | f(X,X) | f(X, skY(Z)))$
$\gamma = \{Z, X\}$

Distribute disjunctions
$(\sim f(X, skY(Z)) | f(X,Z)) \& (\sim f(X, skY(Z)) | \sim f(X,X)) \& (\sim f(X,Z) | f(X,X) | f(X, skY(Z)))$

Convert to CNF
$\{\sim f(X, skY(Z)) | f(X,Z), \sim f(X, skY(Z)) | \sim f(X,X), \sim f(X,Z) | f(X,X) | f(X, skY(Z))\}$

b.

$\forall X \forall Y(q(X,Y) <=> \forall Z(f(Z,X) <=> f(Z,Y)))$

Simplify

$$\begin{aligned}&\forall X \forall Y(\sim q(X,Y)|\forall Z((\sim f(Z,X)|f(Z,Y))\&(\sim f(Z,Y)|f(Z,X))))\\ &\qquad \&(\sim \forall Z((\sim f(Z,X)|f(Z,Y))\&(\sim f(Z,Y)|f(Z,X)))|q(X,Y))\end{aligned}$$

Move negations in

$$\begin{aligned}&\forall X \forall Y(\sim q(X,Y)|\forall Z((\sim f(Z,X)|f(Z,Y))\&(\sim f(Z,Y)|f(Z,X))))\\ &\qquad \&(\exists Z((f(Z,X)\& \sim f(Z,Y))|(f(Z,Y)\& \sim f(Z,X)))|q(X,Y))\end{aligned}$$

Rename variables

$$\begin{aligned}&\forall X \forall Y(\sim q(X,Y)|\forall Z((\sim f(Z,X)|f(Z,Y))\&(\sim f(Z,Y)|f(Z,X))))\\ &\qquad \&(\exists A((f(A,X)\& \sim f(A,Y))|(f(A,Y)\& \sim f(A,X)))|q(X,Y))\end{aligned}$$

Skolemize

$$\begin{aligned}&(\sim q(X,Y)|((\sim f(Z,X)|f(Z,Y))\&(\sim f(Z,Y)|f(Z,X))))\\ &\&(((f(skA(X,Y),X)\& \sim f(skA(X,Y),Y))|(f(skA(X,Y),Y)\& \sim f(skA(X,Y),X)))|q(X,Y))\end{aligned}$$

Distribute disjunctions

$$\begin{aligned}&(\sim q(X,Y)| \sim f(Z,X)|f(Z,Y))\\ &\&(\sim q(X,Y)| \sim f(Z,Y)|f(Z,X))\\ &\&(f(skA(X,Y),X)|f(skA(X,Y),Y)|q(X,Y))\\ &\&(f(skA(X,Y),X)| \sim f(skA(X,Y),X)|q(X,Y))\\ &\&(\sim f(skA(X,Y),Y)|f(skA(X,Y),Y)|q(X,Y))\\ &\&(\sim f(skA(X,Y),Y)| \sim f(skA(X,Y),X)|q(X,Y))\end{aligned}$$

Convert to CNF

$$\begin{aligned}&\{(\sim q(X,Y)| \sim f(Z,X)|f(Z,Y))\\ &,(\sim q(X,Y)| \sim f(Z,Y)|f(Z,X))\\ &,(f(skA(X,Y),X)|f(skA(X,Y),Y)|q(X,Y))\\ &,(f(skA(X,Y),X)| \sim f(skA(X,Y),X)|q(X,Y))\\ &,(\sim f(skA(X,Y),Y)|f(skA(X,Y),Y)|q(X,Y))\\ &,(\sim f(skA(X,Y),Y)| \sim f(skA(X,Y),X)|q(X,Y))\}\end{aligned}$$

3. Aufgabe

Folgendes Herbrand Universum und folgende Herbrand Basis besitzt das Modell.

$$
\begin{aligned}
HU = \{&max,\\
&vater\_von(max),\\
&mutter\_von(max),\\
&vater\_von(vater\_von(max)),\\
&vater\_von(mutter\_von(max)),\\
&mutter\_von(vater\_von(max)),\\
&mutter\_von(mutter\_von(max)),\\
&vater\_von(vater\_von(vater\_von(max))),\\
&vater\_von(vater\_von(mutter\_von(max))),\\
&vater\_von(mutter\_von(vater\_von(max))),\\
&vater\_von(mutter\_von(mutter\_von(max))),\\
&...\}
\end{aligned}
$$

$$
\begin{aligned}
HB = \{&verheiratet(max, max),\\
&verheiratet(max, vater\_von(max)),\\
&verheiratet(vater\_von(max), max),\\
&verheiratet(vater\_von(max), vater\_von(max)),\\
&verheiratet(max, mutter\_von(max)),\\
&verheiratet(vater\_von(max), mutter\_von(max)),\\
&verheiratet(mutter\_von(max), mutter\_von(max)),\\
&verheiratet(mutter\_von(max), max),\\
&verheiratet(mutter\_von(max), vater\_von(max)),\\
&verheiratet(max, vater\_von(vater\_von(max))),\\
&...\}
\end{aligned}
$$

Folgende Herbrand Interpretation mit $verheiratet(vater\_von(max), mutter\_von(max))$ ist möglich:

$$
\begin{aligned}
D = \{&\ddot\smile,\\
&v(\ddot\smile),\\
&m(\ddot\smile),\\
&v(v(\ddot\smile)),\\
&v(m(\ddot\smile)),\\
&...\}
\end{aligned}
$$

$$
\begin{aligned}
F = \{&max \to \ddot\smile,\\
&vater\_von(max) \to v(\ddot\smile),\\
&mutter\_von(max) \to m(\ddot\smile),\\
&vater\_von(vater\_von(max)) \to v(v(\ddot\smile)),\\
&vater\_von(mutter\_von(max)) \to v(m(\ddot\smile)),\\
&...\}
\end{aligned}
$$

$$
\begin{aligned}
R = \{&verheiratet(\ddot\smile, \ddot\smile) \rightarrow FALSE,\\
&verheiratet(\ddot\smile, v(\ddot\smile)) \rightarrow FALSE,\\
&verheiratet(v(\ddot\smile), \ddot\smile) \rightarrow FALSE,\\
&verheiratet(v(\ddot\smile), v(\ddot\smile)) \rightarrow FALSE,\\
&verheiratet(\ddot\smile, m(\ddot\smile)) \rightarrow FALSE,\\
&verheiratet(v(\ddot\smile), m(\ddot\smile)) \rightarrow TRUE,\\
&verheiratet(m(\ddot\smile), m(\ddot\smile)) \rightarrow FALSE,\\
&verheiratet(m(\ddot\smile), \ddot\smile) \rightarrow FALSE,\\
&verheiratet(m(\ddot\smile), v(\ddot\smile)) \rightarrow TRUE,\\
&verheiratet(\ddot\smile, v(v(\ddot\smile))) \rightarrow FALSE,\\
&...\}
\end{aligned}
$$