# 1. Aufgabe

## a.

$$\forall Z \exists Y \forall X (f(X,Y) <=> (f(X,Z) \& \sim f(X,X)))$$

Simplify
$\forall Z \exists Y \forall X (f(X,Y) => (f(X,Z) \& \sim f(X,X))) \& ((f(X,Z) \& \sim f(X,X)) => f(X,Y))$
$\forall Z \exists Y \forall X (\sim f(X,Y) | (f(X,Z) \& \sim f(X,X))) \& (\sim (f(X,Z) \& \sim f(X,X)) | f(X,Y))$

Move negations in
$\forall Z \exists Y \forall X (\sim f(X,Y) | (f(X,Z) \& \sim f(X,X))) \& (\sim f(X,Z) | f(X,X) | f(X,Y))$

Skolemize
$\exists Y \forall X (\sim f(X,Y) | (f(X,Z) \& \sim f(X,X))) \& (\sim f(X,Z) | f(X,X) | f(X,Y))$
$\gamma = \{Z\}$
$\forall X (\sim f(X, skY(Z)) | (f(X,Z) \& \sim f(X,X))) \& (\sim f(X,Z) | f(X,X) | f(X, skY(Z)))$
$\gamma = \{Z\}$
$(\sim f(X, skY(Z)) | (f(X,Z) \& \sim f(X,X))) \& (\sim f(X,Z) | f(X,X) | f(X, skY(Z)))$
$\gamma = \{Z, X\}$

Distribute disjunctions
$(\sim f(X, skY(Z)) | f(X,Z)) \& (\sim f(X, skY(Z)) | \sim f(X,X)) \& (\sim f(X,Z) | f(X,X) | f(X, skY(Z)))$

Convert to CNF
$\{\sim f(X, skY(Z)) | f(X,Z), \sim f(X, skY(Z)) | \sim f(X,X), \sim f(X,Z) | f(X,X) | f(X, skY(Z))\}$

**b.**

$\forall X \forall Y (q(X,Y) <=> \forall Z(f(Z,X) <=> f(Z,Y)))$

Simplify

$$\forall X \forall Y (\sim q(X,Y) | \forall Z((\sim f(Z,X)|f(Z,Y))\&(\sim f(Z,Y)|f(Z,X))))$$
$$\&(\sim \forall Z((\sim f(Z,X)|f(Z,Y))\&(\sim f(Z,Y)|f(Z,X)))|q(X,Y))$$

Move negations in

$$\forall X \forall Y (\sim q(X,Y) | \forall Z((\sim f(Z,X)|f(Z,Y))\&(\sim f(Z,Y)|f(Z,X))))$$
$$\&(\exists Z((f(Z,X)\& \sim f(Z,Y))|(f(Z,Y)\& \sim f(Z,X)))|q(X,Y))$$

Rename variables

$$\forall X \forall Y (\sim q(X,Y) | \forall Z((\sim f(Z,X)|f(Z,Y))\&(\sim f(Z,Y)|f(Z,X))))$$
$$\&(\exists A((f(A,X)\& \sim f(A,Y))|(f(A,Y)\& \sim f(A,X)))|q(X,Y))$$

Skolemize

$$(\sim q(X,Y)|((\sim f(Z,X)|f(Z,Y))\&(\sim f(Z,Y)|f(Z,X))))$$
$$\&(((f(skA(X,Y),X)\& \sim f(skA(X,Y),Y))|(f(skA(X,Y),Y)\& \sim f(skA(X,Y),X)))|q(X,Y))$$

Distribute disjunctions

$$(\sim q(X,Y)| \sim f(Z,X)|f(Z,Y))$$
$$\&(\sim q(X,Y)| \sim f(Z,Y)|f(Z,X))$$
$$\&(f(skA(X,Y),X)|f(skA(X,Y),Y)|q(X,Y))$$
$$\&(f(skA(X,Y),X)| \sim f(skA(X,Y),X)|q(X,Y))$$
$$\&(\sim f(skA(X,Y),Y)|f(skA(X,Y),Y)|q(X,Y))$$
$$\&(\sim f(skA(X,Y),Y)| \sim f(skA(X,Y),X)|q(X,Y))$$

Convert to CNF

$$\{(\sim q(X,Y)| \sim f(Z,X)|f(Z,Y))$$
$$,(\sim q(X,Y)| \sim f(Z,Y)|f(Z,X))$$
$$,(f(skA(X,Y),X)|f(skA(X,Y),Y)|q(X,Y))$$
$$,(f(skA(X,Y),X)| \sim f(skA(X,Y),X)|q(X,Y))$$
$$,(\sim f(skA(X,Y),Y)|f(skA(X,Y),Y)|q(X,Y))$$
$$,(\sim f(skA(X,Y),Y)| \sim f(skA(X,Y),X)|q(X,Y))\}$$

**c.**

$$\forall X \exists Y((p(X,Y) <= \forall X \exists T q(Y,X,T)) => r(Y))$$

Rename variables

$$\forall X \exists Y((p(X,Y) <= \forall A \exists T q(Y,A,T)) => r(Y))$$

Simplify

$$\forall X \exists Y(\sim (p(X,Y) | \sim (\forall A \exists T q(Y,A,T)))|r(Y))$$

Move negations in

$$\forall X \exists Y((\sim p(X,Y) \& (\forall A \exists T q(Y,A,T)))|r(Y))$$

Move quantifiers out

$\forall X \exists Y \forall A \exists T((\sim p(X,Y) \& q(Y,A,T))|r(Y))$

Skolemize

$$((\sim p(X, skY(X)) \& q(skY(X), A, skT(X,A)))|r(skY(X)))$$

Distribute disjunctions

$$(\sim p(X, skY(X))|r(skY(X))) \& (q(skY(X), A, skT(X,A))|r(skY(X)))$$

Convert to CNF

$$\{\sim p(X, skY(X))|r(skY(X)), q(skY(X), A, skT(X,A))|r(skY(X))\}$$

**d.**

$$\forall X \forall Z (p(X,Z) => \exists Y \sim (q(X,Y) | \sim r(Y,Z)))$$

Simplify

$$\forall X \forall Z (\sim p(X,Z) | \exists Y \sim (q(X,Y) | \sim r(Y,Z)))$$

Move negations in

$$\forall X \forall Z (\sim p(X,Z) | \exists Y (\sim q(X,Y) \& r(Y,Z)))$$

Move quantifiers out

$$\forall X \forall Z \exists Y (\sim p(X,Z) | (\sim q(X,Y) \& r(Y,Z)))$$

Skolemize

$$(\sim p(X,Z) | (\sim q(X, skY(X,Z)) \& r(skY(X,Z), Z)))$$

Distribute disjunctions

$$(\sim p(X,Z) | \sim q(X, skY(X,Z))) \& (\sim p(X,Z) | r(skY(X,Z), Z))$$

Convert to CNF

$$\{\sim p(X,Z) | \sim q(X, skY(X,Z)), \sim p(X,Z) | r(skY(X,Z), Z)\}$$

## 3. Aufgabe

Folgendes Herbrand Universum und folgende Herbrand Basis besitzt das Modell.

$$\begin{aligned} HU = \{ & max, \\ & vater\_von(max), \\ & mutter\_von(max), \\ & vater\_von(vater\_von(max)), \\ & vater\_von(mutter\_von(max)), \\ & mutter\_von(vater\_von(max)), \\ & mutter\_von(mutter\_von(max)), \\ & vater\_von(vater\_von(vater\_von(max))), \\ & vater\_von(vater\_von(mutter\_von(max))), \\ & vater\_von(mutter\_von(vater\_von(max))), \\ & vater\_von(mutter\_von(mutter\_von(max))), \\ & ...\} \end{aligned}$$

$$\begin{aligned}HB = \{&verheiratet(max, max),\\
&verheiratet(max, vater_von(max)),\\
&verheiratet(vater_von(max), max),\\
&verheiratet(vater_von(max), vater_von(max)),\\
&verheiratet(max, mutter_von(max)),\\
&verheiratet(vater_von(max), mutter_von(max)),\\
&verheiratet(mutter_von(max), mutter_von(max)),\\
&verheiratet(mutter_von(max), max),\\
&verheiratet(mutter_von(max), vater_von(max)),\\
&verheiratet(max, vater_von(vater_von(max))),\\
&...\}\end{aligned}$$

Folgende Herbrand Interpretation mit $verheiratet(vater\_von(max), mutter\_von(max))$ ist möglich:

$$\begin{aligned}D = \{&\ddot\smile,\\
&v(\ddot\smile),\\
&m(\ddot\smile),\\
&v(v(\ddot\smile)),\\
&v(m(\ddot\smile)),\\
&...\}\end{aligned}$$

$$\begin{aligned}F = \{&max \to \ddot\smile,\\
&vater_von(max) \to v(\ddot\smile),\\
&mutter_von(max) \to m(\ddot\smile),\\
&vater_von(vater_von(max)) \to v(v(\ddot\smile)),\\
&vater_von(mutter_von(max)) \to v(m(\ddot\smile)),\\
&...\}\end{aligned}$$

$$\begin{aligned}R = \{&verheiratet(\ddot\smile, \ddot\smile) \to FALSE,\\
&verheiratet(\ddot\smile, v(\ddot\smile)) \to FALSE,\\
&verheiratet(v(\ddot\smile), \ddot\smile) \to FALSE,\\
&verheiratet(v(\ddot\smile), v(\ddot\smile)) \to FALSE,\\
&verheiratet(\ddot\smile, m(\ddot\smile)) \to FALSE,\\
&verheiratet(v(\ddot\smile), m(\ddot\smile)) \to TRUE,\\
&verheiratet(m(\ddot\smile), m(\ddot\smile)) \to FALSE,\\
&verheiratet(m(\ddot\smile), \ddot\smile) \to FALSE,\\
&verheiratet(m(\ddot\smile), v(\ddot\smile)) \to TRUE,\\
&verheiratet(\ddot\smile, v(v(\ddot\smile))) \to FALSE,\\
&...\}\end{aligned}$$