Time Complexity:

$n = r - p$

Line	Instruction	# of execution times
	Partition(A,p,r)	
1	x = A[r]	1
2	i = p - 1	1
3	**for** j = p to r - 1	n+1
4	**if** A[j] ≤ x	n
5	i = i + 1	n
6	A[i] ↔ A[j]	n
7	A[i + 1] ↔ A[r]	1
8	**return** i + 1	1

Worst, Average & Best:

$T(n) = 1 + 1 + (n+1) + n + n + n + 1 + 1 = 5 + 4n = O(n)$

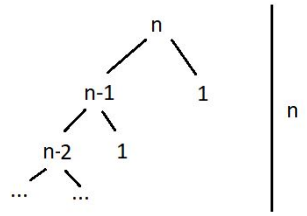
$O(n)$, $\Omega(1)$, $\Theta(n)$

Line	Instruction	# of execution times
	QuickSort(A,p,r)	
1	**if** p < r	1
2	q = Partition(A,p,r)	O(n)
3	QuickSort(A,p,q-1)	T(n/2)
4	QuickSort(A,q+1,r)	T(n/2)

Worst:

$T(n) = 1 + O(n) + T(n-1) + T(1) \simeq O(n) + T(n-1) + T(1)$ 1

$n - i2 = 1 \rightarrow i = 2(n-1) = 2n - 2 \simeq n$



$\sum_{i=0}^{n} n = O(n^2)$

$O(n^2)$

Average & Best:

$T(n) = 1 + O(n) + 2T(n/2) \simeq O(n) + 2T(n/2)$

by the Master method:

$T(1) \rightarrow \Theta(1),\ a = 2,\ b = 2,\ a = b^1\ is\ true$

$T(n) = \Theta(n^1 log\ n) = \Theta(n\ log\ n)$

$\Theta(n\ log\ n)\ ,\ \Omega(n\ log\ n)$

Line	Instruction	# of execution times
	Rand-Parti(A,p,r)	
1	i = Random(p,r)	1
2	A[r] ↔ A[i]	1
3	**return** Partition(A,p,r)	O(n)

Worst, Average & Best:

$T(n) = 1 + 1 + O(n) = O(n)$

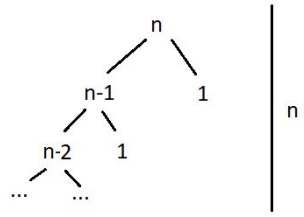
$O(n)\ ,\Omega(n)\ ,\Theta(n)$

Line	Instruction	# of execution times
	Randomized-QS(A,p,r)	
1	**if** p < r	1
2	q = Rand-Parti(A,p,r)	O(n)
3	Randomized-QS(A,p,q-1)	T(n/2)
4	Randomized-QS(A,q+1,r)	T(n/2)

Worst:

$T(n) = 1 + O(n) + T(n-1) + T(1) \simeq O(n) + T(n-1) + T(1)$

$n - i2 = 1 \rightarrow i = 2(n-1) = 2n - 2 \simeq n$



n

$$\sum_{i=0}^{n} n = O(n^2)$$

$O(n^2)$

Average & Best:

$T(n) = 1 + O(n) + 2T(n/2) \simeq O(n) + 2T(n/2)$

by the Master method:

$T(1) \rightarrow \Theta(1),\ a = 2,\ b = 2,\ a = b^1$ *is true*

$T(n) = \Theta(n^1 log\ n) = \Theta(n\ log\ n)$

$\Theta(n\ log\ n)\ ,\ \Omega(n\ log\ n)$

Theoretical treatments:

Prueba	Variante	Estado	Tamaño (n)	Tiempo (ms)
1			10	$O(n\ log\ n)$
2			100	$O(n\ log\ n)$
3			1000	$O(n\ log\ n)$
4		Ascending	10000	$O(n\ log\ n)$
6			10	$O(n^2)$
7			100	$O(n^2)$
8			1000	$O(n^2)$
9		Descending	10000	$O(n^2)$
11			10	$\Theta(n\ log\ n)$
12			100	$\Theta(n\ log\ n)$
13			1000	$\Theta(n\ log\ n)$
14	Normal	Random	10000	$\Theta(n\ log\ n)$
16			10	$\Theta(n\ log\ n)$
17			100	$\Theta(n\ log\ n)$
18			1000	$\Theta(n\ log\ n)$
19		Ascending	10000	$\Theta(n\ log\ n)$
21			10	$\Theta(n\ log\ n)$
22			100	$\Theta(n\ log\ n)$
23			1000	$\Theta(n\ log\ n)$
24		Descending	10000	$\Theta(n\ log\ n)$
26	Random	Random	10	$\Theta(n\ log\ n)$

27			100	$\Theta(n \log n)$
28			1000	$\Theta(n \log n)$
29			10000	$\Theta(n \log n)$

Experimental Unit:
-Quicksort Algorithm

Response Values:
-Execution time of the QuickSort method

Experimental Factors:
- **Studied:**

- Array status.
- Array size.
- Algorithm variant.

- **Not studied:**

-Number of programs being executed
-RAM capacity

Observational Factors:
-Program execution in the computer

Factor Levels:
- **Variant:** Normal, Random
- **Status:** Ascending, Descending, Random.
- **Size:** 10^1 , 10^2 , 10^3 , 10^4

Treatment:

Treatment	Variant	Status	Size(n)	Time (ms)
1	Normal	Ascending	10	
2			100	
3			1000	
4			10000	
6		Descending	10	
7			100	
8			1000	
9			10000	
11		Random	10	

12			100	
13			1000	
14			10000	
16			10	
17			100	
18			1000	
19		Ascending	10000	
21			10	
22			100	
23			1000	
24		Descending	10000	
26			10	
27			100	
28			1000	
29	Random	Random	10000	

1000 repetitions per treatment.

1.b. Hasta ahora las etapas de estudio y diseño de experimentos que se han llevado a cabo hasta el momento son la planeación y realización. Las etapas que faltan son el análisis, la interpretación y el control y conclusiones finales.

1.b. Hitherto the stages of study and experiment design we have completed are planning and realization. The stages that we are missing are analysis, interpretation, and control & final conclusions.

1.c. El objetivo de este programa es comparar dos o más tratamientos, puesto que en este experimento hicimos varios tratamientos para ver cómo se altera la variable de respuesta cada vez y para analizar la varianza utilizando ANOVA. Esto con el fin de conocer el comportamiento del QuickSort y poder sacar una conclusión válida.

1.c. The program objective is to compare two or more treatments, since in this experiment we created many different treatments to see how the response variables change and to analyze the results and the variance using ANOVA. All of this because we want to understand the behavior of the QuickSort algorithm and we want to draw a valid conclusion.

1.d.