OPTIMIZATION. HOMEWORK 3

OSCAR DALMAU

(1) Consider the function $f(x_1, x_2) = (x_1 + x_2^2)^2$. At the point $x^\mathsf{T} = [1, 0]$ we consider the search direction $p^\mathsf{T} = [-1, 1]$. Show that p is a descent direction and find all minimizers of the function.

(2) Find all the values of the parameter a such that $[1, 0]^\mathsf{T}$ is the minimizer or maximizer of the function

$$f(x_1, x_2) = a^3 x_1 e^{x_2} + 2a^2 \log(x_1 + x_2) - (a + 2)x_1 + 8ax_2 + 16x_1x_2.$$

(3) Let $f : \mathbb{R}^n \to \mathbb{R}$ be given by $f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^\mathsf{T}\boldsymbol{Q}\boldsymbol{x} - \boldsymbol{b}^\mathsf{T}\boldsymbol{x}$ with $\boldsymbol{b} \in \mathbb{R}^n$ and $\boldsymbol{Q} \in \mathbb{R}^{n\times n}$ a real symmetric positive definite matrix. Consider the algorithm

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}^k - \beta\alpha_k \nabla f(\boldsymbol{x}_k)$$

where $\alpha_k = \frac{\nabla f(\boldsymbol{x}_k)^\mathsf{T}\nabla f(\boldsymbol{x}_k)}{\nabla f(\boldsymbol{x}_k)^\mathsf{T}\boldsymbol{Q}\nabla f(\boldsymbol{x}_k)}$. Show that $\{\boldsymbol{x}_k\}$ converges to $\boldsymbol{x}^* = \boldsymbol{Q}^{-1}\boldsymbol{b}$ for any initial point $\boldsymbol{x}^0$ if and only if $0 < \beta < 2$.

(4) Find

$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}\in\mathbb{R}^n} f(\boldsymbol{x})$$

where

$$f(\boldsymbol{x}) = \sum_{i=1}^{n}(x_i - y_i)^2 + \lambda\sum_{i=1}^{n-1}(x_{i+1} - x_i)^2$$

using Gradient descent and Newton Methods.

Consider $\lambda \in \{1, 100, 1000\}$

- The entries of vector $\boldsymbol{y}$ are assumed to be known and will be provided in file **yk.txt**. The first line contains the size of the vector, n, and the following lines are the entries of $\boldsymbol{y}$, ie, y_i, $i = 1, \cdots, n$.
- Plot the estimate $\boldsymbol{x}^*$ and $\boldsymbol{y}$ on the same figure, ie, (i, x_i^*), (i, y_i) where $i = 1, 2, 3, \ldots, n$ obtained for each method and for each parameter λ.