# YAGS Yet Another Graph System GAP4 Package

Version 0.0.1 by

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# **Preface**

# 1.1 Welcome to YAGS

Welcome to YAGS.

which is great and have a lot of work over the years.

Our motivation here was this and that.

Our Pourposes and Aim.

authors, contacts

# 1.2 Citing YAGS

Please cite us like this:  $\dots$ 

# 1.3 License and Copyright

GPL V3

# 1.4 More Information

web page, distribution list, github

# **Getting Started**

### 2.1 What is YAGS?

blah, blah, blah

## 2.2 Installing YAGS

blah, blah, blah

## 2.3 Testing the Installation

blah, blah, blah

#### 2.4 A Gentle Tutorial

blah, blah, blah

#### 2.5 An Overview of the Manual

blah, blah, blah

### 2.6 Cheatsheet

blah, blah, blah

# 2.7 ——- Old Sections Bellow —-

YAGS (Yet Another Graph System) is a system designed to aid in the study of graphs. Therefore it provides functions designed to help researchers in the field of graph theory. This chapter is intended as a gentle tutorial on working with YAGS (some knowledge of GAP and the basic use of a computer are assumed). The tutorial is divided as follows:

- Using YAGS
- Definition of a graph
- A taxonomy of graphs
- Creating graphs

gap> RequirePackage("yags");

## 2.8 Using YAGS

YAGS is a GAP package an as such the RequirePackage directive is used to start YAGS

```
Loading YAGS 0.0.1 (Yet Another Graph System),
by R. MacKinney, M.A. Pizana and R. Villarroel-Flores
```

rene@xamanek.izt.uam.mx, map@xamanek.izt.uam.mx, rvf0068@gmail.com

true

Once the package has been loaded help can be obtained at anytime using the GAP help facility. For instance get help on the function *PathGraph*:

# 2.9 Definition of graphs

A graph is defined as follows. A graph G is a set of vertices V and a set of edges (arrows) E,  $G = \{V, E\}$ . The set of edges is a set of tuples of vertices  $(v_i, v_j)$  that belong to  $V, v_i, v_j \in V$  representing that  $v_i, v_j$  are adjacent.

For instance,  $(\{1,2,3,4\},\{(1,3),(2,4),(3,2)\})$  is a graph with four vertices such that vertices 1 and 2 are adjacent to vertex 3 and vertex 2 is adjacent to vertex 4. Visually this can be seen as



The adjacencies can also be represented as a matrix. This would be a boolean matrix M where two vertices i, j are adjacent if M[i, j] = true and not adjacent otherwise.

Given two vertices i, j in graph G we will say that graph G has an **edge**  $\{i, j\}$  if there is an arrow (i, j) and and arrow (j, i).

If a graph G has an arrow that starts and finishes on the same vertex we say that graph G has a loop.



YAGS handles graphs that have arrows, edges and loops. Graphs that, for instance, have multiple arrows between vertices are not handled by YAGS.



## 2.10 A taxonomy of graphs

There are several ways of characterizing graphs. YAGS uses a category system where any graph belongs to a specific category. The following is the list of graph categories in YAGS.

• Graphs: graphs with no particular property.



• Loopless: graphs with no loops.



• Undirected: graphs with no arrows but only edges.



• Oriented: graphs with no edges but only arrows.



• Simple Graphs: graphs with no loops and only edges.



The following figure shows the relationships among categories.

Graphs

Loopless Undirected

Oriented Simple Graphs

Figure 1: Graph Categories

YAGS uses the category of a graph to normalize it. This is helpful, for instance, when we define an undirected graph and inadvertently forget an arrow in its definition. The category of a graph can be given explicitly or implicitly. To do it explicitly the category must be given when creating a graph, as can be seen in the section 2.11. If no category is given the category is assumed to be the *DefaultCategory*. The default category can be changed at any time using the *SetDefaultCategory* function.

Further information regarding categories can be found on chapter 4.

## 2.11 Creating Graphs

There exist several ways to create a graph in YAGS. First, a GAP record can be used. To do so the record has to have either of

- Adjacency List
- Adjacency Matrix

in the graph presented in Section 2.9 the adjacency list would be

and the adjacency matrix

To create a graph YAGS we also need the category the graph belongs to. We give this information to the *Graph* function. For instance to create the graph using the adjacency list we would use the following command:

```
gap> g:=Graph(rec(Category:=OrientedGraphs,Adjacencies:=[[],[4],[1,2],[]]));
Graph( Category := OrientedGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ ], [ 4 ], [ 1, 2 ], [ ] ])
```

This will create a graph g that represents the graph in Section 2.9.



Since the *DefaultCategory* is *SimpleGraphs* when YAGS starts up and the graph we have been using as an example is oriented we must explicitly give the category to YAGS. This is achieved using *Category:=OrientedGraphs* inside the record structure.

The same graph can be created using the function *GraphByAdjacencies* as in

```
gap> g:=GraphByAdjacencies([[],[4],[1,2],[]]:Category:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ ], [ 4 ], [ 1, 2 ], [ ] ] )
```

In this case to explicitly give the Category of the graph we use the construction : Category:=OrientedGraphs inside the function. This construction can be used in any function to explicitly give the category of a graph.

We said previously we can also use the adjacency matrix to create a graph. For instance the command

Creates the same graph. Note that we explicitly give the graph category as before. We also can use the command AdjMatrix as in

If we create the graph using any of the methods so far described omitting the graph category YAGS will create a graph normalized to the *DefaultCategory* which by default is *SimpleGraphs* 

```
gap> g:=GraphByAdjacencies([[],[4],[1,2],[]];
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 3 ], [ 3, 4 ], [ 1, 2 ], [ 2 ] ] )
```

Which creates a graph with only edges



There are many functions to create graphs, some from existing graphs and some create interesting well known graphs.

Among the former we have the function AddEdges which adds edges to an existing graph

```
gap> g:=GraphByAdjacencies([[],[4],[1,2],[]]);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 3 ], [ 3, 4 ], [ 1, 2 ], [ 2 ] ] )
gap> h:=AddEdges(g,[[1,2]]);
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 3 ], [ 1, 3, 4 ], [ 1, 2 ], [ 2 ] ] )
```

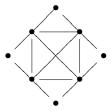
that yields the graph h



Among the latter we have the function SunGraph which takes an integer as argument and returns a fresh copy of a sun graph of the order given as argument.

```
gap> h:=SunGraph(4);
Graph( Category := SimpleGraphs, Order := 8, Size := 14, Adjacencies :=
[ [ 2, 8 ], [ 1, 3, 4, 6, 8 ], [ 2, 4 ], [ 2, 3, 5, 6, 8 ], [ 4, 6 ],
        [ 2, 4, 5, 7, 8 ], [ 6, 8 ], [ 1, 2, 4, 6, 7 ] ] )
```

that produces h as



Further information regarding constructing graphs can be found on chapter 7.3.

# **Cliques**

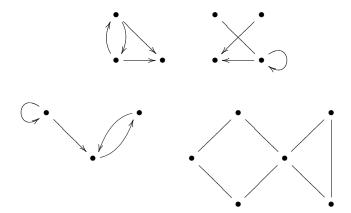
blah, blah, blah.

# **Categories**

# 4.1 Graph Categories

#### **Declaration of:** Graphs

Among them we can find:



#### **Declaration of:** LooplessGraphs

A loop is an arrow that starts and finishes on the same vertex.



Loopless graphs have no such arrows.



#### **Declaration of:** UndirectedGraphs

Given two vertex i, j in graph G we will say that graph G has an **edge**  $\{i, j\}$  if there is an arrow (i, j) and and arrow (j, i).

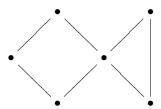
Undirected graphs have no arrows but only edges.

#### **Declaration of:** OrientedGraphs

Oriented graphs have no edges but only arrows.



#### **Declaration of:** SimpleGraphs



The following figure shows the relationships among categories.

Graphs

 $Loopless \ Undirected$ 

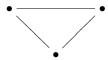
Oriented Simple Graphs

Figure 2: Graph Categories

This relationship is important because when a graph is created it is normalized to the category it belongs. For instance, if we create a graph such as



as a simple graph  $\mathsf{YAGS}$  will normalize the graph as



For further examples see the following section.

# 4.2 Default Category

There are several ways to specify the category in which a new graph will be created. There exists a *Default-Category* which tells YAGS to which category belongs any new graph by default. The *DefaultCategory* can be changed using the following function.

**Declaration of:** SetDefaultGraphCategory

In order to handle graphs with different categories there two functions available.

**Declaration of:** GraphCategory

**Declaration of:** TargetGraphCategory

Finally we can test if a single graph belongs to a given category.

Declaration of: in

# Morphisms of Graphs

There exists several classes of morphisms that can be found on graphs. Moreover, sometimes we want to find a combination of them. For this reason YAGS uses a unique mechanism for dealing with morphisms. This mechanisms allows to find any combination of morphisms using three underlying operations.

# 5.1 Core Operations

The following operations do all the work of finding morphisms that comply with all the properties given in a list. The list of checks that each function receives can have any of the following elements.

- CHK\_METRIC Metric
- CHK\_MONO Mono
- CHK\_FULL Full
- CHK\_EPI Epi
- CHK\_CMPLT Complete
- CHK\_ISO Iso

Additionally it must have at least one of the following.

- CHK\_WEAK Weak
- CHK\_MORPH Morph

These properties are detailed in the next section.

Declaration of: PropertyMorphism

Declaration of: PropertyMorphisms

Declaration of: NextPropertyMorphism

# 5.2 Morphisms

For all the definitions we assume we have a morphism  $\varphi:G\to H\cdot$  The properties for creating morphisms are the following:

Metric A morphism is metric if the distance (see section 8) of any two vertices remains constant

$$d_G(x, y) = d_H(\varphi(x), \varphi(y))$$
.

**Mono** A morphism is mono if two different vertices in G map to two different vertices in H

$$x \neq y \implies \varphi(x) \neq \varphi(y)$$
.

**Full** A morphism is full if every edge in G is mapped to an edge in H.

$$|H| = |G|$$

Not yet implemented.

**Epi** A morphism is Epi if for each vertex in H exist a vertex in G that is mapped from.

$$\forall x \in H \exists x_0 \in G \bullet \varphi(x_0) = x$$

Complete A morphism is complete iff the inverse image of any complete of H is a complete of G.

Iso An isomorphism is a bimorphism which is also complete.

Aditionally they must be one of the following

Weak A morphism is weak if x adjacent to y in G means their mappings are adjacent in H

$$x, y \in G \land x \simeq y \Rightarrow \varphi(x) \simeq \varphi(y)$$
.

**Morph** This is equivalent to *strong*. A morphism is strong if two different vertices in G map to different vertices in H.

$$x, y \in G \land x \sim y \Rightarrow \varphi(x) \sim \varphi(y)$$
.

Note that  $x \neq y \Rightarrow \varphi(x) \neq \varphi(y)$  unless there is a loop in G.

# **Backtrack**

blah, blah, blah.

# YAGS Functions by Topic

A complete list of all YAGS's functions by topic.

#### 7.1 Most Common Functions

```
AddEdges(G, E)
       Returns a new graph obtained from G by adding the list of edges in E.
Adjacency(G, x)
       Returns the list of vertices in G which are adjacent to vertex x.
AutGroupGraph(G)
       Returns the automorphism group of graph G. A synonym is AutomorphismGroup(G).
BoxProduct(G, H);
       Returns the BoxProduct (or cartesian product) of graphs G and H.
{\tt BoxTimesProduct(}\ G , H )
       Returns the BoxTimesProduct (or strong product) of graphs G and H.
Circulant( n, Jumps )
       Returns minimal (1, 2, \dots, n)-invariant graph where vertex 1 is adjacent to vertices in Jumps.
CliqueGraph(G)
CliqueGraph(G, maxNumCli)
       Returns the intersection graph of the (maximal) cliques of G; aborts if maxNumCli cliques are
Cliques(G)
Cliques( G, maxNumCli )
       Returns the list of (maximal) cliques of G; aborts if maxNumCli cliques are found.
ComplementGraph(G)
       Returns the new graph H such that V(H) = V(G) and xy \in E(H) \iff xy \notin E(G).
CompleteGraph(n)
       Returns the graph on n vertices having all possible edges present.
CompleteMultipartiteGraph( n1, n2 [, n3 ...])
       Returns the graph with r \geq 2 parts of orders n1, n2, ... such that each vertex is adjacent exactly
       to all the vertices in the other parts not containing itself.
ConnectedComponents (G)
       Returns the equivalence partition of V(G) corresponding to the equivalence relation reachable in
       G.
CycleGraph(n)
       Returns the cyclic graph on n vertices.
Diameter (G)
       Returns the maximum among the distances between pairs of vertices of G.
```

Link(G, x)

MaxDegree(G)

Order(G)

DiscreteGraph(n) Returns the graph on n vertices with no edges. DisjointUnion( G, H ) Returns the disjoint union of two graphs G and H. Distance (G, x, y)Returns the length of a minimal path connecting x to y in G. Draw(G)Draws the graph G on a new window. Edges (G) Returns the list of edges of graph G. GraphByAdjacencies(AdjList)Returns a new graph having AdjList as its list of adjacencies. GraphByAdjMatrix( Mat ) Returns a new graph created from an adjacency matrix Mat. GraphByCompleteCover( Cover ) Returns the graph where the elements of *Cover* are (the vertex sets of) complete subgraphs.  ${\tt GraphByEdges}(\ L\ )$ Returns the minimal graph such that the pairs in L are edges.  ${\tt GraphByRelation}(\ V,\ Rel\ )$  ${\tt GraphByRelation}(\ n,\ Rel\ )$ Returns a new graph G where  $xy \in E(G)$  iff Rel(x,y)=true. GraphByWalks ( Walk1, Walk2,...) Returns the minimal graph such that Walk1, Walk2, etc are Walks. GraphSum(G, L)Returns the lexicographic sum of a list of graphs L over a graph G. InducedSubgraph( G, V ) Returns the subgraph of graph G induced by the vertex set V. InNeigh( G, x ) Returns the list of in-neighbors of x in G. IntersectionGraph( L ) Returns the graph G where V(G) = L and  $XY \in E(G) \iff X \cap Y \neq \emptyset$ . IsEdge( G , [x,y]) Returns true if [x,y] is an edge of G. IsIsomorphicGraph( G, H ) Returns true when G is isomorphic to H and false otherwise. Join(G, H) Returns the disjoint union of G and H with all the possible edges between G and H added.  ${\tt LineGraph}(\ G\ )$ Returns the intersection graph of the edges of G.

Returns the subgraph induced in G by the neighbors of x.

Returns the maximum degree in graph G.

Returns the number of vertices, of graph G.

```
PathGraph( n )
       Returns the path graph on n vertices.
QuotientGraph( G, Part )
QuotientGraph( G, L1, L2 )
       Returns the quotient graph of graph G given a vertex partition Part, by identifying any two vertices
       in the same part.
RandomGraph( n, p )
RandomGraph(n)
       Returns a random graph of order n with edge probability p (a rational in [0,1]).
RemoveEdges( G, E )
       Returns a new graph created from graph G by removing the edges in list E.
SetDefaultGraphCategory( Catgy )
       Sets the default graph category to Catgy.
Size(G)
       Returns the number of edges of graph G.
TimesProduct( G, H )
       Returns the times product (tensor product) G \times H of two graphs G and H.
TrivialGraph
       The one vertex graph.
VertexDegree(G, x)
       Returns the degree of vertex x in Graph G.
VertexNames(G)
       Returns the list of names of the vertices of G.
WheelGraph(n)
WheelGraph(n, r)
       This is the cone of an n-cycle; when present r is the radius of the wheel.
7.2 Drawing
Coordinates (G)
       Returns the list of coordinates of the vertices of G if they exist; fail otherwise.
Draw(G)
       Draws the graph G on a new window.
{\tt GraphToRaw(}\ {\it FileName} , \ {\it G} )
```

Writes the graph G in raw format to the file FileName.

Updates the coordinates of G from a file FileName in raw format.

Sets the coordinates of the vertices of G, which are used to draw G by Draw(G).

GraphUpdateFromRaw(FileName, G)

SetCoordinates (G, Coord)

## 7.3 Constructing Graphs

AddEdges( G, E )

Returns a new graph obtained from G by adding the list of edges in E.

AddVerticesByAdjacencies( G, NewAdjList )

Returns a new graph obtained from G by adding some vertices with adjacencies described by NewAdjList.

Graph( Rec )

Returns a new graph created from the information in record Rec.

GraphByAdjacencies(AdjList)

Returns a new graph having AdjList as its list of adjacencies.

GraphByAdjMatrix( Mat )

Returns a new graph created from an adjacency matrix Mat.

GraphByCompleteCover( Cover )

Returns the graph where the elements of *Cover* are (the vertex sets of) complete subgraphs.

 ${\tt GraphByEdges(\ \it L\ )}$ 

Returns the minimal graph such that the pairs in L are edges.

 ${\tt GraphByRelation}(\ V,\ Rel\ )$ 

 ${\tt GraphByRelation}(\ n,\ Rel\ )$ 

Returns a new graph G where  $xy \in E(G)$  iff Rel(x,y)=true.

GraphByWalks ( Walk1, Walk2,...)

Returns the minimal graph such that Walk1, Walk2, etc are Walks.

IntersectionGraph( L )

Returns the graph G where V(G) = L and  $XY \in E(G) \iff X \cap Y \neq \emptyset$ .

RandomGraph(n, p)

RandomGraph(n)

Returns a random graph of order n with edge probability p (a rational in [0,1]).

RemoveEdges(G, E)

Returns a new graph created from graph G by removing the edges in list E.

RemoveVertices( G, V )

Returns a new graph created from graph G by removing the vertices in list V.

# 7.4 Families of Graphs

AGraph

A 4-cycle with two pendant vertices on consecutive vertices of the cycle.

AntennaGraph

A HouseGraph with a pendant vertex (antenna) on the roof.

BullGraph

A triangle with two pendant vertices (horns).

ChairGraph

A tree with degree sequence 3,2,1,1,1.

Circulant( n, Jumps )

Returns minimal  $(1, 2, \dots, n)$ -invariant graph where vertex 1 is adjacent to vertices in *Jumps*.

#### ClawGraph

The graph on 4 vertices, 3 edges, and maximum degree 3.

#### CompleteBipartiteGraph( n, m )

Returns the minimal graph such that all vertices in  $\{1 \cdot n\}$  are adjacent to all in  $\{n+1 \cdot n+m\}$ .

#### CompleteGraph(n)

Returns the graph on n vertices having all possible edges present.

#### CompleteMultipartiteGraph( n1, n2 [, n3 ...])

Returns the graph with  $r \geq 2$  parts of orders n1, n2, ... such that each vertex is adjacent exactly to all the vertices in the other parts not containing itself.

#### Cube

The 1-skeleton of Plato's cube.

#### CubeGraph( n )

Returns the underlying graph of the n-hypercube.

#### CycleGraph(n)

Returns the cyclic graph on n vertices.

#### CylinderGraph( b, h )

Returns graph on b(h+1) vertices which is a  $\{4,6\}$ -regular triangulation of the cylinder.

#### DartGraph

A diamond with a pendant vertex and maximum degree 4.

#### DiamondGraph

The graph on 4 vertices and 5 edges.

#### DiscreteGraph(n)

Returns the graph on n vertices with no edges.

#### Dodecahedron

The 1-skeleton of Plato's Dodecahedron.

#### DominoGraph

Two squares glued by an edge.

#### FanGraph(n)

Returns the n-Fan: The join of a vertex and a (n+1)-path.

#### FishGraph

A square and a triangle glued by a vertex.

#### GemGraph

The 3-Fan graph.

#### HouseGraph

A 4-Cycle and a triangle glued by an edge.

#### Icosahedron

The 1-skeleton of Plato's icosahedron.

#### JohnsonGraph( n, r )

Returns a new graph G where V(G) is the set of r-subsets of  $\{1, 2 \dots n\}$ , two of them being adjacent iff their intersection contains exactly r-1 elements.

#### KiteGraph

A diamond with a pendant vertex and maximum degree 3.

#### OctahedralGraph(n)

Returns the (2n-2)-regular graph on 2n vertices.

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#### Octahedron

The 1-skeleton of Plato's octahedron.

#### ParachuteGraph

Returns the suspension of a 4-path with a pendant vertex attached to the south pole.

#### ParapluieGraph

A 3-Fan graph with a 3-path attached to the universal vertex.

#### PathGraph( n )

Returns the path graph on n vertices.

#### PawGraph

A triangle with a pendant vertex.

#### PetersenGraph

The 3-regular graph on 10 vertices having girth 5.

#### RGraph

A square with two pendant vertices attached to the same vertex of the square.

#### SnubDisphenoid

The 1-skeleton of the 84th Johnson solid.

#### SpikyGraph(n)

Returns a complete on n vertices, with an additional complete on n vertices glued to each of its n-1 (n-1)-dimensional faces.

#### SunGraph(n)

Returns a complete graph on n vertices with a zigzaging corona of 2n vertices glued to a n-cycle of the complete graph.

#### Tetrahedron

The 1-skeleton of Plato's tetrahedron.

#### TrivialGraph

The one vertex graph.

```
WheelGraph(n)
```

```
WheelGraph( n, r )
```

This is the cone of an n-cycle; when present r is the radius of the wheel.

# 7.5 Small Graphs

#### ${\tt ConnectedGraphsOfGivenOrder(}\ n$ )

Returns the list of all connected graphs of order n (upto isomorphism).

#### Graph6ToGraph( String )

Returns the graph represented by String which is encoded using Brendan McKay's graph6 format.

#### GraphsOfGivenOrder(n)

Returns the list of all graphs of order n (upto isomorphism).

#### ImportGraph6(Filename)

Returns the list of graphs represented in *Filename* which are encoded using Brendan McKay's graph6 format.

## 7.6 Attributes and Properties

Adjacencies (G)

Returns the list of adjacencies of G: The neighbors of vertex x are listed in position x of that list.

Adjacency(G, x)

Returns the list of vertices in G which are adjacent to vertex x.

AdjMatrix(G)

Returns the Adjacency Matrix of G.

AutGroupGraph(G)

Returns the automorphism group of graph G. A synonym is AutomorphismGroup (G).

ConnectedComponents (G)

Returns the equivalence partition of V(G) corresponding to the equivalence relation **reachable** in G.

Diameter(G)

Returns the maximum among the distances between pairs of vertices of G.

Distance(G, x, y)

Returns the length of a minimal path connecting x to y in G.

 ${\tt DistanceMatrix(}\ {\it G}\ {\tt )}$ 

Returns an  $n \times n$  matrix D, where D[x][y] is the distance between x and y in G.

 ${\tt DistanceSet(}\ G\text{, }A\text{, }B\text{ )}$ 

Returns the set of distances between pairs of vertices in  $A \times B$ .

Distances (G, A, B)

Returns the list of distances between pairs of vertices in  $A \times B$ .

DominatedVertices (G)

Returns the set of dominated vertices of G.

Eccentricity( G, x )

Returns the distance from a vertex x in graph G to its most distant vertex in G.

Edges( G )

Returns the list of edges of graph G.

IsDiamondFree(G)

Returns true if G is free from induced diamonds, false otherwise.

IsEdge( G , [x,y])

Returns true if [x,y] is an edge of G.

IsLoopless(G)

Returns true when G is isomorphic to H and false otherwise.

IsOriented(G)

Returns true if whenever xy is an edge (arrow) of G, yx is not.

IsSimple(G)

Returns true if G contains no loops and no arrows.

IsUndirected(G)

Returns true if whenever xy is an edge (arrow) of G, yx is also an edge of G.

Link(G, x)

Returns the subgraph induced in G by the neighbors of x.

Links(G)

Returns the list of subgraphs of G induced by the neighbors of each vertex of G.

MaxDegree(G)

Returns the maximum degree in graph G.

MinDegree (G)

Returns the minimum degree in graph G.

NumberOfConnectedComponents( G )

Returns the number of connected components of G.

Order(G)

Returns the number of vertices, of graph G.

Radius( G )

Returns the minimal eccentricity among the vertices of graph G.

Size(G)

Returns the number of edges of graph G.

SpanningForest( G )

Returns a spanning forest of G.

SpanningForestEdges( G )

Returns the edges of a spanning forest of G.

VertexDegree( G, x )

Returns the degree of vertex x in Graph G.

VertexDegrees(G)

Returns the list of degrees of the vertices in graph G.

VertexNames(G)

Returns the list of names of the vertices of G.

Vertices (G)

Returns the list [1.. Order(G)].

# 7.7 Unary Operators

ComplementGraph(G)

Returns the new graph H such that V(H) = V(G) and  $xy \in E(H) \iff xy \notin E(G)$ .

CompletelyParedGraph(G)

Returns the graph obtained from G by iteratively removing all dominated vertices.

Cone(G)

Returns a new graph obtained from G by adding a new vertex which is adjacent to all vertices of G

CliqueGraph(G)

CliqueGraph( G, maxNumCli )

Returns the intersection graph of the (maximal) cliques of G; aborts if maxNumCli cliques are found.

DistanceGraph( G, Dist )

Returns a new graph where two vertices are adjacent iff the distance between them belongd to Dist.

InducedSubgraph( G, V )

Returns the subgraph of graph G induced by the vertex set V.

LineGraph(G)

Returns the intersection graph of the edges of G.

ParedGraph(G)

Returns the induced subgraph obtained from G by removing its dominated vertices.

PowerGraph( G, exp )

Returns a new graph where two vertices are neighbors iff their distance in G is less than or equal to exp.

QuotientGraph( G, Part )

QuotientGraph( G, L1, L2 )

Returns the quotient graph of graph G given a vertex partition Part, by identifying any two vertices in the same part.

Suspension(G)

Returns the graph obtained from G by adding two new vertices which are adjacent to every vertex of G but not to each other.

## 7.8 Binary Operators

BoxProduct(G, H);

Returns the BoxProduct (or cartesian product) of graphs G and H.

BoxTimesProduct( G, H )

Returns the BoxTimesProduct (or strong product) of graphs G and H.

Composition( G, H )

Returns the composition G[H] of two graphs G and H.

DisjointUnion( G, H )

Returns the disjoint union of two graphs G and H.

GraphSum(G, L)

Returns the lexicographic sum of a list of graphs L over a graph G.

Join( G, H )

Returns the disjoint union of G and H with all the possible edges between G and H added.

TimesProduct( G, H )

Returns the times product (tensor product)  $G \times H$  of two graphs G and H.

# 7.9 Cliques

```
Basement( G, KnG, x )
```

Basement( G, KnG, V )

Returns the basement of vertex x (vertex set V) of the iterated clique graph KnG with respect to G.

CliqueGraph( G )

CliqueGraph( G, maxNumCli )

Returns the intersection graph of the (maximal) cliques of G; aborts if maxNumCli cliques are found.

CliqueNumber(G)

Returns the order, omega(G), of a maximum clique of G.

Cliques( G )

Cliques (G, maxNumCli)

Returns the list of (maximal) cliques of G; aborts if maxNumCli cliques are found.

Returns true if G is a clique gated graph.

IsCliqueHelly( G )

Returns true if the set of (maximal) cliques G satisfy the Helly property.

IsComplete( G, L )

Returns true if L induces a complete subgraph of G.

IsCompleteGraph(G)

Returns true if graph G is a complete graph, false otherwise.

NumberOfCliques(G)

NumberOfCliques( G, maxNumCli )

Returns the number of (maximal) cliques of G.

## 7.10 Morphisms and Isomorphisms

```
IsIsomorphicGraph( G, H )
```

Returns true when G is isomorphic to H and false otherwise.

IsoMorphism(G, H)

Returns one isomorphism from G to H; fail if there is none.

IsoMorphisms( G, H )

Returns the list of all isomorphism from G to H.

NextIsoMorphism( G, H, F )

Returns the next isomorphism (after F) from G to H.

NextPropertyMorphism(G, H, F, PropList)

Returns the next morphism (after F) from G to H satisfying the list of properties PropList.

PropertyMorphism(G, H, PropList)

Returns the first morphism from G to H satisfying the list of properties PropList.

PropertyMorphisms( G, H, PropList )

Returns all morphisms from G to H satisfying the list of properties PropList.

# 7.11 Graph Categories

#### CopyGraph(G)

Returns a fresh copy of G. Useful to change the category of a graph.

GraphCategory([G, ...]);

For internal use. Returns the minimal common category to a list of graphs.

Graphs()

The category of all graphs that can be represented in YAGS.

in(G, Catgy)

Returns true if graph G belongs to category Catqy and false otherwise.

LooplessGraphs()

The category of all graph that may contain arrows and edges but no loops.

OrientedGraphs()

The category of all graphs that may contain arrows but no edges or loops.

#### SetDefaultGraphCategory( Catgy )

Sets the default graph category to Catgy.

#### SimpleGraphs()

The category of all graphs which may contain edges but no arrows or loops.

#### TargetGraphCategory( [ G, ... ] );

For internal use. Within YAGS methods, returns the graph category to which the new graph will belong.

#### UndirectedGraphs()

The category of all graphs that may contain loops and edges but no arrows.

## 7.12 Digraphs

#### InNeigh( G, x )

Returns the list of in-neighbors of x in G.

#### IsTournament(G)

Returns true if G contains no loops or edges but only arrows and it is maximal w.r.t. this property.

#### IsTransitiveTournament( G )

Returns true if G is a Tournament and whenever xy and yz are arrows, then xz is an arrow too.

#### OutNeigh( G, x )

Returns the list of out-neighbors of x in G.

# 7.13 Groups and Rings

#### CayleyGraph( Grp )

### CayleyGraph( $\mathit{Grp}$ , $\mathit{Elms}$ )

Returns the CayleyGraph of group *Grp*.

#### Circulant( n, Jumps )

Returns minimal  $(1, 2, \dots, n)$ -invariant graph where vertex 1 is adjacent to vertices in *Jumps*.

#### CuadraticRingGraph( Rnq )

Returns a graph H whose vertices are the elements of the ring Rng and  $xy \in E(H) \iff x + z^2 = y$  for some z in Rng.

#### GroupGraph(G, Grp)

#### GroupGraph( G, Grp, Act )

Returns the minimal Grp-invariant (under the action Act) graph containing G.

#### RingGraph( Rng, Elms )

Returns the graph G whose vertices are the elements of the ring Rng such that x is adjacent to y iff x+r=y for some r in Elms.

#### UnitsRingGraph( Rng )

Returns the graph G whose vertices are the elements of Rng such that x is adjacent to y iff x+z=y for some unit z of Rng.

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#### 7.14 Backtrack

```
BackTrack( L, Opts, Chk, Done, Extra )
```

Returns the next solution (after L) to a backtracking combinatorial problem specified by its parameters.

BackTrackBag( Opts, Chk, Done, Extra )

Returns the list of all solutions to a backtracking combinatorial problem specified by its parameters.

#### 7.15 Miscellaneous

```
DumpObject( Obj )
```

For internal use. Dumps all information available for object *Obj*.

EasyExec( Dir, ProgName, InString )

EasyExec( ProgName, InString)

Calls the external program *ProgName* with input string *InString*; returns the output string.

IsBoolean( Obj )

Returns true if object Obj is true or false and false otherwise.

QtfyIsSimple( G )

For internal use. Returns how far is graph G from being simple.

RandomlyPermuted( Obj )

Returns a copy of *Obj* with the order of its elements permuted randomly.

RandomPermutation(n)

Returns a random permutation of the list  $[1,2, \ldots, n]$ .

TimeInSeconds()

Returns the time in seconds since 1970-01-01 00:00:00 UTC as an integer.

UFFind( UFS, x )

For internal use. Implements the find operation on the union-find structure.

UFUnite( UFS, x, y )

For internal use. Implements the unite operation on the union-find structure.

YagsExec( ProgName, InString)

For internal use. Calls external program ProgName located in directory 'YagsDir/bin/' feeding it with InString as input and returning the output of the external program as a string.

# 7.16 Undocumented

```
DeclareQtfyProperty( Name, Filter )
```

For internal use. Declare a quantifiable property.

DumpObject( Obj )

For internal use. Dumps all information available for object *Obj*.

EasyExec( Dir, ProgName, InString )

EasyExec( ProgName, InString)

Calls the external program *ProgName* with input string *InString*; returns the output string.

 ${\tt GraphToRaw(}\ {\it FileName}$  ,  $\ {\it G}$  )

Writes the graph G in raw format to the file FileName.

# ${\tt GraphUpdateFromRaw(} \ \, \textit{FileName} \,, \ \, \textit{G} \, \, \, \, )$

Updates the coordinates of G from a file  $\mathit{FileName}$  in raw format.

#### QtfyIsSimple( G )

For internal use. Returns how far is graph G from being simple.

### YagsExec( ProgName, InString)

For internal use. Calls external program ProgName located in directory 'YagsDir/bin/' feeding it with InString as input and returning the output of the external program as a string.

# YAGS Functions Reference

This chapter contains a complete list of all YAGS's functions, with definitions, in alphabetical order.

```
1 ► AddEdges( G, E )
```

Returns a new graph created from graph G by adding the edges in list E.

```
gap> g:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ])
gap> AddEdges(g,[[1,3]]);
Graph( Category := SimpleGraphs, Order := 4, Size := 5, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3 ] ])
gap> AddEdges(g,[[1,3],[2,4]]);
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ])
```

```
2 \triangleright AddVerticesByAdjacencies( G, NewAdjList )
```

O

Returns a new graph created from graph G by adding as many new vertices as Length (NewAdjList). Each entry in NewAdjList is also a list: the list of neighbors of the corresponding new vertex.

```
gap> g:=PathGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4 ] ] )
gap> AddVerticesByAdjacencies(g,[[1,2],[4,5]]);
Graph( Category := SimpleGraphs, Order := 7, Size := 8, Adjacencies :=
[ [ 2, 6 ], [ 1, 3, 6 ], [ 2, 4 ], [ 3, 5, 7 ], [ 4, 7 ], [ 1, 2 ], [ 4, 5 ] ] )
gap> AddVerticesByAdjacencies(g,[[1,2,7],[4,5]]);
Graph( Category := SimpleGraphs, Order := 7, Size := 9, Adjacencies :=
[ [ 2, 6 ], [ 1, 3, 6 ], [ 2, 4 ], [ 3, 5, 7 ], [ 4, 7 ], [ 1, 2, 7 ], [ 4, 5, 6 ] ] )
```

Returns the adjacency lists of graph G.

```
gap> g:=PathGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> Adjacencies(g);
[ [ 2 ], [ 1, 3 ], [ 2 ] ]
```

 $4 \triangleright Adjacency(G, x)$ 

 $3 \triangleright Adjacencies(G)$ 

Ο

O

Returns the adjacency list of vertex x in G.

```
gap> g:=PathGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> Adjacency(g,1);
[ 2 ]
gap> Adjacency(g,2);
[ 1, 3 ]
```

#### $5 \triangleright AdjMatrix(G)$

Α

Returns the adjacency matrix of graph G.

```
gap> AdjMatrix(CycleGraph(4));
[ [ false, true, false, true ], [ true, false, true, false ],
      [ false, true, false, true ], [ true, false, true, false ] ]
```

#### 6► AGraph

V

A 4-cycle with two pendant vertices on consecutive vertices of the cycle.

```
gap> AGraph;
Graph( Category := SimpleGraphs, Order := 6, Size := 6, Adjacencies :=
[ [ 2 ], [ 1, 3, 5 ], [ 2, 4 ], [ 3, 5 ], [ 2, 4, 6 ], [ 5 ] ] )
```

#### 7 ► AntennaGraph

V

A HouseGraph with a pendant vertex (antenna) on the roof.

```
gap> AntennaGraph;
Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
[ [ 2, 4, 5 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3, 5 ], [ 1, 4, 6 ], [ 5 ] ] )
```

#### $8 \triangleright AutGroupGraph(G)$

Α

Returns the group of automorphisms of the graph G. There is also a synonym for this attribute which is AutomorphismGroup(G).

```
gap> AutGroupGraph(Icosahedron);
Group([ (1,3,2,10,9,12,8,7,5,4)(6,11), (1,7,9)(2,4,12)(3,11,10)(5,8,6) ])
gap> AutomorphismGroup(Icosahedron);
Group([ (1,3,2,10,9,12,8,7,5,4)(6,11), (1,7,9)(2,4,12)(3,11,10)(5,8,6) ])
```

```
9 ▶ BackTrack( L, Opts, Chk, Done, Extra)
```

Ο

Generic, user-customizable backtracking algorithm.

A backtraking algorithm explores a decision tree in search for solutions to a combinatorial problem. The combinatorial problem and the search strategy are specified by the parameters:

L is just a list that BackTrack uses to keep track of solutions and partial solutions. It is usually set to the empty list as a starting point. After a solution is found, it is returned **and** stored in L. This value of L is then used as a starting point to search for the next solution in case BackTrack is called again. Partial solutions are also stored in L during the execution of BackTrack.

Extra may be any object, list, record, etc. BackTrack only uses it to pass this data to the user-defined functions Opts, Chk and Done, therefore offering you a way to share data between your functions.

Opts:=function(L,extra) must return the list of continuation options (childs) one has after some partial solution (node) L has been reached within the decision tree (Opts may use the extra data Extra as needed).

O

Each of the values in the list returned by Opts(L,extra) will be tried as possible continuations of the partial solution L. If Opts(L,extra) always returns the same list, you can put that list in place of the parameter Opts.

Chk:=function(L,extra) must evaluate the partial solution L possibly using the extra data Extra and must return false when it knows that L can not be extended to a solution of the problem. Otherwise it returns true. Chk may assume that L[1..Length(L)-1] already passed the test.

Done:=function(L,extra) returns true if L is already a complete solution and false otherwise. In many combinatorial problems, any partial solution of certain length n is also a solution (and viceversa), so if this is your case, you can put that length in place of the parameter Done.

The following example uses BackTrack in its simplest form to compute derrangements (permutations of a set, where none of the elements appears in its original position).

```
gap> N:=4;;L:=[];;extra:=[];;opts:=[1..N];;done:=N;;
gap> chk:=function(L,extra) local i; i:=Length(L);
           return not L[i] in L{[1..i-1]} and L[i]<> i; end;;
gap> BackTrack(L,opts,chk,done,extra);
[2, 1, 4, 3]
gap> BackTrack(L,opts,chk,done,extra);
[2, 3, 4, 1]
gap> BackTrack(L,opts,chk,done,extra);
[2, 4, 1, 3]
gap> BackTrack(L,opts,chk,done,extra);
[3, 1, 4, 2]
gap> BackTrack(L,opts,chk,done,extra);
[3, 4, 1, 2]
gap> BackTrack(L,opts,chk,done,extra);
[3,4,2,1]
gap> BackTrack(L,opts,chk,done,extra);
[4, 1, 2, 3]
gap> BackTrack(L,opts,chk,done,extra);
[4,3,1,2]
gap> BackTrack(L,opts,chk,done,extra);
[4, 3, 2, 1]
gap> BackTrack(L,opts,chk,done,extra);
fail
```

10 ► BackTrackBag( Opts, Chk, Done, Extra )

Returns the list of all solutions that would be returned one at a time by Backtrack.

The following example computes all derrangements of order 4.

```
gap> N:=4;;
gap> chk:=function(L,extra) local i; i:=Length(L);
>         return not L[i] in L{[1..i-1]} and L[i]<> i; end;;
gap> BackTrackBag([1..N],chk,N,[]);
[[2, 1, 4, 3], [2, 3, 4, 1], [2, 4, 1, 3], [3, 1, 4, 2],
        [3, 4, 1, 2], [3, 4, 2, 1], [4, 1, 2, 3], [4, 3, 1, 2],
        [4, 3, 2, 1]]
```

O

```
11 \blacktriangleright Basement( G, KnG, x )

\blacktriangleright Basement( G, KnG, V )
```

Given a graph G, some iterated clique graph KnG of G and a vertex x of KnG, the operation returns the basement of x with respect to G [Piz04]. Loosely speaking, the basement of x is the set of vertices of G that constitutes the iterated clique x.

```
gap> g:=Icosahedron;;Cliques(g);
[ [ 1, 2, 3 ], [ 1, 2, 6 ], [ 1, 3, 4 ], [ 1, 4, 5 ], [ 1, 5, 6 ],
      [ 4, 5, 7 ], [ 4, 7, 11 ], [ 5, 7, 8 ], [ 7, 8, 12 ], [ 7, 11, 12 ],
      [ 5, 6, 8 ], [ 6, 8, 9 ], [ 8, 9, 12 ], [ 2, 6, 9 ], [ 2, 9, 10 ],
      [ 9, 10, 12 ], [ 2, 3, 10 ], [ 3, 10, 11 ], [ 10, 11, 12 ], [ 3, 4, 11 ] ]
gap> kg:=CliqueGraph(g);; k2g:=CliqueGraph(kg);;
gap> Basement(g,k2g,1);Basement(g,k2g,2);
[ 1, 2, 3, 4, 5, 6 ]
[ 1, 2, 3, 4, 6, 10 ]
```

In its second form, V is a set of vertices of KnG, in that case, the basement is simply the union of the basements of the vertices in V.

```
gap> Basement(g,k2g,[1,2]);
[ 1, 2, 3, 4, 5, 6, 10 ]
```

```
12 ▶ BoxProduct( G, H )
```

Returns the box product,  $G \square H$ , of two graphs G and H (also known as the cartesian product).

The box product is calculated as follows:

For each pair of vertices  $x \in G, y \in H$  we create a vertex (x, y). Given two such vertices (x, y) and (x', y') they are adjacent iff x = x' and  $y \sim y'$  or  $x \sim x'$  and y = y'.

```
gap> g:=PathGraph(3);h:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
gap> gh:=BoxProduct(g,h);
Graph( Category := SimpleGraphs, Order := 12, Size := 20, Adjacencies :=
[ [ 2, 4, 5 ], [ 1, 3, 6 ], [ 2, 4, 7 ], [ 1, 3, 8 ], [ 1, 6, 8, 9 ],
        [ 2, 5, 7, 10 ], [ 3, 6, 8, 11 ], [ 4, 5, 7, 12 ], [ 5, 10, 12 ],
        [ 6, 9, 11 ], [ 7, 10, 12 ], [ 8, 9, 11 ] ] )
gap> VertexNames(gh);
[ [ 1, 1 ], [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 2, 1 ], [ 2, 2 ], [ 2, 3 ],
        [ 2, 4 ], [ 3, 1 ], [ 3, 2 ], [ 3, 3 ], [ 3, 4 ] ]
```

```
13 \blacktriangleright BoxTimesProduct( G, H )
```

Returns the boxtimes product of two graphs G and H,  $G \boxtimes H$  (also known as the strong product).

The boxtimes product is calculated as follows:

For each pair of vertices  $x \in G, y \in H$  we create a vertex (x, y). Given two such vertices (x, y) and (x', y') such that  $(x, y) \neq (x', y')$  they are adjacent iff  $x \simeq x'$  and  $y \simeq y'$ .

O

```
gap> g:=PathGraph(3);h:=CycleGraph(4);
      Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
      [[2],[1,3],[2]])
      Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
      [[2, 4], [1, 3], [2, 4], [1, 3]])
      gap> gh:=BoxTimesProduct(g,h);
      Graph( Category := SimpleGraphs, Order := 12, Size := 36, Adjacencies :=
      [[2, 4, 5, 6, 8], [1, 3, 5, 6, 7], [2, 4, 6, 7, 8], [1, 3, 5, 7, 8],
        [1, 2, 4, 6, 8, 9, 10, 12], [1, 2, 3, 5, 7, 9, 10, 11],
        [2, 3, 4, 6, 8, 10, 11, 12], [1, 3, 4, 5, 7, 9, 11, 12],
        [5, 6, 8, 10, 12], [5, 6, 7, 9, 11], [6, 7, 8, 10, 12],
        [5, 7, 8, 9, 11])
      gap> VertexNames(gh);
      [[1,1],[1,2],[1,3],[1,4],[2,1],[2,2],[2,3],
        [2, 4], [3, 1], [3, 2], [3, 3], [3, 4]]
14► BullGraph
                                                                                       V
   A triangle with two pendant vertices (horns).
      gap> BullGraph;
      Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
      [[2],[1,3,4],[2,4],[2,3,5],[4]])
                                                                                       O
15 ► CayleyGraph( Grp, Elms )
 ► CayleyGraph( Grp )
                                                                                       0
   Returns the graph G whose vertices are the elements of the group Grp such that x is adjacent to y iff
   x*q=y for some q in the list Elms. if Elms is not provided, then the generators of G are used instead.
      gap> grp:=Group((1,2,3),(1,2));
      Group([(1,2,3), (1,2)])
      gap> CayleyGraph(grp);
      Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
      [[3, 4, 5], [3, 5, 6], [1, 2, 6], [1, 5, 6], [1, 2, 4],
        [2, 3, 4]])
      gap> CayleyGraph(grp,[(1,2),(2,3)]);
      Graph( Category := SimpleGraphs, Order := 6, Size := 6, Adjacencies :=
      [[2, 3], [1, 5], [1, 4], [3, 6], [2, 6], [4, 5]])
16 ► ChairGraph
                                                                                       V
   A tree with degree sequence 3,2,1,1,1.
      gap> ChairGraph;
      Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
      [[2], [1, 3, 4], [2], [2, 5], [4]])
```

Returns the graph G whose vertices are [1..n] such that x is adjacent to y iff x+z=y mod n for some z the list of Jumps.

 $17 \triangleright Circulant(n, Jumps)$ 

A

```
gap> Circulant(6,[1,2]);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 2, 3, 5, 6 ], [ 1, 3, 4, 6 ], [ 1, 2, 4, 5 ], [ 2, 3, 5, 6 ],
      [ 1, 3, 4, 6 ], [ 1, 2, 4, 5 ] ] )
```

18► ClawGraph V

The graph on 4 vertices, 3 edges, and maximum degree 3.

```
gap> ClawGraph;
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2, 3, 4 ], [ 1 ], [ 1 ] ] )
```

Returns the intersection graph of all the (maximal) cliques of G.

The additional parameter maxNumCli aborts the computation when maxNumCli cliques are found, even if they are all the cliques of G. If the bound maxNumCli is reached, fail is returned.

```
gap> CliqueGraph(Octahedron);
Graph( Category := SimpleGraphs, Order := 8, Size := 24, Adjacencies :=
[ [ 2, 3, 4, 5, 6, 7 ], [ 1, 3, 4, 5, 6, 8 ], [ 1, 2, 4, 5, 7, 8 ],
        [ 1, 2, 3, 6, 7, 8 ], [ 1, 2, 3, 6, 7, 8 ], [ 1, 2, 4, 5, 7, 8 ],
        [ 1, 3, 4, 5, 6, 8 ], [ 2, 3, 4, 5, 6, 7 ] ] )
gap> CliqueGraph(Octahedron,9);
Graph( Category := SimpleGraphs, Order := 8, Size := 24, Adjacencies :=
[ [ 2, 3, 4, 5, 6, 7 ], [ 1, 3, 4, 5, 6, 8 ], [ 1, 2, 4, 5, 7, 8 ],
        [ 1, 2, 3, 6, 7, 8 ], [ 1, 2, 3, 6, 7, 8 ], [ 1, 2, 4, 5, 7, 8 ],
        [ 1, 3, 4, 5, 6, 8 ], [ 2, 3, 4, 5, 6, 7 ] ] )
gap> CliqueGraph(Octahedron,8);
fail
```

```
20 ► CliqueNumber( G )
```

Returns the order,  $\omega(G)$ , of a maximum clique of G.

```
gap> g:=SunGraph(4);
Graph( Category := SimpleGraphs, Order := 8, Size := 14, Adjacencies :=
[ [ 2, 8 ], [ 1, 3, 4, 6, 8 ], [ 2, 4 ], [ 2, 3, 5, 6, 8 ], [ 4, 6 ],
       [ 2, 4, 5, 7, 8 ], [ 6, 8 ], [ 1, 2, 4, 6, 7 ] ] )
gap> CliqueNumber(g);
4
```

Returns the set of all (maximal) cliques of a graph G. A clique is a maximal complete subgraph. Here, we use the Bron-Kerbosch algorithm [BK73].

In the second form, It stops computing cliques after maxNumCli of them have been found.

```
gap> Cliques(Octahedron);
[ [ 1, 3, 5 ], [ 1, 3, 6 ], [ 1, 4, 5 ], [ 1, 4, 6 ], [ 2, 3, 5 ],
      [ 2, 3, 6 ], [ 2, 4, 5 ], [ 2, 4, 6 ] ]
gap> Cliques(Octahedron,4);
[ [ 1, 3, 5 ], [ 1, 3, 6 ], [ 1, 4, 5 ], [ 1, 4, 6 ] ]
```

```
22 ▶ ComplementGraph( G )
```

Α

Returns the new graph H such that V(H) = V(G) and  $xy \in E(H) \iff xy \notin E(G)$ .

```
gap> g:=ClawGraph;
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2, 3, 4 ], [ 1 ], [ 1 ] ] )
gap> ComplementGraph(g);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ ], [ 3, 4 ], [ 2, 4 ], [ 2, 3 ] ] )
```

#### 23 ► CompleteBipartiteGraph( n, m )

F

Returns the complete bipartite whose parts have order n and m respectively. This is the joint (Zykov sum) of two discrete graphs of order n and m.

```
gap> CompleteBipartiteGraph(2,3);
Graph( Category := SimpleGraphs, Order := 5, Size := 6, Adjacencies :=
[ [ 3, 4, 5 ], [ 3, 4, 5 ], [ 1, 2 ], [ 1, 2 ], [ 1, 2 ] ])
```

```
24 \blacktriangleright CompleteGraph( n )
```

F

Returns the complete graph of order n. A complete graph is a graph where all vertices are connected to each other.

```
gap> CompleteGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
```

```
25 ► CompletelyParedGraph( G )
```

Ο

Returns the completely pared graph of G, which is obtained by repeatedly applying ParedGraph until no more dominated vertices remain.

```
gap> g:=PathGraph(6);
Graph( Category := SimpleGraphs, Order := 6, Size := 5, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4, 6 ], [ 5 ] ] )
gap> CompletelyParedGraph(g);
Graph( Category := SimpleGraphs, Order := 1, Size := 0, Adjacencies :=
[ [ ] ] )
```

```
26 ► CompleteMultipartiteGraph( n1, n2 [, n3 ...])
```

F

Returns the complete multipartite graph where the orders of the parts are n1, n2, ... It is also the Zykov sum of discrete graphs of order n1, n2, ...

```
gap> CompleteMultipartiteGraph(2,2,2);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 3, 4, 5, 6 ], [ 3, 4, 5, 6 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],
        [ 1, 2, 3, 4 ], [ 1, 2, 3, 4 ] ] )
```

```
27 ► CompletesOfGivenOrder( G, Ord )
```

Ο

Returns the list of vertex sets of all complete subgraphs of order Ord of G.

```
gap> g:=SunGraph(4);
Graph( Category := SimpleGraphs, Order := 8, Size := 14, Adjacencies :=
[ [ 2, 8 ], [ 1, 3, 4, 6, 8 ], [ 2, 4 ], [ 2, 3, 5, 6, 8 ], [ 4, 6 ],
        [ 2, 4, 5, 7, 8 ], [ 6, 8 ], [ 1, 2, 4, 6, 7 ] ] )
gap> CompletesOfGivenOrder(g,3);
[ [ 1, 2, 8 ], [ 2, 3, 4 ], [ 2, 4, 6 ], [ 2, 4, 8 ], [ 2, 6, 8 ],
        [ 4, 5, 6 ], [ 4, 6, 8 ], [ 6, 7, 8 ] ]
gap> CompletesOfGivenOrder(g,4);
[ [ 2, 4, 6, 8 ] ]
```

## 28 ► Composition( G, H )

O

Returns the composition G[H] of two graphs G and H.

A composition of graphs is obtained by calculating the GraphSum of G with Order(G) copies of H, G[H] = GraphSum(G, [H, ..., H]).

```
gap> g:=CycleGraph(4);;h:=DiscreteGraph(2);;
gap> Composition(g,h);
Graph( Category := SimpleGraphs, Order := 8, Size := 16, Adjacencies :=
[ [ 3, 4, 7, 8 ], [ 3, 4, 7, 8 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],
        [ 3, 4, 7, 8 ], [ 3, 4, 7, 8 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ] ] )
```

 $29 \triangleright \mathsf{Cone}(G)$ 

Returns the cone of graph G. The cone of G is the graph obtained from G by adding a new vertex which is adjacent to every vertex of G. The new vertex is the first one in the new graph.

```
gap> Cone(CycleGraph(4));
Graph( Category := SimpleGraphs, Order := 5, Size := 8, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 5 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 2, 4 ] ] )
```

#### $30 \triangleright \text{ConnectedComponents}(G)$

A

Returns the connected components of G.

```
31 \triangleright ConnectedGraphsOfGivenOrder(n)
```

О

Returns the list of all connected graphs of order n (upto isomorphism). This operation uses Brendan McKay's data published here:

```
https://cs.anu.edu.au/people/Brendan.McKay/data/graphs.html
```

These data are included with the YAGS distribution in its data directory. Hence this operation simply reads the corresponding file in that directory using ImportGraph6( Filename). Therefore, the integer n must be in the range from 1 upto 9. Data for graphs on 10 vertices is also available, but not included with YAGS, it may not be practical to use that data, but if you would like to try, all you have to do is to copy (and to uncompress) the corresponding file into the directory YAGS-Directory/data.

```
gap> ConnectedGraphsOfGivenOrder(3);
[ Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
        [ [ 3 ], [ 3 ], [ 1, 2 ] ] ), Graph( Category := SimpleGraphs, Order :=
        3, Size := 3, Adjacencies := [ [ 2, 3 ], [ 1, 3 ], [ 1, 2 ] ] ) ]
gap> ConnectedGraphsOfGivenOrder(4);
[ Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
        [ [ 4 ], [ 4 ], [ 4 ], [ 1, 2, 3 ] ] ),
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
        [ [ 3, 4 ], [ 4 ], [ 1 ], [ 1, 2 ] ] ),
```

```
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
    [ [ 3, 4 ], [ 4 ], [ 1, 4 ], [ 1, 2, 3 ] ] ),
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
    [ [ 3, 4 ], [ 3, 4 ], [ 1, 2 ], [ 1, 2 ] ] ),
Graph( Category := SimpleGraphs, Order := 4, Size := 5, Adjacencies :=
    [ [ 3, 4 ], [ 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] ),
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
    [ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] ) ]
gap> Length(ConnectedGraphsOfGivenOrder(9));
261080
gap> ConnectedGraphsOfGivenOrder(10);
#W Unreadable File: /opt/gap4r7/pkg/yags/data/graph10c.g6
fail
```

#### $32 \triangleright \text{Coordinates}(G)$

0

Gets the coordinates of the vertices of G, which are used to draw G by Draw(G). If the coordinates have not been previously set, Coordinates returns fail.

```
gap> g:=CycleGraph(4);;
gap> Coordinates(g);
fail
gap> SetCoordinates(g,[[-10,-10],[-10,20],[20,-10],[20,20]]);
gap> Coordinates(g);
[[-10,-10],[-10,20],[20,-10],[20,20]]
```

#### 33 ► CopyGraph( G )

O

Returns a fresh copy of graph G. Only the order and adjacency information is copied, all other known attributes of G are not. Mainly used to transform a graph from one category to another. The new graph will be forced to comply with the TargetGraphCategory.

```
gap> g:=CompleteGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
  [ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
gap> g1:=CopyGraph(g:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 4, Size := 6, Adjacencies :=
  [ [ 2, 3, 4 ], [ 3, 4 ], [ 4 ], [ ] ] )
gap> CopyGraph(g1:GraphCategory:=SimpleGraphs);
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
  [ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
```

#### 34 ► CuadraticRingGraph( Rng )

Ο

Returns the graph G whose vertices are the elements of Rng such that x is adjacent to y iff  $x+z^2=y$  for some z in Rng.

```
gap> CuadraticRingGraph(ZmodnZ(8));
Graph( Category := SimpleGraphs, Order := 8, Size := 12, Adjacencies :=
[ [ 2, 5, 8 ], [ 1, 3, 6 ], [ 2, 4, 7 ], [ 3, 5, 8 ], [ 1, 4, 6 ],
        [ 2, 5, 7 ], [ 3, 6, 8 ], [ 1, 4, 7 ] ] )
```

The 1-skeleton of Plato's cube.

35 ► Cube V

```
gap> Cube;
Graph( Category := SimpleGraphs, Order := 8, Size := 12, Adjacencies :=
[ [ 2, 3, 5 ], [ 1, 4, 6 ], [ 1, 4, 7 ], [ 2, 3, 8 ], [ 1, 6, 7 ],
       [ 2, 5, 8 ], [ 3, 5, 8 ], [ 4, 6, 7 ] ] )
```

## $36 \triangleright \text{CubeGraph(} n \text{)}$

F

Returns the hypercube of dimension n. This is the box product (cartesian product) of n copies of  $K_2$  (an edge).

```
gap> CubeGraph(3);
Graph( Category := SimpleGraphs, Order := 8, Size := 12, Adjacencies :=
[ [ 2, 3, 5 ], [ 1, 4, 6 ], [ 1, 4, 7 ], [ 2, 3, 8 ], [ 1, 6, 7 ],
[ 2, 5, 8 ], [ 3, 5, 8 ], [ 4, 6, 7 ] ] )
```

#### 37 ► CycleGraph( n )

F

Returns the cyclic graph on n vertices.

```
gap> CycleGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] )
```

```
38 ► CylinderGraph( b, h )
```

F

Returns a cylinder of base b and height h. The order of this graph is b(h+1) and it is constructed by taking h+1 copies of the cyclic graph on b vertices, ordering these cycles linearly and then joining consecutive cycles by a zigzagging (2b)-cycle. This graph is a triangulation of the cylinder where all internal vertices are of degree 6 and the border vertices are of degree 4.

```
gap> g:=CylinderGraph(4,1);
Graph( Category := SimpleGraphs, Order := 8, Size := 16, Adjacencies :=
[ [ 2, 4, 5, 6 ], [ 1, 3, 6, 7 ], [ 2, 4, 7, 8 ], [ 1, 3, 5, 8 ],
        [ 1, 4, 6, 8 ], [ 1, 2, 5, 7 ], [ 2, 3, 6, 8 ], [ 3, 4, 5, 7 ] ] )
gap> g:=CylinderGraph(4,2);
Graph( Category := SimpleGraphs, Order := 12, Size := 28, Adjacencies :=
[ [ 2, 4, 5, 6 ], [ 1, 3, 6, 7 ], [ 2, 4, 7, 8 ], [ 1, 3, 5, 8 ],
        [ 1, 4, 6, 8, 9, 10 ], [ 1, 2, 5, 7, 10, 11 ], [ 2, 3, 6, 8, 11, 12 ],
        [ 3, 4, 5, 7, 9, 12 ], [ 5, 8, 10, 12 ], [ 5, 6, 9, 11 ], [ 6, 7, 10, 12 ],
        [ 7, 8, 9, 11 ] ] )
```

#### 39 ► DartGraph

V

A diamond with a pendant vertex and maximum degree 4.

```
gap> DartGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 6, Adjacencies :=
[ [ 2 ], [ 1, 3, 4, 5 ], [ 2, 4, 5 ], [ 2, 3 ], [ 2, 3 ] ])
```

## 40 ► DeclareQtfyProperty( Name, Filter )

F

For internal use.

Declares a YAGS quantifiable property named *Name* for filter *Filter*. This in turns, declares a boolean GAP property *Name* and an integer GAP attribute *QtfyName*.

The user must provide the method Name(Obj, qtfy). If qtfy is false, the method must return a boolean indicating whether the property holds, otherwise, the method must return a non-negative integer quantifying

how far is the object from satisfying the property. In the latter case, returning 0 actually means that the object does satisfy the property.

```
gap> DeclareQtfyProperty("Is2Regular",Graphs);
       gap> InstallMethod(Is2Regular, "for graphs", true, [Graphs, IsBool], 0,
      > function(G,qtfy)
          local x,count;
           count:=0;
          for x in Vertices(G) do
             if VertexDegree(G,x)<> 2 then
       >
              if not qtfy then
                 return false;
              fi;
                 count:=count+1;
            fi;
           od;
           if not qtfy then return true; fi;
           return count;
       > end);
       gap> Is2Regular(CycleGraph(4));
       true
       gap> QtfyIs2Regular(CycleGraph(4));
       gap> Is2Regular(DiamondGraph);
       false
       gap> QtfyIs2Regular(DiamondGraph);
41 \triangleright Diameter(G)
                                                                                              A
    Returns the maximum among the distances between pairs of vertices of G.
       gap> g:=CycleGraph(5);
       Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
       [[2,5],[1,3],[2,4],[3,5],[1,4]])
       gap> Diameter(g);
       2
                                                                                              V
42 ► DiamondGraph
    The graph on 4 vertices and 5 edges.
       gap> DiamondGraph;
       Graph( Category := SimpleGraphs, Order := 4, Size := 5, Adjacencies :=
       [[2,3,4],[1,3],[1,2,4],[1,3]])
                                                                                              F
43 ▶ DiscreteGraph( n )
    Returns the discrete graph of order n. A discrete graph is a graph without edges.
       gap> DiscreteGraph(4);
       Graph( Category := SimpleGraphs, Order := 4, Size := 0, Adjacencies :=
```

[[],[],[])

```
44 ▶ DisjointUnion( G, H )
```

О

Returns the disjoint union of two graphs G and H,  $G \cup H$ .

```
gap> g:=PathGraph(3);h:=PathGraph(2);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> DisjointUnion(g,h);
Graph( Category := SimpleGraphs, Order := 5, Size := 3, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ], [ 5 ], [ 4 ] ] )
```

#### 45 ▶ Distance( G, x, y )

Ο

Returns the length of a minimal path connecting x to y in G.

```
gap> Distance(CycleGraph(5),1,3);
2
gap> Distance(CycleGraph(5),1,5);
1
```

```
46 ▶ Distances( G, A, B )
```

Ο

Given two lists of vertices A, B of a graph G, Distances returns the list of distances for every pair in the cartesian product of A and B. The order of the vertices in lists A and B affects the order of the list of distances returned.

```
gap> g:=CycleGraph(5);;
gap> Distances(g, [1,3], [2,4]);
[ 1, 2, 1, 1 ]
gap> Distances(g, [3,1], [2,4]);
[ 1, 1, 1, 2 ]
```

## 47 ▶ DistanceGraph( G, Dist )

O

Given a graph G and list of distances Dist, DistanceGraph returns the new graph constructed on the vertices of G where two vertices are adjacent iff the distance (in G) between them belongs to the list Dist.

#### 48 ▶ DistanceMatrix( G )

Α

Returns the distance matrix D of a graph G: D[x][y] is the distance in G from vertex x to vertex y. The matrix may be asymmetric if the graph is not simple. An infinite entry in the matrix means that there is no path between the vertices. Floyd's algorithm is used to compute the matrix.

```
gap> g:=PathGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[[2], [1, 3], [2, 4], [3]])
gap> Display(DistanceMatrix(g));
[[0, 1, 2, 3],
 [ 1, 0, 1, 2],
 [ 2, 1, 0, 1],
 [ 3, 2, 1, 0 ] ]
gap> g:=PathGraph(4:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 4, Size := 3, Adjacencies :=
[[2],[3],[4],[]])
gap> Display(DistanceMatrix(g));
] ]
                                         3],
           0,
                     1,
                                         2],
 [ infinity,
                     0,
                               1,
                                         1],
 [ infinity, infinity,
                               0,
 [ infinity, infinity, infinity,
                                         0]]
```

49 ▶ DistanceSet( G, A, B )

0

Given two subsets of vertices A, B of a graph G, DistanceSet returns the set of distances for every pair in the cartesian product of A and B.

```
gap> g:=CycleGraph(5);;
gap> DistanceSet(g, [1,3], [2,4]);
[ 1, 2 ]
```

#### 50 ► Dodecahedron

V

The 1-skeleton of Plato's Dodecahedron.

```
gap> Dodecahedron;
Graph( Category := SimpleGraphs, Order := 20, Size := 30, Adjacencies :=
[ [ 2, 5, 6 ], [ 1, 3, 7 ], [ 2, 4, 8 ], [ 3, 5, 9 ], [ 1, 4, 10 ],
      [ 1, 11, 15 ], [ 2, 11, 12 ], [ 3, 12, 13 ], [ 4, 13, 14 ], [ 5, 14, 15 ],
      [ 6, 7, 16 ], [ 7, 8, 17 ], [ 8, 9, 18 ], [ 9, 10, 19 ], [ 6, 10, 20 ],
      [ 11, 17, 20 ], [ 12, 16, 18 ], [ 13, 17, 19 ], [ 14, 18, 20 ],
      [ 15, 16, 19 ] ] )
```

#### $51 \triangleright DominatedVertices(G)$

Α

Returns the set of dominated vertices of G.

A vertex x is dominated by another vertex y when the closed neighborhood of x is contained in that of y. However, when there are twin vertices (mutually dominated vertices), exactly one of them (in each equivalent class of mutually dominated vertices) does not appear in the returned set.

```
gap> g1:=PathGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> DominatedVertices(g1);
[ 1, 3 ]
gap> g2:=PathGraph(2);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> DominatedVertices(g2);
[ 2 ]
```

 $52 \triangleright DominoGraph$ 

Two squares glued by an edge.

```
gap> DominoGraph;
   Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
   [ [ 2, 4, 6 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3, 5 ], [ 4, 6 ], [ 1, 5 ] ] )

53▶ Draw( G )
Output

Outpu
```

Takes a graph G and makes a drawing of it in a separate window. The user can then view and modify the drawing and finally save the vertex coordinates of the drawing into the graph G.

Within the separate window, type h to toggle on/off the help menu. Besides the keyword commands indicated in the help menu, the user may also move vertices (by dragging them), move the whole drawing (by dragging the background) and scale the drawing (by using the mouse wheel).

```
gap> Coordinates(Icosahedron);
fail
gap> Draw(Icosahedron);
gap> Coordinates(Icosahedron);
[ [ 29, -107 ], [ 65, -239 ], [ 240, -62 ], [ 78, 79 ], [ -107, 28 ],
        [ -174, -176 ], [ -65, 239 ], [ -239, 62 ], [ -78, -79 ], [ 107, -28 ],
        [ 174, 176 ], [ -29, 107 ] ]
```

This preliminary version, should work fine on GNU/Linux and Mac OS X. For other plataforms, you should probably (at least) set up correctly the variable **drawproc** which should point to the correct external program binary. Java binaries are provided for GNU/Linux, Mac OS X and MS Windows.

```
gap> drawproc;
"/usr/share/gap/pkg/yags/bin/draw/application.linux64/draw"
```

```
54 ► DumpObject( Obj )
```

Dumps all information available for object *Obj*. This information includes to which categories it belongs as well as its type and hashing information used by GAP.

```
gap> DumpObject( true );
  Object( TypeObj := NewType( NewFamily( "BooleanFamily", [ 11 ], [ 11 ] ),
      [ 11, 34 ] ), Categories := [ "IS_BOOL" ] )

55 ► EasyExec( Dir, ProgName, InString )
      □ EasyExec( ProgName, InString )
      □ O
```

Calls external program ProgName located in directory Dir, feeding it with InString as input and returning the output of the external program as a string. Dir must be a directory object or a list of directory objects. If Dir is not provided, ProgName must be in the system's binary PATH. 'fail' is returned if the program could not be located.

```
gap> s:=EasyExec("date","");;
gap> s;
"Sun Nov 9 10:36:16 CST 2014\n"
gap> s:=EasyExec("sort","4\n2\n3\n1");;
gap> s;
"1\n2\n3\n4\n"
```

Currently, this operation is not working on MS Windows.

F

```
56 ► Eccentricity( G, x )
```

Returns the distance from a vertex x in graph G to its most distant vertex in G.

```
gap> g:=PathGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4 ] ] )
gap> Eccentricity(g,1);
4
gap> Eccentricity(g,3);
2
```

#### $57 \triangleright \text{Edges}(G)$

O

Returns the list of edges of graph G in the case of SimpleGraphs.

```
gap> g1:=CompleteGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 3, Adjacencies :=
[ [ 2, 3 ], [ 1, 3 ], [ 1, 2 ] ] )
gap> Edges(g1);
[ [ 1, 2 ], [ 1, 3 ], [ 2, 3 ] ]
```

In the case of UndirectedGraphs, it also returns the loops. While in the other categories, Edges actually does not return the edges, but the loops and arrows of G.

```
gap> g2:=CompleteGraph(3:GraphCategory:=UndirectedGraphs);
Graph( Category := UndirectedGraphs, Order := 3, Size := 6, Adjacencies :=
[ [ 1, 2, 3 ], [ 1, 2, 3 ], [ 1, 2, 3 ] ] )
gap> Edges(g2);
[ [ 1, 1 ], [ 1, 2 ], [ 1, 3 ], [ 2, 2 ], [ 2, 3 ], [ 3, 3 ] ]
gap> g3:=CompleteGraph(3:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 9, Adjacencies :=
[ [ 1, 2, 3 ], [ 1, 2, 3 ], [ 1, 2, 3 ] ] )
gap> Edges(g3);
[ [ 1, 1 ], [ 1, 2 ], [ 1, 3 ], [ 2, 1 ], [ 2, 2 ], [ 2, 3 ], [ 3, 1 ],
        [ 3, 2 ], [ 3, 3 ] ]
```

#### $58 \triangleright \text{FanGraph}(n)$

F

Returns the *n*-Fan: The join of a vertex and a (N+1)-path.

```
gap> FanGraph(4);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4, 6 ],
[ 1, 5 ] ] )
```

## 59 ► FishGraph

V

A square and a triangle glued by a vertex.

```
gap> FishGraph;
Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
[ [ 2, 3, 4, 6 ], [ 1, 3 ], [ 1, 2 ], [ 1, 5 ], [ 4, 6 ], [ 1, 5 ] ] )
```

#### 60 ► GemGraph

V

The 3-Fan graph.

```
gap> GemGraph;
    Graph( Category := SimpleGraphs, Order := 5, Size := 7, Adjacencies :=
    [ [ 2, 3, 4, 5 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4 ] ] )
61 ► Graph( Rec )
```

Returns a new graph created from the record Rec. The record must provide the field Category and either the field Adjacencies or the field AdjMatrix.

```
gap> Graph(rec(Category:=SimpleGraphs,Adjacencies:=[[2],[1]]));
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies := [ [ 2 ], [ 1 ] ] )
gap> Graph(rec(Category:=SimpleGraphs,AdjMatrix:=[[false, true],[true, false]]));
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies := [ [ 2 ], [ 1 ] ] )
```

Its main purpose is to import graphs from files, which could have been previously exported using PrintTo.

```
gap> g:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
gap> PrintTo("aux.g","h1:=",g,";");
gap> Read("aux.g");
gap> h1;
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
```

#### 62 ► Graph6ToGraph( String )

Ο

Returns the graph represented by *String* which is encoded using Brendan McKay's graph6 format. This operation allows us to read data in databases which use this format. Several such databases can be found here:

```
https://cs.anu.edu.au/people/Brendan.McKay/data/graphs.html
```

The graph6 format is described here:

```
https://cs.anu.edu.au/people/Brendan.McKay/data/formats.txt

gap> Graph6ToGraph("D?{");
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 5 ], [ 5 ], [ 5 ], [ 5 ], [ 1, 2, 3, 4 ] ] )
gap> Graph6ToGraph("FUzvW");
Graph( Category := SimpleGraphs, Order := 7, Size := 15, Adjacencies :=
[ [ 3, 4, 5, 6, 7 ], [ 4, 5, 6, 7 ], [ 1, 5, 6, 7 ], [ 1, 2, 6 ],
        [ 1, 2, 3, 7 ], [ 1, 2, 3, 4, 7 ], [ 1, 2, 3, 5, 6 ] ] )
gap> Graph6ToGraph("HUzv~z}");
Graph( Category := SimpleGraphs, Order := 9, Size := 29, Adjacencies :=
[ [ 3, 4, 5, 6, 7, 8, 9 ], [ 4, 5, 6, 7, 8, 9 ], [ 1, 5, 6, 7, 8, 9 ],
        [ 1, 2, 3, 4, 5, 6, 9 ], [ 1, 2, 3, 4, 5, 6 ], [ 1, 2, 3, 4, 5, 6, 7 ] ] )
```

See also ImportGraph6( Filename ).

```
63 ► GraphByAdjacencies ( AdjList )
```

F

Returns a new graph having AdjList as its list of adjacencies. The order of the created graph is Length(A), and the set of neighbors of vertex x is A[x].

```
gap> GraphByAdjacencies([[2],[1,3],[2]]);
      Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
       [[2],[1,3],[2]])
    Note, however, that the graph is forced to comply with the TargetGraphCategory.
      gap> GraphByAdjacencies([[1,2,3],[],[]]);
      Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
       [[2,3],[1],[1]])
64 ► GraphByAdjMatrix( Mat )
                                                                                            \mathbf{F}
    Returns a new graph created from an adjacency matrix Mat. The matrix Mat must be a square boolean
    matrix.
      gap> m:=[ [ false, true, false ], [ true, false, true ], [ false, true, false ] ];;
      gap> g:=GraphByAdjMatrix(m);
      Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
       [[2],[1,3],[2]])
      gap> AdjMatrix(g);
       [ [ false, true, false ], [ true, false, true ], [ false, true, false ] ]
    Note, however, that the graph is forced to comply with the TargetGraphCategory.
      gap> m:=[ [ true, true], [ false, false ] ];;
      gap> g:=GraphByAdjMatrix(m);
      Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies := [ [ 2 ], [ 1 ] ] )
      gap> AdjMatrix(g);
       [ [false, true], [true, false]]
65 ► GraphByCompleteCover( Cover)
                                                                                            F
    Returns the minimal graph where the elements of Cover are (the vertex sets of) complete subgraphs.
      gap> GraphByCompleteCover([[1,2,3,4],[4,6,7]]);
      Graph( Category := SimpleGraphs, Order := 7, Size := 9, Adjacencies :=
       [[2, 3, 4], [1, 3, 4], [1, 2, 4], [1, 2, 3, 6, 7], [], [4, 7],
        [4,6]])
                                                                                            F
66 ► GraphByEdges( L )
    Returns the minimal graph such that the pairs in L are edges.
      gap> GraphByEdges([[1,2],[1,3],[1,4],[4,5]]);
      Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
       [[2, 3, 4], [1], [1], [1, 5], [4]])
    The vertices of the constructed graph range from 1 to the maximum of the numbers appearing in L.
      gap> GraphByEdges([[4,3],[4,5]]);
      Graph( Category := SimpleGraphs, Order := 5, Size := 2, Adjacencies :=
       [[],[],[4],[3,5],[4]])
    Note that GraphByWalks has an even greater functionality.
```

```
67 ► GraphByRelation( V, Rel )
F GraphByRelation( n, Rel )
F
```

Returns a new graph created from a set of vertices V and a binary relation Rel, where  $x \sim y$  iff Rel(x,y)=true. In the second form, n is an integer and V is assumed to be  $\{1, 2, \ldots, n\}$ .

```
gap> Rel:=function(x,y) return Intersection(x,y)<>[]; end;;
gap> GraphByRelation([[1,2,3],[3,4,5],[5,6,7]],Rel);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> GraphByRelation(8,function(x,y) return AbsInt(x-y)<=2; end);
Graph( Category := SimpleGraphs, Order := 8, Size := 13, Adjacencies :=
[ [ 2, 3 ], [ 1, 3, 4 ], [ 1, 2, 4, 5 ], [ 2, 3, 5, 6 ], [ 3, 4, 6, 7 ],
        [ 4, 5, 7, 8 ], [ 5, 6, 8 ], [ 6, 7 ] ] )</pre>
```

```
68 ► GraphByWalks( Walk1, Walk2, ...)
```

 $\mathbf{F}$ 

F

Returns the minimal graph such that Walk1, Walk2, etc are Walks.

```
gap> GraphByWalks([1,2,3,4,1],[1,5,6]);
Graph( Category := SimpleGraphs, Order := 6, Size := 6, Adjacencies :=
[ [ 2, 4, 5 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ], [ 1, 6 ], [ 5 ] ])
```

Walks can be *nested*, which greatly improves the versatility of this function.

```
gap> GraphByWalks([1,[2,3,4],5],[5,6]);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 5 ], [ 1, 2, 4, 5 ], [ 1, 3, 5 ], [ 2, 3, 4, 6 ], [ 5 ] ] )
```

The vertices in the constructed graph range from 1 to the maximum of the numbers appearing in Walk1, Walk2, ... etc.

```
gap> GraphByWalks([4,2],[3,6]);
    Graph( Category := SimpleGraphs, Order := 6, Size := 2, Adjacencies :=
        [ [ ], [ 4 ], [ 6 ], [ 2 ], [ ], [ 3 ] ] )
69 ► GraphCategory( [ G, ... ] )
```

For internal use. Returns the minimal common category to a list of graphs. If the list of graphs is empty, the default category is returned.

The partial order (by inclusion) among graph categories is as follows:

```
SimpleGraphs < UndirectedGraphs < Graphs,

OrientedGraphs < LooplessGraphs < Graphs,

SimpleGraphs < LooplessGraphs < Graphs
```

```
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
        [ [ 2 ], [ 1 ] ] )
        gap> g2:=CompleteGraph(2:GraphCategory:=OrientedGraphs);
        Graph( Category := OrientedGraphs, Order := 2, Size := 1, Adjacencies :=
        [ [ 2 ], [ ] ] )
        gap> g3:=CompleteGraph(2:GraphCategory:=UndirectedGraphs);
        Graph( Category := UndirectedGraphs, Order := 2, Size := 3, Adjacencies :=
        [ [ 1, 2 ], [ 1, 2 ] ] )
        gap> GraphCategory([g1,g2,g3]);
        <Operation "Graphs">
        gap> GraphCategory([g1,g2]);
        <Operation "LooplessGraphs">
        gap> GraphCategory([g1,g3]);
        <Operation "UndirectedGraphs">
        Foraphs()
```

gap> g1:=CompleteGraph(2:GraphCategory:=SimpleGraphs);

Graphs is the most general graph category in YAGS. This category contains all graphs that can be represented in YAGS. A graph in this category may contain loops, arrows and edges (which in YAGS are exactly the same as two opposite arrows between some pair of vertices). This graph category has no parent category.

```
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=SimpleGraphs);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
```

#### 71 ▶ GraphsOfGivenOrder( n )

O

 $\mathbf{C}$ 

Returns the list of all graphs of order n (upto isomorphism). This operation uses Brendan McKay's data published here:

```
https://cs.anu.edu.au/people/Brendan.McKay/data/graphs.html
```

These data are included with the YAGS distribution in its data directory. Hence this operation simply reads the corresponding file in that directory using ImportGraph6(Filename). Therefore, the integer n must be in the range from 1 upto 9. Data for graphs on 10 vertices is also available, but not included with YAGS, it may not be practical to use that data, but if you would like to try, all you have to do is to copy (and to uncompress) the corresponding file into the directory YAGS-Directory/data.

```
gap> GraphsOfGivenOrder(2);
[ Graph( Category := SimpleGraphs, Order := 2, Size := 0, Adjacencies :=
        [ [ ], [ ] ] ), Graph( Category := SimpleGraphs, Order := 2, Size :=
        1, Adjacencies := [ [ 2 ], [ 1 ] ] ) ]
gap> GraphsOfGivenOrder(3);
[ Graph( Category := SimpleGraphs, Order := 3, Size := 0, Adjacencies :=
        [ [ ], [ ], [ ] ] ), Graph( Category := SimpleGraphs, Order :=
        3, Size := 1, Adjacencies := [ [ 3 ], [ 1 ] ] ),
        Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
        [ [ 3 ], [ 3 ], [ 1, 2 ] ] ), Graph( Category := SimpleGraphs, Order :=
        3, Size := 3, Adjacencies := [ [ 2, 3 ], [ 1, 3 ], [ 1, 2 ] ] ) ]
gap> Length(GraphsOfGivenOrder(9));
274668
```

```
gap> GraphsOfGivenOrder(10);
#W Unreadable File: /opt/gap4r7/pkg/yags/data/graph10.g6
fail
```

```
72 \triangleright GraphSum( G, L )
```

Returns the lexicographic sum of a list of graphs L over a graph G.

The lexicographic sum is computed as follows:

Given G, with Order(G) = n and a list of n graphs  $L = [G_1, \ldots, G_n]$ , We take the disjoint union of  $G_1, G_2, \ldots, G_n$  and then we add all the edges between  $G_i$  and  $G_j$  whenever [i, j] is and edge of G.

If L contains holes, the trivial graph is used in place.

```
gap> t:=TrivialGraph;; g:=CycleGraph(4);;
gap> GraphSum(PathGraph(3),[t,g,t]);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 5, 6 ], [ 1, 2, 4, 6 ], [ 1, 3, 5, 6 ],
        [ 1, 2, 4, 6 ], [ 2, 3, 4, 5 ] ] )
gap> GraphSum(PathGraph(3),[,g,]);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 5, 6 ], [ 1, 2, 4, 6 ], [ 1, 3, 5, 6 ],
        [ 1, 2, 4, 6 ], [ 2, 3, 4, 5 ] ] )
```

```
73 ▶ GraphToRaw( FileName, G )
```

О

Converts a YAGS graph G into a raw format (number of vertices, coordinates and adjacency matrix) and writes the converted data to the file FileName. For use by the external program draw (see Draw(G)).

```
gap> g:=CycleGraph(4);;
gap> GraphToRaw("mygraph.raw",g);
```

```
74 \triangleright GraphUpdateFromRaw(FileName, G)
```

Ο

Updates the coordinates of G from a file FileName in raw format. Intended for internal use only.

```
75 ► GroupGraph( G, Grp, Act )
   GroupGraph( G, Grp )
```

Given a graph G, a group Grp and an action Act of Grp in some set S which contains Vertices(G), GroupGraph returns a new graph with vertex set  $\{act(v,g):g\in Grp,v\in Vertices(G)\}$  and edge set  $\{\{act(v,g),act(u,g)\}:g\ inGrp\{u,v\}\in Edges(G)\}$ .

If Act is omited, the standard GAP action OnPoints is used.

```
gap> GroupGraph(GraphByWalks([1,2]),Group([(1,2,3,4,5),(2,5)(3,4)]));
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] )
```

#### 76 ► HouseGraph

V

A 4-Cycle and a triangle glued by an edge.

```
gap> HouseGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 6, Adjacencies :=
[ [ 2, 4, 5 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3, 5 ], [ 1, 4 ] ] )
```

77► Icosahedron V

The 1-skeleton of Plato's icosahedron.

```
gap> Icosahedron;
Graph( Category := SimpleGraphs, Order := 12, Size := 30, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3, 6, 9, 10 ], [ 1, 2, 4, 10, 11 ],
      [ 1, 3, 5, 7, 11 ], [ 1, 4, 6, 7, 8 ], [ 1, 2, 5, 8, 9 ],
      [ 4, 5, 8, 11, 12 ], [ 5, 6, 7, 9, 12 ], [ 2, 6, 8, 10, 12 ],
      [ 2, 3, 9, 11, 12 ], [ 3, 4, 7, 10, 12 ], [ 7, 8, 9, 10, 11 ] ] )
```

```
78 ► ImportGraph6( Filename )
```

Ο

Returns the list of graphs represented in *Filename* which are encoded using Brendan McKay's graph6 format. This operation allows us to read data in databases which use this format. Several such databases can be found here:

```
https://cs.anu.edu.au/people/Brendan.McKay/data/graphs.html
```

The graph6 format is described here:

```
https://cs.anu.edu.au/people/Brendan.McKay/data/formats.txt
```

The following example assumes that you have a file named graph3.g6 in your working directory which encodes graphs in graph6 format; the contents of this file is assumed to be as indicated after the first command in the example.

```
gap> Exec("cat graph3.g6");
B?
B0
BW
Bw
gap> ImportGraph6("graph3.g6");
[ Graph( Category := SimpleGraphs, Order := 3, Size := 0, Adjacencies :=
        [ [ ], [ ] ] ), Graph( Category := SimpleGraphs, Order := 3, Size := 1, Adjacencies := [ [ 3 ], [ ], [ 1 ] ] ),
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
        [ [ 3 ], [ 3 ], [ 1, 2 ] ] ), Graph( Category := SimpleGraphs, Order := 3, Size := 3, Adjacencies := [ [ 2, 3 ], [ 1, 3 ], [ 1, 2 ] ] )]
```

```
79 \triangleright \text{ in}(G, Catgy)
```

Returns true if graph G belongs to category Catqy and false otherwise.

```
80 \blacktriangleright InducedSubgraph( G, V )
```

Returns the subgraph of graph G induced by the vertex set V.

```
gap> g:=CycleGraph(6);
Graph( Category := SimpleGraphs, Order := 6, Size := 6, Adjacencies :=
[ [ 2, 6 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4, 6 ], [ 1, 5 ] ] )
gap> InducedSubgraph(g,[3,4,6]);
Graph( Category := SimpleGraphs, Order := 3, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ], [ ] ] )
```

The order of the elements in V does matter.

```
gap> InducedSubgraph(g,[6,3,4]); Graph( Category := SimpleGraphs, Order := 3, Size := 1, Adjacencies := [ [ ], [ 3 ], [ 2 ] ] )  
81  InNeigh( G, x )
```

Returns the list of in-neighbors of x in G.

```
gap> tt:=CompleteGraph(5:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 5, Size := 10, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 3, 4, 5 ], [ 4, 5 ], [ 5 ], [ ] ] )
gap> InNeigh(tt,3);
[ 1, 2 ]
gap> OutNeigh(tt,3);
[ 4, 5 ]
```

#### $82 \blacktriangleright$ IntersectionGraph( L )

F

O

Returns the intersection graph of the family of sets L. This graph has a vertex for every set in L, and two such vertices are adjacent iff the corresponding sets have non-empty intersection.

```
gap> IntersectionGraph([[1,2,3],[3,4,5],[5,6,7]]);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
```

```
83► IsBoolean( Obj )
```

 $\mathbf{F}$ 

Returns true if object Obj is true or false and false otherwise.

```
gap> IsBoolean( true ); IsBoolean( fail ); IsBoolean ( false );
true
false
true
```

#### $84 \triangleright IsCliqueGated(G)$

Р

Returns true if G is a clique gated graph [HK96].

```
85 ► IsCliqueHelly( G )
```

Р

Returns true if the set of (maximal) cliques G satisfy the Helly property.

The Helly property is defined as follows:

A non-empty family  $\mathcal{F}$  of non-empty sets satisfies the Helly property if every pairwise intersecting subfamily of  $\mathcal{F}$  has a non-empty total intersection.

Here we use the Dragan-Szwarcfiter characterization [Dra89,Szw97] to compute the Helly property.

```
gap> g:=SunGraph(3);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 2, 6 ], [ 1, 3, 4, 6 ], [ 2, 4 ], [ 2, 3, 5, 6 ], [ 4, 6 ],
        [ 1, 2, 4, 5 ] ] )
gap> IsCliqueHelly(g);
false
```

```
86 ► IsComplete( G, L )
```

Ο

Returns true if L induces a complete subgraph of G.

```
gap> IsComplete(DiamondGraph,[1,2,3]);
       true
       gap> IsComplete(DiamondGraph,[1,2,4]);
       false
                                                                                                   Р
87 \blacktriangleright IsCompleteGraph(G)
    Returns true if graph G is a complete graph, false otherwise. In a complete graph every pair of vertices
    is an edge.
                                                                                                   Ρ
88 ► IsDiamondFree( G )
    Returns true if G is free from induced diamonds, false otherwise.
       gap> IsDiamondFree(Cube);
       true
       gap> IsDiamondFree(Octahedron);
       false
                                                                                                   Ο
89 ► IsEdge( G , [x, y] )
    Returns true if [x,y] is an edge of G.
       gap> IsEdge(PathGraph(3),[1,2]);
       true
       gap> IsEdge(PathGraph(3),[1,3]);
       false
90 ► IsIsomorphicGraph( G, H )
                                                                                                   O
    Returns true when G is isomorphic to H and false otherwise.
       gap> g:=PowerGraph(CycleGraph(6),2);;h:=Octahedron;;
       gap> IsIsomorphicGraph(g,h);
       true
91 ▶ IsLoopless( G )
                                                                                                   Ρ
    Returns true if graph G have no loops, false otherwise. Loops are edges from a vertex to itself.
92 ► IsoMorphism(G, H)
                                                                                                   O
    Returns one isomorphism from G to H or fail if none exists. If G has n vertices, an isomorphisms f: G \to H
    is represented as the list F=[f(1), f(2), ..., f(n)].
       gap> g:=CycleGraph(4);;h:=CompleteBipartiteGraph(2,2);;
       gap> f:=IsoMorphism(g,h);
       [1, 3, 2, 4]
    See NextIsoMorphism( G, H, F ).
93 ► IsoMorphisms(G, H)
                                                                                                   O
    Returns the list of all isomorphism from G to H. If G has n vertices, an isomorphisms f: G \to H is
    represented as the list F=[f(1), f(2), \ldots, f(n)].
       gap> g:=CycleGraph(4);;h:=CompleteBipartiteGraph(2,2);;
       gap> IsoMorphisms(g,h);
       [[1, 3, 2, 4], [1, 4, 2, 3], [2, 3, 1, 4], [2, 4, 1, 3],
         [3, 1, 4, 2], [3, 2, 4, 1], [4, 1, 3, 2], [4, 2, 3, 1]]
```

Ρ

Р

Returns true if graph G is an oriented graph, false otherwise. Regardless of the categories that G belongs to, G is oriented if whenever [x,y] is an edge of G, [y,x] is not.

```
95► IsSimple( G )
```

Returns true if graph G is a simple graph, false otherwise. Regardless of the categories that G belongs to, G is simple if and only if G is undirected and loopless.

Returns true if the graph G is simple regardless of its category.

```
96► IsTournament( G )
```

Returns true if G is a tournament.

```
gap> tt:=CompleteGraph(5:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 5, Size := 10, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 3, 4, 5 ], [ 4, 5 ], [ 5 ], [ ] ] )
gap> IsTournament(tt);
true
```

```
97 ► IsTransitiveTournament( G )
```

Returns true if G is a transitive tournament.

```
gap> tt:=CompleteGraph(5:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 5, Size := 10, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 3, 4, 5 ], [ 4, 5 ], [ 5 ], [ ] ] )
gap> IsTransitiveTournament(tt);
true
```

```
98 \blacktriangleright IsUndirected(G)
```

Returns true if graph G is an undirected graph, false otherwise. Regardless of the categories that G belongs to, G is undirected if whenever [x,y] is an edge of G, [y,x] is also an egde of G.

```
99 ► JohnsonGraph( n, r )
```

Returns the Johnson graph J(n, r). The Johnson Graph is the graph whose vertices are r-subset of the set  $\{1, 2, \ldots, n\}$ , two of them being adjacent iff they intersect in exactly r-1 elements.

```
gap> g:=JohnsonGraph(4,2);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 4, 6 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],
        [ 1, 3, 4, 6 ], [ 2, 3, 4, 5 ] ] )
gap> VertexNames(g);
[ [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 2, 3 ], [ 2, 4 ], [ 3, 4 ] ]
```

```
100 \triangleright \text{Join}(G, H)
```

Returns the join graph G + H of G and H (also known as the Zykov sum); it is the graph obtained from the disjoint union of G and H by adding every possible edge from every vertex in G to every vertex in H.

```
gap> g:=DiscreteGraph(2);h:=CycleGraph(4);
       Graph( Category := SimpleGraphs, Order := 2, Size := 0, Adjacencies :=
       [[],[]]
       Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
       [[2, 4], [1, 3], [2, 4], [1, 3]])
       gap> Join(g,h);
       Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
       [[3, 4, 5, 6], [3, 4, 5, 6], [1, 2, 4, 6], [1, 2, 3, 5],
         [1, 2, 4, 6], [1, 2, 3, 5]])
                                                                                          V
101 ► KiteGraph
    A diamond with a pendant vertex and maximum degree 3.
       gap> KiteGraph;
       Graph( Category := SimpleGraphs, Order := 5, Size := 6, Adjacencies :=
       [[2], [1, 3, 4], [2, 4, 5], [2, 3, 5], [3, 4]])
102 \blacktriangleright \text{LineGraph(} G \text{)}
                                                                                          O
    Returns the line graph L(G) of graph G. The line graph is the intersection graph of the edges of G, i.e. the
    vertices of L(G) are the edges of G two of them being adjacent iff they are incident.
       gap> g:=Tetrahedron;
       Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
       [[2, 3, 4], [1, 3, 4], [1, 2, 4], [1, 2, 3]])
       gap> LineGraph(g);
       Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
       [[2, 3, 4, 5], [1, 3, 4, 6], [1, 2, 5, 6], [1, 2, 5, 6],
         [1, 3, 4, 6], [2, 3, 4, 5]])
                                                                                          O
103 ► Link( G, x )
    Returns the subgraph of G induced by the neighbors of x.
       gap> Link(SnubDisphenoid,1);
       Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
       [[2,5],[1,3],[2,4],[3,5],[1,4]])
       gap> Link(SnubDisphenoid,3);
       Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
       [[2,3],[1,4],[1,4],[2,3]])
104 \triangleright Links(G)
                                                                                          Α
    Returns the list of subgraphs of G induced by the neighbors of each vertex of G.
       gap> Links(SnubDisphenoid);
       [ Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
           [[2,5],[1,3],[2,4],[3,5],[1,4]]),
         Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
           [[2,5],[1,3],[2,4],[3,5],[1,4]]),
         Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
           [[2,3],[1,4],[1,4],[2,3]]),
         Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
           [[2,3],[1,4],[1,4],[2,3]]),
         Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
```

```
[ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] ),
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] ),
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 3, 4 ], [ 3, 4 ], [ 1, 2 ], [ 1, 2 ] ] ),
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 3 ], [ 1, 4 ], [ 1, 4 ], [ 2, 3 ] ] ) ]
```

#### 105 ► LooplessGraphs()

 $\mathbf{C}$ 

LooplessGraphs is a graph category in YAGS. A graph in this category may contain arrows and edges but no loops. The parent of this category is Graphs.

```
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=LooplessGraphs);
Graph( Category := LooplessGraphs, Order := 3, Size := 3, Adjacencies :=
[ [ 2 ], [ 1 ], [ 2 ] ] )
```

## 106 ► MaxDegree( G )

Ο

Returns the maximum degree in graph G.

```
gap> g:=GemGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 7, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4 ] ] )
gap> MaxDegree(g);
4
```

## 107 ► MinDegree( G )

Ο

Returns the minimum degree in graph G.

```
gap> g:=GemGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 7, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4 ] ] )
gap> MinDegree(g);
2
```

```
108 ► NextIsoMorphism( G, H, F )
```

O

Returns the next isomorphism (after F) from G to H in the lexicographic order; returns fail if there are no more isomorphisms. If G has n vertices, an isomorphisms  $f:G\to H$  is represented as the list  $F=[f(1), f(2), \ldots, f(n)]$ .

```
gap> g:=CycleGraph(4);;h:=CompleteBipartiteGraph(2,2);;
gap> f:=IsoMorphism(g,h);
[ 1, 3, 2, 4 ]
gap> NextIsoMorphism(g,h,f);
[ 1, 4, 2, 3 ]
gap> NextIsoMorphism(g,h,f);
[ 2, 3, 1, 4 ]
gap> NextIsoMorphism(g,h,f);
[ 2, 4, 1, 3 ]
```

O

```
109 ► NextPropertyMorphism(G, H, F, PropList)
```

Returns the next morphism (in lexicographic order) from G to H satisfying the list of properties PropList starting with (possibly incomplete) morphism F. The morphism found will me returned  ${\bf and}$  stored in F in order to use it as the next starting point, in case NextPropertyMorphism is called again. The operation returns fail if there are no more morphisms of the specified type.

A number of preprogrammed properties are provided by YAGS, and the user may create additional ones. The properties provided are: CHK\_WEAK, CHK\_MORPH, CHK\_METRIC, CHK\_CMPLT, CHK\_MONO and CHK\_EPI.

If G has n vertices and  $f: G \to H$  is a morphism, it is represented as  $F = [f(1), f(2), \ldots, f(n)]$ .

```
gap> g:=CycleGraph(4);;h:=CompleteBipartiteGraph(2,2);;
gap> f:=[];; PropList:=[CHK_MORPH,CHK_MONO];;
gap> NextPropertyMorphism(g,h,f,PropList);
[1, 3, 2, 4]
gap> NextPropertyMorphism(g,h,f,PropList);
[1, 4, 2, 3]
gap> NextPropertyMorphism(g,h,f,PropList);
[2,3,1,4]
gap> NextPropertyMorphism(g,h,f,PropList);
[2, 4, 1, 3]
gap> NextPropertyMorphism(g,h,f,PropList);
[3, 1, 4, 2]
gap> NextPropertyMorphism(g,h,f,PropList);
[3, 2, 4, 1]
gap> NextPropertyMorphism(g,h,f,PropList);
[4, 1, 3, 2]
gap> NextPropertyMorphism(g,h,f,PropList);
[4, 2, 3, 1]
gap> NextPropertyMorphism(g,h,f,PropList);
fail
```

```
110 ► NumberOfCliques( G )

NumberOfCliques( G, maxNumCli )

A
O
```

Returns the number of (maximal) cliques of G. In the second form, It stops computing cliques after maxNum-Cli of them have been counted and returns maxNumCli in case G has maxNumCli or more cliques.

```
gap> NumberOfCliques(Icosahedron);
20
gap> NumberOfCliques(Icosahedron,15);
15
gap> NumberOfCliques(Icosahedron,50);
20
```

This implementation discards the cliques once counted hence, given enough time, it can compute the number of cliques of G even if the set of cliques does not fit in memory.

```
gap> NumberOfCliques(OctahedralGraph(30));
1073741824
```

```
111 ► NumberOfConnectedComponents( G)
```

Returns the number of connected components of G.

Α

#### $112 \triangleright \text{OctahedralGraph}(n)$

F

Return the *n*-dimensional octahedron. This is the complement of *n* copies of  $K_2$  (an edge). It is also the (2n-2)-regular graph on 2n vertices.

```
gap> OctahedralGraph(3);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 3, 4, 5, 6 ], [ 3, 4, 5, 6 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],
[ 1, 2, 3, 4 ], [ 1, 2, 3, 4 ] ] )
```

113 ► Octahedron V

The 1-skeleton of Plato's octahedron.

```
gap> Octahedron;
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 3, 4, 5, 6 ], [ 3, 4, 5, 6 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],
      [ 1, 2, 3, 4 ], [ 1, 2, 3, 4 ] ] )
```

```
114 ▶ Order( G )
```

Returns the number of vertices, of graph G.

```
gap> Order(Icosahedron);
12
```

```
115 ► OrientedGraphs()
```

С

OrientedGraphs is a graph category in YAGS. A graph in this category may contain arrows, but no loops or edges. The parent of this category is LooplessGraphs.

```
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ ], [ 2 ] ] )
```

```
116 \blacktriangleright OutNeigh( G, x )
```

Ο

Returns the list of out-neighbors of x in G.

```
gap> tt:=CompleteGraph(5:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 5, Size := 10, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 3, 4, 5 ], [ 4, 5 ], [ 5 ], [ ] ] )
gap> InNeigh(tt,3);
[ 1, 2 ]
gap> OutNeigh(tt,3);
[ 4, 5 ]
```

## 117 ▶ ParachuteGraph

V

The complement of a ParapluieGraph; The suspension of a 4-path with a pendant vertex attached to the south pole.

```
gap> ParachuteGraph;
Graph( Category := SimpleGraphs, Order := 7, Size := 12, Adjacencies :=
[ [ 2 ], [ 1, 3, 4, 5, 6 ], [ 2, 4, 7 ], [ 2, 3, 5, 7 ], [ 2, 4, 6, 7 ],
       [ 2, 5, 7 ], [ 3, 4, 5, 6 ] ] )
```

## 118 ► ParapluieGraph

V

A 3-Fan graph with a 3-path attached to the universal vertex.

```
gap> ParapluieGraph;
Graph( Category := SimpleGraphs, Order := 7, Size := 9, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4, 5, 6, 7 ], [ 3, 5 ], [ 3, 4, 6 ], [ 3, 5, 7 ],
       [ 3, 6 ] ] )
```

## 119 $\blacktriangleright$ ParedGraph( G )

O

Returns the pared graph of G. This is the induced subgraph obtained from G by removing its dominated vertices. When there are twin vertices (mutually dominated vertices), exactly one of them survives the paring in each equivalent class of mutually dominated vertices.

```
gap> g1:=PathGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3 ] ] )
gap> ParedGraph(g1);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> g2:=PathGraph(2);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> ParedGraph(g2);
Graph( Category := SimpleGraphs, Order := 1, Size := 0, Adjacencies :=
[ [ ] ] )
```

## $120 \triangleright PathGraph(n)$

F

Returns the path graph on n vertices.

```
gap> PathGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3 ] ] )
```

## 121 ► PawGraph

V

The graph on 4 vertices, 4 edges and maximum degree 3: A triangle with a pendant vertex.

```
gap> PawGraph;
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3, 4 ], [ 2, 4 ], [ 2, 3 ] ] )
```

## 122 ▶ PetersenGraph

V

The 3-regular graph on 10 vertices having girth 5.

```
gap> PetersenGraph;
Graph( Category := SimpleGraphs, Order := 10, Size := 15, Adjacencies :=
[ [ 2, 5, 6 ], [ 1, 3, 7 ], [ 2, 4, 8 ], [ 3, 5, 9 ], [ 1, 4, 10 ],
      [ 1, 8, 9 ], [ 2, 9, 10 ], [ 3, 6, 10 ], [ 4, 6, 7 ], [ 5, 7, 8 ] ] )
```

```
123 ▶ PowerGraph( G, exp)
```

О

Returns the DistanceGraph of G using [0, 1, ..., exp] as the list of distances. Note that the distance 0 in the list produces loops in the new graph only when the TargetGraphCategory admits loops.

```
gap> g:=PathGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4 ] ] )
gap> PowerGraph(g,1);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4 ] ] )
gap> PowerGraph(g,1:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 5, Size := 13, Adjacencies :=
[ [ 1, 2 ], [ 1, 2, 3 ], [ 2, 3, 4 ], [ 3, 4, 5 ], [ 4, 5 ] ] )
```

## 124 ▶ PropertyMorphism( G, H, PropList )

O

Returns the first morphism (in lexicographic order) from G to H satisfying the list of properties PropList.

A number of preprogrammed properties are provided by YAGS, and the user may create additional ones. The properties provided are: CHK\_WEAK, CHK\_MORPH, CHK\_METRIC, CHK\_CMPLT, CHK\_MONO and CHK\_EPI.

If G has n vertices and  $f: G \to H$  is a morphism, it is represented as  $F = [f(1), f(2), \ldots, f(n)]$ .

```
gap> g:=CycleGraph(4);;h:=CompleteBipartiteGraph(2,2);;
gap> PropList:=[CHK_MORPH];;
gap> PropertyMorphism(g,h,PropList);
[ 1, 3, 1, 3 ]
```

```
125 ▶ PropertyMorphisms(G, H, PropList)
```

Ο

Returns all morphisms from G to H satisfying the list of properties PropList.

A number of preprogrammed properties are provided by YAGS, and the user may create additional ones. The properties provided are: CHK\_WEAK, CHK\_MORPH, CHK\_METRIC, CHK\_CMPLT, CHK\_MONO and CHK\_EPI.

If G has n vertices and  $f: G \to H$  is a morphism, it is represented as  $F = [f(1), f(2), \ldots, f(n)]$ .

```
gap> g:=CycleGraph(4);;h:=CompleteBipartiteGraph(2,2);;
gap> PropList:=[CHK_WEAK,CHK_MONO];;
gap> PropertyMorphisms(g,h,PropList);
[[ 1, 3, 2, 4 ], [ 1, 4, 2, 3 ], [ 2, 3, 1, 4 ], [ 2, 4, 1, 3 ],
      [ 3, 1, 4, 2 ], [ 3, 2, 4, 1 ], [ 4, 1, 3, 2 ], [ 4, 2, 3, 1 ]]
```

#### $126 \triangleright QtfyIsSimple(G)$

Α

For internal use. Returns how far is graph G from being simple.

```
127 ► QuotientGraph( G, Part )
OutlientGraph( G, L1, L2 )
O
```

Returns the quotient graph of graph G given a vertex partition Part, by identifying any two vertices in the same part. The vertices of the quotient graph are the parts in the partition Part two of them being adjacent iff any vertex in one part is adjacent to any vertex in the other part. Singletons may be omitted in Part.

```
gap> g:=PathGraph(8);;
       gap> QuotientGraph(g, [[1,5,8], [2], [3], [4], [6], [7]]);
       Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
       [[2, 4, 5, 6], [1, 3], [2, 4], [1, 3], [1, 6], [1, 5]])
       gap> QuotientGraph(g,[[1,5,8]]);
       Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
       [[2, 4, 5, 6], [1, 3], [2, 4], [1, 3], [1, 6], [1, 5]])
    In its second form, QuotientGraph identifies each vertex in list L1, with the corresponding vertex in list
    L2. L1 and L2 must have the same length, but any or both of them may have repetitions.
       gap> g:=PathGraph(8);;
       gap> QuotientGraph(g,[[1,7],[4,8]]);
       Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
       [[2,4,6],[1,3],[2,4],[1,3,5],[4,6],[1,5]])
       gap> QuotientGraph(g, [1,4], [7,8]);
       Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
       [[2,4,6],[1,3],[2,4],[1,3,5],[4,6],[1,5]])
128 ▶ Radius( G )
                                                                                        Α
    Returns the minimal eccentricity among the vertices of graph G.
       gap> Radius(PathGraph(5));
                                                                                        F
129 ► RandomGraph(n, p)
  ► RandomGraph( n )
                                                                                        F
    Returns a random graph of order n taking the rational p \in [0,1] as the edge probability.
       gap> RandomGraph(5,1/3);
       Graph( Category := SimpleGraphs, Order := 5, Size := 2, Adjacencies :=
       [[5],[5],[],[],[1,2]])
       gap> RandomGraph(5,2/3);
       Graph( Category := SimpleGraphs, Order := 5, Size := 6, Adjacencies :=
       [[4,5],[3,4,5],[2,4],[1,2,3],[1,2]])
       gap> RandomGraph(5,1/2);
       Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
       [[2,5],[1,3,5],[2],[],[1,2]])
    If p is ommitted, the edge probability is taken to be 1/2.
       gap> RandomGraph(5);
       Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
       [[2,3],[1],[1,4,5],[3,5],[3,4]])
       gap> RandomGraph(5);
       Graph( Category := SimpleGraphs, Order := 5, Size := 3, Adjacencies :=
       [[2,5],[1,4],[],[2],[1]])
130 ► RandomlyPermuted( Obj )
                                                                                        O
```

Returns a copy of Obj with the order of its elements permuted randomly. Currently, the operation is implemented for lists and graphs.

```
gap> RandomlyPermuted([1..9]);
       [ 9, 7, 5, 3, 1, 4, 8, 6, 2 ]
       gap> g:=PathGraph(4);
       Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
       [[2],[1,3],[2,4],[3]])
       gap> RandomlyPermuted(g);
       Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
       [[4],[3,4],[2],[1,2]])
131 ► RandomPermutation(n)
                                                                                        O
    Returns a random permutation of the list [1, 2, \ldots n].
       gap> RandomPermutation(12);
       (1,8,10)(2,7,9,12)(3,5,11)(4,6)
                                                                                        O
132 ► RemoveEdges( G, E )
    Returns a new graph created from graph G by removing the edges in list E.
       gap> g:=CompleteGraph(4);
       Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
       [[2, 3, 4], [1, 3, 4], [1, 2, 4], [1, 2, 3]])
       gap> RemoveEdges(g,[[1,2]]);
       Graph( Category := SimpleGraphs, Order := 4, Size := 5, Adjacencies :=
       [[3,4],[3,4],[1,2,4],[1,2,3]])
       gap> RemoveEdges(g,[[1,2],[3,4]]);
       Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
       [[3, 4], [3, 4], [1, 2], [1, 2]])
133 ► RemoveVertices( G, V )
                                                                                        O
    Returns a new graph created from graph G by removing the vertices in list V.
       gap> g:=PathGraph(5);
       Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
       [[2],[1,3],[2,4],[3,5],[4]])
       gap> RemoveVertices(g,[3]);
       Graph( Category := SimpleGraphs, Order := 4, Size := 2, Adjacencies :=
       [[2],[1],[4],[3]])
       gap> RemoveVertices(g,[1,3]);
       Graph( Category := SimpleGraphs, Order := 3, Size := 1, Adjacencies :=
       [[],[3],[2]])
                                                                                        V
134 ► RGraph
    A square with two pendant vertices attached to the same vertex of the square.
       gap> RGraph;
       Graph( Category := SimpleGraphs, Order := 6, Size := 6, Adjacencies :=
       [[2],[1,3,5,6],[2,4],[3,5],[2,4],[2]])
                                                                                        0
135 ► RingGraph( Rng, Elms)
```

Returns the graph G whose vertices are the elements of the ring Rng such that x is adjacent to y iff x+r=y for some r in Elms.

```
gap> r:=FiniteField(8);Elements(r);
    GF(2^3)
    [ 0*Z(2), Z(2)^0, Z(2^3), Z(2^3)^2, Z(2^3)^3, Z(2^3)^4, Z(2^3)^5, Z(2^3)^6 ]
    gap> RingGraph(r,[Z(2^3),Z(2^3)^4]);
    Graph( Category := SimpleGraphs, Order := 8, Size := 8, Adjacencies :=
        [ [ 3, 6 ], [ 5, 7 ], [ 1, 4 ], [ 3, 6 ], [ 2, 8 ], [ 1, 4 ], [ 2, 8 ],
        [ 5, 7 ] ] )

136► SetCoordinates( G, Coord )

Sets the coordinates of the vertices of G, which are used to draw G by Draw( G ).

gap> g:=CycleGraph(4);;
    gap> SetCoordinates(g,[[-10,-10],[-10,20],[20,-10], [20,20]]);
    gap> Coordinates(g);
    [ [ -10, -10 ], [ -10, 20 ], [ 20, -10 ], [ 20, 20 ] ]
```

## 137 ► SetDefaultGraphCategory( Catgy )

graphs

F

Sets the default graph category to *Categy*. The default graph category is used when constructing new graphs when no other graph category is indicated. New graphs are always forced to comply with the TargetGraph-Category, so loops may be removed, and arrows may replaced by edges or viceversa, depending on the category that the new graph belongs to.

The available graph categories are: SimpleGraphs, OrientedGraphs, UndirectedGraphs, LooplessGraphs, and Graphs.

```
gap> SetDefaultGraphCategory(Graphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[[1, 2], [1], [2]])
gap> SetDefaultGraphCategory(LooplessGraphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := LooplessGraphs, Order := 3, Size := 3, Adjacencies :=
[[2],[1],[2]])
gap> SetDefaultGraphCategory(UndirectedGraphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := UndirectedGraphs, Order := 3, Size := 3, Adjacencies :=
[[1, 2], [1, 3], [2]])
gap> SetDefaultGraphCategory(SimpleGraphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[[2],[1,3],[2]])
gap> SetDefaultGraphCategory(OrientedGraphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := OrientedGraphs, Order := 3, Size := 2, Adjacencies :=
[[2],[],[2]])
```

## 138 ► SimpleGraphs()

 $\mathbf{C}$ 

SimpleGraphs is a graph category in YAGS. A graph in this category may contain edges, but no loops or arrows. The category has two parents: LooplessGraphs and UndirectedGraphs.

```
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=SimpleGraphs);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
```

#### 139 ► Size(G)

Α

Returns the number of edges of graph G.

```
gap> Size(Icosahedron);
30
```

## 140 ► SnubDisphenoid

V

The 1-skeleton of the 84th Johnson solid.

```
gap> SnubDisphenoid;
Graph( Category := SimpleGraphs, Order := 8, Size := 18, Adjacencies :=
[ [ 2, 3, 4, 5, 8 ], [ 1, 3, 6, 7, 8 ], [ 1, 2, 4, 6 ], [ 1, 3, 5, 6 ],
      [ 1, 4, 6, 7, 8 ], [ 2, 3, 4, 5, 7 ], [ 2, 5, 6, 8 ], [ 1, 2, 5, 7 ] ] )
```

## 141 ► SpanningForest( G )

O

Returns a spanning forest of G.

#### 142 ► SpanningForestEdges( G )

Ο

Returns the edges of a spanning forest of G.

```
143 ► SpikyGraph( n )
```

F

The spiky graph is constructed as follows: Take complete graph on n vertices,  $K_N$ , and then, for each the n subsets of  $Vertices(K_n)$  of order n-1, add an additional vertex which is adjacent precisely to this subset of  $Vertices(K_n)$ .

```
gap> SpikyGraph(3);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 4, 6 ], [ 1, 2, 5, 6 ], [ 1, 2 ], [ 1, 3 ],
        [ 2, 3 ] ] )
```

## 144 ► SunGraph( n )

F

Returns the *n*-Sun: A complete graph on *n* vertices,  $K_N$ , with a corona made with a zigzagging 2n-cycle glued to a n-cycle of the  $K_N$ .

```
gap> SunGraph(3);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 2, 6 ], [ 1, 3, 4, 6 ], [ 2, 4 ], [ 2, 3, 5, 6 ], [ 4, 6 ],
        [ 1, 2, 4, 5 ] ])
gap> SunGraph(4);
Graph( Category := SimpleGraphs, Order := 8, Size := 14, Adjacencies :=
[ [ 2, 8 ], [ 1, 3, 4, 6, 8 ], [ 2, 4 ], [ 2, 3, 5, 6, 8 ], [ 4, 6 ],
        [ 2, 4, 5, 7, 8 ], [ 6, 8 ], [ 1, 2, 4, 6, 7 ] ])
```

O

F

```
145 ▶ Suspension( G )
```

Returns the suspension of graph G. The suspension of G is the graph obtained from G by adding two new vertices which are adjacent to every vertex of G but not to each other. The new vertices are the first ones in the new graph.

```
gap> Suspension(CycleGraph(4));
    Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
    [ [ 3, 4, 5, 6 ], [ 3, 4, 5, 6 ], [ 1, 2, 4, 6 ], [ 1, 2, 3, 5 ],
        [ 1, 2, 4, 6 ], [ 1, 2, 3, 5 ] ] )
146 ► TargetGraphCategory( [ G, ... ] )
```

For internal use. Returns the graph category indicated in the *options stack* if any, otherwise if the list of graphs provided is not empty, returns the minimal common graph category for the graphs in the list, else returns the default graph category.

The partial order (by inclusion) among graph categories is as follows:

```
\label{lem:condition} Simple Graphs < Undirected Graphs < Graphs, \\ Oriented Graphs < Loopless Graphs < Graphs \\ Simple Graphs < Loopless Graphs < Graphs \\
```

This function is internally called by all graph constructing operations in YAGS to decide the graph category that the newly constructed graph is going to belong. New graphs are always forced to comply with the TargetGraphCategory, so loops may be removed, and arrows may replaced by edges or viceversa, depending on the category that the new graph belongs to.

The *options stack* is a mechanism provided by GAP to pass implicit parameters and is used by Target-GraphCategory so that the user may indicate the graph category she/he wants for the new graph.

```
gap> SetDefaultGraphCategory(SimpleGraphs);
gap> g1:=CompleteGraph(2);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> g2:=CompleteGraph(2:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ ] ] )
gap> DisjointUnion(g1,g2);
Graph( Category := LooplessGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2 ], [ 1 ], [ 4 ], [ ] ] )
gap> DisjointUnion(g1,g2:GraphCategory:=UndirectedGraphs);
Graph( Category := UndirectedGraphs, Order := 4, Size := 2, Adjacencies :=
[ [ 2 ], [ 1 ], [ 4 ], [ 3 ] ] )
```

In the previous examples, TargetGraphCategory was called internally exactly once for each new graph constructed with the following parameters:

```
gap> TargetGraphCategory();
<Operation "SimpleGraphs">
gap> TargetGraphCategory(:GraphCategory:=OrientedGraphs);
<Operation "OrientedGraphs">
gap> TargetGraphCategory([g1,g2]);
<Operation "LooplessGraphs">
gap> TargetGraphCategory([g1,g2]:GraphCategory:=UndirectedGraphs);
<Operation "UndirectedGraphs">
```

147► Tetrahedron V

The 1-skeleton of Plato's tetrahedron.

```
gap> Tetrahedron;
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
```

```
148 ► TimeInSeconds()
```

Returns the time in seconds since 1970-01-01 00:00:00 UTC as an integer. This is useful to measure execution time. It can also be used to impose time constraints on the execution of algorithms. Note however that the time reported is the 'wall time', not necessarily the time spent in the process you intend to measure.

```
gap> TimeInSeconds();
1415551598
gap> K:=CliqueGraph;;
gap> t1:=TimeInSeconds();NumberOfCliques(K(K(K(K(K(Icosahedron)))));TimeInSeconds()-t1;
1415551608
44644
103
```

Currently, this operation is not working on MS Windows.

```
149 ► TimesProduct( G, H )
```

Returns the times product of two graphs G and H,  $G \times H$  (also known as the tensor product).

The times product is computed as follows:

For each pair of vertices  $x \in G, y \in H$  we create a vertex (x, y). Given two such vertices (x, y) and (x', y') they are adjacent iff  $x \sim x'$  and  $y \sim y'$ .

```
gap> g:=PathGraph(3);h:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
gap> gh:=TimesProduct(g,h);
Graph( Category := SimpleGraphs, Order := 12, Size := 16, Adjacencies :=
[ [ 6, 8 ], [ 5, 7 ], [ 6, 8 ], [ 5, 7 ], [ 2, 4, 10, 12 ], [ 1, 3, 9, 11 ],
        [ 2, 4, 10, 12 ], [ 1, 3, 9, 11 ], [ 6, 8 ], [ 5, 7 ], [ 6, 8 ], [ 5, 7 ] ] )
gap> VertexNames(gh);
[ [ 1, 1 ], [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 2, 1 ], [ 2, 2 ], [ 2, 3 ],
        [ 2, 4 ], [ 3, 1 ], [ 3, 2 ], [ 3, 3 ], [ 3, 4 ] ]
```

150 ► TrivialGraph V

The one vertex graph.

```
gap> TrivialGraph;
Graph( Category := SimpleGraphs, Order := 1, Size := 0, Adjacencies :=
[ [ ] ] )
```

```
151 ► UFFind( UFS, x )
```

For internal use. Implements the find operation on the union-find structure.

F

```
152 ► UFUnite( UFS, x, y )
```

For internal use. Implements the *unite* operation on the *union-find structure*.

```
153 ► UndirectedGraphs()
```

С

UndirectedGraphs is a graph category in YAGS. A graph in this category may contain edges and loops, but no arrows. The parent of this category is Graphs.

```
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=UndirectedGraphs);
Graph( Category := UndirectedGraphs, Order := 3, Size := 3, Adjacencies :=
[ [ 1, 2 ], [ 1, 3 ], [ 2 ] ] )
```

#### 154 ► UnitsRingGraph( Rng )

O

Returns the graph G whose vertices are the elements of Rng such that x is adjacent to y iff x+z=y for some unit z of Rng.

```
gap> UnitsRingGraph(ZmodnZ(8));
Graph( Category := SimpleGraphs, Order := 8, Size := 16, Adjacencies :=
[ [ 2, 4, 6, 8 ], [ 1, 3, 5, 7 ], [ 2, 4, 6, 8 ], [ 1, 3, 5, 7 ],
       [ 2, 4, 6, 8 ], [ 1, 3, 5, 7 ], [ 2, 4, 6, 8 ], [ 1, 3, 5, 7 ] ] )
```

```
155 ► VertexDegree( G, x )
```

O

Returns the degree of vertex x in Graph G.

```
gap> g:=PathGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> VertexDegree(g,1);
1
gap> VertexDegree(g,2);
2
```

## 156 ► VertexDegrees( G )

Ο

Returns the list of degrees of the vertices in graph G.

```
gap> g:=GemGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 7, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4 ] ] )
gap> VertexDegrees(g);
[ 4, 2, 3, 3, 2 ]
```

## 157 ► VertexNames( G )

Α

Return the list of names of the vertices of G. The vertices of a graph in YAGS are always  $\{1, 2, \ldots, Order(G)\}$ , but depending on how the graph was constructed, its vertices may have also some names, that help us identify the origin of the vertices. YAGS will always try to store meaninful names for the vertices. For example, in the case of the LineGraph, the vertex names of the new graph are the edges of the old graph.

O

```
gap> g:=LineGraph(DiamondGraph);
       Graph( Category := SimpleGraphs, Order := 5, Size := 8, Adjacencies :=
       [[2,3,4],[1,3,4,5],[1,2,5],[1,2,5],[2,3,4]])
       gap> VertexNames(g);
       [[1, 2], [1, 3], [1, 4], [2, 3], [3, 4]]
       gap> Edges(DiamondGraph);
       [[1, 2], [1, 3], [1, 4], [2, 3], [3, 4]]
158 ▶ Vertices( G )
                                                                                     0
    Returns the list [1..Order(G)].
       gap> Vertices(Icosahedron);
       [ 1 .. 12 ]
159 ► WheelGraph( n )
                                                                                     Ο
                                                                                     O
  ▶ WheelGraph( n, r )
```

In its first form WheelGraph returns the wheel graph on n+1 vertices. This is the cone of a cycle: a central vertex adjacent to all the vertices of an n-cycle.

```
WheelGraph(5);
gap> Graph( Category := SimpleGraphs, Order := 6, Size := 10, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3, 6 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4, 6 ],
        [ 1, 2, 5 ] ] )
```

In its second form, WheelGraph returns returns the wheel graph, but adding r-1 layers, each layer is a new n-cycle joined to the previous layer by a zigzagging 2n-cycle. This graph is a triangulation of the disk.

```
gap> WheelGraph(5,2);
Graph( Category := SimpleGraphs, Order := 11, Size := 25, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3, 6, 7, 8 ], [ 1, 2, 4, 8, 9 ], [ 1, 3, 5, 9, 10 ],
      [ 1, 4, 6, 10, 11 ], [ 1, 2, 5, 7, 11 ], [ 2, 6, 8, 11 ], [ 2, 3, 7, 9 ],
      [ 3, 4, 8, 10 ], [ 4, 5, 9, 11 ], [ 5, 6, 7, 10 ] ] )
gap> WheelGraph(5,3);
Graph( Category := SimpleGraphs, Order := 16, Size := 40, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3, 6, 7, 8 ], [ 1, 2, 4, 8, 9 ], [ 1, 3, 5, 9, 10 ],
      [ 1, 4, 6, 10, 11 ], [ 1, 2, 5, 7, 11 ], [ 2, 6, 8, 11, 12, 13 ],
      [ 2, 3, 7, 9, 13, 14 ], [ 3, 4, 8, 10, 14, 15 ], [ 4, 5, 9, 11, 15, 16 ],
      [ 5, 6, 7, 10, 12, 16 ], [ 7, 11, 13, 16 ], [ 7, 8, 12, 14 ],
      [ 8, 9, 13, 15 ], [ 9, 10, 14, 16 ], [ 10, 11, 12, 15 ] ] )
```

For internal use. Calls external program ProgName located in directory 'YagsDir/bin/' feeding it with InString as input and returning the output of the external program as a string. 'fail' is returned if the program could not be located.

```
gap> YagsExec("time","");
"1415551127\n"
gap> YagsExec("nauty","l=0$=1dacn=5 g1,2,3. xbzq");
"(4,5)\n(2,3)\n[2,3,4,5,1]\n[\"cb0c\",\"484f264\",\"b0e19f1\"]\n"
```

Currently, this operation is not working on MS Windows.

160 ► YagsExec( ProgName, InString )

# **Bibliography**

- [BK73] Coen Bron and Joep Kerbosch. Finding all cliques of an undirected graph–algorithm 457. Communications of the ACM, 16:575–577, 1973.
- [Dra89] Feodor F. Dragan. Centers of graphs and the Helly property (in Russian). PhD thesis, Moldava State University, Chisinău, Moldava, 1989.
- [HK96] Johann Hagauer and Sandi Klavĉar. Clique-gated graphs. Discrete Mathematics, 161(13):143 149, 1996.
- [Piz04] M. A. Pizaña. Distances and diameters on iterated clique graphs. Discrete Appl. Math., 141(1-3):255–161, 2004.
- [Szw97] Jayme L. Szwarcfiter. Recognizing clique-Helly graphs. Ars Combin., 45:29–32, 1997.

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