YAGS
Yet Another Graph System
GAP4 Package

Version 0.8
by

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Contents

1 Basics

YAGS (Yet Another Graph System) is a system designed to aid in the study of graphs. Therefore it provides functions designed to help researchers in this field. The main goal was, as a start, to be thorough and provide as much functionality as possible, and at a later stage to increase the efficiency of the system. Furthermore, a module on genetic algorithms is provided to allow experiments with graphs to be carried out.

This chapter is intended as a gentle tutorial on working with YAGS (some knowledge of GAP and the basic use of a computer are assumed).

The tutorial is divided as follows:

- Using YAGS
- Definition of a graph
- A taxonomy of graphs
- Creating graphs
- Transforming graphs
- Experimenting on graphs

1.1 Using YAGS

YAGS is a GAP package an as such the *RequirePackage* directive is used to start YAGS

```
gap> RequirePackage("YAGS");

Loading  YAGS 0.01  (Yet Another Graph System),
by  R. MacKinney and M.A. Pizana
rene@xamanek.uam.mx, map@xamanek.uam.mx

true
```

a double semicolon can be used to avoid the banner.

Once the package has been loaded help can be obtained at anytime using the GAP help facility. For instance get help on the function *RandomGraph*:

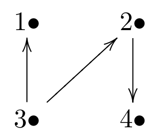
```
gap> ?RandomGraph
Help: Showing ‘yags: RandomGraph’

> RandomGraph( <n>, <p> )                                              F
> RandomGraph( <n> )                                                   F

Returns  a  Random Graph of order <n>. The first form additionally takes a
parameter  <p>,  the probability of an edge to exist. A probability 1 will
return a Complete Graph and a probability 0 a Discrete Graph.
```

```
gap> RandomGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 4, 5 ], [ 4, 5 ], [  ], [ 1, 2, 5 ], [ 1, 2, 4 ] ] )
```

1.2 Definition of graphs

A graph is defined as follows. A graph G is a set of vertices V and a set of edges (arrows) E, $G = \{V{,}E\}$. The set of edges is a set of tuples of vertices (v_i, v_j) that belong to V, $v_i, v_j \in V$ representing that v_i, v_j are adjacent.

For instance, $(\{1,2,3,4\}, \{(1,3),(2,4),(3,2)\})$ is a graph with four vertices such that vertices 1 and 2 are adjacent to vertex 3 and vertex 2 is adjacent to vertex 4. Visually this can be seen as

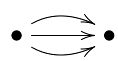


The adjacencies can also be represented as a matrix. This would be a boolean matrix M where two vertices i, j are adjacent if $M[i,j] = \textit{true}$ and not adjacent otherwise.

Given two vertices i, j in graph G we will say that graph G has an **edge** $\{i,j\}$ if there is an arrow (i,j) and and arrow (j,i)·

If a graph G has an arrow that starts and finishes on the same vertex we say that graph G has a loop.

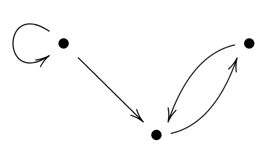
YAGS handles graphs that have arrows, edges and loops. Graphs that, for instance, have multiple arrows between vertices are not handled by YAGS .



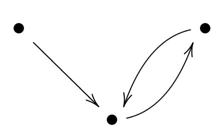
1.3 A taxonomy of graphs

There are several ways of characterizing graphs. YAGS uses a category system where any graph belongs to a specific category. The following is the list of graph categories in YAGS

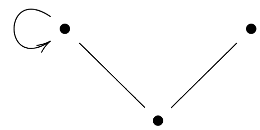
- *Graphs*: graphs with no particular property.



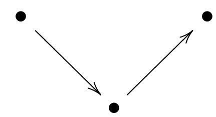
- *Loopless*: graphs with no loops.



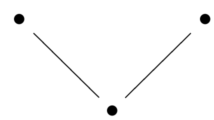
- *Undirected*: graphs with no arrows but only edges.



- *Oriented*: graphs with no edges but only arrows.



- *SimpleGraphs*: graphs with no loops and only edges.



The following figure shows the relationships among categories.

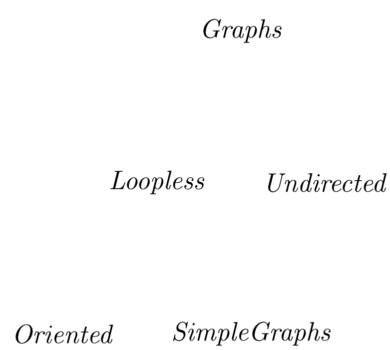


Figure 1: Graph Categories

YAGS uses the category of a graph to normalize it. This is helpful, for instance, when we define an undirected graph and inadvertently forget an arrow in its definition. The category of a graph can be given explicitly or implicitly. To do it explicitly the category must be given when creating a graph, as can be seen in the section 1.4. If no category is given the category is assumed to be the *DefaultCategory*. The default category can be changed at any time using the *SetDefaultCategory* function.

Further information regarding categories can be found on chapter 2.

1.4 Creating Graphs

There exist several ways to create a graph in YAGS. First, a GAP record can be used. To do so the record has to have either of

- Adjacency List
- Adjacency Matrix

in the graph presented in Section 1.2 the adjacency list would be

$$[[], [4], [1, 2], []]$$

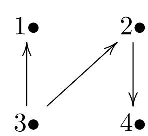
and the adjacency matrix

$$\begin{bmatrix} \mathit{false} & \mathit{false} & \mathit{false} & \mathit{false} \\ \mathit{false} & \mathit{false} & \mathit{false} & \mathit{true} \\ \mathit{true} & \mathit{true} & \mathit{false} & \mathit{false} \\ \mathit{false} & \mathit{false} & \mathit{false} & \mathit{false} \end{bmatrix}$$

To create a graph YAGS we also need the category the graph belongs to. We give this information to the *Graph* function. For instance to create the graph using the adjacency list we would use the following command:

```
gap> g:=Graph(rec(Category:=OrientedGraphs,Adjacencies:=[[],[4],[1,2],[]]));
Graph( Category := OrientedGraphs, Order := 4, Size := 3, Adjacencies :=
[ [  ], [ 4 ], [ 1, 2 ], [  ] ] )
```

This will create a graph g that represents the graph in Section 1.2.



Since the *DefaultCategory* is *SimpleGraphs* when YAGS starts up and the graph we have been using as an example is oriented we must explicitly give the category to YAGS. This is achieved using *Category:=OrientedGraphs* inside the record structure.

The same graph can be created using the function *GraphByAdjacencies* as in

```
gap> g:=GraphByAdjacencies([[],[4],[1,2],[]]:Category:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 4, Size := 3, Adjacencies :=
[ [  ], [ 4 ], [ 1, 2 ], [  ] ] )
```

In this case to explicitly give the Category of the graph we use the construction *:Category:=OrientedGraphs* inside the function. This construction can be used in any function to explicitly give the category of a graph.

We said previously we can also use the adjacency matrix to create a graph. For instance the command

```
gap> g:=Graph(rec(Category:=OrientedGraphs,AdjMatrix:=
         [[false,false,false,false],[false,false,false,true],
         [true,true,false,false],[false,false,false,false]]));
Graph( Category := OrientedGraphs, Order := 4, Size := 3, Adjacencies :=
[ [  ], [ 4 ], [ 1, 2 ], [  ] ] )
```

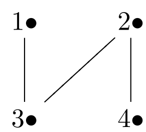
Creates the same graph. Note that we explicitly give the graph category as before. We also can use the command *AdjMatrix* as in

```
gap> g:=AdjMatrix(AdjMatrix:=[[false,false,false,false],
         [false,false,false,true],[true,true,false,false],
         [false,false,false,false]]):Category:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 4, Size := 3, Adjacencies :=
[ [  ], [ 4 ], [ 1, 2 ], [  ] ] )
```

If we create the graph using any of the methods so far described omitting the graph category YAGS will create a graph normalized to the *DefaultCategory* which by default is *SimpleGraphs*

```
gap> g:=GraphByAdjacencies([[],[4],[1,2],[]];
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 3 ], [ 3, 4 ], [ 1, 2 ], [ 2 ] ] )
```

Which creates a graph with only edges

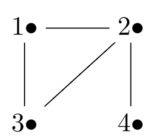


There are many functions to create graphs, some from existing graphs and some create interesting well known graphs.

Among the former we have the function *AddEdges* which adds edges to an existing graph

```
gap> g:=GraphByAdjacencies([[],[4],[1,2],[]]);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 3 ], [ 3, 4 ], [ 1, 2 ], [ 2 ] ] )
gap> h:=AddEdges(g,[[1,2]]);
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 3 ], [ 1, 3, 4 ], [ 1, 2 ], [ 2 ] ] )
```

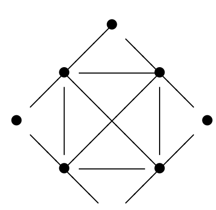
that yields the graph *h*



Among the latter we have the function *SunGraph* which takes an integer as argument and returns a fresh copy of a sun graph of the order given as argument.

```
gap> h:=SunGraph(4);
Graph( Category := SimpleGraphs, Order := 8, Size := 14, Adjacencies :=
[ [ 2, 8 ], [ 1, 3, 4, 6, 8 ], [ 2, 4 ], [ 2, 3, 5, 6, 8 ], [ 4, 6 ],
  [ 2, 4, 5, 7, 8 ], [ 6, 8 ], [ 1, 2, 4, 6, 7 ] ] )
```

that produces *h* as



Further information regarding constructing graphs can be found on chapter 3.

1.5 Transforming graphs

1.6 Experimenting on graphs

Coming soon!

2 Categories

2.1 Graph Categories

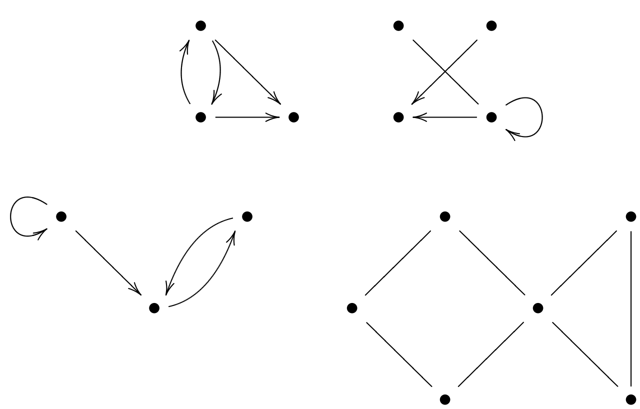
1 ▶ `Graphs( )` C

`Graphs` is the most general graph category in YAGS. This category contains all graphs that can be represented in YAGS. A graph in this category may contain loops, arrows and edges (which in YAGS are exactly the same as two opposite arrows between some pair of vertices). This graph category has no parent category.

```
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=SimpleGraphs);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
```

–map

Among them we can find:



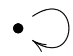
2 ▶ `LooplessGraphs( )` C

`LooplessGraphs` is a graph category in YAGS. A graph in this category may contain arrows and edges but no loops. The parent of this category is `Graphs`

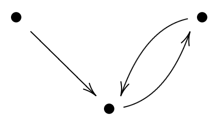
```
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=LooplessGraphs);
Graph( Category := LooplessGraphs, Order := 3, Size := 3, Adjacencies :=
[ [ 2 ], [ 1 ], [ 2 ] ] )
```

–map

A loop is an arrow that starts and finishes on the same vertex.



Loopless graphs have no such arrows.



3 ► `UndirectedGraphs( )` C

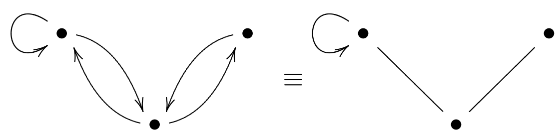
`UndirectedGraphs` is a graph category in YAGS. A graph in this category may contain edges and loops, but no arrows. The parent of this category is `Graphs`

```
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=UndirectedGraphs);
Graph( Category := UndirectedGraphs, Order := 3, Size := 3, Adjacencies :=
[ [ 1, 2 ], [ 1, 3 ], [ 2 ] ] )
```

–map

Given two vertex i, j in graph G we will say that graph G has an **edge** $\{i, j\}$ if there is an arrow (i, j) and and arrow $(j, i)\cdot$

Undirected graphs have no arrows but only edges.



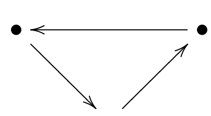
4 ► `OrientedGraphs( )` C

`OrientedGraphs` is a graph category in YAGS. A graph in this category may contain arrows, but no loops or edges. The parent of this category is `LooplessGraphs`.

```
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [  ], [ 2 ] ] )
```

–map

Oriented graphs have no edges but only arrows.

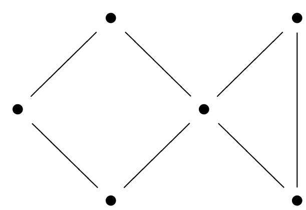


5 ▶ `SimpleGraphs( )` C

`SimpleGraphs` is a graph category in YAGS. A graph in this category may contain edges, but no loops or arrows. The category has two parents: `LooplessGraphs` and `UndirectedGraphs`.

```
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=SimpleGraphs);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
```

–map



The following figure shows the relationships among categories.

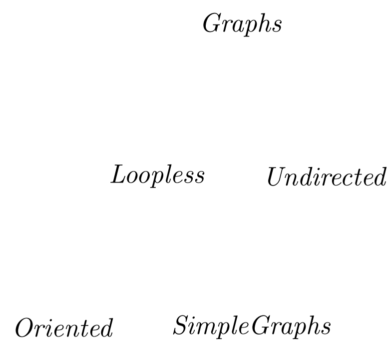
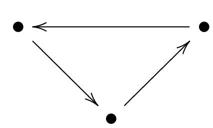
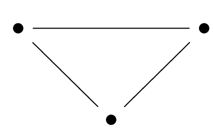


Figure 2: Graph Categories

This relationship is important because when a graph is created it is normalized to the category it belongs. For instance, if we create a graph such as



as a simple graph YAGS will normalize the graph as



For further examples see the following section.

2.2 Default Category

There are several ways to specify the category in which a new graph will be created. There exists a *Default-Category* which tells YAGS to which category belongs any new graph by default. The *DefaultCategory* can be changed using the following function.

1 ▶ `SetDefaultGraphCategory(` C `)` F

Sets the default graphs category to C. The default graph category is used when constructing new graphs when no other graph category is indicated. New graphs are always forced to comply with the `TargetGraph-Category`, so loops may be removed, and arrows may replaced by edges or viceversa, depending on the category that the new graph belongs to.

The available graph categories are: `SimpleGraphs`, `OrientedGraphs`, `UndirectedGraphs`, `LooplessGraphs`, and `Graphs`.

```
gap> SetDefaultGraphCategory(Graphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> SetDefaultGraphCategory(LooplessGraphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := LooplessGraphs, Order := 3, Size := 3, Adjacencies :=
[ [ 2 ], [ 1 ], [ 2 ] ] )
gap> SetDefaultGraphCategory(UndirectedGraphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := UndirectedGraphs, Order := 3, Size := 3, Adjacencies :=
[ [ 1, 2 ], [ 1, 3 ], [ 2 ] ] )
gap> SetDefaultGraphCategory(SimpleGraphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> SetDefaultGraphCategory(OrientedGraphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := OrientedGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [  ], [ 2 ] ] )
```

–map

In order to handle graphs with different categories there two functions available.

2 ▶ `GraphCategory( [`G`, ... ] )` F

For internal use. Returns the minimal common category to a list of graphs. If the list of graphs is empty, the default category is returned.

The partial order (by inclussion) among graph categories is as follows:

$$\texttt{SimpleGraphs} < \texttt{UndirectedGraphs} < \texttt{Graphs},$$

$$\texttt{OrientedGraphs} < \texttt{LooplessGraphs} < \texttt{Graphs}$$

$$\texttt{SimpleGraphs} < \texttt{LooplessGraphs} < \texttt{Graphs}$$

```
gap> g1:=CompleteGraph(2:GraphCategory:=SimpleGraphs);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> g2:=CompleteGraph(2:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [  ] ] )
gap> g3:=CompleteGraph(2:GraphCategory:=UndirectedGraphs);
Graph( Category := UndirectedGraphs, Order := 2, Size := 3, Adjacencies :=
[ [ 1, 2 ], [ 1, 2 ] ] )
gap> GraphCategory([g1,g2,g3]);
<Operation "Graphs">
gap> GraphCategory([g1,g2]);
<Operation "LooplessGraphs">
gap> GraphCategory([g1,g3]);
<Operation "UndirectedGraphs">
```

–map

3 ► `TargetGraphCategory( [`G`, ... ] )` F

For internal use. Returns the graph category indicated in the *options stack* if any, otherwise if the list of graphs provided is not empty, returns the minimal common graph category for the graphs in the list, else returns the default graph category.

The partial order (by inclussion) among graph categories is as follows:

$$\texttt{SimpleGraphs} < \texttt{UndirectedGraphs} < \texttt{Graphs},$$

$$\texttt{OrientedGraphs} < \texttt{LooplessGraphs} < \texttt{Graphs}$$

$$\texttt{SimpleGraphs} < \texttt{LooplessGraphs} < \texttt{Graphs}$$

This function is internally called by all graph constructing operations in YAGS to decide the graph category that the newly constructed graph is going to belong. New graphs are always forced to comply with the `TargetGraphCategory`, so loops may be removed, and arrows may replaced by edges or viceversa, depending on the category that the new graph belongs to.

The *options stack* is a mechanism provided by GAP to pass implicit parameters and is used by `Target-GraphCategory` so that the user may indicate the graph category she/he wants for the new graph.

```
gap> SetDefaultGraphCategory(SimpleGraphs);
gap> g1:=CompleteGraph(2);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> g2:=CompleteGraph(2:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [  ] ] )
gap> DisjointUnion(g1,g2);
Graph( Category := LooplessGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2 ], [ 1 ], [ 4 ], [  ] ] )
gap> DisjointUnion(g1,g2:GraphCategory:=UndirectedGraphs);
Graph( Category := UndirectedGraphs, Order := 4, Size := 2, Adjacencies :=
[ [ 2 ], [ 1 ], [ 4 ], [ 3 ] ] )
```

In the previous examples, `TargetGraphCategory` was called internally exactly once for each new graph constructed with the following parameters:

```
gap> TargetGraphCategory();
<Operation "SimpleGraphs">
gap> TargetGraphCategory(:GraphCategory:=OrientedGraphs);
<Operation "OrientedGraphs">
gap> TargetGraphCategory([g1,g2]);
<Operation "LooplessGraphs">
gap> TargetGraphCategory([g1,g2]:GraphCategory:=UndirectedGraphs);
<Operation "UndirectedGraphs">
```

–map

Finally we can test if a single graph belongs to a given category.

4 ► `in(` G`,` C `)` O

Returns `true` if graph G belongs to category C and `false` otherwise.

–map

3 Constructing graphs

3.1 Primitives

The following functions create new graphs from a variety of sources.

1 ▶ `Graph(` *R* `)` O

Returns a new graph created from the record R. The record must provide the field *Category* and either the field *Adjacencies* or the field *AdjMatrix*

```
gap> Graph(rec(Category:=SimpleGraphs,Adjacencies:=[[2],[1]]));
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies := [ [ 2 ], [ 1 ] ] )
gap> Graph(rec(Category:=SimpleGraphs,AdjMatrix:=[[false, true],[true, false]]));
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies := [ [ 2 ], [ 1 ] ] )
```

Its main purpose is to import graphs from files, which could have been previously exported using `PrintTo`.

```
gap> g:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
gap> PrintTo("aux.g","h1:=",g,";");
gap> Read("aux.g");
gap> h1;
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
```

–map

2 ▶ `GraphByAdjMatrix(` *M* `)` F

Returns a new graph created from an adjacency matrix M. The matrix M must be a square boolean matrix.

```
gap> m:=[ [ false, true, false ], [ true, false, true ], [ false, true, false ] ];;
gap> g:=GraphByAdjMatrix(m);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> AdjMatrix(g);
[ [ false, true, false ], [ true, false, true ], [ false, true, false ] ]
```

Note, however, that the graph is forced to comply with the `TargetGraphCategory`.

```
gap> m:=[ [ true, true], [ false, false ] ];;
gap> g:=GraphByAdjMatrix(m);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies := [ [ 2 ], [ 1 ] ] )
gap> AdjMatrix(g);
[ [ false, true ], [ true, false ] ]
```

–map

3 ► `GraphByAdjacencies( A )` F

Returns a new graph having A as its list of adjacencies. The order of the created graph is `Length(A)`, and the set of neighbors of vertex x is $A[x]$.

```
gap> GraphByAdjacencies([[2],[1,3],[2]]);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
```

Note, however, that the graph is forced to comply with the `TargetGraphCategory`.

```
gap> GraphByAdjacencies([[1,2,3],[],[]]);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2, 3 ], [ 1 ], [ 1 ] ] )
```

–map

4 ► `GraphByCompleteCover( C )` F

Returns the minimal graph where the elements of C are (the vertex sets of) complete subgraphs.

```
gap> GraphByCompleteCover([[1,2,3,4],[4,6,7]]);
Graph( Category := SimpleGraphs, Order := 7, Size := 9, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3, 6, 7 ], [  ], [ 4, 7 ],
  [ 4, 6 ] ] )
```

–map

5 ► `GraphByRelation( V, R )` F
► `GraphByRelation( N, R )` F

Returns a new graph created from a set of vertices V and a binary relation R, where $x \sim y$ iff $R(x,y) = true$. In the second form, N is an integer and V is assumed to be $\{1, 2, \ldots, N\}$.

```
gap> R:=function(x,y) return Intersection(x,y)<>[]; end;;
gap> GraphByRelation([[1,2,3],[3,4,5],[5,6,7]],R);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> GraphByRelation(8,function(x,y) return AbsInt(x-y)<=2; end);
Graph( Category := SimpleGraphs, Order := 8, Size := 13, Adjacencies :=
[ [ 2, 3 ], [ 1, 3, 4 ], [ 1, 2, 4, 5 ], [ 2, 3, 5, 6 ], [ 3, 4, 6, 7 ],
  [ 4, 5, 7, 8 ], [ 5, 6, 8 ], [ 6, 7 ] ] )
```

–map

6 ► `GraphByWalks( walk1, walk2, ... )` F

Returns the minimal graph such that *walk1*, *walk2*, etc are walks.

```
gap> GraphByWalks([1,2,3,4,1],[1,5,6]);
Graph( Category := SimpleGraphs, Order := 6, Size := 6, Adjacencies :=
[ [ 2, 4, 5 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ], [ 1, 6 ], [ 5 ] ] )
```

Walks can be *nested*, which greatly improves the versatility of this function.

```
gap> GraphByWalks([1,[2,3,4],5],[5,6]);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 5 ], [ 1, 2, 4, 5 ], [ 1, 3, 5 ], [ 2, 3, 4, 6 ], [ 5 ] ] )
```

The vertices in the constructed graph range from 1 to the maximum of the numbers appearing in *walk1*, *walk2*, ... etc.

```
gap> GraphByWalks([4,2],[3,6]);
Graph( Category := SimpleGraphs, Order := 6, Size := 2, Adjacencies :=
[ [  ], [ 4 ], [ 6 ], [ 2 ], [  ], [ 3 ] ] )
```

–map

7 ► `IntersectionGraph(` L `)` F

Returns the intersection graph of the family of sets L. This graph has a vertex for every set in L, and two such vertices are adjacent iff the corresponding sets have non-empty intersection.

```
gap> IntersectionGraph([[1,2,3],[3,4,5],[5,6,7]]);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
```

–map

The following functions create graphs from existing graphs

8 ► `CopyGraph(` G `)` O

Returns a fresh copy of graph G. Only the order and adjacency information is copied, all other known attributes of G are not. Mainly used to transform a graph from one category to another. The new graph will be forced to comply with the `TargetGraphCategory`.

```
gap> g:=CompleteGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
gap> g1:=CopyGraph(g:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 3, 4 ], [ 4 ], [  ] ] )
gap> CopyGraph(g1:GraphCategory:=SimpleGraphs);
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
```

–map

9 ► `InducedSubgraph(` G, V `)` O

Returns the subgraph of graph G induced by the vertex set V.

```
gap> g:=CycleGraph(6);
Graph( Category := SimpleGraphs, Order := 6, Size := 6, Adjacencies :=
[ [ 2, 6 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4, 6 ], [ 1, 5 ] ] )
gap> InducedSubgraph(g,[3,4,6]);
Graph( Category := SimpleGraphs, Order := 3, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ], [  ] ] )
```

The order of the elements in V does matter.

```
gap> InducedSubgraph(g,[6,3,4]);
Graph( Category := SimpleGraphs, Order := 3, Size := 1, Adjacencies :=
[ [  ], [ 3 ], [ 2 ] ] )
```

–map

10 ► `RemoveVertices(` G, V `)` O

Returns a new graph created from graph G by removing the vertices in list V.

```
gap> g:=PathGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4 ] ] )
gap> RemoveVertices(g,[3]);
Graph( Category := SimpleGraphs, Order := 4, Size := 2, Adjacencies :=
[ [ 2 ], [ 1 ], [ 4 ], [ 3 ] ] )
gap> RemoveVertices(g,[1,3]);
Graph( Category := SimpleGraphs, Order := 3, Size := 1, Adjacencies :=
[ [  ], [ 3 ], [ 2 ] ] )
```

–map

11 ▶ `AddEdges(` G, E `)` O

Returns a new graph created from graph G by adding the edges in list E.

```
gap> g:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
gap> AddEdges(g,[[1,3]]);
Graph( Category := SimpleGraphs, Order := 4, Size := 5, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3 ] ] )
gap> AddEdges(g,[[1,3],[2,4]]);
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
```

–map

12 ▶ `RemoveEdges(` G, E `)` O

Returns a new graph created from graph G by removing the edges in list E.

```
gap> g:=CompleteGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
gap> RemoveEdges(g,[[1,2]]);
Graph( Category := SimpleGraphs, Order := 4, Size := 5, Adjacencies :=
[ [ 3, 4 ], [ 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
gap> RemoveEdges(g,[[1,2],[3,4]]);
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 3, 4 ], [ 3, 4 ], [ 1, 2 ], [ 1, 2 ] ] )
```

–map

13 ▶ `CliqueGraph(` G `)` A
▶ `CliqueGraph(` G, m `)` O

Returns the intersection graph of all the (maximal) cliques of G.

The additional parameter m aborts the computation when m cliques are found, even if they are all the cliques of G. If the bound m is reached, *fail* is returned.

```
gap> CliqueGraph(Octahedron);
Graph( Category := SimpleGraphs, Order := 8, Size := 24, Adjacencies :=
[ [ 2, 3, 4, 5, 6, 7 ], [ 1, 3, 4, 5, 6, 8 ], [ 1, 2, 4, 5, 7, 8 ],
  [ 1, 2, 3, 6, 7, 8 ], [ 1, 2, 3, 6, 7, 8 ], [ 1, 2, 4, 5, 7, 8 ],
  [ 1, 3, 4, 5, 6, 8 ], [ 2, 3, 4, 5, 6, 7 ] ] )
gap> CliqueGraph(Octahedron,9);
Graph( Category := SimpleGraphs, Order := 8, Size := 24, Adjacencies :=
[ [ 2, 3, 4, 5, 6, 7 ], [ 1, 3, 4, 5, 6, 8 ], [ 1, 2, 4, 5, 7, 8 ],
  [ 1, 2, 3, 6, 7, 8 ], [ 1, 2, 3, 6, 7, 8 ], [ 1, 2, 4, 5, 7, 8 ],
  [ 1, 3, 4, 5, 6, 8 ], [ 2, 3, 4, 5, 6, 7 ] ] )
gap> CliqueGraph(Octahedron,8);
fail
```

–map

3.2 Families

The following functions return well known graphs. Most of them can be found in Brandstadt, Le and Spinrad.

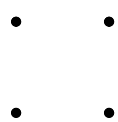
1 ► `DiscreteGraph(` *n* `)` F

Returns the discrete graph of order n. A discrete graph is a graph without edges.

```
gap> DiscreteGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 0, Adjacencies :=
[ [  ], [  ], [  ], [  ] ] )
```

–map

4-Discrete Graph



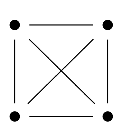
2 ► `CompleteGraph(` *n* `)` F

Returns the complete graph of order n. A complete graph is a graph where all vertices are connected to each other.

```
gap> CompleteGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
```

–map

4-Complete Graph



3 ► `PathGraph(` *n* `)` F

Returns the path graph on n vertices.

```
gap> PathGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3 ] ] )
```

–map

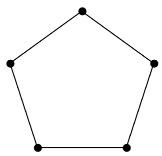
4-Path Graph

4 ► `CycleGraph( n )` F

Returns the cyclic graph on n vertices.

```
gap> CycleGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] )
```

–map

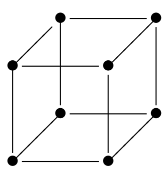
5-Cycle Graph 

5 ► `CubeGraph( n )` F

Returns the hypercube of dimension n. This is the box product (cartesian product) of n copies of K_2 (an edge).

```
gap> CubeGraph(3);
Graph( Category := SimpleGraphs, Order := 8, Size := 12, Adjacencies :=
[ [ 2, 3, 5 ], [ 1, 4, 6 ], [ 1, 4, 7 ], [ 2, 3, 8 ], [ 1, 6, 7 ],
[ 2, 5, 8 ], [ 3, 5, 8 ], [ 4, 6, 7 ] ] )
```

-map

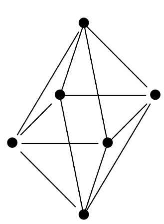
3-Cube Graph 

6 ► `OctahedralGraph( n )` F

Return the n-dimensional octahedron. This is the complement of n copies of K_2 (an edge). It is also the *(2n-2)*-regular graph on $2n$ vertices.

```
gap> OctahedralGraph(3);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 3, 4, 5, 6 ], [ 3, 4, 5, 6 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],
[ 1, 2, 3, 4 ], [ 1, 2, 3, 4 ] ] )
```

–map

3-Octahedral Graph 

7 ► `JohnsonGraph(` n, r `)` F

Returns the Johnson graph $J(n,r)\cdot$ A Johnson Graph is a graph constructed as follows. Each vertex represents a subset of the set $\{1,\ldots,n\}$ with cardinality $r\cdot$

$$V(J(n,r)) = \{X \subset \{1,\ldots,n\}||X| = r\}$$

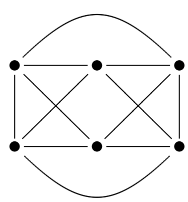
and there is an edge between two vertices if and only if the cardinality of the intersection of the sets they represent is $r-1$

$$X \sim X' \text{ iff } |X \cup X'| = r - 1\cdot$$

```
gap> JohnsonGraph(4,2);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 4, 6 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],
 [ 1, 3, 4, 6 ], [ 2, 3, 4, 5 ] ] )
```

–map

4,2-Johnson Graph



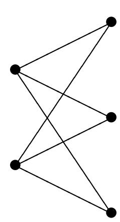
8 ► `CompleteBipartiteGraph(` n, m `)` F

Returns the complete bipartite whose parts have order n and m respectively. This is the joint (Zykov sum) of two discrete graphs of order n and m.

```
gap> CompleteBipartiteGraph(2,3);
Graph( Category := SimpleGraphs, Order := 5, Size := 6, Adjacencies :=
[ [ 3, 4, 5 ], [ 3, 4, 5 ], [ 1, 2 ], [ 1, 2 ], [ 1, 2 ] ] )
```

–map

2,3-Complete Bipartite Graph

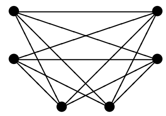


9 ► `CompleteMultipartiteGraph(` *n1* , *n2* `[,` *n3* `...] )` F

Returns the complete multipartite graph where the orders of the parts are *n1*, *n2*, ... It is also the Zykov sum of discrete graphs of order *n1*, *n2*, ...

```
gap> CompleteMultipartiteGraph(2,2,2);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 3, 4, 5, 6 ], [ 3, 4, 5, 6 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],
 [ 1, 2, 3, 4 ], [ 1, 2, 3, 4 ] ] )
```

–map

2,2,2-Complete Multipartite Graph 

10 ▶ `RandomGraph( n, p )` F
▶ `RandomGraph( n )` F

Returns a random graph of order n taking the rational $p \in [0, 1]$ as the edge probability.

```
gap> RandomGraph(5,1/3);
Graph( Category := SimpleGraphs, Order := 5, Size := 2, Adjacencies :=
[ [ 5 ], [ 5 ], [  ], [  ], [ 1, 2 ] ] )
gap> RandomGraph(5,2/3);
Graph( Category := SimpleGraphs, Order := 5, Size := 6, Adjacencies :=
[ [ 4, 5 ], [ 3, 4, 5 ], [ 2, 4 ], [ 1, 2, 3 ], [ 1, 2 ] ] )
gap> RandomGraph(5,1/2);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2, 5 ], [ 1, 3, 5 ], [ 2 ], [  ], [ 1, 2 ] ] )
```

If p is ommited, the edge probability is taken to be 1/2.

```
gap> RandomGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 2, 3 ], [ 1 ], [ 1, 4, 5 ], [ 3, 5 ], [ 3, 4 ] ] )
gap> RandomGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 3, Adjacencies :=
[ [ 2, 5 ], [ 1, 4 ], [  ], [ 2 ], [ 1 ] ] )
```

–map

5-Random Graph

11 ▶ `WheelGraph( N )` O
▶ `WheelGraph( N, Radius )` O

In its first form `WheelGraph` returns the wheel graph on *N+1* vertices. This is the cone of a cycle: a central vertex adjacent to all the vertices of an N-cycle

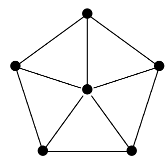
```
WheelGraph(5);
gap> Graph( Category := SimpleGraphs, Order := 6, Size := 10, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3, 6 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4, 6 ],
  [ 1, 2, 5 ] ] )
```

In its second form, `WheelGraph` returns returns the wheel graph, but adding *Radius-1* layers, each layer is a new N-cycle joined to the previous layer by a zigzagging *2N*-cycle. This graph is a triangulation of the disk.

```
gap> WheelGraph(5,2);
Graph( Category := SimpleGraphs, Order := 11, Size := 25, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3, 6, 7, 8 ], [ 1, 2, 4, 8, 9 ], [ 1, 3, 5, 9, 10 ],
  [ 1, 4, 6, 10, 11 ], [ 1, 2, 5, 7, 11 ], [ 2, 6, 8, 11 ], [ 2, 3, 7, 9 ],
  [ 3, 4, 8, 10 ], [ 4, 5, 9, 11 ], [ 5, 6, 7, 10 ] ] )
gap> WheelGraph(5,3);
Graph( Category := SimpleGraphs, Order := 16, Size := 40, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3, 6, 7, 8 ], [ 1, 2, 4, 8, 9 ], [ 1, 3, 5, 9, 10 ],
```

```
[ 1, 4, 6, 10, 11 ], [ 1, 2, 5, 7, 11 ], [ 2, 6, 8, 11, 12, 13 ],
[ 2, 3, 7, 9, 13, 14 ], [ 3, 4, 8, 10, 14, 15 ], [ 4, 5, 9, 11, 15, 16 ],
[ 5, 6, 7, 10, 12, 16 ], [ 7, 11, 13, 16 ], [ 7, 8, 12, 14 ],
[ 8, 9, 13, 15 ], [ 9, 10, 14, 16 ], [ 10, 11, 12, 15 ] ] )
```

–map

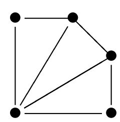
Wheel Graph of Order 5 

12 ► `FanGraph(` N `)` F

Returns the N-Fan: The join of a vertex and a *(N+1)*-path.

```
gap> FanGraph(4);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4, 6 ],
[ 1, 5 ] ] )
```

–map

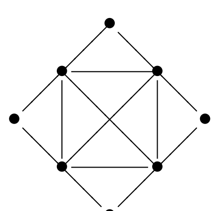
4-Fan Graph 

13 ► `SunGraph(` N `)` F

Returns the N-Sun: A complete graph on N vertices, K_N, with a corona made with a zigzagging *2N*-cycle glued to a N-cyle of the K_N.

```
gap> SunGraph(3);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 2, 6 ], [ 1, 3, 4, 6 ], [ 2, 4 ], [ 2, 3, 5, 6 ], [ 4, 6 ],
  [ 1, 2, 4, 5 ] ] )
gap> SunGraph(4);
Graph( Category := SimpleGraphs, Order := 8, Size := 14, Adjacencies :=
[ [ 2, 8 ], [ 1, 3, 4, 6, 8 ], [ 2, 4 ], [ 2, 3, 5, 6, 8 ], [ 4, 6 ],
  [ 2, 4, 5, 7, 8 ], [ 6, 8 ], [ 1, 2, 4, 6, 7 ] ] )
```

–map

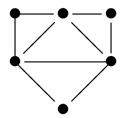
4-Sun Graph 

14 ► `SpikyGraph(` N `)` F

The spiky graph is constructed as follows: Take complete graph on N vertices, K_N, and then, for each the N subsets of *Vertices*(K_n) of order *N-1*, add an additional vertex which is adjacent precisely to this subset of *Vertices*(K_n).

```
gap> SpikyGraph(3);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 4, 6 ], [ 1, 2, 5, 6 ], [ 1, 2 ], [ 1, 3 ],
  [ 2, 3 ] ] )
```

–map

3-Spiky Graph 

15 ► `TrivialGraph` V

The one vertex graph.

```
gap> TrivialGraph;
Graph( Category := SimpleGraphs, Order := 1, Size := 0, Adjacencies :=
[ [  ] ] )
```

–map

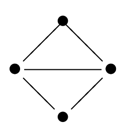
Trivial Graph

16 ► `DiamondGraph` V

The graph on 4 vertices and 5 edges.

```
gap> DiamondGraph;
Graph( Category := SimpleGraphs, Order := 4, Size := 5, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3 ] ] )
```

–map

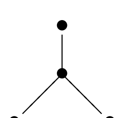
Diamond Graph 

17 ► `ClawGraph` V

The graph on 4 vertices, 3 edges, and maximum degree 3.

```
gap> ClawGraph;
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2, 3, 4 ], [ 1 ], [ 1 ], [ 1 ] ] )
```

–map

Claw Graph 

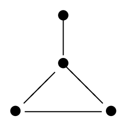
18 ► `PawGraph` V

The graph on 4 vertices, 4 edges and maximum degree 3: A triangle with a pendant vertex.

```
gap> PawGraph;
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3, 4 ], [ 2, 4 ], [ 2, 3 ] ] )
```

–map

Paw Graph



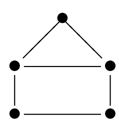
19 ► `HouseGraph` V

A 4-Cycle and a triangle glued by an edge.

```
gap> HouseGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 6, Adjacencies :=
[ [ 2, 4, 5 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3, 5 ], [ 1, 4 ] ] )
```

–map

House Graph



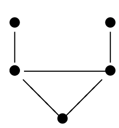
20 ► `BullGraph` V

A triangle with two pendant vertices (horns).

```
gap> BullGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 2 ], [ 1, 3, 4 ], [ 2, 4 ], [ 2, 3, 5 ], [ 4 ] ] )
```

–map

Bull Graph

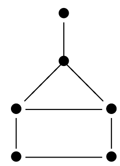


21 ► `AntennaGraph` V

A `HouseGraph` with a pendant vertex (antenna) on the roof.

```
gap> AntennaGraph;
Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
[ [ 2, 4, 5 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3, 5 ], [ 1, 4, 6 ], [ 5 ] ] )
```

–map

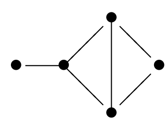
Antenna Graph 

22 ► `KiteGraph` V

A diamond with a pending vertex and maximum degree 3.

```
gap> KiteGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 6, Adjacencies :=
[ [ 2 ], [ 1, 3, 4 ], [ 2, 4, 5 ], [ 2, 3, 5 ], [ 3, 4 ] ] )
```

–map

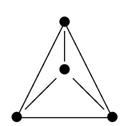
Kite Graph 

23 ► `Tetrahedron` V

The 1-skeleton of Plato's tetrahedron.

```
gap> Tetrahedron;
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
```

–map

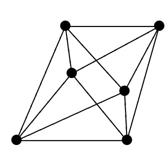
Tetrahedron 

24 ► `Octahedron` V

The 1-skeleton of Plato's octahedron.

```
gap> Octahedron;
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 3, 4, 5, 6 ], [ 3, 4, 5, 6 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],
  [ 1, 2, 3, 4 ], [ 1, 2, 3, 4 ] ] )
```

–map

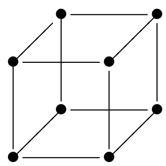
Octahedron 

25 ► `Cube` V

The 1-skeleton of Plato's cube.

```
gap> Cube;
Graph( Category := SimpleGraphs, Order := 8, Size := 12, Adjacencies :=
[ [ 2, 3, 5 ], [ 1, 4, 6 ], [ 1, 4, 7 ], [ 2, 3, 8 ], [ 1, 6, 7 ],
  [ 2, 5, 8 ], [ 3, 5, 8 ], [ 4, 6, 7 ] ] )
```

–map

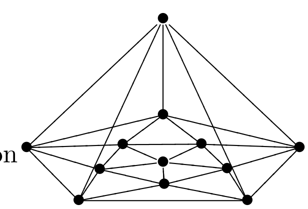
Cube Graph 

26 ► `Icosahedron` V

The 1-skeleton of Plato's icosahedron.

```
gap> Icosahedron;
Graph( Category := SimpleGraphs, Order := 12, Size := 30, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3, 6, 9, 10 ], [ 1, 2, 4, 10, 11 ],
   [ 1, 3, 5, 7, 11 ], [ 1, 4, 6, 7, 8 ], [ 1, 2, 5, 8, 9 ],
   [ 4, 5, 8, 11, 12 ], [ 5, 6, 7, 9, 12 ], [ 2, 6, 8, 10, 12 ],
   [ 2, 3, 9, 11, 12 ], [ 3, 4, 7, 10, 12 ], [ 7, 8, 9, 10, 11 ] ] )
```

–map

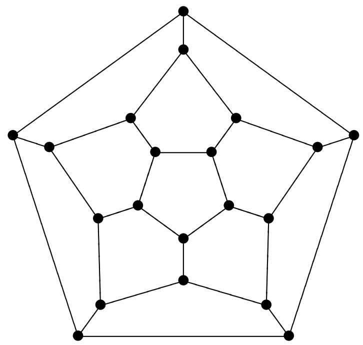
Icosahedron 

27 ► `Dodecahedron` V

The 1-skeleton of Plato's Dodecahedron.

```
gap> Dodecahedron;
Graph( Category := SimpleGraphs, Order := 20, Size := 30, Adjacencies :=
[ [ 2, 5, 6 ], [ 1, 3, 7 ], [ 2, 4, 8 ], [ 3, 5, 9 ], [ 1, 4, 10 ],
   [ 1, 11, 15 ], [ 2, 11, 12 ], [ 3, 12, 13 ], [ 4, 13, 14 ], [ 5, 14, 15 ],
   [ 6, 7, 16 ], [ 7, 8, 17 ], [ 8, 9, 18 ], [ 9, 10, 19 ], [ 6, 10, 20 ],
   [ 11, 17, 20 ], [ 12, 16, 18 ], [ 13, 17, 19 ], [ 14, 18, 20 ],
   [ 15, 16, 19 ] ] )
```

–map

Dodecahedron 

3.3 Unary operations

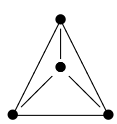
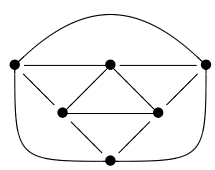
These are operations that can be performed over graphs.

1 ► **LineGraph(<G>)** O

Returns the line graph ¡L(G)¿ of graph ¡G¿. The line graph is the intersection graph of the edges of ¡G¿, ¡i.e.¿ the vertices of $L(G)$ are the edges of ¡G¿ two of them being adjacent iff they are incident.

```
gap> g:=Tetrahedron;
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
gap> LineGraph(g);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 4, 6 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],
  [ 1, 3, 4, 6 ], [ 2, 3, 4, 5 ] ] )
```

–map

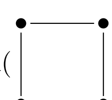
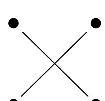
LineGraph() = 

2 ► **ComplementGraph(<G>)** A

Computes the complement of graph ¡G¿. The complement of a graph is created as follows: Create a graph ¡G'¿ with same vertices of ¡G¿. For each ¡x¿, ¡y¿ $\in$ ¡G¿ if ¡x¿ $\nsim$ ¡y¿ in ¡G¿ then ¡x¿ $\sim$ ¡y¿ in ¡G'¿

```
gap> g:=ClawGraph;
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2, 3, 4 ], [ 1 ], [ 1 ], [ 1 ] ] )
gap> ComplementGraph(g);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [  ], [ 3, 4 ], [ 2, 4 ], [ 2, 3 ] ] )
```

–map

ComplementGraph() = 

3 ► `QuotientGraph( <G>, <P> )` O
► `QuotientGraph( <G>, <L1>, <L2> )` O

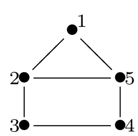
Returns the quotient graph of graph ¡G¿ given a vertex partition ¡P¿, by identifying any two vertices in the same part. The vertices of the quotient graph are the parts in the partition ¡P¿ two of them being adjacent iff any vertex in one part is adjacent to any vertex in the other part. Singletons may be omited in P.

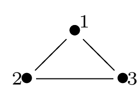
```
gap> g:=PathGraph(8);;
gap> QuotientGraph(g,[[1,5,8],[2],[3],[4],[6],[7]]);
Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
[ [ 2, 4, 5, 6 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ], [ 1, 6 ], [ 1, 5 ] ] )
gap> QuotientGraph(g,[[1,5,8]]);
Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
[ [ 2, 4, 5, 6 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ], [ 1, 6 ], [ 1, 5 ] ] )
```

In its second form, `QuotientGraph` identifies each vertex in list ¡L1¿, with the corresponding vertex in list ¡L2¿. ¡L1¿ and ¡L2¿ must have the same length, but any or both of them may have repetitions.

```
gap> g:=PathGraph(8);;
gap> QuotientGraph(g,[[1,7],[4,8]]);
Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
[ [ 2, 4, 6 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3, 5 ], [ 4, 6 ], [ 1, 5 ] ] )
gap> QuotientGraph(g,[1,4],[7,8]);
Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
[ [ 2, 4, 6 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3, 5 ], [ 4, 6 ], [ 1, 5 ] ] )
```

–map

QuotientGraph(

$, [2,3], [4,5]) =$ 

3.4 Binary operations

These are binary operations that can be performed over graphs.

1 ► `BoxProduct(` G, H `)` O

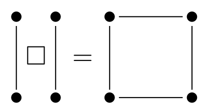
Returns the box product, $G \square H$, of two graphs G and H (also known as the cartesian product).

The box product is calculated as follows:

For each pair of vertices $g \in G, h \in H$ we create a vertex (g,h). Given two such vertices (g,h) and (g',h') they are adjacent *iff* $g = g'$ and $h \sim h'$ or $g \sim g'$ and $h = h'$.

```
gap> g1:=PathGraph(3);g2:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
gap> g1g2:=BoxProduct(g1,g2);
Graph( Category := SimpleGraphs, Order := 12, Size := 20, Adjacencies :=
[ [ 2, 4, 5 ], [ 1, 3, 6 ], [ 2, 4, 7 ], [ 1, 3, 8 ], [ 1, 6, 8, 9 ],
  [ 2, 5, 7, 10 ], [ 3, 6, 8, 11 ], [ 4, 5, 7, 12 ], [ 5, 10, 12 ],
  [ 6, 9, 11 ], [ 7, 10, 12 ], [ 8, 9, 11 ] ] )
gap> VertexNames(g1g2);
[ [ 1, 1 ], [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 2, 1 ], [ 2, 2 ], [ 2, 3 ],
  [ 2, 4 ], [ 3, 1 ], [ 3, 2 ], [ 3, 3 ], [ 3, 4 ] ]
```

–map



2 ► `TimesProduct(` G, H `)` O

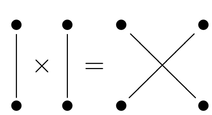
Returns the times product of two graphs G and H, $G \times H$ (also known as the tensor product).

The times product is computed as follows:

For each pair of vertices $g \in G, h \in H$ we create a vertex (g, h). Given two such vertices (g, h) and (g', h') they are adjacent *iff* $g \sim g'$ and $h \sim h'$.

```
gap> g1:=PathGraph(3);g2:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
gap> g1g2:=TimesProduct(g1,g2);
Graph( Category := SimpleGraphs, Order := 12, Size := 16, Adjacencies :=
[ [ 6, 8 ], [ 5, 7 ], [ 6, 8 ], [ 5, 7 ], [ 2, 4, 10, 12 ], [ 1, 3, 9, 11 ],
  [ 2, 4, 10, 12 ], [ 1, 3, 9, 11 ], [ 6, 8 ], [ 5, 7 ], [ 6, 8 ], [ 5, 7 ] ] )
gap> VertexNames(g1g2);
[ [ 1, 1 ], [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 2, 1 ], [ 2, 2 ], [ 2, 3 ],
  [ 2, 4 ], [ 3, 1 ], [ 3, 2 ], [ 3, 3 ], [ 3, 4 ] ]
```

–map



3 ► `BoxTimesProduct(` G, H `)` O

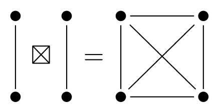
Returns the boxtimes product of two graphs G and H, $G \boxtimes H$ (also known as the strong product).

The box times product is calculated as follows:

For each pair of vertices $g \in G, h \in H$ we create a vertex (g, h). Given two such vertices (g, h) and (g', h') such that $(g, h) \neq (g', h')$ they are adjacent *iff* $g \simeq g'$ and $h \simeq h'$.

```
gap> g1:=PathGraph(3);g2:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
gap> g1g2:=BoxTimesProduct(g1,g2);
Graph( Category := SimpleGraphs, Order := 12, Size := 36, Adjacencies :=
[ [ 2, 4, 5, 6, 8 ], [ 1, 3, 5, 6, 7 ], [ 2, 4, 6, 7, 8 ], [ 1, 3, 5, 7, 8 ],
  [ 1, 2, 4, 6, 8, 9, 10, 12 ], [ 1, 2, 3, 5, 7, 9, 10, 11 ],
  [ 2, 3, 4, 6, 8, 10, 11, 12 ], [ 1, 3, 4, 5, 7, 9, 11, 12 ],
  [ 5, 6, 8, 10, 12 ], [ 5, 6, 7, 9, 11 ], [ 6, 7, 8, 10, 12 ],
  [ 5, 7, 8, 9, 11 ] ] )
gap> VertexNames(g1g2);
[ [ 1, 1 ], [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 2, 1 ], [ 2, 2 ], [ 2, 3 ],
  [ 2, 4 ], [ 3, 1 ], [ 3, 2 ], [ 3, 3 ], [ 3, 4 ] ]
```

–map



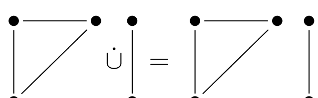
In the previous examples k^2 (*i.e.* the complete graph or order two) was chosen because it better pictures how the operators work.

4 ► `DisjointUnion(` G, H `)` O

Returns the disjoint union of two graphs G and H, $G \dot{\cup} H$.

```
gap> g1:=PathGraph(3);g2:=PathGraph(2);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> DisjointUnion(g1,g2);
Graph( Category := SimpleGraphs, Order := 5, Size := 3, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ], [ 5 ], [ 4 ] ] )
```

–map



5 ► `Join(` G, H `)` O

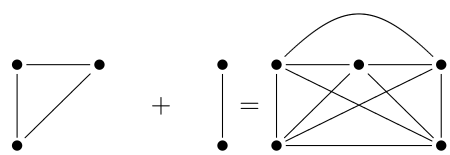
Returns the result of joining graph G and H, $G + H$ (also known as the Zykov sum).

Joining graphs is computed as follows:

First, we obtain the disjoint union of graphs G and H. Second, for each vertex $g \in G$ we add an edge to each vertex $h \in H$.

```
gap> g1:=DiscreteGraph(2);g2:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 2, Size := 0, Adjacencies :=
[ [  ], [  ] ] )
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
gap> Join(g1,g2);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 3, 4, 5, 6 ], [ 3, 4, 5, 6 ], [ 1, 2, 4, 6 ], [ 1, 2, 3, 5 ],
  [ 1, 2, 4, 6 ], [ 1, 2, 3, 5 ] ] )
```

–map



6 ▶ `GraphSum( ` G`, ` L ` )` O

Returns the lexicographic sum of a list of graphs L over a graph G.

The lexicographic sum is computed as follows:

Given G, with $Order(G) = n$ and a list of n graphs $L = [G_1, \ldots, G_n]$, We take the disjoint union of $G_1, G_2, \ldots, G_n$ and then we add all the edges between G_i and G_j whenever $[i, j]$ is and edge of G.

If L contains holes, the trivial graph is used in place.

```
gap> t:=TrivialGraph;; g:=CycleGraph(4);;
gap> GraphSum(PathGraph(3),[t,g,t]);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 5, 6 ], [ 1, 2, 4, 6 ], [ 1, 3, 5, 6 ],
  [ 1, 2, 4, 6 ], [ 2, 3, 4, 5 ] ] )
gap> GraphSum(PathGraph(3),[,g,]);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 5, 6 ], [ 1, 2, 4, 6 ], [ 1, 3, 5, 6 ],
  [ 1, 2, 4, 6 ], [ 2, 3, 4, 5 ] ] )
```

–map

7 ▶ `Composition( ` G`, ` H ` )` O

Returns the composition $G[H]$ of two graphs G and H.

A composition of graphs is obtained by calculating the GraphSum of G with *Order(G)* copies of H, $G[H] = \textit{GraphSum}(G, [H, \ldots, H])$.

```
gap> g1:=CycleGraph(4);;g2:=DiscreteGraph(2);;
gap> Composition(g1,g2);
Graph( Category := SimpleGraphs, Order := 8, Size := 16, Adjacencies :=
[ [ 3, 4, 7, 8 ], [ 3, 4, 7, 8 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],
  [ 3, 4, 7, 8 ], [ 3, 4, 7, 8 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ] ] )
```

–map

4 Inspecting Graphs

4.1 Atributes and properties of graphs

The following are functions to obtain atributes and properties of graphs.

1 ▶ `AdjMatrix(` G `)` A

Returns the adjacency matrix of graph G.

```
gap> AdjMatrix(CycleGraph(4));
[ [ false, true, false, true ], [ true, false, true, false ],
  [ false, true, false, true ], [ true, false, true, false ] ]
```

–map

2 ▶ `Order(` G `)` A

Returns the number of vertices, of graph G.

```
gap> Order(Icosahedron);
12
```

–map

3 ▶ `Size(` G `)` A

Returns the number of edges of graph G.

```
gap> Size(Icosahedron);
30
```

–map

4 ▶ `VertexNames(` G `)` A

Return the list of names of the vertices of G. The vertices of a graph in YAGS are always $\{1, 2, \ldots, Order(G)\}$, but depending on how the graph was constructed, its vertices may have also some *names*, that help us identify the origin of the vertices. YAGS will always try to store meaninful names for the vertices. For example, in the case of the LineGraph, the vertex names of the new graph are the edges of the old graph.

```
gap> g:=LineGraph(DiamondGraph);
Graph( Category := SimpleGraphs, Order := 5, Size := 8, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4, 5 ], [ 1, 2, 5 ], [ 1, 2, 5 ], [ 2, 3, 4 ] ] )
gap> VertexNames(g);
[ [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 2, 3 ], [ 3, 4 ] ]
gap> Edges(DiamondGraph);
[ [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 2, 3 ], [ 3, 4 ] ]
```

–map

5 ▶ `IsCompleteGraph(` G `)` P

Returns `true` if graph G is a complete graph, `false` otherwise. In a complete graph every pair of vertices is an edge.

–map

6 ▶ `IsLoopless(` G `)` P

Returns `true` if graph G have no loops, `false` otherwise. Loops are edges from a vertex to itself.

–map

7 ▶ `IsUndirected(` G `)` P

Returns `true` if graph G is an undirected graph, `false` otherwise. Regardless of the categories that G belongs to, G is undirected if whenever `[x,y]` is an edge of G, `[y,x]` is also an egde of G.

–map

8 ▶ `IsOriented(` G `)` P
▶ `QtfyIsOriented(` G `)` A

Returns `true` if graph G is an oriented graph, `false` otherwise. Regardless of the categories that G belongs to, G is oriented if whenever `[x,y]` is an edge of G, `[y,x]` is not.

–map

9 ▶ `CliqueNumber(` G `)` A

Returns the order, $\omega(G)$, of a maximum clique of G.

```
gap> g:=SunGraph(4);
Graph( Category := SimpleGraphs, Order := 8, Size := 14, Adjacencies :=
[ [ 2, 8 ], [ 1, 3, 4, 6, 8 ], [ 2, 4 ], [ 2, 3, 5, 6, 8 ], [ 4, 6 ],
  [ 2, 4, 5, 7, 8 ], [ 6, 8 ], [ 1, 2, 4, 6, 7 ] ] )
gap> CliqueNumber(g);
4
```

–map

10 ▶ `Cliques(` G `)` A
▶ `Cliques(` G, m `)` O

Returns the set of all (maximal) cliques of a graph G. A clique is a maximal complete subgraph. Here, we use the Bron-Kerbosch algorithm [BK73].

In the second form, It stops computing cliques after m of them have been found.

```
gap> Cliques(Octahedron);
[ [ 1, 3, 5 ], [ 1, 3, 6 ], [ 1, 4, 5 ], [ 1, 4, 6 ], [ 2, 3, 5 ],
  [ 2, 3, 6 ], [ 2, 4, 5 ], [ 2, 4, 6 ] ]
gap> Cliques(Octahedron,4);
[ [ 1, 3, 5 ], [ 1, 3, 6 ], [ 1, 4, 5 ], [ 1, 4, 6 ] ]
```

–map

11 ▶ `IsCliqueHelly(` G `)` P

Returns `true` if the set of (maximal) cliques G satisfy the *Helly* property.

The Helly property is defined as follows:

A non-empty family $\mathcal{F}$ of non-empty sets satisfies the Helly property if every pairwise intersecting subfamily of $\mathcal{F}$ has a non-empty total intersection.

Here we use the Dragan-Szwarcfiter characterization [Dra89,Szw97] to compute the Helly property.

```
gap> g:=SunGraph(3);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 2, 6 ], [ 1, 3, 4, 6 ], [ 2, 4 ], [ 2, 3, 5, 6 ], [ 4, 6 ],
  [ 1, 2, 4, 5 ] ] )
gap> IsCliqueHelly(g);
false
```

–map

4.2 Information about graphs

The following functions give information regarding graphs.

1 ▶ `IsSimple(` G `)` P

Returns `true` if graph G is a simple graph, `false` otherwise. Regardless of the categories that G belongs to, G is simple if and only if G is undirected and loopless.

Returns `true` if the graph G is simple regardless of its category.

–map

2 ▶ `QtfyIsSimple(` G `)` A

For internal use. Returns how far is graph G from being simple.

–map

3 ▶ `Adjacency(` G, v `)` O

Returns the adjacency list of vertex v in G.

```
gap> g:=PathGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> Adjacency(g,1);
[ 2 ]
gap> Adjacency(g,2);
[ 1, 3 ]
```

–map

4 ▶ `Adjacencies(` G `)` O

Returns the adjacency lists of graph G.

```
gap> g:=PathGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> Adjacencies(g);
[ [ 2 ], [ 1, 3 ], [ 2 ] ]
```

–map

5 ▶ `VertexDegree(` G, v `)` O

Returns the degree of vertex v in Graph G.

```
gap> g:=PathGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> VertexDegree(g,1);
1
gap> VertexDegree(g,2);
2
```

–map

6 ▶ `VertexDegrees(` G `)` O

Returns the list of degrees of the vertices in graph G.

```
gap> g:=GemGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 7, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4 ] ] )
gap> VertexDegrees(g);
[ 4, 2, 3, 3, 2 ]
```

–map

7 ▶ `Edges(` G `)` O

Returns the list of edges of graph G in the case of `SimpleGraphs`.

```
gap> g1:=CompleteGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 3, Adjacencies :=
[ [ 2, 3 ], [ 1, 3 ], [ 1, 2 ] ] )
gap> Edges(g1);
[ [ 1, 2 ], [ 1, 3 ], [ 2, 3 ] ]
```

In the case of `UndirectedGraphs`, it also returns the loops. While in the other categories, `Edges` actually does not return the edges, but the loops and arrows of G.

```
gap> g2:=CompleteGraph(3:GraphCategory:=UndirectedGraphs);
Graph( Category := UndirectedGraphs, Order := 3, Size := 6, Adjacencies :=
[ [ 1, 2, 3 ], [ 1, 2, 3 ], [ 1, 2, 3 ] ] )
gap> Edges(g2);
[ [ 1, 1 ], [ 1, 2 ], [ 1, 3 ], [ 2, 2 ], [ 2, 3 ], [ 3, 3 ] ]
gap> g3:=CompleteGraph(3:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 9, Adjacencies :=
[ [ 1, 2, 3 ], [ 1, 2, 3 ], [ 1, 2, 3 ] ] )
gap> Edges(g3);
[ [ 1, 1 ], [ 1, 2 ], [ 1, 3 ], [ 2, 1 ], [ 2, 2 ], [ 2, 3 ], [ 3, 1 ],
  [ 3, 2 ], [ 3, 3 ] ]
```

–map

8 ▶ `CompletesOfGivenOrder(` G, o `)` O

This operation finds all complete subgraphs of order o in graph G.

```
gap> g:=SunGraph(4);
Graph( Category := SimpleGraphs, Order := 8, Size := 14, Adjacencies :=
[ [ 2, 8 ], [ 1, 3, 4, 6, 8 ], [ 2, 4 ], [ 2, 3, 5, 6, 8 ], [ 4, 6 ],
  [ 2, 4, 5, 7, 8 ], [ 6, 8 ], [ 1, 2, 4, 6, 7 ] ] )
gap> CompletesOfGivenOrder(g,3);
[ [ 1, 2, 8 ], [ 2, 3, 4 ], [ 2, 4, 6 ], [ 2, 4, 8 ], [ 2, 6, 8 ],
  [ 4, 5, 6 ], [ 4, 6, 8 ], [ 6, 7, 8 ] ]
gap> CompletesOfGivenOrder(g,4);
[ [ 2, 4, 6, 8 ] ]
```

–map

4.3 Distances

These are functions that measure distances between graphs.

1 ► `Distance(` G, x, y `)` O

Returns the length of a minimal path connecting x to y in G.

```
gap> Distance(CycleGraph(5),1,3);
2
gap> Distance(CycleGraph(5),1,5);
1
```

–map

2 ► `DistanceMatrix(` G `)` A

Returns the distance matrix D of a graph G: D[x][y] is the distance in G from vertex x to vertex y. The matrix may be asymmetric if the graph is not simple. An infinite entry in the matrix means that there is no path between the vertices. Floyd's algorithm is used to compute the matrix.

```
gap> g:=PathGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3 ] ] )
gap> Display(DistanceMatrix(g));
[ [  0,  1,  2,  3 ],
  [  1,  0,  1,  2 ],
  [  2,  1,  0,  1 ],
  [  3,  2,  1,  0 ] ]
gap> g:=PathGraph(4:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2 ], [ 3 ], [ 4 ], [  ] ] )
gap> Display(DistanceMatrix(g));
[ [         0,         1,         2,         3 ],
  [  infinity,         0,         1,         2 ],
  [  infinity,  infinity,         0,         1 ],
  [  infinity,  infinity,  infinity,         0 ] ]
```

–map

3 ► `Diameter(` G `)` A

Returns the maximum among the distances between pairs of vertices of G.

```
gap> g:=CycleGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] )
gap> Diameter(g);
2
```

–map

4 ▶ `Excentricity(` G, x `)` F

Returns the distance from a vertex x in graph G to its most distant vertex in G.

```
gap> g:=PathGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4 ] ] )
gap> Excentricity(g,1);
4
gap> Excentricity(g,3);
2
```

–map

5 ▶ `Radius(` G `)` A

Returns the minimal excentricity among the vertices of graph G.

```
gap> Radius(PathGraph(5));
2
```

–map

6 ▶ `Distances(` G, A, B `)` O

Given two lists of vertices A, B of a graph G, `Distances` returns the list of distances for every pair in the cartesian product of A and B. The order of the vertices in lists A and B affects the order of the list of distances returned.

```
gap> g:=CycleGraph(5);;
gap> Distances(g, [1,3], [2,4]);
[ 1, 2, 1, 1 ]
gap> Distances(g, [3,1], [2,4]);
[ 1, 1, 1, 2 ]
```

–map

7 ▶ `DistanceSet(` G, A, B `)` O

Given two subsets of vertices A, B of a graph G, `DistanceSet` returns the set of distances for every pair in the cartesian product of A and B.

```
gap> g:=CycleGraph(5);;
gap> DistanceSet(g, [1,3], [2,4]);
[ 1, 2 ]
```

–map

8 ▶ `DistanceGraph(` G, D `)` O

Given a graph G and list of distances D, `DistanceGraph` returns the new graph constructed on the vertices of G where two vertices are adjacent iff the distance (in G) between them belongs to the list D.

```
gap> g:=CycleGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] )
gap> DistanceGraph(g,[2]);
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 3, 4 ], [ 4, 5 ], [ 1, 5 ], [ 1, 2 ], [ 2, 3 ] ] )
gap> DistanceGraph(g,[1,2]);
Graph( Category := SimpleGraphs, Order := 5, Size := 10, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 4, 5 ], [ 1, 2, 4, 5 ], [ 1, 2, 3, 5 ],
  [ 1, 2, 3, 4 ] ] )
```

–map

9 ► `PowerGraph(` G, e `)` O

Returns the `DistanceGraph` of G using `[0, 1, ...,` e`]` as the list of distances. Note that the distance `0` in the list produces loops in the new graph only when the `TargetGraphCategory` admits loops.

```
gap> g:=PathGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4 ] ] )
gap> PowerGraph(g,1);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4 ] ] )
gap> PowerGraph(g,1:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 5, Size := 13, Adjacencies :=
[ [ 1, 2 ], [ 1, 2, 3 ], [ 2, 3, 4 ], [ 3, 4, 5 ], [ 4, 5 ] ] )
```

–map

5 Morphisms of Graphs

There exists several classes of morphisms that can be found on graphs. Moreover, sometimes we want to find a combination of them. For this reason YAGS uses a unique mechanism for dealing with morphisms. This mechanisms allows to find any combination of morphisms using three underlying operations.

5.1 Core Operations

The following operations do all the work of finding morphisms that comply with all the properties given in a list. The list of checks that each function receives can have any of the following elements.

- CHQ_METRIC *Metric*
- CHQ_MONO *Mono*
- CHQ_FULL *Full*
- CHQ_EPI *Epi*
- CHQ_CMPLT *Complete*
- CHQ_ISO *Iso*

Additionally it must have at least one of the following.

- CHQ_WEAK *Weak*
- CHQ_MORPH *Morph*

These properties are detailed in the next section.

1 ► `PropertyMorphism(` *G1* , *G2* , *c* `)` O

Returns the first morphisms (in lexicographic order) from *G1* to *G2* satisfying the list of properties *c*

A number of preprogrammed properties are provided by YAGS, and the user may create additional ones. The properties provided are: `CHK_WEAK`, `CHK_MORPH`, `CHK_METRIC`, `CHK_CMPLT`, `CHK_MONO` and `CHK_EPI`.

If *G1* has n vertices and $f : G1 \to G2$ is a morphism, it is represented as `[f(1), f(2), ..., f(n)]`.

```
gap> g1:=CycleGraph(4);;g2:=CompleteBipartiteGraph(2,2);;
gap> c:=[CHK_MORPH];;
gap> PropertyMorphism(g1,g2,c);
[ 1, 3, 1, 3 ]
```

–map

2 ► `PropertyMorphisms(` *G1* , *G2* , *c* `)` O

Returns all morphisms from *G1* to *G2* satisfying the list of properties *c*

A number of preprogrammed properties are provided by YAGS, and the user may create additional ones. The properties provided are: `CHK_WEAK`, `CHK_MORPH`, `CHK_METRIC`, `CHK_CMPLT`, `CHK_MONO` and `CHK_EPI`.

If *G1* has n vertices and $f : G1 \to G2$ is a morphism, it is represented as `[f(1), f(2), ..., f(n)]`.

```
gap> g1:=CycleGraph(4);;g2:=CompleteBipartiteGraph(2,2);;
gap> c:=[CHK_WEAK,CHK_MONO];;
gap> PropertyMorphisms(g1,g2,c);
[ [ 1, 3, 2, 4 ], [ 1, 4, 2, 3 ], [ 2, 3, 1, 4 ], [ 2, 4, 1, 3 ],
  [ 3, 1, 4, 2 ], [ 3, 2, 4, 1 ], [ 4, 1, 3, 2 ], [ 4, 2, 3, 1 ] ]
```

–map

3 ▶ `NextPropertyMorphism(` *G1*, *G2*, *m*, *c* `)` O

Returns the next morphisms (in lexicographic order) from *G1* to *G2* satisfying the list of properties *c* starting with (possibly incomplete) morphism *m*. The morphism found will me returned **and** stored in *m* in order to use it as the next starting point, in case `NextPropertyMorphism` is called again. The operation returns `fail` if there are no more morphisms of the specified type.

A number of preprogrammed properties are provided by YAGS, and the user may create additional ones. The properties provided are: `CHK_WEAK`, `CHK_MORPH`, `CHK_METRIC`, `CHK_CMPLT`, `CHK_MONO` and `CHK_EPI`.

If *G1* has n vertices and $f : G1 \to G2$ is a morphism, it is represented as `[f(1), f(2), ..., f(n)]`.

```
gap> g1:=CycleGraph(4);;g2:=CompleteBipartiteGraph(2,2);;
gap> m:=[];; c:=[CHK_MORPH,CHK_MONO];;
gap> NextPropertyMorphism(g1,g2,m,c);
[ 1, 3, 2, 4 ]
gap> NextPropertyMorphism(g1,g2,m,c);
[ 1, 4, 2, 3 ]
gap> NextPropertyMorphism(g1,g2,m,c);
[ 2, 3, 1, 4 ]
gap> NextPropertyMorphism(g1,g2,m,c);
[ 2, 4, 1, 3 ]
gap> NextPropertyMorphism(g1,g2,m,c);
[ 3, 1, 4, 2 ]
gap> NextPropertyMorphism(g1,g2,m,c);
[ 3, 2, 4, 1 ]
gap> NextPropertyMorphism(g1,g2,m,c);
[ 4, 1, 3, 2 ]
gap> NextPropertyMorphism(g1,g2,m,c);
[ 4, 2, 3, 1 ]
gap> NextPropertyMorphism(g1,g2,m,c);
fail
```

–map

5.2 Morphisms

For all the definitions we assume we have a morphism $\varphi : G \to H\cdot$ The properties for creating morphisms are the following:

Metric A morphism is metric if the distance (see section 6) of any two vertices remains constant

$$d_G(x, y) = d_H(\varphi(x), \varphi(y))\cdot$$

Mono A morphism is mono if two different vertices in G map to two different vertices in H

$$x \neq y \implies \varphi(x) \neq \varphi(y)\cdot$$

Full A morphism is full if every edge in G is mapped to an edge in H.

$$|H| = |G|\cdot$$

Not yet implemented.

Epi A morphism is Epi if for each vertex in H exist a vertex in G that is mapped from.

$$\forall x \in H \exists x_0 \in G \bullet \varphi(x_0) = x$$

Complete A morphism is complete iff the inverse image of any complete of H is a complete of $G\cdot$

Iso An isomorphism is a bimorphism which is also complete.

Aditionally they must be one of the following

Weak A morphism is weak if x adjacent to y in G means their mappings are adjacent in H

$$x, y \in G \wedge x \simeq y \Rightarrow \varphi(x) \simeq \varphi(y)\cdot$$

Morph This is equivalent to *strong*. A morphism is strong if two different vertices in G map to different vertices in H.

$$x, y \in G \wedge x \sim y \Rightarrow \varphi(x) \sim \varphi(y)\cdot$$

Note that $x \neq y \Rightarrow \varphi(x) \neq \varphi(y)$ unless there is a loop in $G\cdot$

6 Other Functions

Here we keep a complete list of all of YAGS's functions not mentioned elsewhere.

1 ▶ `AddEdges(` G, E `)` O

Returns a new graph created from graph G by adding the edges in list E.

```
gap> g:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
gap> AddEdges(g,[[1,3]]);
Graph( Category := SimpleGraphs, Order := 4, Size := 5, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3 ] ] )
gap> AddEdges(g,[[1,3],[2,4]]);
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
```

–map

2 ▶ `AddVerticesByAdjacencies(` G, *NewAdjList* `)` O

Returns a new graph created from graph G by adding as many new vertices as `Length(`*NewAdjList*`)`. Each entry in *NewAdjList* is also a list: the list of neighbors of the corresponding new vertex.

```
gap> g:=PathGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4 ] ] )
gap> AddVerticesByAdjacencies(g,[[1,2],[4,5]]);
Graph( Category := SimpleGraphs, Order := 7, Size := 8, Adjacencies :=
[ [ 2, 6 ], [ 1, 3, 6 ], [ 2, 4 ], [ 3, 5, 7 ], [ 4, 7 ], [ 1, 2 ], [ 4, 5 ] ] )
gap> AddVerticesByAdjacencies(g,[[1,2,7],[4,5]]);
Graph( Category := SimpleGraphs, Order := 7, Size := 9, Adjacencies :=
[ [ 2, 6 ], [ 1, 3, 6 ], [ 2, 4 ], [ 3, 5, 7 ], [ 4, 7 ], [ 1, 2, 7 ], [ 4, 5, 6 ] ] )
```

–map

3 ▶ `Adjacencies(` G `)` O

Returns the adjacency lists of graph G.

```
gap> g:=PathGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> Adjacencies(g);
[ [ 2 ], [ 1, 3 ], [ 2 ] ]
```

–map

4 ▶ `Adjacency(` *G*, *v* `)` O

Returns the adjacency list of vertex v in G.

```
gap> g:=PathGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> Adjacency(g,1);
[ 2 ]
gap> Adjacency(g,2);
[ 1, 3 ]
```

–map

5 ▶ `AdjMatrix(` *G* `)` A

Returns the adjacency matrix of graph G.

```
gap> AdjMatrix(CycleGraph(4));
[ [ false, true, false, true ], [ true, false, true, false ],
  [ false, true, false, true ], [ true, false, true, false ] ]
```

–map

6 ▶ `AGraph` V

A 4-cycle with two pendant vertices on consecutive vertices of the cycle.

```
gap> AGraph;
Graph( Category := SimpleGraphs, Order := 6, Size := 6, Adjacencies :=
[ [ 2 ], [ 1, 3, 5 ], [ 2, 4 ], [ 3, 5 ], [ 2, 4, 6 ], [ 5 ] ] )
```

–map

7 ▶ `AntennaGraph` V

A `HouseGraph` with a pendant vertex (antenna) on the roof.

```
gap> AntennaGraph;
Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
[ [ 2, 4, 5 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3, 5 ], [ 1, 4, 6 ], [ 5 ] ] )
```

–map

FIXME AutomorphismGroup

8 ▶ `BackTrack(` *L*, *opts*, *chk*, *done*, *extra* `)` O

Generic, user-customizable backtracking algorithm.

A backtraking algorithm explores a decision tree in search for solutions to a combinatorial problem. The combinatorial problem and the search strategy are specified by the parameters:

L is just a list that `BackTrack` uses to keep track of solutions and partial solutions. It is usually set to the empty list as a starting point. After a solution is found, it is returned **and** stored in L. This value of L is then used as a starting point to search for the next solution in case `BackTrack` is called again. Partial solutions are also stored in L during the execution of `BackTrack`.

extra may be any object, list, record, etc. `BackTrack` only uses it to pass this data to the user-defined functions *opts*, *chk* and *done*, therefore offering you a way to share data between your functions.

opts`:=function(L,extra)` must return the list of continuation options (childs) one has after some partial solution (node) L has been reached within the decision tree (*opts* may use the extra data *extra* as needed). Each of the values in the list returned by *opts*`(L,extra)` will be tried as possible continuations of the partial solution L. If *opts*`(L,extra)` always returns the same list, you can put that list in place of the parameter *opts*.

chk`:=function(L,extra)` must evaluate the partial solution L possibly using the extra data *extra* and must return `false` when it knows that L can not be extended to a solution of the problem. Otherwise it returns `true`. *chk* may assume that L`[1..Length(L)-1]` already passed the test.

done`:=function(L,extra)` returns `true` if L is already a complete solution and `false` otherwise. In many combinatorial problems, any partial solution of certain length N is also a solution (and viceversa), so if this is your case, you can put that length in place of the parameter *done*.

The following example uses `BackTrack` in its simplest form to compute derrangements (permutations of a set, where none of the elements appears in its original position).

```
gap> N:=4;;L:=[];;extra:=[];;opts:=[1..N];;done:=N;;
gap> chk:=function(L,extra) local i; i:=Length(L);
>           return not L[i] in L{[1..i-1]} and L[i]<> i; end;;
gap> BackTrack(L,opts,chk,done,extra);
[ 2, 1, 4, 3 ]
gap> BackTrack(L,opts,chk,done,extra);
[ 2, 3, 4, 1 ]
gap> BackTrack(L,opts,chk,done,extra);
[ 2, 4, 1, 3 ]
gap> BackTrack(L,opts,chk,done,extra);
[ 3, 1, 4, 2 ]
gap> BackTrack(L,opts,chk,done,extra);
[ 3, 4, 1, 2 ]
gap> BackTrack(L,opts,chk,done,extra);
[ 3, 4, 2, 1 ]
gap> BackTrack(L,opts,chk,done,extra);
[ 4, 1, 2, 3 ]
gap> BackTrack(L,opts,chk,done,extra);
[ 4, 3, 1, 2 ]
gap> BackTrack(L,opts,chk,done,extra);
[ 4, 3, 2, 1 ]
gap> BackTrack(L,opts,chk,done,extra);
fail
```

–map

9 ► `BackTrackBag(` *opts*, *chk*, *done*, *extra* `)` O

Returns the list of all solutions that would be returned one at a time by `Backtrack`.

The following example computes all derrangements of order 4.

```
gap> N:=4;;
gap> chk:=function(L,extra) local i; i:=Length(L);
>           return not L[i] in L{[1..i-1]} and L[i]<> i; end;;
gap> BackTrackBag([1..N],chk,N,[]);
[ [ 2, 1, 4, 3 ], [ 2, 3, 4, 1 ], [ 2, 4, 1, 3 ], [ 3, 1, 4, 2 ],
  [ 3, 4, 1, 2 ], [ 3, 4, 2, 1 ], [ 4, 1, 2, 3 ], [ 4, 3, 1, 2 ],
  [ 4, 3, 2, 1 ] ]
```

–map

10 ► `Basement(` G, KnG, x `)` O
► `Basement(` G, KnG, V `)` O

Given a graph G, some iterated clique graph KnG of G and a vertex x of KnG, the operation computes the *basement* of x with respect to G [Piz04]. Loosely speaking, the basement of x is the set of vertices of G that constitutes the iterated clique x.

```
gap> g:=Icosahedron;;Cliques(g);
[ [ 1, 2, 3 ], [ 1, 2, 6 ], [ 1, 3, 4 ], [ 1, 4, 5 ], [ 1, 5, 6 ],
  [ 4, 5, 7 ], [ 4, 7, 11 ], [ 5, 7, 8 ], [ 7, 8, 12 ], [ 7, 11, 12 ],
  [ 5, 6, 8 ], [ 6, 8, 9 ], [ 8, 9, 12 ], [ 2, 6, 9 ], [ 2, 9, 10 ],
  [ 9, 10, 12 ], [ 2, 3, 10 ], [ 3, 10, 11 ], [ 10, 11, 12 ], [ 3, 4, 11 ] ]
gap> kg:=CliqueGraph(g);; k2g:=CliqueGraph(kg);;
gap> Basement(g,k2g,1);Basement(g,k2g,2);
[ 1, 2, 3, 4, 5, 6 ]
[ 1, 2, 3, 4, 6, 10 ]
```

In its second form, V is a set of vertices of KnG, in that case, the basement is simply the union of the basements of the vertices in V.

```
gap> Basement(g,k2g,[1,2]);
[ 1, 2, 3, 4, 5, 6, 10 ]
```

–map

11 ► `BoxProduct(` G, H `)` O

Returns the box product, $G \square H$, of two graphs G and H (also known as the cartesian product).

The box product is calculated as follows:

For each pair of vertices $g \in G, h \in H$ we create a vertex (g,h). Given two such vertices (g,h) and (g',h') they are adjacent *iff* $g = g'$ and $h \sim h'$ or $g \sim g'$ and $h = h'$.

```
gap> g1:=PathGraph(3);g2:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
gap> g1g2:=BoxProduct(g1,g2);
Graph( Category := SimpleGraphs, Order := 12, Size := 20, Adjacencies :=
[ [ 2, 4, 5 ], [ 1, 3, 6 ], [ 2, 4, 7 ], [ 1, 3, 8 ], [ 1, 6, 8, 9 ],
  [ 2, 5, 7, 10 ], [ 3, 6, 8, 11 ], [ 4, 5, 7, 12 ], [ 5, 10, 12 ],
  [ 6, 9, 11 ], [ 7, 10, 12 ], [ 8, 9, 11 ] ] )
gap> VertexNames(g1g2);
[ [ 1, 1 ], [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 2, 1 ], [ 2, 2 ], [ 2, 3 ],
  [ 2, 4 ], [ 3, 1 ], [ 3, 2 ], [ 3, 3 ], [ 3, 4 ] ]
```

–map

12 ► `BoxTimesProduct(` G, H `)` O

Returns the boxtimes product of two graphs G and H, $G \boxtimes H$ (also known as the strong product).

The box times product is calculated as follows:

For each pair of vertices $g \in G, h \in H$ we create a vertex (g,h). Given two such vertices (g,h) and (g',h') such that $(g,h) \neq (g',h')$ they are adjacent *iff* $g \simeq g'$ and $h \simeq h'$.

```
gap> g1:=PathGraph(3);g2:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
gap> g1g2:=BoxTimesProduct(g1,g2);
Graph( Category := SimpleGraphs, Order := 12, Size := 36, Adjacencies :=
[ [ 2, 4, 5, 6, 8 ], [ 1, 3, 5, 6, 7 ], [ 2, 4, 6, 7, 8 ], [ 1, 3, 5, 7, 8 ],
  [ 1, 2, 4, 6, 8, 9, 10, 12 ], [ 1, 2, 3, 5, 7, 9, 10, 11 ],
  [ 2, 3, 4, 6, 8, 10, 11, 12 ], [ 1, 3, 4, 5, 7, 9, 11, 12 ],
  [ 5, 6, 8, 10, 12 ], [ 5, 6, 7, 9, 11 ], [ 6, 7, 8, 10, 12 ],
  [ 5, 7, 8, 9, 11 ] ] )
gap> VertexNames(g1g2);
[ [ 1, 1 ], [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 2, 1 ], [ 2, 2 ], [ 2, 3 ],
  [ 2, 4 ], [ 3, 1 ], [ 3, 2 ], [ 3, 3 ], [ 3, 4 ] ]
```

–map

13 ► `BullGraph` V

A triangle with two pendant vertices (horns).

```
gap> BullGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 2 ], [ 1, 3, 4 ], [ 2, 4 ], [ 2, 3, 5 ], [ 4 ] ] )
```

–map

14 ► `CayleyGraph(` *Grp*, *elms* `)` O
► `CayleyGraph(` *Grp* `)` O

Returns the graph G whose vertices are the elements of the group *Grp* such that x is adjacent to y iff $x * g = y$ for some g in the list *elms*. if *elms* is not provided, then the generators of G are used instead.

```
gap> grp:=Group((1,2,3),(1,2));
Group([ (1,2,3), (1,2) ])
gap> CayleyGraph(grp);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 3, 4, 5 ], [ 3, 5, 6 ], [ 1, 2, 6 ], [ 1, 5, 6 ], [ 1, 2, 4 ],
  [ 2, 3, 4 ] ] )
gap> CayleyGraph(grp,[(1,2),(2,3)]);
Graph( Category := SimpleGraphs, Order := 6, Size := 6, Adjacencies :=
[ [ 2, 3 ], [ 1, 5 ], [ 1, 4 ], [ 3, 6 ], [ 2, 6 ], [ 4, 5 ] ] )
```

–map

15 ► `ChairGraph` V

A tree with degree sequence 3,2,1,1,1.

```
gap> ChairGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3, 4 ], [ 2 ], [ 2, 5 ], [ 4 ] ] )
```

–map

16 ▶ `Circulant( n, jumps )` O

Returns the graph G whose vertices are [1..n] such that x is adjacent to y iff x+z=y mod n for some z the list of *jumps*

```
gap> Circulant(6,[1,2]);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 2, 3, 5, 6 ], [ 1, 3, 4, 6 ], [ 1, 2, 4, 5 ], [ 2, 3, 5, 6 ],
  [ 1, 3, 4, 6 ], [ 1, 2, 4, 5 ] ] )
```

–map

17 ▶ `ClawGraph` V

The graph on 4 vertices, 3 edges, and maximum degree 3.

```
gap> ClawGraph;
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2, 3, 4 ], [ 1 ], [ 1 ], [ 1 ] ] )
```

–map

18 ▶ `CliqueGraph( G )` A
▶ `CliqueGraph( G, m )` O

Returns the intersection graph of all the (maximal) cliques of G.

The additional parameter m aborts the computation when m cliques are found, even if they are all the cliques of G. If the bound m is reached, *fail* is returned.

```
gap> CliqueGraph(Octahedron);
Graph( Category := SimpleGraphs, Order := 8, Size := 24, Adjacencies :=
[ [ 2, 3, 4, 5, 6, 7 ], [ 1, 3, 4, 5, 6, 8 ], [ 1, 2, 4, 5, 7, 8 ],
  [ 1, 2, 3, 6, 7, 8 ], [ 1, 2, 3, 6, 7, 8 ], [ 1, 2, 4, 5, 7, 8 ],
  [ 1, 3, 4, 5, 6, 8 ], [ 2, 3, 4, 5, 6, 7 ] ] )
gap> CliqueGraph(Octahedron,9);
Graph( Category := SimpleGraphs, Order := 8, Size := 24, Adjacencies :=
[ [ 2, 3, 4, 5, 6, 7 ], [ 1, 3, 4, 5, 6, 8 ], [ 1, 2, 4, 5, 7, 8 ],
  [ 1, 2, 3, 6, 7, 8 ], [ 1, 2, 3, 6, 7, 8 ], [ 1, 2, 4, 5, 7, 8 ],
  [ 1, 3, 4, 5, 6, 8 ], [ 2, 3, 4, 5, 6, 7 ] ] )
gap> CliqueGraph(Octahedron,8);
fail
```

–map

19 ▶ `CliqueNumber( G )` A

Returns the order, $\omega(G)$, of a maximum clique of G.

```
gap> g:=SunGraph(4);
Graph( Category := SimpleGraphs, Order := 8, Size := 14, Adjacencies :=
[ [ 2, 8 ], [ 1, 3, 4, 6, 8 ], [ 2, 4 ], [ 2, 3, 5, 6, 8 ], [ 4, 6 ],
  [ 2, 4, 5, 7, 8 ], [ 6, 8 ], [ 1, 2, 4, 6, 7 ] ] )
gap> CliqueNumber(g);
4
```

–map

20 ► `Cliques(` G `)` A
► `Cliques(` G`,` m `)` O

Returns the set of all (maximal) cliques of a graph G. A clique is a maximal complete subgraph. Here, we use the Bron-Kerbosch algorithm [BK73].

In the second form, It stops computing cliques after m of them have been found.

```
gap> Cliques(Octahedron);
[ [ 1, 3, 5 ], [ 1, 3, 6 ], [ 1, 4, 5 ], [ 1, 4, 6 ], [ 2, 3, 5 ],
  [ 2, 3, 6 ], [ 2, 4, 5 ], [ 2, 4, 6 ] ]
gap> Cliques(Octahedron,4);
[ [ 1, 3, 5 ], [ 1, 3, 6 ], [ 1, 4, 5 ], [ 1, 4, 6 ] ]
```

–map

21 ► `ComplementGraph(` G `)` A

Computes the complement of graph G. The complement of a graph is created as follows: Create a graph G' with same vertices of G. For each x, $y \in G$ if $x \nsim y$ in G then $x \sim y$ in G'

```
gap> g:=ClawGraph;
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2, 3, 4 ], [ 1 ], [ 1 ], [ 1 ] ] )
gap> ComplementGraph(g);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [  ], [ 3, 4 ], [ 2, 4 ], [ 2, 3 ] ] )
```

–map

22 ► `CompleteBipartiteGraph(` n`,` m `)` F

Returns the complete bipartite whose parts have order n and m respectively. This is the joint (Zykov sum) of two discrete graphs of order n and m.

```
gap> CompleteBipartiteGraph(2,3);
Graph( Category := SimpleGraphs, Order := 5, Size := 6, Adjacencies :=
[ [ 3, 4, 5 ], [ 3, 4, 5 ], [ 1, 2 ], [ 1, 2 ], [ 1, 2 ] ] )
```

–map

23 ► `CompleteGraph(` n `)` F

Returns the complete graph of order n. A complete graph is a graph where all vertices are connected to each other.

```
gap> CompleteGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
```

–map

24 ► `CompletelyParedGraph(` G `)` O

Returns the completely pared graph of G, which is obtained by repeatedly applying `ParedGraph` until no more dominated vertices remain.

```
gap> g:=PathGraph(6);
Graph( Category := SimpleGraphs, Order := 6, Size := 5, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4, 6 ], [ 5 ] ] )
gap> CompletelyParedGraph(g);
Graph( Category := SimpleGraphs, Order := 1, Size := 0, Adjacencies :=
[ [  ] ] )
```

–map

25 ▶ `CompleteMultipartiteGraph(` *n1* `,` *n2* `[,` *n3* `...] )` F

Returns the complete multipartite graph where the orders of the parts are *n1*, *n2*, ... It is also the Zykov sum of discrete graphs of order *n1*, *n2*, ...

```
gap> CompleteMultipartiteGraph(2,2,2);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 3, 4, 5, 6 ], [ 3, 4, 5, 6 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],
 [ 1, 2, 3, 4 ], [ 1, 2, 3, 4 ] ] )
```

–map

26 ▶ `CompletesOfGivenOrder(` G`,` o `)` O

This operation finds all complete subgraphs of order o in graph G.

```
gap> g:=SunGraph(4);
Graph( Category := SimpleGraphs, Order := 8, Size := 14, Adjacencies :=
[ [ 2, 8 ], [ 1, 3, 4, 6, 8 ], [ 2, 4 ], [ 2, 3, 5, 6, 8 ], [ 4, 6 ],
  [ 2, 4, 5, 7, 8 ], [ 6, 8 ], [ 1, 2, 4, 6, 7 ] ] )
gap> CompletesOfGivenOrder(g,3);
[ [ 1, 2, 8 ], [ 2, 3, 4 ], [ 2, 4, 6 ], [ 2, 4, 8 ], [ 2, 6, 8 ],
  [ 4, 5, 6 ], [ 4, 6, 8 ], [ 6, 7, 8 ] ]
gap> CompletesOfGivenOrder(g,4);
[ [ 2, 4, 6, 8 ] ]
```

–map

27 ▶ `Composition(` G`,` H `)` O

Returns the composition $G[H]$ of two graphs G and H.

A composition of graphs is obtained by calculating the GraphSum of G with *Order(G)* copies of H, $G[H] = \mathit{GraphSum}(G, [H, \dots, H])$.

```
gap> g1:=CycleGraph(4);;g2:=DiscreteGraph(2);;
gap> Composition(g1,g2);
Graph( Category := SimpleGraphs, Order := 8, Size := 16, Adjacencies :=
[ [ 3, 4, 7, 8 ], [ 3, 4, 7, 8 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],
  [ 3, 4, 7, 8 ], [ 3, 4, 7, 8 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ] ] )
```

–map

28 ▶ `Cone(` G `)` O

Returns the cone of graph G. The cone of G is the graph obtained from G by adding a new vertex which is adjacent to every vertex of G. The new vertex is the first one in the new graph.

```
gap> Cone(CycleGraph(4));
Graph( Category := SimpleGraphs, Order := 5, Size := 8, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 5 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 2, 4 ] ] )
```

–map

29 ▶ `ConnectedComponents(` G `)` A

Returns the connected components of G.

–map

30 ▶ `Coordinates(` G `)` O

Gets the coordinates of the vertices of G, which are used to draw G by `Draw(` G `)`. If the coordinates have not been previously set, `Coordinates` returns *fail*.

```
gap> g:=CycleGraph(4);;
gap> Coordinates(g);
fail
gap> SetCoordinates(g,[[-10,-10 ],[-10,20],[20,-10 ], [20,20]]);
gap> Coordinates(g);
[ [ -10, -10 ], [ -10, 20 ], [ 20, -10 ], [ 20, 20 ] ]
```

–map

31 ▶ `CopyGraph(` G `)` O

Returns a fresh copy of graph G. Only the order and adjacency information is copied, all other known attributes of G are not. Mainly used to transform a graph from one category to another. The new graph will be forced to comply with the `TargetGraphCategory`.

```
gap> g:=CompleteGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
gap> g1:=CopyGraph(g:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 3, 4 ], [ 4 ], [  ] ] )
gap> CopyGraph(g1:GraphCategory:=SimpleGraphs);
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
```

–map

32 ▶ `CuadraticRingGraph(` *Rng* `)` O

Returns the graph G whose vertices are the elements of *Rng* such that x is adjacent to y iff x+zˆ2=y for some z in *Rng*

```
gap> CuadraticRingGraph(ZmodnZ(8));
Graph( Category := SimpleGraphs, Order := 8, Size := 12, Adjacencies :=
[ [ 2, 5, 8 ], [ 1, 3, 6 ], [ 2, 4, 7 ], [ 3, 5, 8 ], [ 1, 4, 6 ],
  [ 2, 5, 7 ], [ 3, 6, 8 ], [ 1, 4, 7 ] ] )
```

–map

33 ▶ `Cube` V

The 1-skeleton of Plato's cube.

```
gap> Cube;
Graph( Category := SimpleGraphs, Order := 8, Size := 12, Adjacencies :=
[ [ 2, 3, 5 ], [ 1, 4, 6 ], [ 1, 4, 7 ], [ 2, 3, 8 ], [ 1, 6, 7 ],
  [ 2, 5, 8 ], [ 3, 5, 8 ], [ 4, 6, 7 ] ] )
```

–map

34 ▶ `CubeGraph( n )` F

Returns the hypercube of dimension n. This is the box product (cartesian product) of n copies of K_2 (an edge).

```
gap> CubeGraph(3);
Graph( Category := SimpleGraphs, Order := 8, Size := 12, Adjacencies :=
[ [ 2, 3, 5 ], [ 1, 4, 6 ], [ 1, 4, 7 ], [ 2, 3, 8 ], [ 1, 6, 7 ],
[ 2, 5, 8 ], [ 3, 5, 8 ], [ 4, 6, 7 ] ] )
```

-map

35 ▶ `CycleGraph( n )` F

Returns the cyclic graph on n vertices.

```
gap> CycleGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] )
```

–map

36 ▶ `CylinderGraph( Base, Height )` F

Returns a cylinder of base *Base* and height *Height*. The order of this graph is *Base*(Height+1)* and it is constructed by taking *Height+1* copies of the cyclic graph on *Base* vertices, ordering these cycles linearly and then joining consecutive cycles by a zigzagging *2*Base*-cycle. This graph is a triangulation of the cylinder where all internal vertices are of degree 6 and the border vertices are of degree 4.

```
gap> g:=CylinderGraph(4,1);
Graph( Category := SimpleGraphs, Order := 8, Size := 16, Adjacencies :=
[ [ 2, 4, 5, 6 ], [ 1, 3, 6, 7 ], [ 2, 4, 7, 8 ], [ 1, 3, 5, 8 ],
  [ 1, 4, 6, 8 ], [ 1, 2, 5, 7 ], [ 2, 3, 6, 8 ], [ 3, 4, 5, 7 ] ] )
gap> g:=CylinderGraph(4,2);
Graph( Category := SimpleGraphs, Order := 12, Size := 28, Adjacencies :=
[ [ 2, 4, 5, 6 ], [ 1, 3, 6, 7 ], [ 2, 4, 7, 8 ], [ 1, 3, 5, 8 ],
  [ 1, 4, 6, 8, 9, 10 ], [ 1, 2, 5, 7, 10, 11 ], [ 2, 3, 6, 8, 11, 12 ],
  [ 3, 4, 5, 7, 9, 12 ], [ 5, 8, 10, 12 ], [ 5, 6, 9, 11 ], [ 6, 7, 10, 12 ],
  [ 7, 8, 9, 11 ] ] )
```

–map

37 ▶ `DartGraph` V

A diamond with a pending vertex and maximum degree 4.

```
gap> DartGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 6, Adjacencies :=
[ [ 2 ], [ 1, 3, 4, 5 ], [ 2, 4, 5 ], [ 2, 3 ], [ 2, 3 ] ] )
```

–map

38 ► `DeclareQtfyProperty(` *Name*, *Filter* `)` F

For internal use.

Declares a YAGS quantifiable property named *Name* for filter *Filter*. This in turns, declares a boolean GAP property *Name* and an integer GAP attribute *QtfyName*.

The user must provide the method *Name*(O, *qtfy*). If *qtfy* is false, the method must return a boolean indicating whether the property holds, otherwise, the method must return a non-negative integer quantifying how far is the object from satisfying the property. In the latter case, returning 0 actually means that the object does satisfy the property.

```
gap> DeclareQtfyProperty("Is2Regular",Graphs);
gap> InstallMethod(Is2Regular,"for graphs",true,[Graphs,IsBool],0,
> function(G,qtfy)
>   local x,count;
>   count:=0;
>   for x in Vertices(G) do
>     if VertexDegree(G,x)<> 2 then
>       if not qtfy then
>         return false;
>       fi;
>         count:=count+1;
>     fi;
>   od;
>   if not qtfy then return true; fi;
>   return count;
> end);
gap> Is2Regular(CycleGraph(4));
true
gap> QtfyIs2Regular(CycleGraph(4));
0
gap> Is2Regular(DiamondGraph);
false
gap> QtfyIs2Regular(DiamondGraph);
2
```

–map

39 ► `Diameter(` G `)` A

Returns the maximum among the distances between pairs of vertices of G.

```
gap> g:=CycleGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] )
gap> Diameter(g);
2
```

–map

40 ► `DiamondGraph` V

The graph on 4 vertices and 5 edges.

```
gap> DiamondGraph;
Graph( Category := SimpleGraphs, Order := 4, Size := 5, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3 ] ] )
```

–map

41 ▶ `DiscreteGraph(` n `)` F

Returns the discrete graph of order n. A discrete graph is a graph without edges.

```
gap> DiscreteGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 0, Adjacencies :=
[ [  ], [  ], [  ], [  ] ] )
```

–map

42 ▶ `DisjointUnion(` G, H `)` O

Returns the disjoint union of two graphs G and H, $G \dot{\cup} H$.

```
gap> g1:=PathGraph(3);g2:=PathGraph(2);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> DisjointUnion(g1,g2);
Graph( Category := SimpleGraphs, Order := 5, Size := 3, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ], [ 5 ], [ 4 ] ] )
```

–map

43 ▶ `Distance(` G, x, y `)` O

Returns the length of a minimal path connecting x to y in G.

```
gap> Distance(CycleGraph(5),1,3);
2
gap> Distance(CycleGraph(5),1,5);
1
```

–map

44 ▶ `DistanceGraph(` G, D `)` O

Given a graph G and list of distances D, `DistanceGraph` returns the new graph constructed on the vertices of G where two vertices are adjacent iff the distance (in G) between them belongs to the list D.

```
gap> g:=CycleGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] )
gap> DistanceGraph(g,[2]);
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 3, 4 ], [ 4, 5 ], [ 1, 5 ], [ 1, 2 ], [ 2, 3 ] ] )
gap> DistanceGraph(g,[1,2]);
Graph( Category := SimpleGraphs, Order := 5, Size := 10, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 4, 5 ], [ 1, 2, 4, 5 ], [ 1, 2, 3, 5 ],
  [ 1, 2, 3, 4 ] ] )
```

–map

45 ▶ `DistanceMatrix(` G `)` A

Returns the distance matrix D of a graph G: D[x][y] is the distance in G from vertex x to vertex y. The matrix may be asymmetric if the graph is not simple. An infinite entry in the matrix means that there is no path between the vertices. Floyd's algorithm is used to compute the matrix.

```
gap> g:=PathGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3 ] ] )
gap> Display(DistanceMatrix(g));
[ [  0,  1,  2,  3 ],
  [  1,  0,  1,  2 ],
  [  2,  1,  0,  1 ],
  [  3,  2,  1,  0 ] ]
gap> g:=PathGraph(4:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2 ], [ 3 ], [ 4 ], [  ] ] )
gap> Display(DistanceMatrix(g));
[ [         0,         1,         2,         3 ],
  [  infinity,         0,         1,         2 ],
  [  infinity,  infinity,         0,         1 ],
  [  infinity,  infinity,  infinity,         0 ] ]
```

–map

46 ▶ `Distances(` G, A, B `)` O

Given two lists of vertices A, B of a graph G, `Distances` returns the list of distances for every pair in the cartesian product of A and B. The order of the vertices in lists A and B affects the order of the list of distances returned.

```
gap> g:=CycleGraph(5);;
gap> Distances(g, [1,3], [2,4]);
[ 1, 2, 1, 1 ]
gap> Distances(g, [3,1], [2,4]);
[ 1, 1, 1, 2 ]
```

–map

47 ▶ `DistanceSet(` G, A, B `)` O

Given two subsets of vertices A, B of a graph G, `DistanceSet` returns the set of distances for every pair in the cartesian product of A and B.

```
gap> g:=CycleGraph(5);;
gap> DistanceSet(g, [1,3], [2,4]);
[ 1, 2 ]
```

–map

48 ▶ `Dodecahedron` V

The 1-skeleton of Plato's Dodecahedron.

```
gap> Dodecahedron;
Graph( Category := SimpleGraphs, Order := 20, Size := 30, Adjacencies :=
[ [ 2, 5, 6 ], [ 1, 3, 7 ], [ 2, 4, 8 ], [ 3, 5, 9 ], [ 1, 4, 10 ],
  [ 1, 11, 15 ], [ 2, 11, 12 ], [ 3, 12, 13 ], [ 4, 13, 14 ], [ 5, 14, 15 ],
  [ 6, 7, 16 ], [ 7, 8, 17 ], [ 8, 9, 18 ], [ 9, 10, 19 ], [ 6, 10, 20 ],
  [ 11, 17, 20 ], [ 12, 16, 18 ], [ 13, 17, 19 ], [ 14, 18, 20 ],
  [ 15, 16, 19 ] ] )
```

–map

49 ▶ `DominatedVertices( G )` A

Returns the set of dominated vertices of G.

A vertex x is dominated by another vertex y when the closed neighborhood of x is contained in that of y. However, when there are twin vertices (mutually dominated vertices), exactly one of them (in each equivalent class of mutually dominated vertices) does not appear in the returned set.

```
gap> g1:=PathGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> DominatedVertices(g1);
[ 1, 3 ]
gap> g2:=PathGraph(2);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> DominatedVertices(g2);
[ 2 ]
```

–map

50 ▶ `DominoGraph` V

Two squares glued by an edge.

```
gap> DominoGraph;
Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
[ [ 2, 4, 6 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3, 5 ], [ 4, 6 ], [ 1, 5 ] ] )
```

–map

51 ▶ `Draw( G )` O

Takes a graph G and makes a drawing of it in a separate window. The user can then view and modify the drawing and finaly save the vertex coordinates of the drawing into the graph G.

Within the separate window, type h to toggle on/off the help menu. Besides the keyword commands indicated in the help menu, the user may also move vertices (by dragging them), move the whole drawing (by dragging the background) and scale the drawing (by using the mouse wheel).

```
gap> Coordinates(Icosahedron);
fail
gap> Draw(Icosahedron);
gap> Coordinates(Icosahedron);
[ [ 29, -107 ], [ 65, -239 ], [ 240, -62 ], [ 78, 79 ], [ -107, 28 ],
  [ -174, -176 ], [ -65, 239 ], [ -239, 62 ], [ -78, -79 ], [ 107, -28 ],
  [ 174, 176 ], [ -29, 107 ] ]
```

This preliminary version, should work fine on GNU/Linux. For other plataforms, you should probably (at least) set up correctly the variable `drawproc` which should point to the correct external program binary. Java binaries are provided for GNU/Linux, Mac OS X and Windows.

```
gap> drawproc;
"/usr/share/gap/pkg/yags/bin/draw/application.linux64/draw"
```

–map

52 ▶ `DumpObject( ` *O* ` )` O

Dumps all information available for object O. This information includes to which categories it belongs as well as its type and hashing information used by GAP.

```
gap> DumpObject( true );
Object( TypeObj := NewType( NewFamily( "BooleanFamily", [ 11 ], [ 11 ] ),
[ 11, 34 ] ), Categories := [ "IS_BOOL" ] )
```

–map

53 ▶ `EasyExec( ` *dir* `, ` *progname* `, ` *instring* ` )` O
▶ `EasyExec( ` *progname* `, ` *instring* ` )` O

Calls external program *prog* located in directory *dir*, feeding it with *instring* as input and returning the output of the external program as a string. *dir* must be a directory object or a list of diretory objects. If *dir* is not provided, *prog* must be in the system's binary PATH. 'fail' is returned if the program could not be located.

```
gap> s:=EasyExec("date","");;
gap> s;
"Sun Nov  9 10:36:16 CST 2014\n"
gap> s:=EasyExec("sort","4\n2\n3\n1");;
gap> s;
"1\n2\n3\n4\n"
```

–map

54 ▶ `Edges( ` *G* ` )` O

Returns the list of edges of graph G in the case of `SimpleGraphs`.

```
gap> g1:=CompleteGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 3, Adjacencies :=
[ [ 2, 3 ], [ 1, 3 ], [ 1, 2 ] ] )
gap> Edges(g1);
[ [ 1, 2 ], [ 1, 3 ], [ 2, 3 ] ]
```

In the case of `UndirectedGraphs`, it also returns the loops. While in the other categories, `Edges` actually does not return the edges, but the loops and arrows of G.

```
gap> g2:=CompleteGraph(3:GraphCategory:=UndirectedGraphs);
Graph( Category := UndirectedGraphs, Order := 3, Size := 6, Adjacencies :=
[ [ 1, 2, 3 ], [ 1, 2, 3 ], [ 1, 2, 3 ] ] )
gap> Edges(g2);
[ [ 1, 1 ], [ 1, 2 ], [ 1, 3 ], [ 2, 2 ], [ 2, 3 ], [ 3, 3 ] ]
gap> g3:=CompleteGraph(3:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 9, Adjacencies :=
[ [ 1, 2, 3 ], [ 1, 2, 3 ], [ 1, 2, 3 ] ] )
gap> Edges(g3);
[ [ 1, 1 ], [ 1, 2 ], [ 1, 3 ], [ 2, 1 ], [ 2, 2 ], [ 2, 3 ], [ 3, 1 ],
  [ 3, 2 ], [ 3, 3 ] ]
```

–map

55 ▶ `Excentricity(` G, x `)` F

Returns the distance from a vertex x in graph G to its most distant vertex in G.

```
gap> g:=PathGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4 ] ] )
gap> Excentricity(g,1);
4
gap> Excentricity(g,3);
2
```

–map

56 ▶ `FanGraph(` N `)` F

Returns the N-Fan: The join of a vertex and a *(N+1)*-path.

```
gap> FanGraph(4);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4, 6 ],
[ 1, 5 ] ] )
```

–map

57 ▶ `FishGraph` V

A square and a triangle glued by a vertex.

```
gap> FishGraph;
Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
[ [ 2, 3, 4, 6 ], [ 1, 3 ], [ 1, 2 ], [ 1, 5 ], [ 4, 6 ], [ 1, 5 ] ] )
```

–map

58 ▶ `GemGraph` V

The 3-Fan graph.

```
gap> GemGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 7, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4 ] ] )
```

–map

59 ▶ `Graph(` R `)` O

Returns a new graph created from the record R. The record must provide the field *Category* and either the field *Adjacencies* or the field *AdjMatrix*

```
gap> Graph(rec(Category:=SimpleGraphs,Adjacencies:=[[2],[1]]));
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies := [ [ 2 ], [ 1 ] ] )
gap> Graph(rec(Category:=SimpleGraphs,AdjMatrix:=[[false, true],[true, false]]));
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies := [ [ 2 ], [ 1 ] ] )
```

Its main purpose is to import graphs from files, which could have been previously exported using `PrintTo`.

```
gap> g:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
gap> PrintTo("aux.g","h1:=",g,";");
gap> Read("aux.g");
gap> h1;
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
```

–map

60 ► `GraphByAdjacencies(` A `)` F

Returns a new graph having A as its list of adjacencies. The order of the created graph is `Length(A)`, and the set of neighbors of vertex x is $A[x]$.

```
gap> GraphByAdjacencies([[2],[1,3],[2]]);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
```

Note, however, that the graph is forced to comply with the `TargetGraphCategory`.

```
gap> GraphByAdjacencies([[1,2,3],[],[]]);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2, 3 ], [ 1 ], [ 1 ] ] )
```

–map

61 ► `GraphByAdjMatrix(` M `)` F

Returns a new graph created from an adjacency matrix M. The matrix M must be a square boolean matrix.

```
gap> m:=[ [ false, true, false ], [ true, false, true ], [ false, true, false ] ];;
gap> g:=GraphByAdjMatrix(m);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> AdjMatrix(g);
[ [ false, true, false ], [ true, false, true ], [ false, true, false ] ]
```

Note, however, that the graph is forced to comply with the `TargetGraphCategory`.

```
gap> m:=[ [ true, true], [ false, false ] ];;
gap> g:=GraphByAdjMatrix(m);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies := [ [ 2 ], [ 1 ] ] )
gap> AdjMatrix(g);
[ [ false, true ], [ true, false ] ]
```

–map

62 ► `GraphByCompleteCover(` C `)` F

Returns the minimal graph where the elements of C are (the vertex sets of) complete subgraphs.

```
gap> GraphByCompleteCover([[1,2,3,4],[4,6,7]]);
Graph( Category := SimpleGraphs, Order := 7, Size := 9, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3, 6, 7 ], [  ], [ 4, 7 ],
  [ 4, 6 ] ] )
```

–map

63 ► `GraphByEdges( L )` F

Returns the minimal graph such that the pairs in L are edges.

```
gap> GraphByEdges([[1,2],[1,3],[1,4],[4,5]]);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2, 3, 4 ], [ 1 ], [ 1 ], [ 1, 5 ], [ 4 ] ] )
```

The vertices of the constructed graph range from 1 to the maximum of the numbers appearing in L.

```
gap> GraphByEdges([[4,3],[4,5]]);
Graph( Category := SimpleGraphs, Order := 5, Size := 2, Adjacencies :=
[ [  ], [  ], [ 4 ], [ 3, 5 ], [ 4 ] ] )
```

Note that `GraphByWalks` has an even greater functionality.

–map

64 ► `GraphByRelation( V, R )` F
► `GraphByRelation( N, R )` F

Returns a new graph created from a set of vertices V and a binary relation R, where $x \sim y$ iff $R(x, y) = true$. In the second form, N is an integer and V is assumed to be $\{1, 2, \ldots, N\}$.

```
gap> R:=function(x,y) return Intersection(x,y)<>[]; end;;
gap> GraphByRelation([[1,2,3],[3,4,5],[5,6,7]],R);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> GraphByRelation(8,function(x,y) return AbsInt(x-y)<=2; end);
Graph( Category := SimpleGraphs, Order := 8, Size := 13, Adjacencies :=
[ [ 2, 3 ], [ 1, 3, 4 ], [ 1, 2, 4, 5 ], [ 2, 3, 5, 6 ], [ 3, 4, 6, 7 ],
  [ 4, 5, 7, 8 ], [ 5, 6, 8 ], [ 6, 7 ] ] )
```

–map

65 ► `GraphByWalks( walk1, walk2, ... )` F

Returns the minimal graph such that *walk1*, *walk2*, etc are walks.

```
gap> GraphByWalks([1,2,3,4,1],[1,5,6]);
Graph( Category := SimpleGraphs, Order := 6, Size := 6, Adjacencies :=
[ [ 2, 4, 5 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ], [ 1, 6 ], [ 5 ] ] )
```

Walks can be *nested*, which greatly improves the versatility of this function.

```
gap> GraphByWalks([1,[2,3,4],5],[5,6]);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 5 ], [ 1, 2, 4, 5 ], [ 1, 3, 5 ], [ 2, 3, 4, 6 ], [ 5 ] ] )
```

The vertices in the constructed graph range from 1 to the maximum of the numbers appearing in *walk1*, *walk2*, ... etc.

```
gap> GraphByWalks([4,2],[3,6]);
Graph( Category := SimpleGraphs, Order := 6, Size := 2, Adjacencies :=
[ [  ], [ 4 ], [ 6 ], [ 2 ], [  ], [ 3 ] ] )
```

–map

66 ▶ `GraphCategory( [`G`, ... ] )` F

For internal use. Returns the minimal common category to a list of graphs. If the list of graphs is empty, the default category is returned.

The partial order (by inclussion) among graph categories is as follows:

$$\texttt{SimpleGraphs} < \texttt{UndirectedGraphs} < \texttt{Graphs},$$

$$\texttt{OrientedGraphs} < \texttt{LooplessGraphs} < \texttt{Graphs}$$

$$\texttt{SimpleGraphs} < \texttt{LooplessGraphs} < \texttt{Graphs}$$

```
gap> g1:=CompleteGraph(2:GraphCategory:=SimpleGraphs);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> g2:=CompleteGraph(2:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [  ] ] )
gap> g3:=CompleteGraph(2:GraphCategory:=UndirectedGraphs);
Graph( Category := UndirectedGraphs, Order := 2, Size := 3, Adjacencies :=
[ [ 1, 2 ], [ 1, 2 ] ] )
gap> GraphCategory([g1,g2,g3]);
<Operation "Graphs">
gap> GraphCategory([g1,g2]);
<Operation "LooplessGraphs">
gap> GraphCategory([g1,g3]);
<Operation "UndirectedGraphs">
```

–map

67 ▶ `Graphs( )` C

`Graphs` is the most general graph category in YAGS. This category contains all graphs that can be represented in YAGS. A graph in this category may contain loops, arrows and edges (which in YAGS are exactly the same as two opposite arrows between some pair of vertices). This graph category has no parent category.

```
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=SimpleGraphs);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
```

–map

68 ▶ `GraphSum( `G`, `L` )` O

Returns the lexicographic sum of a list of graphs L over a graph G.

The lexicographic sum is computed as follows:

Given G, with $Order(G) = n$ and a list of n graphs $L = [G_1, \ldots, G_n]$, We take the disjoint union of $G_1, G_2, \ldots, G_n$ and then we add all the edges between G_i and G_j whenever $[i, j]$ is and edge of G.

If L contains holes, the trivial graph is used in place.

```
gap> t:=TrivialGraph;; g:=CycleGraph(4);;
gap> GraphSum(PathGraph(3),[t,g,t]);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 5, 6 ], [ 1, 2, 4, 6 ], [ 1, 3, 5, 6 ],
  [ 1, 2, 4, 6 ], [ 2, 3, 4, 5 ] ] )
gap> GraphSum(PathGraph(3),[,g,]);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 5, 6 ], [ 1, 2, 4, 6 ], [ 1, 3, 5, 6 ],
  [ 1, 2, 4, 6 ], [ 2, 3, 4, 5 ] ] )
```

–map

69 ► `GraphToRaw(` *filename*, *G* `)` O

Converts a YAGS graph G into a raw format (number of vertices, coordinates and adjacency matrix) and writes the converted data to the file *filename*. For use by the external program `draw` (see `Draw(`G`)`).

```
gap> g:=CycleGraph(4);;
gap> GraphToRaw("mygraph.raw",g);
```

–map

70 ► `GraphUpdateFromRaw(` *filename*, *G* `)` O

Updates the coordinates of G from a file *filename* in raw format. Intended for internal use only.

–map

71 ► `GroupGraph(` *G*, *Grp*, *act* `)` O
► `GroupGraph(` *G*, *Grp* `)` O

Given a graph G, a group *Grp* and an action *act* of *Grp* in some set S which contains Vertices(G), GroupGraph returns a new graph with vertex set $\{act(v,g) : g \in Grp, v \in Vertices(G)\}$ and edge set $\{\{act(v,g), act(u,g)\} : g\ inGrp\{u,v\} \in Edges(G)\}$.

If *act* is omited, the standard GAP action `OnPoints` is used.

```
gap> GroupGraph(GraphByWalks([1,2]),Group([(1,2,3,4,5),(2,5)(3,4)]));
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] )
```

–map

72 ► `HouseGraph` V

A 4-Cycle and a triangle glued by an edge.

```
gap> HouseGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 6, Adjacencies :=
[ [ 2, 4, 5 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3, 5 ], [ 1, 4 ] ] )
```

–map

73 ► `Icosahedron` V

The 1-skeleton of Plato's icosahedron.

```
gap> Icosahedron;
Graph( Category := SimpleGraphs, Order := 12, Size := 30, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3, 6, 9, 10 ], [ 1, 2, 4, 10, 11 ],
  [ 1, 3, 5, 7, 11 ], [ 1, 4, 6, 7, 8 ], [ 1, 2, 5, 8, 9 ],
  [ 4, 5, 8, 11, 12 ], [ 5, 6, 7, 9, 12 ], [ 2, 6, 8, 10, 12 ],
  [ 2, 3, 9, 11, 12 ], [ 3, 4, 7, 10, 12 ], [ 7, 8, 9, 10, 11 ] ] )
```

–map

74 ► `in(` G`,` C `)` O

Returns `true` if graph G belongs to category C and `false` otherwise.

–map

75 ► `InducedSubgraph(` G`,` V `)` O

Returns the subgraph of graph G induced by the vertex set V.

```
gap> g:=CycleGraph(6);
Graph( Category := SimpleGraphs, Order := 6, Size := 6, Adjacencies :=
[ [ 2, 6 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4, 6 ], [ 1, 5 ] ] )
gap> InducedSubgraph(g,[3,4,6]);
Graph( Category := SimpleGraphs, Order := 3, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ], [  ] ] )
```

The order of the elements in V does matter.

```
gap> InducedSubgraph(g,[6,3,4]);
Graph( Category := SimpleGraphs, Order := 3, Size := 1, Adjacencies :=
[ [  ], [ 3 ], [ 2 ] ] )
```

–map

76 ► `InNeigh(` G`,` x `)` O

Returns the list of in-neighbors of x in G.

```
gap> tt:=CompleteGraph(5:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 5, Size := 10, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 3, 4, 5 ], [ 4, 5 ], [ 5 ], [  ] ] )
gap> InNeigh(tt,3);
[ 1, 2 ]
gap> OutNeigh(tt,3);
[ 4, 5 ]
```

–map

77 ► `IntersectionGraph(` L `)` F

Returns the intersection graph of the family of sets L. This graph has a vertex for every set in L, and two such vertices are adjacent iff the corresponding sets have non-empty intersection.

```
gap> IntersectionGraph([[1,2,3],[3,4,5],[5,6,7]]);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
```

–map

78 ▶ `IsBoolean( O )` F

Returns `true` if object O is `true` or `false` and `false` otherwise.

```
gap> IsBoolean( true ); IsBoolean( fail ); IsBoolean ( false );
true
false
true
```

–map

79 ▶ `IsCliqueGated( G )` P

Returns `true` if G is a clique gated graph [HK96].

–map

80 ▶ `IsCliqueHelly( G )` P

Returns `true` if the set of (maximal) cliques G satisfy the *Helly* property.

The Helly property is defined as follows:

A non-empty family $\mathcal{F}$ of non-empty sets satisfies the Helly property if every pairwise intersecting subfamily of $\mathcal{F}$ has a non-empty total intersection.

Here we use the Dragan-Szwarcfiter characterization [Dra89,Szw97] to compute the Helly property.

```
gap> g:=SunGraph(3);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 2, 6 ], [ 1, 3, 4, 6 ], [ 2, 4 ], [ 2, 3, 5, 6 ], [ 4, 6 ],
  [ 1, 2, 4, 5 ] ] )
gap> IsCliqueHelly(g);
false
```

–map

81 ▶ `IsComplete( G, L )` O

Returns true if L induces a complete subgraph of G.

```
gap> IsComplete(DiamondGraph,[1,2,3]);
true
gap> IsComplete(DiamondGraph,[1,2,4]);
false
```

–map

82 ▶ `IsCompleteGraph( G )` P

Returns `true` if graph G is a complete graph, `false` otherwise. In a complete graph every pair of vertices is an edge.

–map

83 ▶ `IsDiamondFree( G )` P

Returns `true` if G is free from induced diamonds, `false` otherwise.

```
gap> IsDiamondFree(Cube);
true
gap> IsDiamondFree(Octahedron);
false
```

–map

84 ► `IsEdge(` G `, [x, y] )` O

Returns true if [x,y] is an edge of G.

```
gap> IsEdge(PathGraph(3),[1,2]);
true
gap> IsEdge(PathGraph(3),[1,3]);
false
```

–map

85 ► `IsIsomorphicGraph(` G, H `)` O

Returns `true` when G is isomorphic to H and `false` otherwise.

```
gap> g:=PowerGraph(CycleGraph(6),2);;h:=Octahedron;;
gap> IsIsomorphicGraph(g,h);
true
```

–map

86 ► `IsLoopless(` G `)` P

Returns `true` if graph G have no loops, `false` otherwise. Loops are edges from a vertex to itself.

–map

87 ► `IsoMorphism(` G, H `)` O
► `NextIsoMorphism(` G, H, f `)` O

`IsoMorphism` returns one isomorphism from G to H. `NextIsoMorphism` returns the next isomorphism from G to H in the lexicographic order, it returns `fail` if there are no more isomorphisms. If G has n vertices, an isomorphisms $f : G \to H$ is represented as the list `[f(1), f(2), ..., f(n)]`.

```
gap> g:=CycleGraph(4);;h:=CompleteBipartiteGraph(2,2);;
gap> f:=IsoMorphism(g,h);
[ 1, 3, 2, 4 ]
gap> NextIsoMorphism(g,h,f);
[ 1, 4, 2, 3 ]
gap> NextIsoMorphism(g,h,f);
[ 2, 3, 1, 4 ]
gap> NextIsoMorphism(g,h,f);
[ 2, 4, 1, 3 ]
```

–map

88 ► `IsoMorphisms(` G, H `)` O

Returns the list of all isomorphism from G to H. If G has n vertices, an isomorphisms $f : G \to H$ is represented as the list `[f(1), f(2), ..., f(n)]`.

```
gap> g:=CycleGraph(4);;h:=CompleteBipartiteGraph(2,2);;
gap> IsoMorphisms(g,h);
[ [ 1, 3, 2, 4 ], [ 1, 4, 2, 3 ], [ 2, 3, 1, 4 ], [ 2, 4, 1, 3 ],
  [ 3, 1, 4, 2 ], [ 3, 2, 4, 1 ], [ 4, 1, 3, 2 ], [ 4, 2, 3, 1 ] ]
```

–map

89 ▶ `IsOriented( G )` P
▶ `QtfyIsOriented( G )` A

Returns `true` if graph G is an oriented graph, `false` otherwise. Regardless of the categories that G belongs to, G is oriented if whenever `[x,y]` is an edge of G, `[y,x]` is not.

–map

90 ▶ `IsSimple( G )` P

Returns `true` if graph G is a simple graph, `false` otherwise. Regardless of the categories that G belongs to, G is simple if and only if G is undirected and loopless.

Returns `true` if the graph G is simple regardless of its category.

–map

91 ▶ `IsTournament( G )` P

Returns `true` if G is a tournament.

```
gap> tt:=CompleteGraph(5:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 5, Size := 10, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 3, 4, 5 ], [ 4, 5 ], [ 5 ], [  ] ] )
gap> IsTournament(tt);
true
```

–map

92 ▶ `IsTransitiveTournament( G )` P

Returns `true` if G is a transitive tournament.

```
gap> tt:=CompleteGraph(5:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 5, Size := 10, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 3, 4, 5 ], [ 4, 5 ], [ 5 ], [  ] ] )
gap> IsTransitiveTournament(tt);
true
```

–map

93 ▶ `IsUndirected( G )` P

Returns `true` if graph G is an undirected graph, `false` otherwise. Regardless of the categories that G belongs to, G is undirected if whenever `[x,y]` is an edge of G, `[y,x]` is also an egde of G.

–map

94 ▶ `JohnsonGraph( n, r )` F

Returns the Johnson graph $J(n,r)\cdot$ A Johnson Graph is a graph constructed as follows. Each vertex represents a subset of the set $\{1,\ldots,n\}$ with cardinality $r\cdot$

$$V(J(n,r)) = \{X \subset \{1,\ldots,n\} || X| = r\}$$

and there is an edge between two vertices if and only if the cardinality of the intersection of the sets they represent is $r-1$

$$X \sim X' \text{ iff } |X \cup X'| = r-1\cdot$$

```
gap> JohnsonGraph(4,2);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 4, 6 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],
  [ 1, 3, 4, 6 ], [ 2, 3, 4, 5 ] ] )
```

–map

95 ▶ `Join(` G, H `)` O

Returns the result of joining graph G and H, $G + H$ (also known as the Zykov sum).

Joining graphs is computed as follows:

First, we obtain the disjoint union of graphs G and H. Second, for each vertex $g \in G$ we add an edge to each vertex $h \in H$.

```
gap> g1:=DiscreteGraph(2);g2:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 2, Size := 0, Adjacencies :=
[ [  ], [  ] ] )
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
gap> Join(g1,g2);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 3, 4, 5, 6 ], [ 3, 4, 5, 6 ], [ 1, 2, 4, 6 ], [ 1, 2, 3, 5 ],
  [ 1, 2, 4, 6 ], [ 1, 2, 3, 5 ] ] )
```

–map

96 ▶ `KiteGraph` V

A diamond with a pending vertex and maximum degree 3.

```
gap> KiteGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 6, Adjacencies :=
[ [ 2 ], [ 1, 3, 4 ], [ 2, 4, 5 ], [ 2, 3, 5 ], [ 3, 4 ] ] )
```

–map

97 ▶ `LineGraph(` G `)` O

Returns the line graph *L(G)* of graph G. The line graph is the intersection graph of the edges of G, *i.e.* the vertices of $L(G)$ are the edges of G two of them being adjacent iff they are incident.

```
gap> g:=Tetrahedron;
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
gap> LineGraph(g);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 4, 6 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],
  [ 1, 3, 4, 6 ], [ 2, 3, 4, 5 ] ] )
```

–map

98 ▶ `Link(` G, x `)` O

Returns the subgraph of G induced by the neighbors of x.

```
gap> Link(SnubDisphenoid,1);
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] )
gap> Link(SnubDisphenoid,3);
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 3 ], [ 1, 4 ], [ 1, 4 ], [ 2, 3 ] ] )
```

–map

99 ▶ `Links(` *G* `)` A

Returns the list of subgraphs of G induced by the neighbors of each vertex of G.

```
gap> Links(SnubDisphenoid);
[ Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
    [ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] ),
  Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
    [ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] ),
  Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
    [ [ 2, 3 ], [ 1, 4 ], [ 1, 4 ], [ 2, 3 ] ] ),
  Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
    [ [ 2, 3 ], [ 1, 4 ], [ 1, 4 ], [ 2, 3 ] ] ),
  Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
    [ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] ),
  Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
    [ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] ),
  Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
    [ [ 3, 4 ], [ 3, 4 ], [ 1, 2 ], [ 1, 2 ] ] ),
  Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
    [ [ 2, 3 ], [ 1, 4 ], [ 1, 4 ], [ 2, 3 ] ] ) ]
```

–map

100 ▶ `LooplessGraphs( )` C

`LooplessGraphs` is a graph category in YAGS. A graph in this category may contain arrows and edges but no loops. The parent of this category is `Graphs`

```
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=LooplessGraphs);
Graph( Category := LooplessGraphs, Order := 3, Size := 3, Adjacencies :=
[ [ 2 ], [ 1 ], [ 2 ] ] )
```

–map

101 ▶ `MaxDegree(` *G* `)` O

Returns the maximum degree in graph G.

```
gap> g:=GemGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 7, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4 ] ] )
gap> MaxDegree(g);
4
```

–map

102 ▶ `MinDegree(` G `)` O

Returns the minimum degree in graph G.

```
gap> g:=GemGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 7, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4 ] ] )
gap> MinDegree(g);
2
```

–map

103 ▶ `NextPropertyMorphism(` $G1$, $G2$, m, c `)` O

Returns the next morphisms (in lexicographic order) from $G1$ to $G2$ satisfying the list of properties c starting with (possibly incomplete) morphism m. The morphism found will me returned **and** stored in m in order to use it as the next starting point, in case `NextPropertyMorphism` is called again. The operation returns `fail` if there are no more morphisms of the specified type.

A number of preprogrammed properties are provided by YAGS, and the user may create additional ones. The properties provided are: `CHK_WEAK`, `CHK_MORPH`, `CHK_METRIC`, `CHK_CMPLT`, `CHK_MONO` and `CHK_EPI`.

If $G1$ has n vertices and $f : G1 \rightarrow G2$ is a morphism, it is represented as `[f(1), f(2), ..., f(n)]`.

```
gap> g1:=CycleGraph(4);;g2:=CompleteBipartiteGraph(2,2);;
gap> m:=[];; c:=[CHK_MORPH,CHK_MONO];;
gap> NextPropertyMorphism(g1,g2,m,c);
[ 1, 3, 2, 4 ]
gap> NextPropertyMorphism(g1,g2,m,c);
[ 1, 4, 2, 3 ]
gap> NextPropertyMorphism(g1,g2,m,c);
[ 2, 3, 1, 4 ]
gap> NextPropertyMorphism(g1,g2,m,c);
[ 2, 4, 1, 3 ]
gap> NextPropertyMorphism(g1,g2,m,c);
[ 3, 1, 4, 2 ]
gap> NextPropertyMorphism(g1,g2,m,c);
[ 3, 2, 4, 1 ]
gap> NextPropertyMorphism(g1,g2,m,c);
[ 4, 1, 3, 2 ]
gap> NextPropertyMorphism(g1,g2,m,c);
[ 4, 2, 3, 1 ]
gap> NextPropertyMorphism(g1,g2,m,c);
fail
```

–map

104 ▶ `NumberOfCliques(` G `)` A
▶ `NumberOfCliques(` G, m `)` O

Returns the number of (maximal) cliques of G. In the second form, It stops computing cliques after m of them have been counted and returns m in case G has m or more cliques.

```
gap> NumberOfCliques(Icosahedron);
20
gap> NumberOfCliques(Icosahedron,15);
15
gap> NumberOfCliques(Icosahedron,50);
20
```

This implementation discards the cliques once counted hence, given enough time, it can compute the number of cliques of G even if the set of cliques does not fit in memory.

```
gap> NumberOfCliques(OctahedralGraph(30));
1073741824
```

–map

105 ▶ `NumberOfConnectedComponents(` G `)` A

Returns the number of connected components of G.

–map

106 ▶ `OctahedralGraph(` n `)` F

Return the n-dimensional octahedron. This is the complement of n copies of K_2 (an edge). It is also the *(2n-2)*-regular graph on $2n$ vertices.

```
gap> OctahedralGraph(3);
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 3, 4, 5, 6 ], [ 3, 4, 5, 6 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],
[ 1, 2, 3, 4 ], [ 1, 2, 3, 4 ] ] )
```

–map

107 ▶ `Octahedron` V

The 1-skeleton of Plato's octahedron.

```
gap> Octahedron;
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 3, 4, 5, 6 ], [ 3, 4, 5, 6 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],
  [ 1, 2, 3, 4 ], [ 1, 2, 3, 4 ] ] )
```

–map

108 ▶ `Order(` G `)` A

Returns the number of vertices, of graph G.

```
gap> Order(Icosahedron);
12
```

–map

109 ▶ `OrientedGraphs( )` C

`OrientedGraphs` is a graph category in YAGS. A graph in this category may contain arrows, but no loops or edges. The parent of this category is `LooplessGraphs`.

```
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [  ], [ 2 ] ] )
```

–map

110 ► `OutNeigh(` G, x `)` O

Returns the list of out-neighbors of x in G.

```
gap> tt:=CompleteGraph(5:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 5, Size := 10, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 3, 4, 5 ], [ 4, 5 ], [ 5 ], [  ] ] )
gap> InNeigh(tt,3);
[ 1, 2 ]
gap> OutNeigh(tt,3);
[ 4, 5 ]
```

–map

111 ► `ParachuteGraph` V

The complement of a `ParapluieGraph`; The suspension of a 4-path with a pendant vertex attached to the south pole.

```
gap> ParachuteGraph;
Graph( Category := SimpleGraphs, Order := 7, Size := 12, Adjacencies :=
[ [ 2 ], [ 1, 3, 4, 5, 6 ], [ 2, 4, 7 ], [ 2, 3, 5, 7 ], [ 2, 4, 6, 7 ],
  [ 2, 5, 7 ], [ 3, 4, 5, 6 ] ] )
```

–map

112 ► `ParapluieGraph` V

A 3-Fan graph with a 3-path attached to the universal vertex.

```
gap> ParapluieGraph;
Graph( Category := SimpleGraphs, Order := 7, Size := 9, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4, 5, 6, 7 ], [ 3, 5 ], [ 3, 4, 6 ], [ 3, 5, 7 ],
  [ 3, 6 ] ] )
```

–map

113 ► `ParedGraph(` G `)` O

Returns the pared graph of G. This is the induced subgraph obtained from G by removing its dominated vertices. When there are twin vertices (mutually dominated vertices), exactly one of them survives the paring in each equivalent class of mutually dominated vertices.

```
gap> g1:=PathGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3 ] ] )
gap> ParedGraph(g1);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> g2:=PathGraph(2);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> ParedGraph(g2);
Graph( Category := SimpleGraphs, Order := 1, Size := 0, Adjacencies :=
[ [  ] ] )
```

–map

114 ▶ `PathGraph(` *n* `)` F

Returns the path graph on n vertices.

```
gap> PathGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3 ] ] )
```

–map

115 ▶ `PawGraph` V

The graph on 4 vertices, 4 edges and maximum degree 3: A triangle with a pendant vertex.

```
gap> PawGraph;
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3, 4 ], [ 2, 4 ], [ 2, 3 ] ] )
```

–map

116 ▶ `PetersenGraph` V

The 3-regular graph on 10 vertices having girth 5.

```
gap> PetersenGraph;
Graph( Category := SimpleGraphs, Order := 10, Size := 15, Adjacencies :=
[ [ 2, 5, 6 ], [ 1, 3, 7 ], [ 2, 4, 8 ], [ 3, 5, 9 ], [ 1, 4, 10 ],
  [ 1, 8, 9 ], [ 2, 9, 10 ], [ 3, 6, 10 ], [ 4, 6, 7 ], [ 5, 7, 8 ] ] )
```

–map

117 ▶ `PowerGraph(` *G*`,` *e* `)` O

Returns the `DistanceGraph` of G using `[0, 1, ..., `e`]` as the list of distances. Note that the distance `0` in the list produces loops in the new graph only when the `TargetGraphCategory` admits loops.

```
gap> g:=PathGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4 ] ] )
gap> PowerGraph(g,1);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4 ] ] )
gap> PowerGraph(g,1:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 5, Size := 13, Adjacencies :=
[ [ 1, 2 ], [ 1, 2, 3 ], [ 2, 3, 4 ], [ 3, 4, 5 ], [ 4, 5 ] ] )
```

–map

118 ▶ `PropertyMorphism(` *G1*, *G2*, *c* `)` O

Returns the first morphisms (in lexicographic order) from *G1* to *G2* satisfying the list of properties *c*

A number of preprogrammed properties are provided by YAGS, and the user may create additional ones. The properties provided are: `CHK_WEAK`, `CHK_MORPH`, `CHK_METRIC`, `CHK_CMPLT`, `CHK_MONO` and `CHK_EPI`.

If *G1* has n vertices and $f : G1 \to G2$ is a morphism, it is represented as `[f(1), f(2), ..., f(n)]`.

```
gap> g1:=CycleGraph(4);;g2:=CompleteBipartiteGraph(2,2);;
gap> c:=[CHK_MORPH];;
gap> PropertyMorphism(g1,g2,c);
[ 1, 3, 1, 3 ]
```

–map

119 ▶ `PropertyMorphisms(` *G1*, *G2*, *c* `)` O

Returns all morphisms from *G1* to *G2* satisfying the list of properties *c*

A number of preprogrammed properties are provided by YAGS, and the user may create additional ones. The properties provided are: `CHK_WEAK`, `CHK_MORPH`, `CHK_METRIC`, `CHK_CMPLT`, `CHK_MONO` and `CHK_EPI`.

If *G1* has n vertices and $f : G1 \to G2$ is a morphism, it is represented as `[f(1), f(2), ..., f(n)]`.

```
gap> g1:=CycleGraph(4);;g2:=CompleteBipartiteGraph(2,2);;
gap> c:=[CHK_WEAK,CHK_MONO];;
gap> PropertyMorphisms(g1,g2,c);
[ [ 1, 3, 2, 4 ], [ 1, 4, 2, 3 ], [ 2, 3, 1, 4 ], [ 2, 4, 1, 3 ],
  [ 3, 1, 4, 2 ], [ 3, 2, 4, 1 ], [ 4, 1, 3, 2 ], [ 4, 2, 3, 1 ] ]
```

–map

120 ▶ `QtfyIsSimple(` *G* `)` A

For internal use. Returns how far is graph *G* from being simple.

–map

121 ▶ `QuotientGraph(` *G*, *P* `)` O
▶ `QuotientGraph(` *G*, *L1*, *L2* `)` O

Returns the quotient graph of graph *G* given a vertex partition *P*, by identifying any two vertices in the same part. The vertices of the quotient graph are the parts in the partition *P* two of them being adjacent iff any vertex in one part is adjacent to any vertex in the other part. Singletons may be omited in P.

```
gap> g:=PathGraph(8);;
gap> QuotientGraph(g,[[1,5,8],[2],[3],[4],[6],[7]]);
Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
[ [ 2, 4, 5, 6 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ], [ 1, 6 ], [ 1, 5 ] ] )
gap> QuotientGraph(g,[[1,5,8]]);
Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
[ [ 2, 4, 5, 6 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ], [ 1, 6 ], [ 1, 5 ] ] )
```

In its second form, `QuotientGraph` identifies each vertex in list *L1*, with the corresponding vertex in list *L2*. *L1* and *L2* must have the same length, but any or both of them may have repetitions.

```
gap> g:=PathGraph(8);;
gap> QuotientGraph(g,[[1,7],[4,8]]);
Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
[ [ 2, 4, 6 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3, 5 ], [ 4, 6 ], [ 1, 5 ] ] )
gap> QuotientGraph(g,[1,4],[7,8]);
Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
[ [ 2, 4, 6 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3, 5 ], [ 4, 6 ], [ 1, 5 ] ] )
```

–map

122 ▶ `Radius(` G `)` A

Returns the minimal excentricity among the vertices of graph G.

```
gap> Radius(PathGraph(5));
2
```

–map

123 ▶ `RandomGraph(` n, p `)` F
▶ `RandomGraph(` n `)` F

Returns a random graph of order n taking the rational $p \in [0,1]$ as the edge probability.

```
gap> RandomGraph(5,1/3);
Graph( Category := SimpleGraphs, Order := 5, Size := 2, Adjacencies :=
[ [ 5 ], [ 5 ], [  ], [  ], [ 1, 2 ] ] )
gap> RandomGraph(5,2/3);
Graph( Category := SimpleGraphs, Order := 5, Size := 6, Adjacencies :=
[ [ 4, 5 ], [ 3, 4, 5 ], [ 2, 4 ], [ 1, 2, 3 ], [ 1, 2 ] ] )
gap> RandomGraph(5,1/2);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2, 5 ], [ 1, 3, 5 ], [ 2 ], [  ], [ 1, 2 ] ] )
```

If p is ommited, the edge probability is taken to be 1/2.

```
gap> RandomGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 2, 3 ], [ 1 ], [ 1, 4, 5 ], [ 3, 5 ], [ 3, 4 ] ] )
gap> RandomGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 3, Adjacencies :=
[ [ 2, 5 ], [ 1, 4 ], [  ], [ 2 ], [ 1 ] ] )
```

–map

124 ▶ `RandomlyPermuted(` *Obj* `)` O

Returns a copy of *Obj* with the order of its elements permuted randomly. Currently, the operation is implemented for lists and graphs.

```
gap> RandomlyPermuted([1..9]);
[ 9, 7, 5, 3, 1, 4, 8, 6, 2 ]
gap> g:=PathGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3 ] ] )
gap> RandomlyPermuted(g);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 4 ], [ 3, 4 ], [ 2 ], [ 1, 2 ] ] )
```

–map

125 ▶ `RandomPermutation(` *N* `)` O

Returns a random permutation of the list *[1..N]*

```
gap> RandomPermutation(12);
(1,8,10)(2,7,9,12)(3,5,11)(4,6)
```

–map

126 ▶ `RemoveEdges(` *G*, *E* `)` O

Returns a new graph created from graph G by removing the edges in list E.

```
gap> g:=CompleteGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
gap> RemoveEdges(g,[[1,2]]);
Graph( Category := SimpleGraphs, Order := 4, Size := 5, Adjacencies :=
[ [ 3, 4 ], [ 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
gap> RemoveEdges(g,[[1,2],[3,4]]);
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 3, 4 ], [ 3, 4 ], [ 1, 2 ], [ 1, 2 ] ] )
```

–map

127 ▶ `RemoveVertices(` *G*, *V* `)` O

Returns a new graph created from graph G by removing the vertices in list V.

```
gap> g:=PathGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4 ] ] )
gap> RemoveVertices(g,[3]);
Graph( Category := SimpleGraphs, Order := 4, Size := 2, Adjacencies :=
[ [ 2 ], [ 1 ], [ 4 ], [ 3 ] ] )
gap> RemoveVertices(g,[1,3]);
Graph( Category := SimpleGraphs, Order := 3, Size := 1, Adjacencies :=
[ [  ], [ 3 ], [ 2 ] ] )
```

–map

128 ▶ `RGraph` V

A square with two pendant vertices attached to the same vertex of the square.

```
gap> RGraph;
Graph( Category := SimpleGraphs, Order := 6, Size := 6, Adjacencies :=
[ [ 2 ], [ 1, 3, 5, 6 ], [ 2, 4 ], [ 3, 5 ], [ 2, 4 ], [ 2 ] ] )
```

–map

129 ▶ `RingGraph(` *Rng*, *elms* `)` O

Returns the graph G whose vertices are the elements of the ring *Rng* such that x is adjacent to y iff x+r=y for some r in *elms*.

```
gap> r:=FiniteField(8);Elements(r);
GF(2^3)
[ 0*Z(2), Z(2)^0, Z(2^3), Z(2^3)^2, Z(2^3)^3, Z(2^3)^4, Z(2^3)^5, Z(2^3)^6 ]
gap> RingGraph(r,[Z(2^3),Z(2^3)^4]);
Graph( Category := SimpleGraphs, Order := 8, Size := 8, Adjacencies :=
[ [ 3, 6 ], [ 5, 7 ], [ 1, 4 ], [ 3, 6 ], [ 2, 8 ], [ 1, 4 ], [ 2, 8 ],
  [ 5, 7 ] ] )
```

–map

130 ▶ `SetCoordinates(` *G*, *Coord* `)` O

Sets the coordinates of the vertices of G, which are used to draw G by `Draw(` *G* `)`.

```
gap> g:=CycleGraph(4);;
gap> SetCoordinates(g,[[-10,-10 ],[-10,20],[20,-10 ], [20,20]]);
gap> Coordinates(g);
[ [ -10, -10 ], [ -10, 20 ], [ 20, -10 ], [ 20, 20 ] ]
```

–map

131 ▶ `SetDefaultGraphCategory(` *C* `)` F

Sets the default graphs category to C. The default graph category is used when constructing new graphs when no other graph category is indicated. New graphs are always forced to comply with the `TargetGraph-Category`, so loops may be removed, and arrows may replaced by edges or viceversa, depending on the category that the new graph belongs to.

The available graph categories are: `SimpleGraphs`, `OrientedGraphs`, `UndirectedGraphs`, `LooplessGraphs`, and `Graphs`.

```
gap> SetDefaultGraphCategory(Graphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> SetDefaultGraphCategory(LooplessGraphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := LooplessGraphs, Order := 3, Size := 3, Adjacencies :=
[ [ 2 ], [ 1 ], [ 2 ] ] )
gap> SetDefaultGraphCategory(UndirectedGraphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := UndirectedGraphs, Order := 3, Size := 3, Adjacencies :=
[ [ 1, 2 ], [ 1, 3 ], [ 2 ] ] )
gap> SetDefaultGraphCategory(SimpleGraphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> SetDefaultGraphCategory(OrientedGraphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := OrientedGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [  ], [ 2 ] ] )
```

–map

132 ▶ `SimpleGraphs( )` C

`SimpleGraphs` is a graph category in YAGS. A graph in this category may contain edges, but no loops or arrows. The category has two parents: `LooplessGraphs` and `UndirectedGraphs`.

```
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=SimpleGraphs);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
```

–map

133 ► `Size(` G `)` A

Returns the number of edges of graph G.

```
gap> Size(Icosahedron);
30
```

–map

134 ► `SnubDisphenoid` V

The 1-skeleton of the 84th Johnson solid.

```
gap> SnubDisphenoid;
Graph( Category := SimpleGraphs, Order := 8, Size := 18, Adjacencies :=
[ [ 2, 3, 4, 5, 8 ], [ 1, 3, 6, 7, 8 ], [ 1, 2, 4, 6 ], [ 1, 3, 5, 6 ],
  [ 1, 4, 6, 7, 8 ], [ 2, 3, 4, 5, 7 ], [ 2, 5, 6, 8 ], [ 1, 2, 5, 7 ] ] )
```

–map

135 ► `SpanningForest(` G `)` O

Returns a spanning forest of G.

–map

136 ► `SpanningForestEdges(` G `)` O

Returns the edges of a spanning forest of G.

–map

137 ► `SpikyGraph(` N `)` F

The spiky graph is constructed as follows: Take complete graph on N vertices, K_N, and then, for each the N subsets of *Vertices*(K_n) of order *N-1*, add an additional vertex which is adjacent precisely to this subset of *Vertices*(K_n).

```
gap> SpikyGraph(3);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 4, 6 ], [ 1, 2, 5, 6 ], [ 1, 2 ], [ 1, 3 ],
  [ 2, 3 ] ] )
```

–map

138 ► `SunGraph(` N `)` F

Returns the N-Sun: A complete graph on N vertices, K_N, with a corona made with a zigzagging *2N*-cycle glued to a N-cyle of the K_N.

```
gap> SunGraph(3);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 2, 6 ], [ 1, 3, 4, 6 ], [ 2, 4 ], [ 2, 3, 5, 6 ], [ 4, 6 ],
  [ 1, 2, 4, 5 ] ] )
gap> SunGraph(4);
Graph( Category := SimpleGraphs, Order := 8, Size := 14, Adjacencies :=
[ [ 2, 8 ], [ 1, 3, 4, 6, 8 ], [ 2, 4 ], [ 2, 3, 5, 6, 8 ], [ 4, 6 ],
  [ 2, 4, 5, 7, 8 ], [ 6, 8 ], [ 1, 2, 4, 6, 7 ] ] )
```

–map

139 ▶ `Suspension(` G `)` O

Returns the suspension of graph G. The suspension of G is the graph obtained from G by adding two new vertices which are adjacent to every vertex of G but not to each other. The new vertices are the first ones in the new graph.

```
gap> Suspension(CycleGraph(4));
Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 3, 4, 5, 6 ], [ 3, 4, 5, 6 ], [ 1, 2, 4, 6 ], [ 1, 2, 3, 5 ],
  [ 1, 2, 4, 6 ], [ 1, 2, 3, 5 ] ] )
```

–map

140 ▶ `TargetGraphCategory( [`G`, ... ] )` F

For internal use. Returns the graph category indicated in the *options stack* if any, otherwise if the list of graphs provided is not empty, returns the minimal common graph category for the graphs in the list, else returns the default graph category.

The partial order (by inclussion) among graph categories is as follows:

$$\texttt{SimpleGraphs} < \texttt{UndirectedGraphs} < \texttt{Graphs},$$

$$\texttt{OrientedGraphs} < \texttt{LooplessGraphs} < \texttt{Graphs}$$

$$\texttt{SimpleGraphs} < \texttt{LooplessGraphs} < \texttt{Graphs}$$

This function is internally called by all graph constructing operations in YAGS to decide the graph category that the newly constructed graph is going to belong. New graphs are always forced to comply with the `TargetGraphCategory`, so loops may be removed, and arrows may replaced by edges or viceversa, depending on the category that the new graph belongs to.

The *options stack* is a mechanism provided by GAP to pass implicit parameters and is used by `TargetGraphCategory` so that the user may indicate the graph category she/he wants for the new graph.

```
gap> SetDefaultGraphCategory(SimpleGraphs);
gap> g1:=CompleteGraph(2);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> g2:=CompleteGraph(2:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [  ] ] )
gap> DisjointUnion(g1,g2);
Graph( Category := LooplessGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2 ], [ 1 ], [ 4 ], [  ] ] )
gap> DisjointUnion(g1,g2:GraphCategory:=UndirectedGraphs);
Graph( Category := UndirectedGraphs, Order := 4, Size := 2, Adjacencies :=
[ [ 2 ], [ 1 ], [ 4 ], [ 3 ] ] )
```

In the previous examples, `TargetGraphCategory` was called internally exactly once for each new graph constructed with the following parameters:

```
gap> TargetGraphCategory();
<Operation "SimpleGraphs">
gap> TargetGraphCategory(:GraphCategory:=OrientedGraphs);
<Operation "OrientedGraphs">
gap> TargetGraphCategory([g1,g2]);
<Operation "LooplessGraphs">
gap> TargetGraphCategory([g1,g2]:GraphCategory:=UndirectedGraphs);
<Operation "UndirectedGraphs">
```

–map

141 ► `Tetrahedron` V

The 1-skeleton of Plato's tetrahedron.

```
gap> Tetrahedron;
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
```

–map

142 ► `TimeInSeconds( )` O

Returns the time in seconds since 1970-01-01 00:00:00 UTC as an integer. This is useful to measure execution time. It can also be used to impose time constraints on the execution of algorithms. Note however that the time reported is the 'wall time', not necessarily the time spent in the process you intend to measure.

```
gap> TimeInSeconds();
1415551598
gap> K:=CliqueGraph;;
gap>  t1:=TimeInSeconds();NumberOfCliques(K(K(K(K(Icosahedron)))));TimeInSeconds()-t1;
1415551608
44644
103
```

–map

143 ► `TimesProduct(` G, H `)` O

Returns the times product of two graphs G and H, $G \times H$ (also known as the tensor product).

The times product is computed as follows:

For each pair of vertices $g \in G, h \in H$ we create a vertex (g,h). Given two such vertices (g,h) and (g',h') they are adjacent *iff* $g \sim g'$ and $h \sim h'$.

```
gap> g1:=PathGraph(3);g2:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
gap> g1g2:=TimesProduct(g1,g2);
Graph( Category := SimpleGraphs, Order := 12, Size := 16, Adjacencies :=
[ [ 6, 8 ], [ 5, 7 ], [ 6, 8 ], [ 5, 7 ], [ 2, 4, 10, 12 ], [ 1, 3, 9, 11 ],
  [ 2, 4, 10, 12 ], [ 1, 3, 9, 11 ], [ 6, 8 ], [ 5, 7 ], [ 6, 8 ], [ 5, 7 ] ] )
gap> VertexNames(g1g2);
[ [ 1, 1 ], [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 2, 1 ], [ 2, 2 ], [ 2, 3 ],
  [ 2, 4 ], [ 3, 1 ], [ 3, 2 ], [ 3, 3 ], [ 3, 4 ] ]
```

–map

144 ▶ `TrivialGraph` V

The one vertex graph.

```
gap> TrivialGraph;
Graph( Category := SimpleGraphs, Order := 1, Size := 0, Adjacencies :=
[ [  ] ] )
```

–map

145 ▶ `UFFind(` *UFS*, *x* `)` F

For internal use. Implements the *find* operation on the *union-find structure*.

–map

146 ▶ `UFUnite(` *UFS*, *x*, *y* `)` F

For internal use. Implements the *unite* operation on the *union-find structure*.

–map

147 ▶ `UndirectedGraphs( )` C

`UndirectedGraphs` is a graph category in YAGS. A graph in this category may contain edges and loops, but no arrows. The parent of this category is `Graphs`

```
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=UndirectedGraphs);
Graph( Category := UndirectedGraphs, Order := 3, Size := 3, Adjacencies :=
[ [ 1, 2 ], [ 1, 3 ], [ 2 ] ] )
```

–map

148 ▶ `UnitsRingGraph(` *Rng* `)` O

Returns the graph G whose vertices are the elements of *Rng* such that x is adjacent to y iff x+z=y for some unit z of *Rng*

```
gap> UnitsRingGraph(ZmodnZ(8));
Graph( Category := SimpleGraphs, Order := 8, Size := 16, Adjacencies :=
[ [ 2, 4, 6, 8 ], [ 1, 3, 5, 7 ], [ 2, 4, 6, 8 ], [ 1, 3, 5, 7 ],
  [ 2, 4, 6, 8 ], [ 1, 3, 5, 7 ], [ 2, 4, 6, 8 ], [ 1, 3, 5, 7 ] ] )
```

–map

149 ▶ `VertexDegree(` *G*, *v* `)` O

Returns the degree of vertex v in Graph G.

```
gap> g:=PathGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> VertexDegree(g,1);
1
gap> VertexDegree(g,2);
2
```

–map

150 ▶ `VertexDegrees(` G `)` O

Returns the list of degrees of the vertices in graph G.

```
gap> g:=GemGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 7, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4 ] ] )
gap> VertexDegrees(g);
[ 4, 2, 3, 3, 2 ]
```

–map

151 ▶ `VertexNames(` G `)` A

Return the list of names of the vertices of G. The vertices of a graph in YAGS are always $\{1, 2, \ldots, Order(G)\}$, but depending on how the graph was constructed, its vertices may have also some *names*, that help us identify the origin of the vertices. YAGS will always try to store meaninful names for the vertices. For example, in the case of the LineGraph, the vertex names of the new graph are the edges of the old graph.

```
gap> g:=LineGraph(DiamondGraph);
Graph( Category := SimpleGraphs, Order := 5, Size := 8, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4, 5 ], [ 1, 2, 5 ], [ 1, 2, 5 ], [ 2, 3, 4 ] ] )
gap> VertexNames(g);
[ [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 2, 3 ], [ 3, 4 ] ]
gap> Edges(DiamondGraph);
[ [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 2, 3 ], [ 3, 4 ] ]
```

–map

152 ▶ `Vertices(` G `)` O

Returns the list [1..Order(G)].

```
gap> Vertices(Icosahedron);
[ 1 .. 12 ]
```

–map

153 ▶ `WheelGraph(` N `)` O

▶ `WheelGraph(` N`,` *Radius* `)` O

In its first form `WheelGraph` returns the wheel graph on *N+1* vertices. This is the cone of a cycle: a central vertex adjacent to all the vertices of an N-cycle

```
WheelGraph(5);
gap> Graph( Category := SimpleGraphs, Order := 6, Size := 10, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3, 6 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4, 6 ],
  [ 1, 2, 5 ] ] )
```

In its second form, `WheelGraph` returns returns the wheel graph, but adding *Radius-1* layers, each layer is a new N-cycle joined to the previous layer by a zigzagging *2N*-cycle. This graph is a triangulation of the disk.

```
gap> WheelGraph(5,2);
Graph( Category := SimpleGraphs, Order := 11, Size := 25, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3, 6, 7, 8 ], [ 1, 2, 4, 8, 9 ], [ 1, 3, 5, 9, 10 ],
  [ 1, 4, 6, 10, 11 ], [ 1, 2, 5, 7, 11 ], [ 2, 6, 8, 11 ], [ 2, 3, 7, 9 ],
  [ 3, 4, 8, 10 ], [ 4, 5, 9, 11 ], [ 5, 6, 7, 10 ] ] )
gap> WheelGraph(5,3);
Graph( Category := SimpleGraphs, Order := 16, Size := 40, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3, 6, 7, 8 ], [ 1, 2, 4, 8, 9 ], [ 1, 3, 5, 9, 10 ],
  [ 1, 4, 6, 10, 11 ], [ 1, 2, 5, 7, 11 ], [ 2, 6, 8, 11, 12, 13 ],
  [ 2, 3, 7, 9, 13, 14 ], [ 3, 4, 8, 10, 14, 15 ], [ 4, 5, 9, 11, 15, 16 ],
  [ 5, 6, 7, 10, 12, 16 ], [ 7, 11, 13, 16 ], [ 7, 8, 12, 14 ],
  [ 8, 9, 13, 15 ], [ 9, 10, 14, 16 ], [ 10, 11, 12, 15 ] ] )
```

–map

154 ▶ `YagsExec(` *progname* , *instring* `)` O

For internal use. Calls external program *prog* located in directory '*yags-dir*/bin/' feeding it with *instring* as input and returning the output of the external program as a string. 'fail' is returned if the program could not be located.

```
gap> YagsExec("time","");
"1415551127\n"
gap> YagsExec("nauty","l=0$=1dacn=5 g1,2,3. xbzq");
"(4,5)\n(2,3)\n[2,3,4,5,1]\n[\"cb0c\",\"484f264\",\"b0e19f1\"]\n"
```

–map

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Index

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