### **Yet Another Graph System**

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YAGS - Yet Another Graph System

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### **Preface**

Ejemplo de cita: [12]

#### 1.1 Disclaimer

THIS IS NOT AN OFFICIAL RELEASE YET, this is a version in development. This particular version, 0.0.1, changes from one day to another without warning and even without a change in the version number. Also, the operations and global variables can still change name or even disappear without warning. No commitment is made at the moment concerning compatibility of this version of the software with any future version.

As of this writing (16/Feb/2016) there are only two trustable chapters in this manual: Appendixes 'YAGS Functions by Topic' and 'YAGS Functions Reference'; also the file cheatsheet-yags.pdf (within directory: YAGSDIR/doc/) may be useful. All other chapters may contain errors, broken links and misleading information (with higher probability).

The first official version will be 0.0.2 and is scheduled to be ready this year (2016), so come back soon.

#### 1.2 Welcome to YAGS

YAGS - Yet Another Graph System is a computing system for dealing with graphs, in the sense of Graph Theory (not bar graphs, pie charts nor graphs of functions). Hence our graphs are ordered pairs G = (V, E), where V is a finite set of vertices and E is a finite set of edges which are (ordered or unordered) pairs of vertices.

YAGS was initiated by M.A. Pizaña in May 2003, and soon incorporated the work of R. MacKinney-Romero and R. Villarroel-Flores.

Our motivation here was this and that.

Our Pourposes and Aim.

authors, contacts

### 1.3 Citing YAGS

If you publish a result and you used YAGS during your research, please cite us as you would normally do with a research paper:

```
R. MacKinney-Romero, M.A. Pizaña and R. Villarroel-Flores.
YAGS - Yet Another Graph System, Version 0.0.1 (2016)
http://xamanek.izt.uam.mx/yags/

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Villarroel-Flores}, title = {YAGS - Yet Another Graph System, Version 0.0.1},
year = {2016}, note = {http://xamanek.izt.uam.mx/yags/}, }
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#### 1.4 Copyright

YAGS - Yet Another Graph System

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For details, see the file GPL in the installation directory of YAGS typically under GAP-Dir/pkg/yags/GPL or see http://www.gnu.org/licenses/gpl-3.0.html.

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#### **1.6** More Information

More information about YAGS can be found on its official web page:

'http://xamanek.izt.uam.mx/yags/'

You can receive notifications about YAGS (i.e. new releases, bug fixes, etc.) by subscribing to its email distribution list:

'http://xamanek.izt.uam.mx/cgi-bin/mailman/listinfo/yagsnews/'

If you are a developer, you may contribute to our project on public repository:

'https://github.com/yags/main/'

## **Getting Started**

- 2.1 What is YAGS?
- 2.2 Installing YAGS
- **2.3** Testing the Installation
- 2.4 A Gentle Tutorial
- 2.5 An Overview of the Manual
- 2.6 Cheatsheet

## Cliques

- 3.1 Cliques and Clique Number
- 3.2 Clique Graphs
- 3.3 Basements, Stars and Neckties
- 3.4 Clique Behavior

## **Graph Categories**

- **4.1** The Default Graph Category
- **4.2** The Target Graph Category
- 4.3 Changing the Target Graph Category Temporaryly
- 4.4 Digraphs, Tournaments, etc.

## **Morphisms of Graphs**

- 5.1 A Quick Start
- **5.2** Main Procedures
- **5.3** User-Defined Types of Morphisms
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## **Backtracking**

- **6.1** A Simple Example
- 6.2 How Does it Work?
- **6.3** Backtracking in Depth

### Appendix A

### **YAGS** Functions by Topic

- **A.1** Most Common Functions
- A.2 Drawing
- **A.3** Constructing Graphs
- **A.4** Families of Graphs
- A.5 Small Graphs
- A.6 Attributes and Properties
- **A.7** Unary Operators
- **A.8** Binary Operators
- A.9 Cliques

Functions dealing with cliques.

- Basement( G, KnG, x )
  Basement( G, KnG, V )
  Returns the basement of vertex x (vertex set V) of the iterated clique graph KnG with respect to
- CliqueGraph( G )
   CliqueGraph( G, maxNumCli )
   Returns the intersection graph of the (maximal) cliques of G; aborts if maxNumCli cliques are found.
- CliqueNumber( G ) Returns the order,  $\omega(G)$ , of a maximum clique of G.

```
• Cliques( G )
Cliques( G, maxNumCli )
Returns the list of (maximal) cliques of G; aborts if maxNumCli cliques are found.
```

- CompletesOfGivenOrder( G, Ord )
  Returns the list of vertex sets of all complete subgraphs of order Ord of G.
- IsCliqueGated( G )
  Returns 'true' if G is a clique gated graph.
- IsCliqueHelly( *G* )
  Returns 'true' if the set of (maximal) cliques *G* satisfy the *Helly* property.
- IsComplete( *G*, *L* )
  Returns 'true' if *L* induces a complete subgraph of *G*.
- IsCompleteGraph( *G* )
  Returns 'true' if graph *G* is a complete graph, 'false' otherwise.
- NumberOfCliques( G )
  NumberOfCliques( G, maxNumCli )
  Returns the number of (maximal) cliques of G.

#### A.10 Morphisms and Isomorphisms

### **A.11 Graphs Categories**

- A.12 Digraphs
- A.13 Groups and Rings
- A.14 Backtracking
- A.15 Miscellaneous
- A.16 Undocumented

### Appendix B

### **YAGS** Functions Reference

This chapter contains a complete list of all YAGS's functions, with definitions, in alphabetical order.

#### **B.1** Primera seccion

#### B.1.1 Order

```
\triangleright Order(G) (attribute)
```

Returns the number of vertices, of graph G.

```
gap> Order(Icosahedron);
12
```

#### **B.1.2** AddEdges

```
\triangleright AddEdges (G, E) (operation)
```

Returns a new graph created from graph G by adding the edges in list E.

```
Example

gap> g:=CycleGraph(4);

Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=

[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ])

gap> AddEdges(g, [[1,3]]);

Graph( Category := SimpleGraphs, Order := 4, Size := 5, Adjacencies :=

[ [ 2, 3, 4 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3 ] ])

gap> AddEdges(g, [[1,3], [2,4]]);

Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=

[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ])
```

#### **B.1.3** AddVerticesByAdjacencies

```
▷ AddVerticesByAdjacencies(G, NewAdjList)
```

(operation)

Returns a new graph created from graph G by adding as many new vertices as Length(NewAdjList). Each entry in NewAdjList is also a list: the list of neighbors of the corresponding new vertex.

```
Example

gap> g:=PathGraph(5);

Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=

[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4 ] ] )

gap> AddVerticesByAdjacencies(g,[[1,2],[4,5]]);

Graph( Category := SimpleGraphs, Order := 7, Size := 8, Adjacencies :=

[ [ 2, 6 ], [ 1, 3, 6 ], [ 2, 4 ], [ 3, 5, 7 ], [ 4, 7 ], [ 1, 2 ], [ 4, 5 ] ] )

gap> AddVerticesByAdjacencies(g,[[1,2,7],[4,5]]);

Graph( Category := SimpleGraphs, Order := 7, Size := 9, Adjacencies :=

[ [ 2, 6 ], [ 1, 3, 6 ], [ 2, 4 ], [ 3, 5, 7 ], [ 4, 7 ], [ 1, 2, 7 ], [ 4, 5, 6 ] ] )
```

#### **B.1.4** Adjacencies

Returns the adjacency lists of graph G.

```
gap> g:=PathGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> Adjacencies(g);
[ [ 2 ], [ 1, 3 ], [ 2 ] ]
```

#### **B.1.5** Adjacency

```
\triangleright Adjacency (G, x) (operation)
```

Returns the adjacency list of vertex x in G.

```
gap> g:=PathGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> Adjacency(g,1);
[ 2 ]
gap> Adjacency(g,2);
[ 1, 3 ]
```

#### **B.1.6** AdjMatrix

```
ightharpoonup AdjMatrix(G) (attribute)
```

Returns the adjacency matrix of graph G.

(attribute)

#### **B.1.7** AGraph

▷ AGraph (global variable)

A 4-cycle with two pendant vertices on consecutive vertices of the cycle.

```
Example

gap> AGraph;

Graph( Category := SimpleGraphs, Order := 6, Size := 6, Adjacencies :=

[ [ 2 ], [ 1, 3, 5 ], [ 2, 4 ], [ 3, 5 ], [ 2, 4, 6 ], [ 5 ] ])
```

#### **B.1.8** AntennaGraph

A HouseGraph with a pendant vertex (antenna) on the roof.

```
Example

gap> AntennaGraph;

Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=

[ [ 2, 4, 5 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3, 5 ], [ 1, 4, 6 ], [ 5 ] ])
```

#### **B.1.9** AutGroupGraph

\indextt{AutomorphismGroup} Returns the group of automorphisms of the graph G. There is also a synonym for this attribute which is AutomorphismGroup(G).

```
Example

gap> AutGroupGraph(Icosahedron);

Group([ (1,3,2,10,9,12,8,7,5,4)(6,11), (1,7,9)(2,4,12)(3,11,10)(5,8,6) ])

gap> AutomorphismGroup(Icosahedron);

Group([ (1,3,2,10,9,12,8,7,5,4)(6,11), (1,7,9)(2,4,12)(3,11,10)(5,8,6) ])
```

#### **B.1.10** BackTrack

```
▷ BackTrack(L, Opts, Chk, Done, Extra) (operation)
```

Generic, user-customizable backtracking algorithm. A backtraking algorithm explores a decision tree in search for solutions to a combinatorial problem. The combinatorial problem and the search strategy are specified by the parameters: L is just a list that BackTrack uses to keep track of solutions and partial solutions. It is usually set to the empty list as a starting point. After a solution is found, it is returned \*and\* stored in L. This value of L is then used as a starting point to search for the next solution in case BackTrack is called again. Partial solutions are also stored in L during the execution of BackTrack. Extra may be any object, list, record, etc. BackTrack only uses it to pass this data to the user-defined functions Opts, Chk and Done, therefore offering you a way to share data between your functions. Opts:=function(L,extra) must return the list of continuation options (childs) one has after some partial solution (node) L has been reached within the decision tree (Opts may use the extra data Extra as needed). Each of the values in the list returned by Opts (L,extra) will be tried as possible continuations of the partial solution L. If Opts (L,extra) always returns the same

list, you can put that list in place of the parameter Opts. Chk:=function(L,extra) must evaluate the partial solution L possibly using the extra data Extra and must return false when it knows that L can not be extended to a solution of the problem. Otherwise it returns true. Chk may assume that  $L\{[1.Length(L)-1]\}$  already passed the test. Done:=function(L,extra) returns true if L is already a complete solution and false otherwise. In many combinatorial problems, any partial solution of certain length n is also a solution (and viceversa), so if this is your case, you can put that length in place of the parameter Done. The following example uses BackTrack in its simplest form to compute derrangements (permutations of a set, where none of the elements appears in its original position).

```
Example
gap> N:=4;;L:=[];;extra:=[];;opts:=[1..N];;done:=N;;
gap> chk:=function(L,extra) local i; i:=Length(L);
           return not L[i] in L\{[1..i-1]\} and L[i] \Leftrightarrow i; end;;
gap> BackTrack(L,opts,chk,done,extra);
[2, 1, 4, 3]
gap> BackTrack(L,opts,chk,done,extra);
[2,3,4,1]
gap> BackTrack(L,opts,chk,done,extra);
[2, 4, 1, 3]
gap> BackTrack(L,opts,chk,done,extra);
[3, 1, 4, 2]
gap> BackTrack(L,opts,chk,done,extra);
[3, 4, 1, 2]
gap> BackTrack(L,opts,chk,done,extra);
[3, 4, 2, 1]
gap> BackTrack(L,opts,chk,done,extra);
[4, 1, 2, 3]
gap> BackTrack(L,opts,chk,done,extra);
[4,3,1,2]
gap> BackTrack(L,opts,chk,done,extra);
[4, 3, 2, 1]
gap> BackTrack(L,opts,chk,done,extra);
fail
```

#### **B.1.11** BackTrackBag

```
▷ BackTrackBag(Opts, Chk, Done, Extra) (operation)
```

Returns the list of all solutions that would be returned one at a time by Backtrack. The following example computes all derrangements of order 4.

```
gap> N:=4;;
gap> chk:=function(L,extra) local i; i:=Length(L);
> return not L[i] in L{[1..i-1]} and L[i]<> i; end;;
gap> BackTrackBag([1..N],chk,N,[]);
[[2, 1, 4, 3], [2, 3, 4, 1], [2, 4, 1, 3], [3, 1, 4, 2],
[3, 4, 1, 2], [3, 4, 2, 1], [4, 1, 2, 3], [4, 3, 1, 2],
[4, 3, 2, 1]]
```

#### **B.1.12** Basement

```
ightharpoonup Basement(G, KnG, x) (operation)

ightharpoonup (operation)
```

Given a graph G, some iterated clique graph KnG of G and a vertex x of KnG, the operation returns the basement of x with respect to G [14]. Loosely speaking, the basement of x is the set of vertices of G that constitutes the iterated clique x.

```
Example

gap> g:=Icosahedron;;Cliques(g);

[[1, 2, 3], [1, 2, 6], [1, 3, 4], [1, 4, 5], [1, 5, 6],
    [4, 5, 7], [4, 7, 11], [5, 7, 8], [7, 8, 12], [7, 11, 12],
    [5, 6, 8], [6, 8, 9], [8, 9, 12], [2, 6, 9], [2, 9, 10],
    [9, 10, 12], [2, 3, 10], [3, 10, 11], [10, 11, 12], [3, 4, 11]]

gap> kg:=CliqueGraph(g);; k2g:=CliqueGraph(kg);;
gap> Basement(g,k2g,1);Basement(g,k2g,2);
[1, 2, 3, 4, 5, 6]
[1, 2, 3, 4, 6, 10]
```

In its second form, V is a set of vertices of KnG, in that case, the basement is simply the union of the basements of the vertices in V.

```
gap> Basement(g,k2g,[1,2]);
[ 1, 2, 3, 4, 5, 6, 10 ]
```

#### **B.1.13** Boundary Vertices

```
▷ BoundaryVertices(G)
```

(attribute)

When G is a compact surface, it returns the list of vertices in the boundary (of the triangulation) of the surface. That is, the list of vertices of G that have links isomorphic to a path. It returns fail if G is not a compact surface.

```
gap> BoundaryVertices(WheelGraph(4,2));
[ 6, 7, 8, 9 ]
gap> BoundaryVertices(Octahedron);
[ ]
```

#### **B.1.14** BoxProduct

```
▷ BoxProduct(G, H) (operation)
```

Returns the box product, G \$\square\$ H, of two graphs G and H (also known as the cartesian product). The box product is calculated as follows: For each pair of vertices  $x \in G$ ,  $y \in H$  we create a vertex \$(x,y)\$. Given two such vertices \$(x,y)\$ and \$(x',y')\$ they are adjacent iff \$x = x\$ and \$y \sim x \times x^2 \text{ and } y = y'\$.

```
Example
gap> g:=PathGraph(3);h:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
```

```
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
gap> gh:=BoxProduct(g,h);
Graph( Category := SimpleGraphs, Order := 12, Size := 20, Adjacencies :=
[ [ 2, 4, 5 ], [ 1, 3, 6 ], [ 2, 4, 7 ], [ 1, 3, 8 ], [ 1, 6, 8, 9 ],
        [ 2, 5, 7, 10 ], [ 3, 6, 8, 11 ], [ 4, 5, 7, 12 ], [ 5, 10, 12 ],
        [ 6, 9, 11 ], [ 7, 10, 12 ], [ 8, 9, 11 ] ] )
gap> VertexNames(gh);
[ [ 1, 1 ], [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 2, 1 ], [ 2, 2 ], [ 2, 3 ],
        [ 2, 4 ], [ 3, 1 ], [ 3, 2 ], [ 3, 3 ], [ 3, 4 ] ]
```

#### **B.1.15** BoxTimesProduct

```
▷ BoxTimesProduct(G, H)
```

(operation)

Returns the boxtimes product of two graphs G and H, G \$\boxtimes\$ H (also known as the strong product). The boxtimes product is calculated as follows: For each pair of vertices \$x \in G, y \in H\$ we create a vertex \$(x,y)\$. Given two such vertices \$(x,y)\$ and \$(x',y')\$ such that \$(x,y) \neq (x',y')\$ they are adjacent iff \$x \simeq x'\$ and \$y \simeq y'\$.

```
Example
gap> g:=PathGraph(3);h:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[[2],[1,3],[2]])
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[[2, 4], [1, 3], [2, 4], [1, 3]])
gap> gh:=BoxTimesProduct(g,h);
Graph( Category := SimpleGraphs, Order := 12, Size := 36, Adjacencies :=
[[2, 4, 5, 6, 8], [1, 3, 5, 6, 7], [2, 4, 6, 7, 8], [1, 3, 5, 7, 8],
 [1, 2, 4, 6, 8, 9, 10, 12], [1, 2, 3, 5, 7, 9, 10, 11],
 [2, 3, 4, 6, 8, 10, 11, 12], [1, 3, 4, 5, 7, 9, 11, 12],
 [5, 6, 8, 10, 12], [5, 6, 7, 9, 11], [6, 7, 8, 10, 12],
 [5,7,8,9,11])
gap> VertexNames(gh);
[[1, 1], [1, 2], [1, 3], [1, 4], [2, 1], [2, 2], [2, 3],
  [2, 4], [3, 1], [3, 2], [3, 3], [3, 4]]
```

#### **B.1.16** BullGraph

▷ BullGraph (global variable)

A triangle with two pendant vertices (horns).

#### **B.1.17** CayleyGraph

Returns the graph G whose vertices are the elements of the group Grp such that x is adjacent to y iff x\*g=y for some g in the list Elms. if Elms is not provided, then the generators of G are used instead.

```
gap> grp:=Group((1,2,3),(1,2));
Group([ (1,2,3), (1,2) ])
gap> CayleyGraph(grp);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 3, 4, 5 ], [ 3, 5, 6 ], [ 1, 2, 6 ], [ 1, 5, 6 ], [ 1, 2, 4 ],
        [ 2, 3, 4 ] ] )
gap> CayleyGraph(grp,[(1,2),(2,3)]);
Graph( Category := SimpleGraphs, Order := 6, Size := 6, Adjacencies :=
[ [ 2, 3 ], [ 1, 5 ], [ 1, 4 ], [ 3, 6 ], [ 2, 6 ], [ 4, 5 ] ] )
```

#### **B.1.18** ChairGraph

A tree with degree sequence 3,2,1,1,1.

#### **B.1.19** Circulant

```
▷ Circulant(n, Jumps) (operation)
```

Returns the graph G whose vertices are [1..n] such that x is adjacent to y iff  $x+z=y \mod n$  for some z the list of Jumps.

```
Example

gap> Circulant(6,[1,2]);

Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=

[ [ 2, 3, 5, 6 ], [ 1, 3, 4, 6 ], [ 1, 2, 4, 5 ], [ 2, 3, 5, 6 ],

[ 1, 3, 4, 6 ], [ 1, 2, 4, 5 ] ])
```

#### **B.1.20** ClawGraph

▷ ClawGraph (global variable)

The graph on 4 vertices, 3 edges, and maximum degree 3.

```
gap> ClawGraph;
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2, 3, 4 ], [ 1 ], [ 1 ] ] )
```

#### **B.1.21** CliqueGraph

```
▷ CliqueGraph(G)
▷ CliqueGraph(G, maxNumCli) (operation)
```

Returns the intersection graph of all the (maximal) cliques of G. The additional parameter maxNumCli aborts the computation when maxNumCli cliques are found, even if they are all the cliques of G. If the bound maxNumCli is reached, fail is returned.

```
gap> CliqueGraph(Octahedron);
Graph( Category := SimpleGraphs, Order := 8, Size := 24, Adjacencies :=
[ [ 2, 3, 4, 5, 6, 7 ], [ 1, 3, 4, 5, 6, 8 ], [ 1, 2, 4, 5, 7, 8 ],
        [ 1, 2, 3, 6, 7, 8 ], [ 1, 2, 3, 6, 7, 8 ], [ 1, 2, 4, 5, 7, 8 ],
        [ 1, 3, 4, 5, 6, 8 ], [ 2, 3, 4, 5, 6, 7 ] ] )
gap> CliqueGraph(Octahedron,9);
Graph( Category := SimpleGraphs, Order := 8, Size := 24, Adjacencies :=
[ [ 2, 3, 4, 5, 6, 7 ], [ 1, 3, 4, 5, 6, 8 ], [ 1, 2, 4, 5, 7, 8 ],
        [ 1, 2, 3, 6, 7, 8 ], [ 1, 2, 3, 6, 7, 8 ], [ 1, 2, 4, 5, 7, 8 ],
        [ 1, 3, 4, 5, 6, 8 ], [ 2, 3, 4, 5, 6, 7 ] ] )
gap> CliqueGraph(Octahedron,8);
fail
```

#### **B.1.22** CliqueNumber

```
\triangleright CliqueNumber(G) (attribute)
```

Returns the order,  $\sigma(G)$ , of a maximum clique of G.

```
gap> g:=SunGraph(4);
Graph( Category := SimpleGraphs, Order := 8, Size := 14, Adjacencies :=
[ [ 2, 8 ], [ 1, 3, 4, 6, 8 ], [ 2, 4 ], [ 2, 3, 5, 6, 8 ], [ 4, 6 ],
        [ 2, 4, 5, 7, 8 ], [ 6, 8 ], [ 1, 2, 4, 6, 7 ] ] )
gap> CliqueNumber(g);
4
```

#### **B.1.23** Cliques

```
ightharpoonup Cliques(G) (attribute)

ightharpoonup Cliques(G, maxNumCli) (operation)
```

Returns the set of all (maximal) cliques of a graph G. A clique is a maximal complete subgraph. Here, we use the Bron-Kerbosch algorithm [1]. In the second form, It stops computing cliques after maxNumCli of them have been found.

```
Example

gap> Cliques(Octahedron);

[ [ 1, 3, 5 ], [ 1, 3, 6 ], [ 1, 4, 5 ], [ 1, 4, 6 ], [ 2, 3, 5 ],

        [ 2, 3, 6 ], [ 2, 4, 5 ], [ 2, 4, 6 ] ]

gap> Cliques(Octahedron,4);

[ [ 1, 3, 5 ], [ 1, 3, 6 ], [ 1, 4, 5 ], [ 1, 4, 6 ] ]
```

#### **B.1.24** ClockworkGraph

```
▷ ClockworkGraph(NNFSList) (operation)
▷ ClockworkGraph(NNFSList, rank) (operation)
▷ ClockworkGraph(NNFSList, Perm) (operation)
▷ ClockworkGraph(NNFSList, rank, Perm) (operation)
```

Returns the clockwork graph [10][12] specified by its parameters. A clockwork graph consists of two parts: the crown and the core, both of them are cyclically segmented. When not specified, the rank is assumed to be 2 and the return permutation, Perm, is assumed to be trivial, let us assume this is our case. Consider the following examples:

```
Example

gap> ClockworkGraph([[0],[0],[0],[0]]);

Graph( Category := SimpleGraphs, Order := 12, Size := 28, Adjacencies :=

[ [ 2, 3, 4, 10, 12 ], [ 1, 3, 5, 11, 12 ], [ 1, 2, 4, 5 ], [ 1, 3, 5, 6, 7 ],

      [ 2, 3, 4, 6, 8 ], [ 4, 5, 7, 8 ], [ 4, 6, 8, 9, 10 ], [ 5, 6, 7, 9, 11 ],

      [ 7, 8, 10, 11 ], [ 1, 7, 9, 11, 12 ], [ 2, 8, 9, 10, 12 ], [ 1, 2, 10, 11 ] ])

gap> ClockworkGraph([[1],[1],[1]]);

Graph( Category := SimpleGraphs, Order := 12, Size := 32, Adjacencies :=

[ [ 2, 3, 4, 10, 12 ], [ 1, 3, 5, 11, 12 ], [ 1, 2, 4, 5, 6, 12 ], [ 1, 3, 5, 6, 7 ],

      [ 2, 3, 4, 6, 8 ], [ 3, 4, 5, 7, 8, 9 ], [ 4, 6, 8, 9, 10 ], [ 5, 6, 7, 9, 11 ],

      [ 6, 7, 8, 10, 11, 12 ], [ 1, 7, 9, 11, 12 ], [ 2, 8, 9, 10, 12 ],

[ 1, 2, 3, 9, 10, 11 ] ])
```

In both cases, the crown is the subgraph induced by the vertices  $\{1,2,4,5,7,8,10,11\}$ and the core is induced by  $\{3,6,9,12\}$ . Also in both cases the cyclic segmentations (partitions) of the crown and the core are  $\{\1,2\\},\1,0,11\\}$ and  ${{3}},{6},{12}}$  respectively. The number of segmentes s is specified by s:=Length(NNFSList) which is 4 in these cases. The crown is isomorphic to BoxProduct(CycleGraph(s), Completegraph(rank)): All the crown segments are complete subgraphs and the vertices of cyclically consecutive segments are joined by a perfect matching. The adjacencies between crown and core vertices are simple to describe: Cyclically intercalate crown and core segments, making each core vertex adjacent to the vertices in the previous and the following crown segments. Hence in our examples vertex 3 is adjacent to vertices 1 and 2 (previous segment), but also 4 and 5 (following segment). Note that since the segmentations and intercalations are cyclic, we have that vertex 12 is adjacent to 10 and 11, but also to 1 and 2. Finally the edges between core vertices are as follows: first each core segment is a complete subgraph; the vertices within each core segment are linearly ordered and for vertex number t in segment number s there is a non-negative integer NNFSList [s] [t] which specifies, the Number of Neighbors in the Following core Segment for that vertex (hence the name NNFSList) (Since the vertices in core segments are linearly ordered, it is enough to specify the number of neighbors in the following segment and the first ones of those are selected as the neighbors). Hence in our two examples above, each core segment consists of exactly one vertex. In the first example each core segment is adjacent to no vertex in the following segment (e.g. 3 is not adjacent to 6) but in the second one, each core segment is adjacent to exactly one vertex in the following segment (e.g. 3 is adjacent to 6). A more complicated example should be now mostly self-explanatory:

```
gap> ClockworkGraph([[2],[0,1,3],[0,1,1],[1]]);
Graph( Category := SimpleGraphs, Order := 16, Size := 59, Adjacencies :=
```

```
[ [ 2, 3, 4, 14, 16 ], [ 1, 3, 5, 15, 16 ], [ 1, 2, 4, 5, 6, 7, 16 ], [ 1, 3, 5, 6, 7, 8, 9 ], [ 2, 3, 4, 6, 7, 8, 10 ], [ 3, 4, 5, 7, 8, 9, 10 ], [ 3, 4, 5, 6, 8, 9, 10, 11 ], [ 4, 5, 6, 7, 9, 10, 11, 12, 13 ], [ 4, 6, 7, 8, 10, 11, 12, 13, 14 ], [ 5, 6, 7, 8, 9, 11, 12, 13, 15 ], [ 7, 8, 9, 10, 12, 13, 14, 15 ], [ 8, 9, 10, 11, 13, 14, 15, 16 ], [ 8, 9, 10, 11, 12, 14, 15, 16 ], [ 1, 9, 11, 12, 13, 15, 16 ], [ 2, 10, 11, 12, 13, 14, 16 ], [ 1, 2, 3, 12, 13, 14, 15 ] ])
```

The crown and core segmentations are  $\{\{1,2\},\{4,5\},\{9,10\},\{14,15\}\}\$  and  $\{\{3}\}, \{6,7,8\}, \{11,12,13\}, \{16\}\}$  respectively and the adjacencies specified by the NNFSList are: 3 is adjacent to 6 and 7; 6 is adjacent to none (in the following core segment); 7 is adjacent to 11; 8 to 11, 12 and 13; 11 to none; 12 to 16; 13 to 16 and 16 to 3. When rank and/or Perm are specified, they have the following effects: rank (which must be at least 2) is the number of vertices in each crown segment, and Perm (which must belong to SymmetricGroup(rank)) specifies the perfect matching joining the vertices in the last crown segment with the vertices in the first crown segment: The k-th vertex in the last crown segment  $k\in \{1,2,\ldots\}$  is made adjacent to the \$Perm(k)\$-th vertex of the first crown segment. A number of requisites are put forward in the literature for a graph to be a clockwork graph but this operation does not enforce those conditions, on the contrary, it tries to make sense of the data provided as much as possible. For instance NNFSList:=[[2], [2], [2], [2]] would be inconsistent since there are not enough vertices in each core segment to provide for the required 2 neighbors. However, the result is just the same as with NNFSList:=[[1],[1],[1]]. The requisites that are mandatory are exactly these: the rank must be at least 2, Perm must belong to SymmetricGroup(rank), NNFSList must be a list of lists of non-negative integers, and the number of segments (= Length(NNFSList)) must be at least 3. A call to ClockworkGraph which fails to conform to these requisites will produce an error. Clockwork graphs have been very useful in constructing examples and counter-examples in clique graph theory. In particular, they have been used to construct examples of clique-periodic graphs of all possible periods [3], clique-divergent graphs of linear and polynomial growth rate [8][10], clique-convergent graphs whose period is not invariant under removal of dominated vertices [4], clique-convergent graphs which become clique-divergent by just gluing a 4-cycle to a vertex [5], rank-divergent graphs [13], etc.

#### **B.1.25** ComplementGraph

```
▷ ComplementGraph(G)
```

(attribute)

Returns the new graph H such that V(H)=V(G) and  $xy\in E(H) \in E(G)$ .

```
gap> g:=ClawGraph;
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2, 3, 4 ], [ 1 ], [ 1 ] ] )
gap> ComplementGraph(g);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ ], [ 3, 4 ], [ 2, 4 ], [ 2, 3 ] ] )
```

#### **B.1.26** CompleteBipartiteGraph

```
▷ CompleteBipartiteGraph(n, m)
```

(function)

Returns the complete bipartite whose parts have order n and m respectively. This is the joint (Zykov sum) of two discrete graphs of order n and m.

```
Example

gap> CompleteBipartiteGraph(2,3);

Graph( Category := SimpleGraphs, Order := 5, Size := 6, Adjacencies :=

[ [ 3, 4, 5 ], [ 3, 4, 5 ], [ 1, 2 ], [ 1, 2 ], [ 1, 2 ] ])
```

#### **B.1.27** CompleteGraph

```
▷ CompleteGraph(n)
```

(function)

Returns the complete graph of order n. A complete graph is a graph where all vertices are connected to each other.

```
gap> CompleteGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
```

#### **B.1.28** CompletelyParedGraph

```
▷ CompletelyParedGraph(G)
```

(operation)

Returns the completely pared graph of G, which is obtained by repeatedly applying ParedGraph until no more dominated vertices remain.

```
gap> g:=PathGraph(6);
Graph( Category := SimpleGraphs, Order := 6, Size := 5, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4, 6 ], [ 5 ] ] )
gap> CompletelyParedGraph(g);
Graph( Category := SimpleGraphs, Order := 1, Size := 0, Adjacencies :=
[ [ ] ] )
```

#### **B.1.29** CompleteMultipartiteGraph

```
▷ CompleteMultipartiteGraph(n1, n2)
```

(function)

Returns the complete multipartite graph where the orders of the parts are n1, n2, ... It is also the Zykov sum of discrete graphs of order n1, n2, ...

```
Example

gap> CompleteMultipartiteGraph(2,2,2);

Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=

[ [ 3, 4, 5, 6 ], [ 3, 4, 5, 6 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],

[ 1, 2, 3, 4 ], [ 1, 2, 3, 4 ] ] )
```

#### **B.1.30** CompletesOfGivenOrder

```
▷ CompletesOfGivenOrder(G, Ord)
```

(operation)

Returns the list of vertex sets of all complete subgraphs of order Ord of G.

```
gap> g:=SunGraph(4);
Graph( Category := SimpleGraphs, Order := 8, Size := 14, Adjacencies :=
[ [ 2, 8 ], [ 1, 3, 4, 6, 8 ], [ 2, 4 ], [ 2, 3, 5, 6, 8 ], [ 4, 6 ],
        [ 2, 4, 5, 7, 8 ], [ 6, 8 ], [ 1, 2, 4, 6, 7 ] ] )
gap> CompletesOfGivenOrder(g,3);
[ [ 1, 2, 8 ], [ 2, 3, 4 ], [ 2, 4, 6 ], [ 2, 4, 8 ], [ 2, 6, 8 ],
        [ 4, 5, 6 ], [ 4, 6, 8 ], [ 6, 7, 8 ] ]
gap> CompletesOfGivenOrder(g,4);
[ [ 2, 4, 6, 8 ] ]
```

#### **B.1.31** Composition

```
\triangleright Composition(G, H)
```

(operation)

Returns the composition G[H] of two graphs G and H. A composition of graphs is obtained by calculating the GraphSum of G with Order(G) copies of H,  $G[H] = GraphSum(G, [H, \dots, H])$ .

```
Example

gap> g:=CycleGraph(4);;h:=DiscreteGraph(2);;

gap> Composition(g,h);

Graph( Category := SimpleGraphs, Order := 8, Size := 16, Adjacencies := [ [ 3, 4, 7, 8 ], [ 3, 4, 7, 8 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ], [ 3, 4, 7, 8 ], [ 3, 4, 7, 8 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ] ] )
```

#### **B.1.32** Cone

```
\triangleright Cone(G) (operation)
```

Returns the cone of graph G. The cone of G is the graph obtained from G by adding a new vertex which is adjacent to every vertex of G. The new vertex is the first one in the new graph.

#### **B.1.33** ConnectedComponents

▷ ConnectedComponents(G)

(attribute)

Returns the connected components of G.

#### **B.1.34** ConnectedGraphsOfGivenOrder

ightharpoonup ConnectedGraphsOfGivenOrder(n)

(operation)

Returns the list of all connected order isomorgraphs of (upto This operation uses Brendan McKay's data published here: \URL{https://cs.anu.edu.au/people/Brendan.McKay/data/graphs.html}. These data are included with the YAGS distribution in its data directory. Hence this operation simply reads the corresponding

file in that directory using ImportGraph6 ( Filename ). Therefore, the integer n must be in the range from 1 upto 9. Data for graphs on 10 vertices is also available, but not included with YAGS, it may not be practical to use that data, but if you would like to try, all you have to do is to copy (and to uncompress) the corresponding file into the directory YAGS-Directory/data.

```
_ Example _
gap> ConnectedGraphsOfGivenOrder(3);
[ Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
   [[3], [3], [1, 2]]), Graph(Category := SimpleGraphs, Order :=
   3, Size := 3, Adjacencies := [ [ 2, 3 ], [ 1, 3 ], [ 1, 2 ] ] ) ]
gap> ConnectedGraphsOfGivenOrder(4);
[ Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
    [[4],[4],[4],[1,2,3]]),
 Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
    [[3, 4], [4], [1], [1, 2]]),
 Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
    [[3, 4], [4], [1, 4], [1, 2, 3]]),
 Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
    [[3, 4], [3, 4], [1, 2], [1, 2]]),
 Graph( Category := SimpleGraphs, Order := 4, Size := 5, Adjacencies :=
    [[3, 4], [3, 4], [1, 2, 4], [1, 2, 3]]),
 Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
    [[2, 3, 4], [1, 3, 4], [1, 2, 4], [1, 2, 3]])]
gap> Length(ConnectedGraphsOfGivenOrder(9));
gap> ConnectedGraphsOfGivenOrder(10);
#W Unreadable File: /opt/gap4r7/pkg/yags/data/graph10c.g6
```

#### **B.1.35** Coordinates

```
\triangleright Coordinates (G) (operation)
```

Gets the coordinates of the vertices of G, which are used to draw G by Draw(G). If the coordinates have not been previously set, Coordinates returns fail.

```
gap> g:=CycleGraph(4);;
gap> Coordinates(g);
fail
gap> SetCoordinates(g,[[-10,-10],[-10,20],[20,-10], [20,20]]);
gap> Coordinates(g);
[[-10,-10],[-10,20],[20,-10],[20,20]]
```

#### B.1.36 CopyGraph

```
\triangleright CopyGraph(G) (operation)
```

Returns a fresh copy of graph G. Only the order and adjacency information is copied, all other known attributes of G are not. Mainly used to transform a graph from one category to another. The new graph will be forced to comply with the TargetGraphCategory.

```
gap> g:=CompleteGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
gap> g1:=CopyGraph(g:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 3, 4 ], [ 4 ], [ ] ] )
gap> CopyGraph(g1:GraphCategory:=SimpleGraphs);
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
```

#### **B.1.37** Cube

The 1-skeleton of Plato's cube.

```
Example

gap> Cube;

Graph( Category := SimpleGraphs, Order := 8, Size := 12, Adjacencies :=

[ [ 2, 3, 5 ], [ 1, 4, 6 ], [ 1, 4, 7 ], [ 2, 3, 8 ], [ 1, 6, 7 ],

[ 2, 5, 8 ], [ 3, 5, 8 ], [ 4, 6, 7 ] ])
```

#### **B.1.38** CubeGraph

```
    □ CubeGraph(n) (function)
```

Returns the hypercube of dimension n. This is the box product (cartesian product) of n copies of  $K_2$  (an edge).

```
Example

gap> CubeGraph(3);

Graph( Category := SimpleGraphs, Order := 8, Size := 12, Adjacencies :=

[ [ 2, 3, 5 ], [ 1, 4, 6 ], [ 1, 4, 7 ], [ 2, 3, 8 ], [ 1, 6, 7 ],

[ 2, 5, 8 ], [ 3, 5, 8 ], [ 4, 6, 7 ] ] )
```

#### **B.1.39** CycleGraph

```
▷ CycleGraph(n) (function)
```

Returns the cyclic graph on n vertices.

#### **B.1.40** CylinderGraph

```
▷ CylinderGraph(b, h)
```

(function)

Returns a cylinder of base b and height h. The order of this graph is b(h+1) and it is constructed by taking h+1 copies of the cyclic graph on b vertices, ordering these cycles linearly and then joining consecutive cycles by a zigzagging (2b)-cycle. This graph is a triangulation of the cylinder where all internal vertices are of degree 6 and the border vertices are of degree 4.

```
gap> g:=CylinderGraph(4,1);
Graph( Category := SimpleGraphs, Order := 8, Size := 16, Adjacencies :=
[ [ 2, 4, 5, 6 ], [ 1, 3, 6, 7 ], [ 2, 4, 7, 8 ], [ 1, 3, 5, 8 ],
        [ 1, 4, 6, 8 ], [ 1, 2, 5, 7 ], [ 2, 3, 6, 8 ], [ 3, 4, 5, 7 ] ] )
gap> g:=CylinderGraph(4,2);
Graph( Category := SimpleGraphs, Order := 12, Size := 28, Adjacencies :=
[ [ 2, 4, 5, 6 ], [ 1, 3, 6, 7 ], [ 2, 4, 7, 8 ], [ 1, 3, 5, 8 ],
        [ 1, 4, 6, 8, 9, 10 ], [ 1, 2, 5, 7, 10, 11 ], [ 2, 3, 6, 8, 11, 12 ],
        [ 3, 4, 5, 7, 9, 12 ], [ 5, 8, 10, 12 ], [ 5, 6, 9, 11 ], [ 6, 7, 10, 12 ],
        [ 7, 8, 9, 11 ] ] )
```

#### **B.1.41** DartGraph

A diamond with a pendant vertex and maximum degree 4.

```
Example

gap> DartGraph;

Graph( Category := SimpleGraphs, Order := 5, Size := 6, Adjacencies :=

[ [ 2 ], [ 1, 3, 4, 5 ], [ 2, 4, 5 ], [ 2, 3 ], [ 2, 3 ] ])
```

#### **B.1.42** DeclareQtfyProperty

```
▷ DeclareQtfyProperty(Name, Filter)
```

(function)

For internal use. Declares a YAGS quantifiable property named Name for filter Filter. This in turns, declares a boolean GAP property Name and an integer GAP attribute QtfyName. The user must provide the method Name(Obj, qtfy). If qtfy is false, the method must return a boolean indicating whether the property holds, otherwise, the method must return a non-negative integer quantifying how far is the object from satisfying the property. In the latter case, returning 0 actually means that the object does satisfy the property.

```
> if not qtfy then return true; fi;
> return count;
> end);
gap> Is2Regular(CycleGraph(4));
true
gap> QtfyIs2Regular(CycleGraph(4));
0
gap> Is2Regular(DiamondGraph);
false
gap> QtfyIs2Regular(DiamondGraph);
2
```

#### **B.1.43** Diameter

 $\triangleright$  Diameter(G) (attribute)

Returns the maximum among the distances between pairs of vertices of G.

```
gap> g:=CycleGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] )
gap> Diameter(g);
2
```

#### **B.1.44** DiamondGraph

▷ DiamondGraph (global variable)

The graph on 4 vertices and 5 edges.

#### **B.1.45** DiscreteGraph

```
▷ DiscreteGraph(n) (function)
```

Returns the discrete graph of order n. A discrete graph is a graph without edges.

```
gap> DiscreteGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 0, Adjacencies :=
[ [ ], [ ], [ ] ] )
```

#### **B.1.46** DisjointUnion

```
▷ DisjointUnion(G, H) (operation)
```

Returns the disjoint union of two graphs G and H, G \$\dotUNKNOWNEntity(cup)\$ H.

```
gap> g:=PathGraph(3);h:=PathGraph(2);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> DisjointUnion(g,h);
Graph( Category := SimpleGraphs, Order := 5, Size := 3, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ], [ 5 ], [ 4 ] ] )
```

#### **B.1.47** Distance

```
\triangleright Distance(G, x, y) (operation)
```

Returns the length of a minimal path connecting x to y in G.

```
gap> Distance(CycleGraph(5),1,3);
2
gap> Distance(CycleGraph(5),1,5);
1
```

#### **B.1.48** Distances

```
\triangleright Distances (G, A, B) (operation)
```

Given two lists of vertices A, B of a graph G, Distances returns the list of distances for every pair in the cartesian product of A and B. The order of the vertices in lists A and B affects the order of the list of distances returned.

```
gap> g:=CycleGraph(5);;
gap> Distances(g, [1,3], [2,4]);
[ 1, 2, 1, 1 ]
gap> Distances(g, [3,1], [2,4]);
[ 1, 1, 1, 2 ]
```

#### **B.1.49** DistanceGraph

```
▷ DistanceGraph(G, Dist) (operation)
```

Given a graph G and list of distances Dist, DistanceGraph returns the new graph constructed on the vertices of G where two vertices are adjacent iff the distance (in G) between them belongs to the list Dist.

```
Example
gap> g:=CycleGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] )
gap> DistanceGraph(g,[2]);
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 3, 4 ], [ 4, 5 ], [ 1, 5 ], [ 1, 2 ], [ 2, 3 ] ] )
gap> DistanceGraph(g,[1,2]);
```

```
Graph( Category := SimpleGraphs, Order := 5, Size := 10, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3, 4, 5 ], [ 1, 2, 4, 5 ], [ 1, 2, 3, 5 ],
      [ 1, 2, 3, 4 ] ] )
```

#### **B.1.50** DistanceMatrix

```
    DistanceMatrix(G) (attribute)
```

Returns the distance matrix D of a graph G: D[x][y] is the distance in G from vertex x to vertex y. The matrix may be asymmetric if the graph is not simple. An infinite entry in the matrix means that there is no path between the vertices. Floyd's algorithm is used to compute the matrix.

```
____ Example -
gap> g:=PathGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[[2],[1,3],[2,4],[3]])
gap> Display(DistanceMatrix(g));
[[0, 1, 2, 3],
 [ 1, 0, 1, 2],
 [ 2, 1, 0, 1],
 [ 3, 2, 1, 0]]
gap> g:=PathGraph(4:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 4, Size := 3, Adjacencies :=
[[2],[3],[4],[]])
gap> Display(DistanceMatrix(g));
[ [
          0,
                     1,
                                         3],
                                         2],
 infinity,
                     Ο,
                               1,
   infinity,
                                         1],
              infinity,
                               0,
   infinity,
              infinity,
                        infinity,
                                         0]]
```

#### **B.1.51** DistanceSet

```
▷ DistanceSet(G, A, B) (operation)
```

Given two subsets of vertices A, B of a graph G, DistanceSet returns the set of distances for every pair in the cartesian product of A and B.

```
gap> g:=CycleGraph(5);;
gap> DistanceSet(g, [1,3], [2,4]);
[ 1, 2 ]
```

#### **B.1.52** Dodecahedron

▷ Dodecahedron (global variable)

The 1-skeleton of Plato's Dodecahedron.

```
Example

gap> Dodecahedron;

Graph( Category := SimpleGraphs, Order := 20, Size := 30, Adjacencies :=

[ [ 2, 5, 6 ], [ 1, 3, 7 ], [ 2, 4, 8 ], [ 3, 5, 9 ], [ 1, 4, 10 ],

[ 1, 11, 15 ], [ 2, 11, 12 ], [ 3, 12, 13 ], [ 4, 13, 14 ], [ 5, 14, 15 ],
```

```
[ 6, 7, 16 ], [ 7, 8, 17 ], [ 8, 9, 18 ], [ 9, 10, 19 ], [ 6, 10, 20 ], [ 11, 17, 20 ], [ 12, 16, 18 ], [ 13, 17, 19 ], [ 14, 18, 20 ], [ 15, 16, 19 ] ])
```

#### **B.1.53** Dominated Vertices

```
▷ DominatedVertices(G)
```

(attribute)

Returns the set of dominated vertices of G. A vertex x is dominated by another vertex y when the closed neighborhood of x is contained in that of y. However, when there are twin vertices (mutually dominated vertices), exactly one of them (in each equivalent class of mutually dominated vertices) does not appear in the returned set.

```
gap> g1:=PathGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> DominatedVertices(g1);
[ 1, 3 ]
gap> g2:=PathGraph(2);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> DominatedVertices(g2);
[ 2 ]
```

#### **B.1.54** DominoGraph

▷ DominoGraph

(global variable)

Two squares glued by an edge.

#### **B.1.55** Draw

```
▷ Draw(G) (operation)
```

Takes a graph G and makes a drawing of it in a separate window. The user can then view and modify the drawing and finally save the vertex coordinates of the drawing into the graph G. Within the separate window, type h to toggle on/off the help menu. Besides the keyword commands indicated in the help menu, the user may also move vertices (by dragging them), move the whole drawing (by dragging the background) and scale the drawing (by using the mouse wheel).

```
gap> Coordinates(Icosahedron);
fail
gap> Draw(Icosahedron);
gap> Coordinates(Icosahedron);
[ [ 29, -107 ], [ 65, -239 ], [ 240, -62 ], [ 78, 79 ], [ -107, 28 ],
```

```
[ -174, -176 ], [ -65, 239 ], [ -239, 62 ], [ -78, -79 ], [ 107, -28 ], [ 174, 176 ], [ -29, 107 ] ]
```

Draw() uses an external java program (included with YAGS) and hence, may not work on some platforms. Current version has been tested successfully on GNU/Linux, Mac OS X and Windows7. For other platforms (specially 32-bit platforms), you should probably (at least) set up correctly the variables YAGSInfo.Draw.prog and YAGSInfo.Draw.opts. The former is a strings representing the external binary program path and name; the latter is a list of strings representing the required command line options. Java binaries are provided for 32 and 64 bit versions of GNU/Linux (which also works for Mac OS X) and of MS Windows.

```
gap> YAGSInfo.Draw.prog; YAGSInfo.Draw.opts;

"/usr/share/gap/pkg/yags/bin/draw/application.linux64/draw"

[ ]
```

#### **B.1.56** DumpObject

```
▷ DumpObject(Obj) (operation)
```

Dumps all information available for object *Obj*. This information includes to which categories it belongs as well as its type and hashing information used by GAP.

```
gap> DumpObject( true );
Object( TypeObj := NewType( NewFamily( "BooleanFamily", [ 11 ], [ 11 ] ),
[ 11, 34 ] ), Categories := [ "IS_BOOL" ] )
```

#### B.1.57 EasyExec

```
▷ EasyExec(Dir, ProgName, InString) (operation)
▷ EasyExec(ProgName, InString) (operation)
```

Calls external program *ProgName* located in directory *Dir*, feeding it with *InString* as input and returning the output of the external program as a string. *Dir* must be a directory object or a list of directory objects. If *Dir* is not provided, *ProgName* must be in the system's binary PATH. fail is returned if the program could not be located.

```
gap> s:=EasyExec("date","");;
gap> s;
"Sun Nov 9 10:36:16 CST 2014\n"
gap> s:=EasyExec("sort","4\n2\n3\n1");;
gap> s;
"1\n2\n3\n4\n"
```

Currently, this operation is not working on MS Windows.

#### **B.1.58** Eccentricity

```
\triangleright Eccentricity(G, x) (function)
```

Returns the distance from a vertex x in graph G to its most distant vertex in G.

```
gap> g:=PathGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4 ] ] )
gap> Eccentricity(g,1);
4
gap> Eccentricity(g,3);
2
```

#### **B.1.59 Edges**

 $\triangleright$  Edges (G) (operation)

Returns the list of edges of graph G in the case of SimpleGraphs.

```
gap> g1:=CompleteGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 3, Adjacencies :=
[ [ 2, 3 ], [ 1, 3 ], [ 1, 2 ] ] )
gap> Edges(g1);
[ [ 1, 2 ], [ 1, 3 ], [ 2, 3 ] ]
```

In the case of UndirectedGraphs, it also returns the loops. While in the other categories, Edges actually does not return the edges, but the loops and arrows of G.

```
Example
gap> g2:=CompleteGraph(3:GraphCategory:=UndirectedGraphs);
Graph( Category := UndirectedGraphs, Order := 3, Size := 6, Adjacencies :=
[[1, 2, 3], [1, 2, 3], [1, 2, 3]])
gap> Edges(g2);
[[1, 1], [1, 2], [1, 3], [2, 2], [2, 3], [3, 3]]
gap> g3:=CompleteGraph(3:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 9, Adjacencies :=
[[1, 2, 3], [1, 2, 3], [1, 2, 3]])
gap> Edges(g3);
[[1, 1], [1, 2], [1, 3], [2, 1], [2, 2], [2, 3], [3, 1],
[3, 2], [3, 3]]
```

#### **B.1.60** EquivalenceRepresentatives

```
⊳ EquivalenceRepresentatives(L, Eqiv)
```

(operation)

Returns a sublist of L, which is a complete list of representatives of L under the equivalent relation Equiv.

```
gap> L:=[10,2,6,5,9,7,3,1,4,8];
[ 10, 2, 6, 5, 9, 7, 3, 1, 4, 8 ]
gap> EquivalenceRepresentatives(L,function(x,y) return (x mod 4)=(y mod 4); end);
[ 10, 5, 7, 4 ]
gap> L:=Links(SnubDisphenoid);;Length(L);
8
gap> L:=EquivalenceRepresentatives(L,IsIsomorphicGraph);;Length(L);
```

```
2
gap> L;
[ Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
       [ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] ),
    Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
       [ [ 2, 3 ], [ 1, 4 ], [ 1, 4 ], [ 2, 3 ] ] ) ]
```

#### **B.1.61** FanGraph

Returns the n-Fan: The join of a vertex and a (n+1)-path.

```
gap> FanGraph(4);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4, 6 ],
[ 1, 5 ] ] )
```

#### B.1.62 FishGraph

A square and a triangle glued by a vertex.

```
Example

gap> FishGraph;

Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=

[ [ 2, 3, 4, 6 ], [ 1, 3 ], [ 1, 2 ], [ 1, 5 ], [ 4, 6 ], [ 1, 5 ] ])
```

#### **B.1.63** GemGraph

The 3-Fan graph.

#### **B.1.64** Girth

```
ightharpoonup Girth(G) (attribute)
```

Returns the length of the minimum induced cycle in G. At this time, this works only when G belongs to the graph categories SimpleGraphs or UndirectedGraphs. If G has loops, its girth is 1 by definition.

```
gap> Girth(Octahedron);
3
gap> Girth(PetersenGraph);
```

```
gap> Girth(Cube);
4
gap> Girth(PathGraph(5));
infinity
gap> g:=AddEdges(CycleGraph(4),[[3,3]]:GraphCategory:=UndirectedGraphs);
Graph( Category := UndirectedGraphs, Order := 4, Size := 5, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 3, 4 ], [ 1, 3 ] ] )
gap> Girth(g);
1
```

## **B.1.65** Graph

```
    □ Graph(Rec) (operation)
```

Returns a new graph created from the record Rec. The record must provide the field Category and either the field Adjacencies or the field AdjMatrix.

```
Example
gap> Graph(rec(Category:=SimpleGraphs,Adjacencies:=[[2],[1]]));
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies := [ [ 2 ], [ 1 ] ] )
gap> Graph(rec(Category:=SimpleGraphs,AdjMatrix:=[[false, true],[true, false]]));
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies := [ [ 2 ], [ 1 ] ] )
```

Its main purpose is to import graphs from files, which could have been previously exported using PrintTo.

```
Example
gap> g:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ])
gap> PrintTo("aux.g", "h1:=",g,";");
gap> Read("aux.g");
gap> h1;
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ])
```

## **B.1.66** GraphAttributeStatistics

```
▷ GraphAttributeStatistics(OrderList, ProbList, Attribute) (function)
```

Returns statistics for graph attribute Attribute. For each of the orders n in OrderList and for each of the probabilities p in ProbList this function generates 100 random graphs of order n and edge probability p and then evaluates the graph attribute Attribute on each of them. The function then returns statistical data on these experiments. The form in which the statistical data is reported depend on a number of issues and is best explained by examples. First let us consider the case where Attribute is a Boolean attribute (always returns true or false) and where OrderList and ProbList consist of a unique value. In this case, the respective lists may be replaced by the corresponding unique values on invocation:

This tells us that 43 of the 100 examined random graphs resulted to be clique-Helly; The random sample was constructed using graphs of order 10 and edge probability 1/2. Now we can specify a list of probabilities to be examined:

The last example tells us that, for graphs on 10 vertices, the property IsCliqueHelly is least probable to be true for graphs with edge probabilities 5/10 6/10 and 7/10, being 6/10 the probability that reaches the minimum in the random sample. Note that the 36 in the previous example does not match the 43 in the first one, this is to be expected as the statistics are compiled from a random sample of graphs. Also, note that in the previous example, 900 random graphs where generated and examined. We can also specify a list of orders to consider:

```
Example
gap> GraphAttributeStatistics([10,12..20],1/10*[1..9],IsCliqueHelly);
[ [ 100, 100, 91, 63, 30, 23, 39, 65, 99 ], [ 100, 98, 81, 35, 4, 2, 20, 63, 98 ]
    , [ 100, 95, 49, 15, 1, 2, 13, 51, 98 ], [ 99, 82, 39, 3, 0, 2, 9, 42, 97 ],
  [ 100, 86, 15, 0, 0, 0, 7, 32, 93 ], [ 100, 69, 5, 0, 0, 0, 3, 24, 90 ] ]
gap> Display(last);
                                30,
                                                          99],
[ [ 100,
           100,
                   91,
                          63,
                                       23,
                                             39,
                                                   65,
                         35,
                                 4,
                                       2,
                                             20,
    100,
                                                   63.
                                                          98],
  Γ
            98,
                   81,
                                                          98],
  100,
            95,
                   49,
                          15,
                                        2,
                                             13,
                                                   51,
                                 1,
      99,
            82,
                   39,
                           3,
                                 0,
                                        2,
                                              9,
                                                   42,
                                                          97],
  Γ
    100,
            86,
                   15,
                          Ο,
                                 Ο,
                                       Ο,
                                              7,
                                                   32,
                                                          93],
  Γ
     100,
                          0,
                                                   24,
                                                          90 ] ]
            69,
                    5,
                                        0,
                                 0.
                                              З.
```

Which tell us that the observed bimodal distribution is even more pronounced when the order of the graphs considered grows. In the case of a non-Boolean attribute GraphAttributeStatistics() reports the values that Attribute took on the sample as well as the number of times that each of these values where obtained:

```
Example gap> GraphAttributeStatistics(10,1/2,Diameter); [ [ 2, 26 ], [ 3, 60 ], [ 4, 8 ], [ 6, 1 ], [ infinity, 5 ] ]
```

The returned statistics mean that among the 100 generated random graphs on 10 vertices with edge probability 1/2, there were 26 graphs with diameter 2, 60 graphs of diameter 3, 8 of 4, 1 of 6 and 5 graphs which were not connected. Now it should be evident the format of the returned statistics when we specify a list of probabilities and/or a list of orders to be considered for a non-Boolean Attribute:

```
Example

gap> GraphAttributeStatistics(10,1/5*[1..4],Diameter);

[[ [ 3, 3 ], [ 4, 5 ], [ 5, 9 ], [ 6, 3 ], [ 7, 2 ], [ infinity, 78 ] ],

[ [ 2, 8 ], [ 3, 55 ], [ 4, 19 ], [ 5, 3 ], [ infinity, 15 ] ],

[ [ 2, 73 ], [ 3, 26 ], [ 4, 1 ] ], [ [ 2, 100 ] ] ]

gap> GraphAttributeStatistics([10,12,14],1/5*[1..4],Diameter);

[[ [ [ 4, 8 ], [ 5, 7 ], [ 6, 3 ], [ infinity, 82 ] ],

[ [ 2, 3 ], [ 3, 64 ], [ 4, 15 ], [ 5, 3 ], [ infinity, 15 ] ],

[ [ 2, 69 ], [ 3, 30 ], [ infinity, 1 ] ], [ [ 2, 100 ] ] ],

[ [ [ 3, 1 ], [ 4, 11 ], [ 5, 13 ], [ 6, 7 ], [ 7, 3 ], [ 8, 2 ],

[ infinity, 63 ] ],

[ [ 2, 8 ], [ 3, 69 ], [ 4, 18 ], [ 5, 2 ], [ infinity, 3 ] ],

[ [ 2, 79 ], [ 3, 21 ] ], [ [ 2, 100 ] ] ],
```

```
[[[3, 1], [4, 15], [5, 13], [6, 5], [7, 4], [8, 3],
[infinity, 59]], [[2, 6], [3, 82], [4, 9], [infinity, 3]],
[[2, 86], [3, 14]], [[2, 100]]]
```

## B.1.67 Graph6ToGraph

```
▷ Graph6ToGraph(String)
```

(operation)

Returns the represented String which encoded Brengraph by is using McKay's graph6 format. operation allows us This to read databases which use this format. Several such databases can be found here: The graph6 format is \URL{https://cs.anu.edu.au/people/Brendan.McKay/data/graphs.html}. described here: \URL{https://cs.anu.edu.au/people/Brendan.McKay/data/formats.txt}.

```
gap> Graph6ToGraph("D?{");
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 5 ], [ 5 ], [ 5 ], [ 1, 2, 3, 4 ] ] )
gap> Graph6ToGraph("FUzvW");
Graph( Category := SimpleGraphs, Order := 7, Size := 15, Adjacencies :=
[ [ 3, 4, 5, 6, 7 ], [ 4, 5, 6, 7 ], [ 1, 5, 6, 7 ], [ 1, 2, 6 ],
        [ 1, 2, 3, 7 ], [ 1, 2, 3, 4, 7 ], [ 1, 2, 3, 5, 6 ] ] )
gap> Graph6ToGraph("HUzv~z}");
Graph( Category := SimpleGraphs, Order := 9, Size := 29, Adjacencies :=
[ [ 3, 4, 5, 6, 7, 8, 9 ], [ 4, 5, 6, 7, 8, 9 ], [ 1, 5, 6, 7, 8, 9 ],
        [ 1, 2, 6, 7, 8, 9 ], [ 1, 2, 3, 4, 5, 6, 7 ] ] )
```

See also ImportGraph6( Filename ).

#### **B.1.68** GraphByAdjacencies

```
ightarrow GraphByAdjacencies(AdjList)
```

(function)

Returns a new graph having AdjList as its list of adjacencies. The order of the created graph is Length(A), and the set of neighbors of vertex x is A[x].

```
Example

gap> GraphByAdjacencies([[2],[1,3],[2]]);

Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=

[[2],[1,3],[2]])
```

Note, however, that the graph is forced to comply with the TargetGraphCategory.

```
gap> GraphByAdjacencies([[1,2,3],[],[]]);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2, 3 ], [ 1 ], [ 1 ] ])
```

## B.1.69 GraphByAdjMatrix

```
▷ GraphByAdjMatrix(Mat)
```

(function)

Returns a new graph created from an adjacency matrix Mat. The matrix Mat must be a square boolean matrix.

```
Example
gap> m:=[ [ false, true, false ], [ true, false, true ], [ false, true, false ] ];;
gap> g:=GraphByAdjMatrix(m);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> AdjMatrix(g);
[ [ false, true, false ], [ true, false, true ], [ false, true, false ] ]
```

Note, however, that the graph is forced to comply with the TargetGraphCategory.

```
Example
gap> m:=[[ true, true], [ false, false ] ];;
gap> g:=GraphByAdjMatrix(m);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies := [ [ 2 ], [ 1 ] ] )
gap> AdjMatrix(g);
[ [ false, true ], [ true, false ] ]
```

## **B.1.70** GraphByCompleteCover

```
▷ GraphByCompleteCover(Cover)
```

(function)

Returns the minimal graph where the elements of *Cover* are (the vertex sets of) complete subgraphs.

```
Example

gap> GraphByCompleteCover([[1,2,3,4],[4,6,7]]);

Graph( Category := SimpleGraphs, Order := 7, Size := 9, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3, 6, 7 ], [ ], [ 4, 7 ],
[ 4, 6 ] ] )
```

#### **B.1.71** GraphByEdges

```
▷ GraphByEdges(L)
```

(function)

Returns the minimal graph such that the pairs in L are edges.

```
Example

gap> GraphByEdges([[1,2],[1,3],[1,4],[4,5]]);

Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2, 3, 4 ], [ 1 ], [ 1 ], [ 1, 5 ], [ 4 ] ] )
```

The vertices of the constructed graph range from 1 to the maximum of the numbers appearing in L.

Note that GraphByWalks has an even greater functionality.

## **B.1.72** GraphByRelation

```
▷ GraphByRelation(V, Rel)

▷ GraphByRelation(n, Rel)

(function)

(function)
```

Returns a new graph created from a set of vertices V and a binary relation Rel, where  $x\sim y$  iff Rel(x,y)=true. In the second form, n is an integer and V is assumed to be  $1, 2, \ldots, N$ .

```
Example

gap> Rel:=function(x,y) return Intersection(x,y)<>[]; end;;

gap> GraphByRelation([[1,2,3],[3,4,5],[5,6,7]],Rel);

Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=

[ [ 2 ], [ 1, 3 ], [ 2 ] ])

gap> GraphByRelation(8,function(x,y) return AbsInt(x-y)<=2; end);

Graph( Category := SimpleGraphs, Order := 8, Size := 13, Adjacencies :=

[ [ 2, 3 ], [ 1, 3, 4 ], [ 1, 2, 4, 5 ], [ 2, 3, 5, 6 ], [ 3, 4, 6, 7 ],

[ 4, 5, 7, 8 ], [ 5, 6, 8 ], [ 6, 7 ] ])
```

## **B.1.73** GraphByWalks

```
    □ GraphByWalks(Walk1, Walk2, ...) (function)
```

Returns the minimal graph such that Walk1, Walk2, etc are Walks.

```
Example

gap> GraphByWalks([1,2,3,4,1],[1,5,6]);

Graph( Category := SimpleGraphs, Order := 6, Size := 6, Adjacencies :=

[ [ 2, 4, 5 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ], [ 1, 6 ], [ 5 ] ])
```

Walks can be nested, which greatly improves the versatility of this function.

```
Example

gap> GraphByWalks([1,[2,3,4],5],[5,6]);

Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=

[ [ 2, 3, 4 ], [ 1, 3, 5 ], [ 1, 2, 4, 5 ], [ 1, 3, 5 ], [ 2, 3, 4, 6 ], [ 5 ] ] )
```

The vertices in the constructed graph range from 1 to the maximum of the numbers appearing in Walk1, Walk2, ... etc.

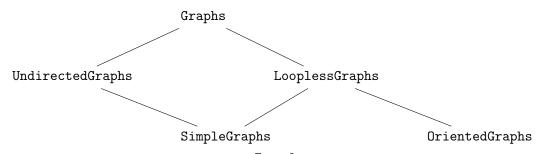
```
Example gap> GraphByWalks([4,2],[3,6]);
Graph( Category := SimpleGraphs, Order := 6, Size := 2, Adjacencies := [ [ ], [ 4 ], [ 6 ], [ 2 ], [ ], [ 3 ] ])
```

#### **B.1.74** GraphCategory

```
▷ GraphCategory([G, ...])

(function)
```

For internal use. Returns the minimal common category to a list of graphs. If the list of graphs is empty, the default category is returned. The partial order (by inclusion) among graph categories is as follows:



```
_ Example
gap> g1:=CompleteGraph(2:GraphCategory:=SimpleGraphs);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[[2],[1]])
gap> g2:=CompleteGraph(2:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 2, Size := 1, Adjacencies :=
[[2],[]])
gap> g3:=CompleteGraph(2:GraphCategory:=UndirectedGraphs);
Graph( Category := UndirectedGraphs, Order := 2, Size := 3, Adjacencies :=
[[1, 2], [1, 2]])
gap> GraphCategory([g1,g2,g3]);
<Operation "Graphs">
gap> GraphCategory([g1,g2]);
<Operation "LooplessGraphs">
gap> GraphCategory([g1,g3]);
<Operation "UndirectedGraphs">
```

## **B.1.75 Graphs**

 $\triangleright$  Graphs (G) (function)

Graphs is the most general graph category in YAGS. This category contains all graphs that can be represented in YAGS. A graph in this category may contain loops, arrows and edges (which in YAGS are exactly the same as two opposite arrows between some pair of vertices). This graph category has no parent category.

```
Example

gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);

Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=

[ [ 1, 2 ], [ 1 ], [ 2 ] ])

gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=SimpleGraphs);

Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=

[ [ 2 ], [ 1, 3 ], [ 2 ] ])
```

#### **B.1.76** GraphsOfGivenOrder

```
▷ GraphsOfGivenOrder(n)
```

(operation)

Returns the list of all graphs of order *n* (upto isomorphism). This operation uses Brendan McKay's data published here: \URL{https://cs.anu.edu.au/people/Brendan.McKay/data/graphs.html}. These data are included with the YAGS distribution in its data directory. Hence this operation simply reads the corresponding file in that directory using ImportGraph6( *Filename* ). Therefore, the integer *n* 

must be in the range from 1 upto 9. Data for graphs on 10 vertices is also available, but not included with YAGS, it may not be practical to use that data, but if you would like to try, all you have to do is to copy (and to uncompress) the corresponding file into the directory YAGS-Directory/data.

```
Example
gap> GraphsOfGivenOrder(2);
[ Graph( Category := SimpleGraphs, Order := 2, Size := 0, Adjacencies :=
    [ [ ], [ ] ]), Graph( Category := SimpleGraphs, Order := 2, Size :=
   1, Adjacencies := [ [ 2 ], [ 1 ] ] ) ]
gap> GraphsOfGivenOrder(3);
[ Graph( Category := SimpleGraphs, Order := 3, Size := 0, Adjacencies :=
    [[], [], []]), Graph(Category := SimpleGraphs, Order :=
   3, Size := 1, Adjacencies := [ [ 3 ], [ ], [ 1 ] ]),
 Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
    [[3], [3], [1, 2]]), Graph(Category := SimpleGraphs, Order :=
   3, Size := 3, Adjacencies := [ [ 2, 3 ], [ 1, 3 ], [ 1, 2 ] ] ) ]
gap> Length(GraphsOfGivenOrder(9));
274668
gap> GraphsOfGivenOrder(10);
#W Unreadable File: /opt/gap4r7/pkg/yags/data/graph10.g6
```

## B.1.77 GraphSum

```
ightharpoonup GraphSum(G, L) (operation)
```

Returns the lexicographic sum of a list of graphs L over a graph G. The lexicographic sum is computed as follows: Given G, with G0=n\$ and a list of G1 graphs  $L = [G_1, \ldots, G_n]$ \$, We take the disjoint union of  $G_1,G_2, \ldots, G_n$ \$ and then we add all the edges between  $G_i$ \$ and  $G_j$ \$ whenever [i,j]\$ is and edge of G5. If L contains holes, the trivial graph is used in place.

## **B.1.78** GraphToRaw

```
▷ GraphToRaw(FileName, G) (operation)
```

Converts a YAGS graph G into a raw format (number of vertices, coordinates and adjacency matrix) and writes the converted data to the file FileName. For use by the external program draw (see Draw(G)).

```
gap> g:=CycleGraph(4);;
gap> GraphToRaw("mygraph.raw",g);
Example
```

## **B.1.79** GraphUpdateFromRaw

```
  □ GraphUpdateFromRaw(FileName, G) (operation)
```

Updates the coordinates of G from a file FileName in raw format. Intended for internal use only.

## **B.1.80** GroupGraph

```
ightharpoonup GroupGraph(G, Grp, Act) (operation)

ightharpoonup GroupGraph(G, Grp) (operation)
```

Given a graph G, a group Grp and an action Act of Grp in some set S which contains Srp Vertices G, Group Grp returns a new graph with vertex set  $\frac{c}{act(v,g): g \in Grp, v \in G$ 

```
Example

gap> GroupGraph(GraphByWalks([1,2]),Group([(1,2,3,4,5),(2,5)(3,4)]));

Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=

[[2,5],[1,3],[2,4],[3,5],[1,4]])
```

## **B.1.81** HararyToMcKay

Returns the McKay's index of a Harary's graph specification Spec and viceversa. Frank Harary published in his book [7], a list af all 208 simple graphs of order upto 6 (upto isomorphism). Each of them had a label (which we call Harary's graph specification) of the form [n, m, s] where n is the number of vertices, m is the number of edges, and s is a consecutive integer which uniquely identifies the graph from the others with the same n and m. On the other hand, Brendan McKay published data sets containing a list of all graphs of order upto 10 (also upto isomorphism), here: \URL{https://cs.anu.edu.au/people/Brendan.McKay/data/graphs.html} Each graph in these data sets appears in some specific position (which we call McKay's index). We found it convenient to have an automated way to convert from Harary's graph specifications to McKay's indexes and viceversa.

```
gap> HararyToMcKay([1,0,1]);
1
gap> HararyToMcKay([1,0,2]);
fail
gap> HararyToMcKay([5,5,2]);
31
gap> HararyToMcKay([5,5,3]);
34
gap> HararyToMcKay([5,5,5]);
30
gap> HararyToMcKay([5,5,6]);
45
gap> HararyToMcKay([5,5,7]);
fail
```

```
gap> HararyToMcKay([6,15,1]);
208
gap> HararyToMcKay([6,15,2]);
fail
gap> List([1..208],McKayToHarary);
[[ 1, 0, 1 ], [ 2, 0, 1 ], [ 2, 1, 1 ], [ 3, 0, 1 ], [ 3, 1, 1 ],
        [ 3, 2, 1 ], [ 3, 3, 1 ], [ 4, 0, 1 ], [ 4, 1, 1 ], [ 4, 2, 1 ],
        [ 4, 3, 3 ], [ 4, 2, 2 ], [ 4, 3, 1 ], [ 4, 3, 2 ], [ 4, 4, 1 ],

--- many more lines here ---

[ 6, 10, 10 ], [ 6, 10, 7 ], [ 6, 11, 3 ], [ 6, 12, 1 ], [ 6, 13, 1 ],
        [ 6, 11, 7 ], [ 6, 11, 9 ], [ 6, 11, 8 ], [ 6, 12, 4 ], [ 6, 12, 5 ],
        [ 6, 13, 2 ], [ 6, 14, 1 ], [ 6, 15, 1 ] ]
```

## **B.1.82** HouseGraph

▷ HouseGraph (global variable)

A 4-Cycle and a triangle glued by an edge.

```
gap> HouseGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 6, Adjacencies :=
[ [ 2, 4, 5 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3, 5 ], [ 1, 4 ] ] )
```

#### **B.1.83** Icosahedron

▷ Icosahedron (global variable)

The 1-skeleton of Plato's icosahedron.

```
Example

gap> Icosahedron;

Graph( Category := SimpleGraphs, Order := 12, Size := 30, Adjacencies :=

[ [ 2, 3, 4, 5, 6 ], [ 1, 3, 6, 9, 10 ], [ 1, 2, 4, 10, 11 ],

[ 1, 3, 5, 7, 11 ], [ 1, 4, 6, 7, 8 ], [ 1, 2, 5, 8, 9 ],

[ 4, 5, 8, 11, 12 ], [ 5, 6, 7, 9, 12 ], [ 2, 6, 8, 10, 12 ],

[ 2, 3, 9, 11, 12 ], [ 3, 4, 7, 10, 12 ], [ 7, 8, 9, 10, 11 ] ])
```

## **B.1.84** ImportGraph6

```
▷ ImportGraph6(Filename)
```

(operation)

Returns the list of graphs represented in Filename which are encoded using Brendan McKay's graph6 format. This operation allows us to read data in databases which use this format. Several such databases be found here: can \URL{https://cs.anu.edu.au/people/Brendan.McKay/data/graphs.html}. The graph6 format is described here: \URL{https://cs.anu.edu.au/people/Brendan.McKay/data/formats.txt}. The following example assumes that you have a file named graph3.g6 in your working directory which encodes graphs in graph6 format; the contents of this file is assumed to be as indicated after the first command in the example.

```
gap> Example
gap> Exec("cat graph3.g6");
B?
B0
BW
Bw
gap> ImportGraph6("graph3.g6");
[ Graph( Category := SimpleGraphs, Order := 3, Size := 0, Adjacencies :=
        [ [ ], [ ] ] ), Graph( Category := SimpleGraphs, Order := 3, Size := 1, Adjacencies := [ [ 3 ], [ ], [ 1 ] ] ),
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
        [ [ 3 ], [ 3 ], [ 1, 2 ] ] ), Graph( Category := SimpleGraphs, Order := 3, Size := 3, Adjacencies := [ [ 2, 3 ], [ 1, 3 ], [ 1, 2 ] ] )]
```

#### B.1.85 in

```
\triangleright in(G, Catgy) (operation)
```

Returns true if graph G belongs to category Catgy and false otherwise.

```
##FIXME **** poner ejemplo.
```

## **B.1.86** InducedSubgraph

```
▷ InducedSubgraph(G, V)
```

(operation)

Returns the subgraph of graph G induced by the vertex set V.

```
gap> g:=CycleGraph(6);
Graph( Category := SimpleGraphs, Order := 6, Size := 6, Adjacencies :=
[ [ 2, 6 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4, 6 ], [ 1, 5 ] ] )
gap> InducedSubgraph(g,[3,4,6]);
Graph( Category := SimpleGraphs, Order := 3, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ], [ ] ] )
```

The order of the elements in V does matter.

```
gap> InducedSubgraph(g,[6,3,4]);
Graph( Category := SimpleGraphs, Order := 3, Size := 1, Adjacencies :=
[ [ ], [ 3 ], [ 2 ] ] )
```

#### B.1.87 InNeigh

```
\triangleright InNeigh(G, x) (operation)
```

Returns the list of in-neighbors of x in G.

```
Example

gap> tt:=CompleteGraph(5:GraphCategory:=OrientedGraphs);

Graph( Category := OrientedGraphs, Order := 5, Size := 10, Adjacencies :=

[ [ 2, 3, 4, 5 ], [ 3, 4, 5 ], [ 4, 5 ], [ 5 ], [ ] ] )
```

```
gap> InNeigh(tt,3);
[ 1, 2 ]
gap> OutNeigh(tt,3);
[ 4, 5 ]
```

#### **B.1.88** Interior Vertices

```
▷ InteriorVertices(G)
```

(attribute)

When G is a compact surface, it returns the list of vertices in the interior (of the triangulation) of the surface. That is, the list of vertices of G that have links isomorphic to a cycle. It returns fail if G is not a compact surface.

```
gap> InteriorVertices(WheelGraph(4,2));
[ 1, 2, 3, 4, 5 ]
gap> InteriorVertices(Octahedron);
[ 1, 2, 3, 4, 5, 6 ]
```

## **B.1.89** IntersectionGraph

```
▷ IntersectionGraph(L)
```

(function)

Returns the intersection graph of the family of sets L. This graph has a vertex for every set in L, and two such vertices are adjacent iff the corresponding sets have non-empty intersection.

#### **B.1.90** IsBoolean

```
\triangleright IsBoolean(0bj) (function)
```

Returns true if object Obj is true or false and false otherwise.

```
gap> IsBoolean( true ); IsBoolean( fail ); IsBoolean ( false );
true
false
true
```

## **B.1.91** IsCliqueGated

```
\triangleright IsCliqueGated(G) (property)
```

Returns true if G is a clique gated graph [6].

## **B.1.92** IsCliqueHelly

```
▷ IsCliqueHelly(G) (property)
```

Returns true if the set of (maximal) cliques G satisfy the *Helly* property. The Helly property is defined as follows: A non-empty family  $Cal{F}$  of non-empty sets satisfies the Helly property if every pairwise intersecting subfamily of  $Cal{F}$  has a non-empty total intersection. Here we use the Dragan-Szwarcfiter characterization [2][15] to compute the Helly property.

```
gap> g:=SunGraph(3);
Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=
[ [ 2, 6 ], [ 1, 3, 4, 6 ], [ 2, 4 ], [ 2, 3, 5, 6 ], [ 4, 6 ],
        [ 1, 2, 4, 5 ] ] )
gap> IsCliqueHelly(g);
false
```

## **B.1.93** IsCompactSurface

```
\triangleright IsCompactSurface(G) (property)
```

Returns true if every link of G is either an n-cycle, for  $n\neq 4$  or an m-path, for  $m\neq 2$ . (not necessarily the same n/m for all vertices); it returns false otherwise. This notion correspond to Whitney triangulations of compact surfaces [11] in which the (maximal) cliques of the graph are exactly the triangles of the triangulation.

```
gap> IsCompactSurface(Icosahedron);
true
gap> IsCompactSurface(RemoveVertices(Icosahedron,[1]));
true
gap> IsCompactSurface(WheelGraph(4,2));
true
gap> IsCompactSurface(Tetrahedron);
false
gap> IsCompactSurface(CompleteGraph(2));
false
gap> IsCompactSurface(CompleteGraph(3));
true
gap> IsCompactSurface(CompleteGraph(4));
false
```

Topologically, the difference between a surface and a compact surface is that the points of a surface always have a open neighborhood homeomorphic to an open disk, whereas a compact surface may also contain points with open neighborhoods homeomorphic to a closed half-plane.

#### **B.1.94** IsComplete

```
\triangleright IsComplete(G, L) (operation)
```

Returns true if *L* induces a complete subgraph of *G*.

```
gap> IsComplete(DiamondGraph,[1,2,3]);
true
gap> IsComplete(DiamondGraph,[1,2,4]);
false
```

## **B.1.95** IsCompleteGraph

```
▷ IsCompleteGraph(G) (property)
```

Returns true if graph G is a complete graph, false otherwise. In a complete graph every pair of vertices is an edge.

#### **B.1.96** IsDiamondFree

```
\triangleright IsDiamondFree(G) (property)
```

Returns true if G is free from induced diamonds, false otherwise.

```
gap> IsDiamondFree(Cube);
true
gap> IsDiamondFree(Octahedron);
false
```

## **B.1.97 IsEdge**

```
\triangleright IsEdge(G, x, y) (operation)
\triangleright IsEdge(G[, x, y>]) (operation)
```

Returns true if [x,y] is an edge of G.

```
gap> IsEdge(PathGraph(3),1,2);
true
gap> IsEdge(PathGraph(3),[1,2]);
true
gap> IsEdge(PathGraph(3),1,3);
false
gap> IsEdge(PathGraph(3),[1,3]);
false
```

The first form, IsEdge(G, x, y), is a bit faster and hence more suitable for use in algoritms which make extensive use of this operation. On the other hand, the first form does no error checking at all, and hence, it may produce an error where the second form returns false (for instance when x is not a vertex of G). The second form is therefore a bit slower, but more robust.

```
gap> IsEdge(PathGraph(3),[7,3]);
false
gap> IsEdge(PathGraph(3),7,3);
Error, List Element: <list>[7] must have an assigned value in
return AdjMatrix( G )[x][y]; called from
```

```
<function "unknown">( <arguments> )
  called from read-eval loop at line 4 of *stdin*
  you can <return;> after assigning a value
  brk>
```

## **B.1.98** IsIsomorphicGraph

▷ IsIsomorphicGraph(G, H)

(operation)

Returns true when G is isomorphic to H and false otherwise.

```
gap> g:=PowerGraph(CycleGraph(6),2);;h:=Octahedron;;
gap> IsIsomorphicGraph(g,h);
true
```

## **B.1.99** IsLocallyConstant

▷ IsLocallyConstant(G)

(property)

Returns true if all the links of G are isomorphic to each other; false otherwise.

```
gap> IsLocallyConstant(PathGraph(2));
true
gap> IsLocallyConstant(PathGraph(3));
false
gap> IsLocallyConstant(CompleteGraph(3));
true
gap> IsLocallyConstant(CycleGraph(4));
true
gap> IsLocallyConstant(Icosahedron);
true
gap> IsLocallyConstant(TorusGraph(5,4));
true
gap> IsLocallyConstant(WheelGraph(4,2));
false
gap> IsLocallyConstant(SnubDisphenoid);
false
```

## **B.1.100** IsLocallyH

```
\triangleright IsLocallyH(G, H)
```

(operation)

Returns true if all the links of G are isomorphic to H; false otherwise.

```
gap> IsLocallyH(Octahedron,CycleGraph(4));
true
gap> IsLocallyH(Octahedron,CycleGraph(5));
false
gap> IsLocallyH(Icosahedron,CycleGraph(5));
true
```

```
gap> IsLocallyH(TorusGraph(4,4),CycleGraph(6));
true
```

## **B.1.101** IsLoopless

```
\triangleright IsLoopless(G) (property)
```

Returns true if graph G have no loops, false otherwise. Loops are edges from a vertex to itself.

## **B.1.102** IsoMorphism

```
\triangleright IsoMorphism(G, H) (operation)
```

Returns one isomorphism from G to H or fail if none exists. If G has n vertices, an isomorphisms f: G rightarrow f: G is represented as the list  $f=[f(1), f(2), \ldots, f(n)]$ .

```
gap> g:=CycleGraph(4);;h:=CompleteBipartiteGraph(2,2);;
gap> f:=IsoMorphism(g,h);
[ 1, 3, 2, 4 ]
```

See NextIsoMorphism(G, H, F).

## **B.1.103** IsoMorphisms

```
\triangleright IsoMorphisms(G, H) (operation)
```

Returns the list of all isomorphism from G to H. If G has n vertices, an isomorphisms f: G-rightarrow H is represented as the list  $F = [f(1), f(2), \ldots, f(n)]$ .

```
Example

gap> g:=CycleGraph(4);;h:=CompleteBipartiteGraph(2,2);;

gap> IsoMorphisms(g,h);

[[1,3,2,4],[1,4,2,3],[2,3,1,4],[2,4,1,3],
      [3,1,4,2],[3,2,4,1],[4,1,3,2],[4,2,3,1]]
```

## **B.1.104** IsOriented

```
▷ IsOriented(G)
(property)
```

Returns true if graph G is an oriented graph, false otherwise. Regardless of the categories that G belongs to, G is oriented if whenever [x,y] is an edge of G, [y,x] is not.

## **B.1.105 IsSimple**

```
\triangleright IsSimple(G) (property)
```

Returns true if graph G is a simple graph, false otherwise. Regardless of the categories that G belongs to, G is simple if and only if G is undirected and loopless. Returns true if the graph G is simple regardless of its category.

#### **B.1.106** IsSurface

```
▷ IsSurface(G) (property)
```

Returns true if every link of G is an n-cycle, for  $n \neq 4$  (not necessarily the same n for all vertices); false otherwise. This notion correspond to Whitney triangulations of (closed) surfaces [11] in which the (maximal) cliques of the graph are exactly the triangles of the triangulation.

```
gap> IsSurface(SnubDisphenoid);
true
gap> IsSurface(Icosahedron);
true
gap> IsSurface(RemoveVertices(Icosahedron,[1]));
false
gap> IsSurface(TorusGraph(4,5));
true
gap> IsSurface(WheelGraph(4,2));
false
gap> IsSurface(Tetrahedron);
false
```

Topologically, the difference between a (closed) surface and a compact surface is that the points of a surface always have a open neighborhood homeomorphic to an open disk, whereas a compact surface may also contain points with open neighborhoods homeomorphic to a closed half-plane.

#### **B.1.107** IsTournament

```
▷ IsTournament(G) (property)
```

Returns true if G is a tournament.

```
gap> tt:=CompleteGraph(5:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 5, Size := 10, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 3, 4, 5 ], [ 4, 5 ], [ 5 ], [ ] ] )
gap> IsTournament(tt);
true
```

#### **B.1.108** IsTransitiveTournament

```
▷ IsTransitiveTournament(G) (property)
```

Returns true if G is a transitive tournament.

```
gap> tt:=CompleteGraph(5:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 5, Size := 10, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 3, 4, 5 ], [ 4, 5 ], [ 5 ], [ ] ] )
gap> IsTransitiveTournament(tt);
true
```

#### **B.1.109** IsUndirected

```
▷ IsUndirected(G) (property)
```

Returns true if graph G is an undirected graph, false otherwise. Regardless of the categories that G belongs to, G is undirected if whenever [x,y] is an edge of G, [y,x] is also an egde of G.

## **B.1.110** JohnsonGraph

```
\triangleright JohnsonGraph(n, r) (function)
```

Returns the Johnson graph J(n,r). The Johnson Graph is the graph whose vertices are r-subset of the set  $\{1, 2, \ldots, n\}$ , two of them being adjacent iff they intersect in exactly r-1 elements.

```
Example

gap> g:=JohnsonGraph(4,2);

Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=

[ [ 2, 3, 4, 5 ], [ 1, 3, 4, 6 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],

[ 1, 3, 4, 6 ], [ 2, 3, 4, 5 ] ])

gap> VertexNames(g);

[ [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 2, 3 ], [ 2, 4 ], [ 3, 4 ] ]
```

## **B.1.111** Join

```
\triangleright Join(G, H) (operation)
```

 $\inf \{Zykov \ sum\}$  Returns the join graph G + H of G and H (also known as the  $Zykov \ sum$ ); it is the graph obtained from the disjoint union of G and H by adding every possible edge from every vertex in G to every vertex in H.

```
Example

gap> g:=DiscreteGraph(2);h:=CycleGraph(4);

Graph( Category := SimpleGraphs, Order := 2, Size := 0, Adjacencies :=
[ [ ], [ ] ] )

Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )

gap> Join(g,h);

Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=
[ [ 3, 4, 5, 6 ], [ 3, 4, 5, 6 ], [ 1, 2, 4, 6 ], [ 1, 2, 3, 5 ],
[ 1, 2, 4, 6 ], [ 1, 2, 3, 5 ] ] )
```

#### **B.1.112 KiteGraph**

```
▷ KiteGraph (global variable)
```

A diamond with a pendant vertex and maximum degree 3.

## **B.1.113** LineGraph

```
    □ LineGraph(G) (operation)
```

Returns the line graph L(G) of graph G. The line graph is the intersection graph of the edges of G, i.e. the vertices of L(G) are the edges of G two of them being adjacent iff they are incident.

#### **B.1.114** Link

```
\triangleright Link(G, x) (operation)
```

Returns the subgraph of G induced by the neighbors of x.

```
Example

gap> Link(SnubDisphenoid,1);

Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=

[ [ 2, 5 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 4 ] ] )

gap> Link(SnubDisphenoid,3);

Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=

[ [ 2, 3 ], [ 1, 4 ], [ 1, 4 ], [ 2, 3 ] ] )
```

#### **B.1.115** Links

ightharpoonup Links (G) (attribute)

Returns the list of subgraphs of G induced by the neighbors of each vertex of G.

```
Example
gap> Links(SnubDisphenoid);
[ Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
   [[2,5],[1,3],[2,4],[3,5],[1,4]]),
 Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
   [[2,5],[1,3],[2,4],[3,5],[1,4]]),
 Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
   [[2,3],[1,4],[1,4],[2,3]]),
 Graph (Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
   [[2,3],[1,4],[1,4],[2,3]]),
 Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
   [[2,5],[1,3],[2,4],[3,5],[1,4]]),
 Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
   [[2,5],[1,3],[2,4],[3,5],[1,4]]),
 Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
   [[3, 4], [3, 4], [1, 2], [1, 2]]),
 Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
   [[2,3],[1,4],[1,4],[2,3]])]
```

## **B.1.116** LooplessGraphs

```
\triangleright LooplessGraphs(G) (function)
```

LooplessGraphs is a graph category in YAGS. A graph in this category may contain arrows and edges but no loops. The parent of this category is Graphs.

```
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=LooplessGraphs);
Graph( Category := LooplessGraphs, Order := 3, Size := 3, Adjacencies :=
[ [ 2 ], [ 1 ], [ 2 ] ] )
```

## **B.1.117** MaxDegree

Returns the maximum degree in graph G.

```
gap> g:=GemGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 7, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4 ] ] )
gap> MaxDegree(g);
4
```

## **B.1.118** MinDegree

```
ightharpoonup MinDegree (G) (operation)
```

Returns the minimum degree in graph G.

```
gap> g:=GemGraph;
Graph( Category := SimpleGraphs, Order := 5, Size := 7, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4 ] ] )
gap> MinDegree(g);
2
```

## **B.1.119** NextIsoMorphism

```
\triangleright NextIsoMorphism(G, H, F) (operation)
```

Returns the next isomorphism (after F) from G to H in the lexicographic order; returns fail if there are no more isomorphisms. If G has n vertices, an isomorphisms f: G-rightarrow f is represented as the list f=[f(1), f(2), ..., f(n)].

```
gap> g:=CycleGraph(4);;h:=CompleteBipartiteGraph(2,2);;
gap> f:=IsoMorphism(g,h);
[ 1, 3, 2, 4 ]
gap> NextIsoMorphism(g,h,f);
```

```
[ 1, 4, 2, 3 ]
gap> NextIsoMorphism(g,h,f);
[ 2, 3, 1, 4 ]
gap> NextIsoMorphism(g,h,f);
[ 2, 4, 1, 3 ]
```

## **B.1.120** NextPropertyMorphism

```
▷ NextPropertyMorphism(G, H, F, PropList)
```

(operation)

```
_{-} Example
gap> g:=CycleGraph(4);;h:=CompleteBipartiteGraph(2,2);;
gap> f:=[];; PropList:=[CHK_MORPH,CHK_MONO];;
gap> NextPropertyMorphism(g,h,f,PropList);
[1, 3, 2, 4]
gap> NextPropertyMorphism(g,h,f,PropList);
[1, 4, 2, 3]
gap> NextPropertyMorphism(g,h,f,PropList);
[ 2, 3, 1, 4 ]
gap> NextPropertyMorphism(g,h,f,PropList);
[2, 4, 1, 3]
gap> NextPropertyMorphism(g,h,f,PropList);
[3, 1, 4, 2]
gap> NextPropertyMorphism(g,h,f,PropList);
[3, 2, 4, 1]
gap> NextPropertyMorphism(g,h,f,PropList);
[4, 1, 3, 2]
gap> NextPropertyMorphism(g,h,f,PropList);
[4, 2, 3, 1]
gap> NextPropertyMorphism(g,h,f,PropList);
fail
```

## **B.1.121** NumberOfCliques

Returns the number of (maximal) cliques of G. In the second form, It stops computing cliques after maxNumCli of them have been counted and returns maxNumCli in case G has maxNumCli or more cliques.

```
gap> NumberOfCliques(Icosahedron);
20
gap> NumberOfCliques(Icosahedron,15);
15
gap> NumberOfCliques(Icosahedron,50);
20
```

This implementation discards the cliques once counted hence, given enough time, it can compute the number of cliques of G even if the set of cliques does not fit in memory.

```
gap> NumberOfCliques(OctahedralGraph(30));
1073741824
```

## **B.1.122** NumberOfConnectedComponents

▷ NumberOfConnectedComponents(G)

(attribute)

Returns the number of connected components of G.

## **B.1.123** OctahedralGraph

```
▷ OctahedralGraph(n)
```

(function)

Return the *n*-dimensional octahedron. This is the complement of *n* copies of  $K_2$  (an edge). It is also the (2n-2)-regular graph on 2n vertices.

```
Example

gap> OctahedralGraph(3);

Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=

[ [ 3, 4, 5, 6 ], [ 3, 4, 5, 6 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],

[ 1, 2, 3, 4 ], [ 1, 2, 3, 4 ] ])
```

#### **B.1.124** Octahedron

▷ Octahedron (global variable)

The 1-skeleton of Plato's octahedron.

```
Example

gap> Octahedron;

Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=

[ [ 3, 4, 5, 6 ], [ 3, 4, 5, 6 ], [ 1, 2, 5, 6 ], [ 1, 2, 5, 6 ],

[ 1, 2, 3, 4 ], [ 1, 2, 3, 4 ] ])
```

#### **B.1.125** Order

```
\triangleright Order(G) (attribute)
```

Returns the number of vertices, of graph G.

(function)

```
gap> Order(Icosahedron);
12
```

#### **B.1.126** Orientations

```
\triangleright Orientations(G) (operation)
```

Returns the list of all the oriented graphs that are obtained from G by replacing (in every possible way) each edge [x,y] of G by one arrow: either [x,y] or [y,x]. In each of these orientations Loops are removed and existing arrows of G are left untouched. Note that this operation will use time and memory which is exponential on the number of edges of G.

```
gap> g:=GraphByWalks([1,1,2,3,1,3,2]:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 6, Adjacencies :=
[ [ 1, 2, 3 ], [ 3 ], [ 1, 2 ] ] )
gap> Orientations(g);
[ Graph( rec( Category := OrientedGraphs, Order := 3, Size :=
        3, Adjacencies := [ [ 2 ], [ ], [ 1, 2 ] ] ) ),
Graph( rec( Category := OrientedGraphs, Order := 3, Size :=
        3, Adjacencies := [ [ 2 ], [ 3 ], [ 1 ] ] ) ),
Graph( rec( Category := OrientedGraphs, Order := 3, Size :=
        3, Adjacencies := [ [ 2, 3 ], [ ], [ 2 ] ] ) ),
Graph( rec( Category := OrientedGraphs, Order := 3, Size :=
        3, Adjacencies := [ [ 2, 3 ], [ 3 ], [ ] ] ) )]
gap> Length(Orientations(Octahedron));
4096
```

Note that Orientations(G) returns a list of graphs, each of them in the category OrientedGraphs regardless of the TargetGraphCategory.

## **B.1.127** OrientedGraphs

```
▷ OrientedGraphs(G)
```

OrientedGraphs is a graph category in YAGS. A graph in this category may contain arrows, but no loops or edges. The parent of this category is LooplessGraphs.

```
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[ [ 1, 2 ], [ 1 ], [ 2 ] ] )
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ ], [ 2 ] ] )
```

#### **B.1.128** OutNeigh

```
\triangleright OutNeigh(G, x) (operation)
```

Returns the list of out-neighbors of x in G.

```
Example
gap> tt:=CompleteGraph(5:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 5, Size := 10, Adjacencies :=
[ [ 2, 3, 4, 5 ], [ 3, 4, 5 ], [ 4, 5 ], [ 5 ], [ ] ] )
gap> InNeigh(tt,3);
[ 1, 2 ]
gap> OutNeigh(tt,3);
[ 4, 5 ]
```

## **B.1.129** PaleyTournament

```
▷ PaleyTournament(prime)
```

(operation)

Returns the Paley tournament associated with prime number *prime*. *prime* must be congruent to 3 mod 4. The Paley tournament is the oriented circulant whose *jumps* are the all squares of the ring \$\Z\_p\$.

```
Example
gap> Filtered([1..30],x \rightarrow 0=((x-3) mod \overline{4}) and IsPrime(x));
[ 3, 7, 11, 19, 23 ]
gap> PaleyTournament(3);PaleyTournament(7);PaleyTournament(11);
Graph( Category := OrientedGraphs, Order := 3, Size := 3, Adjacencies :=
[[2],[3],[1]])
Graph( Category := OrientedGraphs, Order := 7, Size := 21, Adjacencies :=
[[2, 3, 5], [3, 4, 6], [4, 5, 7], [1, 5, 6], [2, 6, 7],
  [1, 3, 7], [1, 2, 4]])
Graph( Category := OrientedGraphs, Order := 11, Size := 55, Adjacencies :=
[[2, 4, 5, 6, 10], [3, 5, 6, 7, 11], [1, 4, 6, 7, 8],
  [2, 5, 7, 8, 9], [3, 6, 8, 9, 10], [4, 7, 9, 10, 11],
  [1, 5, 8, 10, 11], [1, 2, 6, 9, 11], [1, 2, 3, 7, 10],
  [2, 3, 4, 8, 11], [1, 3, 4, 5, 9]])
gap> PaleyTournament(5);
fail
```

Note that PaleyTournament(prime) returns a graph in the category OrientedGraphs regardless of the TargetGraphCategory.

## **B.1.130** ParachuteGraph

```
▷ ParachuteGraph
```

(global variable)

The complement of a ParapluieGraph; The suspension of a 4-path with a pendant vertex attached to the south pole.

## **B.1.131** ParapluieGraph

▶ ParapluieGraph (global variable)

A 3-Fan graph with a 3-path attached to the universal vertex.

```
gap> ParapluieGraph;
Graph( Category := SimpleGraphs, Order := 7, Size := 9, Adjacencies :=
[[2], [1, 3], [2, 4, 5, 6, 7], [3, 5], [3, 4, 6], [3, 5, 7],
[3, 6]])
```

## **B.1.132** ParedGraph

```
    ParedGraph(G) (operation)
```

Returns the pared graph of G. This is the induced subgraph obtained from G by removing its dominated vertices. When there are twin vertices (mutually dominated vertices), exactly one of them survives the paring in each equivalent class of mutually dominated vertices.

```
gap> g1:=PathGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3 ] ] )
gap> ParedGraph(g1);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> g2:=PathGraph(2);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> ParedGraph(g2);
Graph( Category := SimpleGraphs, Order := 1, Size := 0, Adjacencies :=
[ [ ] ] )
```

## **B.1.133** PathGraph

▷ PathGraph(n) (function)

Returns the path graph on n vertices.

```
gap> PathGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3 ] ] )
```

#### **B.1.134** PawGraph

▶ PawGraph (global variable)

The graph on 4 vertices, 4 edges and maximum degree 3: A triangle with a pendant vertex.

Example

```
gap> PawGraph;
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3, 4 ], [ 2, 4 ], [ 2, 3 ] ] )
```

## **B.1.135** PetersenGraph

▶ PetersenGraph (global variable)

The 3-regular graph on 10 vertices having girth 5.

```
Example

gap> PetersenGraph;

Graph( Category := SimpleGraphs, Order := 10, Size := 15, Adjacencies :=

[ [ 2, 5, 6 ], [ 1, 3, 7 ], [ 2, 4, 8 ], [ 3, 5, 9 ], [ 1, 4, 10 ],

[ 1, 8, 9 ], [ 2, 9, 10 ], [ 3, 6, 10 ], [ 4, 6, 7 ], [ 5, 7, 8 ] ] )
```

## **B.1.136** PowerGraph

```
\triangleright PowerGraph(G, exp) (operation)
```

Returns the DistanceGraph of G using  $[0, 1, \ldots, exp]$  as the list of distances. Note that the distance 0 in the list produces loops in the new graph only when the TargetGraphCategory admits loops.

```
Example

gap> g:=PathGraph(5);

Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=

[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4 ] ] )

gap> PowerGraph(g,1);

Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=

[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4 ] ] )

gap> PowerGraph(g,1:GraphCategory:=Graphs);

Graph( Category := Graphs, Order := 5, Size := 13, Adjacencies :=

[ [ 1, 2 ], [ 1, 2, 3 ], [ 2, 3, 4 ], [ 3, 4, 5 ], [ 4, 5 ] ] )
```

#### **B.1.137** PropertyMorphism

```
    ▷ PropertyMorphism(G, H, PropList) (operation)
```

Returns the first morphism (in lexicographic order) from G to H satisfying the list of properties PropList. A number of preprogrammed properties are provided by YAGS, and the user may create additional ones. The properties provided are:  $CHK\_WEAK$ ,  $CHK\_MORPH$ ,  $CHK\_METRIC$ ,  $CHK\_CMPLT$ ,  $CHK\_MONO$  and  $CHK\_EPI$ . If G has n vertices and f:G is a morphism, it is represented as  $F = [f(1), f(2), \ldots, f(n)]$ .

```
Example
gap> g:=CycleGraph(4);;h:=CompleteBipartiteGraph(2,2);;
gap> PropList:=[CHK_MORPH];;
gap> PropertyMorphism(g,h,PropList);
[ 1, 3, 1, 3 ]
```

## **B.1.138** PropertyMorphisms

```
    ▷ PropertyMorphisms(G, H, PropList) (operation)
```

Returns all morphisms from G to H satisfying the list of properties PropList. A number of preprogrammed properties are provided by YAGS, and the user may create additional ones. The properties provided are: CHK\_WEAK, CHK\_MORPH, CHK\_METRIC, CHK\_CMPLT, CHK\_MONO and CHK\_EPI. If G has n vertices and G rightarrow G is a morphism, it is represented as G is G and G is a morphism, it is represented as G is G and G is a morphism, it is represented as G is G in G and G is a morphism, it is represented as G is G is G in G in G in G is G in G in G in G is G in G is G in G in

```
Example

gap> g:=CycleGraph(4);;h:=CompleteBipartiteGraph(2,2);;

gap> PropList:=[CHK_WEAK,CHK_MONO];;

gap> PropertyMorphisms(g,h,PropList);

[[1,3,2,4],[1,4,2,3],[2,3,1,4],[2,4,1,3],
    [3,1,4,2],[3,2,4,1],[4,1,3,2],[4,2,3,1]]
```

## **B.1.139 QtfyIsSimple**

```
    □ QtfyIsSimple(G) (attribute)
```

For internal use. Returns how far is graph G from being simple.

## **B.1.140** QuadraticRingGraph

```
    □ QuadraticRingGraph(Rng) (operation)
```

Returns the graph G whose vertices are the elements of Rng such that x is adjacent to y iff  $x+z^2=y$  for some z in Rng.

```
Example

gap> QuadraticRingGraph(ZmodnZ(8));

Graph( Category := SimpleGraphs, Order := 8, Size := 12, Adjacencies :=

[ [ 2, 5, 8 ], [ 1, 3, 6 ], [ 2, 4, 7 ], [ 3, 5, 8 ], [ 1, 4, 6 ],

[ 2, 5, 7 ], [ 3, 6, 8 ], [ 1, 4, 7 ] ] )
```

## **B.1.141** QuotientGraph

```
▷ QuotientGraph(G, Part) (operation)
▷ QuotientGraph(G, L1, L2) (operation)
```

Returns the quotient graph of graph G given a vertex partition Part, by identifying any two vertices in the same part. The vertices of the quotient graph are the parts in the partition Part two of them being adjacent iff any vertex in one part is adjacent to any vertex in the other part. Singletons may be omitted in Part.

```
Example

gap> g:=PathGraph(8);;
gap> QuotientGraph(g,[[1,5,8],[2],[3],[4],[6],[7]]);
Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
[ [ 2, 4, 5, 6 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ], [ 1, 6 ], [ 1, 5 ] ])
gap> QuotientGraph(g,[[1,5,8]]);
Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
[ [ 2, 4, 5, 6 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ], [ 1, 6 ], [ 1, 5 ] ])
```

In its second form, QuotientGraph identifies each vertex in list L1, with the corresponding vertex in list L2. L1 and L2 must have the same length, but any or both of them may have repetitions.

```
Example

gap> g:=PathGraph(8);;
gap> QuotientGraph(g,[[1,7],[4,8]]);

Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
[ [ 2, 4, 6 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3, 5 ], [ 4, 6 ], [ 1, 5 ] ])
gap> QuotientGraph(g,[1,4],[7,8]);

Graph( Category := SimpleGraphs, Order := 6, Size := 7, Adjacencies :=
[ [ 2, 4, 6 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3, 5 ], [ 4, 6 ], [ 1, 5 ] ])
```

## **B.1.142** Radius

 $\triangleright$  Radius(G) (attribute)

Returns the minimal eccentricity among the vertices of graph G.

```
gap> Radius(PathGraph(5));
2
```

#### **B.1.143** RandomCirculant

```
▷ RandomCirculant(n) (operation)
▷ RandomCirculant(n, k) (operation)
▷ RandomCirculant(n, p) (operation)
```

Returns a circulant on n vertices with its jumps selected randomly. In its third form, each possible jump has probability p of being selected. In its second form, when k is a positive integer, exactly k jumps are selected (provided there are at least k possible jumps to select from). The first form is equivalent to specifying p=1/2.

```
_ Example _
gap> RandomCirculant(11,2);
Graph( Category := SimpleGraphs, Order := 11, Size := 22, Adjacencies :=
[ [4, 6, 7, 9], [5, 7, 8, 10], [6, 8, 9, 11], [1, 7, 9, 10], [2, 8, 10, 1], [1, 7, 9, 10], [2, 8, 10, 1], [1, 7, 9, 10], [2, 8, 10, 1], [1, 7, 9, 10], [2, 8, 10, 1], [1, 7, 9, 10], [2, 8, 10, 1], [1, 7, 9, 10], [2, 8, 10, 1], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8, 10], [2, 8,
     [1, 3, 9, 11], [1, 2, 4, 10], [2, 3, 5, 11], [1, 3, 4, 6], [2, 4, 5, 7],
     [3, 5, 6, 8]])
gap> RandomCirculant(11,2);
Graph( Category := SimpleGraphs, Order := 11, Size := 22, Adjacencies :=
[[2, 4, 9, 11], [1, 3, 5, 10], [2, 4, 6, 11], [1, 3, 5, 7], [2, 4, 6, 8],
     [3, 5, 7, 9], [4, 6, 8, 10], [5, 7, 9, 11], [1, 6, 8, 10], [2, 7, 9, 11],
     [1, 3, 8, 10])
gap> RandomCirculant(11,1/2);
Graph( Category := SimpleGraphs, Order := 11, Size := 11, Adjacencies :=
[[2, 11], [1, 3], [2, 4], [3, 5], [4, 6], [5, 7], [6, 8], [7, 9],
      [8, 10], [9, 11], [1, 10])
gap> RandomCirculant(11,1/2);
Graph( Category := SimpleGraphs, Order := 11, Size := 44, Adjacencies :=
[[2, 3, 4, 5, 8, 9, 10, 11], [1, 3, 4, 5, 6, 9, 10, 11],
     [ 1, 2, 4, 5, 6, 7, 10, 11 ], [ 1, 2, 3, 5, 6, 7, 8, 11 ], [ 1, 2, 3, 4, 6, 7, $, 9 ],
     [2, 3, 4, 5, 7, 8, 9, 10], [3, 4, 5, 6, 8, 9, 10, 11], [1, 4, 5, 6, 7, 9, 10, 11],
     [1, 2, 5, 6, 7, 8, 10, 11], [1, 2, 3, 6, 7, 8, 9, 11], [1, 2, 3, 4, 7, 8, 9, 10]
])
```

```
gap> RandomCirculant(11,1/2);
Graph( Category := SimpleGraphs, Order := 11, Size := 33, Adjacencies :=
[ [ 3, 4, 6, 7, 9, 10 ], [ 4, 5, 7, 8, 10, 11 ], [ 1, 5, 6, 8, 9, 11 ],
      [ 1, 2, 6, 7, 9, 10 ], [ 2, 3, 7, 8, 10, 11 ], [ 1, 3, 4, 8, 9, 11 ],
      [ 1, 2, 4, 5, 9, 10 ], [ 2, 3, 5, 6, 10, 11 ], [ 1, 3, 4, 6, 7, 11 ],
      [ 1, 2, 4, 5, 7, 8 ], [ 2, 3, 5, 6, 8, 9 ] ] )
```

## **B.1.144** RandomGraph

```
ightharpoonup RandomGraph(n, p) (function) 
ightharpoonup RandomGraph(n) (function)
```

Returns a random graph of order n taking the rational  $p \in [0,1]$  as the edge probability.

```
gap> RandomGraph(5,1/3);
Graph( Category := SimpleGraphs, Order := 5, Size := 2, Adjacencies :=
[ [ 5 ], [ 5 ], [ ], [ 1, 2 ] ] )
gap> RandomGraph(5,2/3);
Graph( Category := SimpleGraphs, Order := 5, Size := 6, Adjacencies :=
[ [ 4, 5 ], [ 3, 4, 5 ], [ 2, 4 ], [ 1, 2, 3 ], [ 1, 2 ] ] )
gap> RandomGraph(5,1/2);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2, 5 ], [ 1, 3, 5 ], [ 2 ], [ ], [ 1, 2 ] ] )
```

If p is ommited, the edge probability is taken to be 1/2.

```
gap> RandomGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 5, Adjacencies :=
[ [ 2, 3 ], [ 1 ], [ 1, 4, 5 ], [ 3, 5 ], [ 3, 4 ] ] )
gap> RandomGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 3, Adjacencies :=
[ [ 2, 5 ], [ 1, 4 ], [ ], [ 2 ], [ 1 ] ] )
```

## **B.1.145** RandomPermutation

▷ RandomPermutation(n)

(operation)

Returns a random permutation of the list  $[1, 2, \ldots n]$ .

```
gap> RandomPermutation(12);
(1,8,10)(2,7,9,12)(3,5,11)(4,6)
```

#### **B.1.146** RandomSubset

```
ightharpoonup RandomSubset(Set) (operation)

ightharpoonup RandomSubset(Set, k) (operation)

ightharpoonup RandomSubset(Set, p) (operation)
```

Returns a random subset of the set Set. When the positive integer k is provided, the returned subset has k elements (or fail if Set does not have at least k elements). When the probability p is provided, each element of Set has probability p of being selected for inclusion in the returned subset. When k and p are both missing, it is equivalent to specifying p=1/2. In the ambiguous case when the second parameter is 1, it is interpreted as the value of k.

```
Example
gap> RandomSubset([1..10],5);
[7, 3, 10, 6, 4]
gap> RandomSubset([1..10],5);
[3, 7, 6, 9, 10]
gap> RandomSubset([1..10],5);
[3, 9, 7, 2, 6]
gap> RandomSubset([1..10],5);
[1, 2, 4, 3, 9]
gap> RandomSubset([1..10],1/2);
[ 1, 3, 7, 10 ]
gap> RandomSubset([1..10],1/2);
[ 1, 2, 5, 6, 7, 8, 10 ]
gap> RandomSubset([1..10],1/2);
[4, 5, 8, 10]
gap> RandomSubset([1..10],1/2);
[ 1, 4, 10 ]
```

Even if this operation is intended to be applied to sets, it does not impose this condition on its operand, and can be applied to lists as well.

```
gap> RandomSubset([1,3,2,2,3,2,1]);
[ 1, 3, 2, 2, 2 ]
gap> RandomSubset([1,3,2,2,3,2,1]);
[ 2, 2 ]
```

## **B.1.147** RandomlyPermuted

```
▷ RandomlyPermuted(Obj)
```

(operation)

Returns a copy of Obj with the order of its elements permuted randomly. Currently, the operation is implemented for lists and graphs.

```
gap> RandomlyPermuted([1..9]);
[ 9, 7, 5, 3, 1, 4, 8, 6, 2 ]
gap> g:=PathGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3 ] ])
gap> RandomlyPermuted(g);
Graph( Category := SimpleGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 4 ], [ 3, 4 ], [ 2 ], [ 1, 2 ] ])
```

## **B.1.148** RemoveEdges

```
▷ RemoveEdges(G, E)
```

Returns a new graph created from graph G by removing the edges in list E.

```
gap> g:=CompleteGraph(4);
Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
gap> RemoveEdges(g,[[1,2]]);
Graph( Category := SimpleGraphs, Order := 4, Size := 5, Adjacencies :=
[ [ 3, 4 ], [ 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )
gap> RemoveEdges(g,[[1,2],[3,4]]);
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 3, 4 ], [ 3, 4 ], [ 1, 2 ], [ 1, 2 ] ] )
```

#### **B.1.149** RemoveVertices

```
\triangleright RemoveVertices(G, V)
```

(operation)

Returns a new graph created from graph G by removing the vertices in list V.

```
gap> g:=PathGraph(5);
Graph( Category := SimpleGraphs, Order := 5, Size := 4, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 4 ] ] )
gap> RemoveVertices(g,[3]);
Graph( Category := SimpleGraphs, Order := 4, Size := 2, Adjacencies :=
[ [ 2 ], [ 1 ], [ 4 ], [ 3 ] ] )
gap> RemoveVertices(g,[1,3]);
Graph( Category := SimpleGraphs, Order := 3, Size := 1, Adjacencies :=
[ [ ], [ 3 ], [ 2 ] ] )
```

## B.1.150 RGraph

▷ RGraph (global variable)

A square with two pendant vertices attached to the same vertex of the square.

## **B.1.151** RingGraph

```
▷ RingGraph(Rng, Elms)
```

(operation)

Returns the graph G whose vertices are the elements of the ring Rng such that x is adjacent to y iff x+r=y for some r in Elms.

```
Example

gap> r:=FiniteField(8);Elements(r);

GF(2^3)

[ 0*Z(2), Z(2)^0, Z(2^3), Z(2^3)^2, Z(2^3)^3, Z(2^3)^4, Z(2^3)^5, Z(2^3)^6 ]

gap> RingGraph(r,[Z(2^3),Z(2^3)^4]);

Graph( Category := SimpleGraphs, Order := 8, Size := 8, Adjacencies :=
```

```
[[3,6],[5,7],[1,4],[3,6],[2,8],[1,4],[2,8],
[5,7]])
```

#### **B.1.152** SetCoordinates

```
▷ SetCoordinates(G, Coord)
```

(operation)

Sets the coordinates of the vertices of G, which are used to draw G by Draw(G).

```
Example

gap> g:=CycleGraph(4);;

gap> SetCoordinates(g,[[-10,-10],[-10,20],[20,-10], [20,20]]);

gap> Coordinates(g);

[[-10, -10], [-10, 20], [20, -10], [20, 20]]
```

## **B.1.153** SetDefaultGraphCategory

```
▷ SetDefaultGraphCategory(Catgy)
```

(function)

Sets the default graph category to *Catgy*. The default graph category is used when constructing new graphs when no other graph category is indicated. New graphs are always forced to comply with the TargetGraphCategory, so loops may be removed, and arrows may replaced by edges or viceversa, depending on the category that the new graph belongs to. The available graph categories are: SimpleGraphs, OrientedGraphs, UndirectedGraphs, LooplessGraphs, and Graphs.

```
Example
gap> SetDefaultGraphCategory(Graphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=
[[1,2],[1],[2]])
gap> SetDefaultGraphCategory(LooplessGraphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := LooplessGraphs, Order := 3, Size := 3, Adjacencies :=
[[2],[1],[2]])
gap> SetDefaultGraphCategory(UndirectedGraphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := UndirectedGraphs, Order := 3, Size := 3, Adjacencies :=
[[1,2],[1,3],[2]])
gap> SetDefaultGraphCategory(SimpleGraphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[[2],[1,3],[2]])
gap> SetDefaultGraphCategory(OrientedGraphs);
gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]);
Graph( Category := OrientedGraphs, Order := 3, Size := 2, Adjacencies :=
[[2],[],[2]])
```

#### **B.1.154** SimpleGraphs

```
▷ SimpleGraphs(G)
```

SimpleGraphs is a graph category in YAGS. A graph in this category may contain edges, but no loops or arrows. The category has two parents: LooplessGraphs and UndirectedGraphs.

```
Example

gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);

Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=

[ [ 1, 2 ], [ 1 ], [ 2 ] ] )

gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=SimpleGraphs);

Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=

[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
```

#### **B.1.155** Size

```
\triangleright Size(G) (attribute)
```

Returns the number of edges of graph G.

```
gap> Size(Icosahedron);
30
```

## **B.1.156** SnubDisphenoid

▷ SnubDisphenoid

(global variable)

The 1-skeleton of the 84th Johnson solid.

```
Example

gap> SnubDisphenoid;

Graph( Category := SimpleGraphs, Order := 8, Size := 18, Adjacencies :=

[[2, 3, 4, 5, 8], [1, 3, 6, 7, 8], [1, 2, 4, 6], [1, 3, 5, 6],

[1, 4, 6, 7, 8], [2, 3, 4, 5, 7], [2, 5, 6, 8], [1, 2, 5, 7]])
```

## **B.1.157** SpanningForest

▷ SpanningForest(G)

(operation)

Returns a spanning forest of G.

## **B.1.158** SpanningForestEdges

```
▷ SpanningForestEdges(G)
```

(operation)

Returns the edges of a spanning forest of G.

## B.1.159 SpikyGraph

```
▷ SpikyGraph(n)

(function)
```

The spiky graph is constructed as follows: Take complete graph on n vertices,  $K_N$ , and then, for each the n subsets of  $\operatorname{Vertices}(K_n)$  of order n-1, add an additional vertex which is adjacent precisely to this subset of  $\operatorname{Vertices}(K_n)$ .

```
Example

gap> SpikyGraph(3);

Graph( Category := SimpleGraphs, Order := 6, Size := 9, Adjacencies :=

[ [ 2, 3, 4, 5 ], [ 1, 3, 4, 6 ], [ 1, 2, 5, 6 ], [ 1, 2 ], [ 1, 3 ],

[ 2, 3 ] ] )
```

## B.1.160 SunGraph

```
\triangleright SunGraph(n) (function)
```

Returns the n-Sun: A complete graph on n vertices,  $K_N$ , with a corona made with a zigzagging 2n-cycle glued to a n-cycle of the  $K_N$ .

## **B.1.161** Suspension

```
\triangleright Suspension(G) (operation)
```

Returns the suspension of graph G. The suspension of G is the graph obtained from G by adding two new vertices which are adjacent to every vertex of G but not to each other. The new vertices are the first ones in the new graph.

```
Example

gap> Suspension(CycleGraph(4));

Graph( Category := SimpleGraphs, Order := 6, Size := 12, Adjacencies :=

[[3, 4, 5, 6], [3, 4, 5, 6], [1, 2, 4, 6], [1, 2, 3, 5],

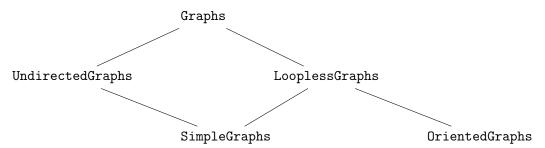
[1, 2, 4, 6], [1, 2, 3, 5]])
```

#### **B.1.162** TargetGraphCategory

```
▷ TargetGraphCategory([G, ...])

(function)
```

For internal use. Returns the graph category indicated in the *options stack* if any, otherwise if the list of graphs provided is not empty, returns the minimal common graph category for the graphs in the list, else returns the default graph category. The partial order (by inclusion) among graph categories is as follows:



This function is internally called by all graph constructing operations in YAGS to decide the graph category that the newly constructed graph is going to belong. New graphs are always forced to comply with the TargetGraphCategory, so loops may be removed, and arrows may replaced by edges or viceversa, depending on the category that the new graph belongs to.

The *options stack* is a mechanism provided by GAP to pass implicit parameters and is used by TargetGraphCategory so that the user may indicate the graph category she/he wants for the new graph.

```
gap> SetDefaultGraphCategory(SimpleGraphs);
gap> g1:=CompleteGraph(2);
Graph( Category := SimpleGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ 1 ] ] )
gap> g2:=CompleteGraph(2:GraphCategory:=OrientedGraphs);
Graph( Category := OrientedGraphs, Order := 2, Size := 1, Adjacencies :=
[ [ 2 ], [ ] ] )
gap> DisjointUnion(g1,g2);
Graph( Category := LooplessGraphs, Order := 4, Size := 3, Adjacencies :=
[ [ 2 ], [ 1 ], [ 4 ], [ ] ] )
gap> DisjointUnion(g1,g2:GraphCategory:=UndirectedGraphs);
Graph( Category := UndirectedGraphs, Order := 4, Size := 2, Adjacencies :=
[ [ 2 ], [ 1 ], [ 4 ], [ 3 ] ] )
```

In the previous examples, TargetGraphCategory was called internally exactly once for each new graph constructed with the following parameters:

```
gap> TargetGraphCategory();
<Operation "SimpleGraphs">
gap> TargetGraphCategory(:GraphCategory:=OrientedGraphs);
<Operation "OrientedGraphs">
gap> TargetGraphCategory([g1,g2]);
<Operation "LooplessGraphs">
gap> TargetGraphCategory([g1,g2]:GraphCategory:=UndirectedGraphs);
<Operation "UndirectedGraphs">
```

## **B.1.163** Tetrahedron

□ Tetrahedron
 (global variable)

The 1-skeleton of Plato's tetrahedron.

gap> Tetrahedron;

Graph( Category := SimpleGraphs, Order := 4, Size := 6, Adjacencies := [ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ] )

#### **B.1.164** TimeInSeconds

```
▷ TimeInSeconds()

(operation)
```

Returns the time in seconds since 1970-01-01 00:00:00 UTC as an integer. This is useful to measure execution time. It can also be used to impose time constraints on the execution of algorithms. Note however that the time reported is the *wall time*, not necessarily the time spent in the process you intend to measure.

```
gap> TimeInSeconds();
1415551598
gap> K:=CliqueGraph;;
gap> t1:=TimeInSeconds();NumberOfCliques(K(K(K(K(Icosahedron)))));TimeInSeconds()-t1;
1415551608
44644
103
```

Currently, this operation is not working on MS Windows.

#### **B.1.165** TimesProduct

```
\triangleright TimesProduct(G, H) (operation)
```

Returns the times product of two graphs G and H, G \$\times\$ H (also known as the tensor product). The times product is computed as follows: For each pair of vertices \$x \in G, y \in H\$ we create a vertex \$(x,y)\$. Given two such vertices \$(x,y)\$ and \$(x',y')\$ they are adjacent iff \$x \times x'\$ and \$y \times y'\$.

```
Example
gap> g:=PathGraph(3);h:=CycleGraph(4);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
Graph( Category := SimpleGraphs, Order := 4, Size := 4, Adjacencies :=
[ [ 2, 4 ], [ 1, 3 ], [ 2, 4 ], [ 1, 3 ] ] )
gap> gh:=TimesProduct(g,h);
Graph( Category := SimpleGraphs, Order := 12, Size := 16, Adjacencies :=
[ [ 6, 8 ], [ 5, 7 ], [ 6, 8 ], [ 5, 7 ], [ 2, 4, 10, 12 ], [ 1, 3, 9, 11 ],
        [ 2, 4, 10, 12 ], [ 1, 3, 9, 11 ], [ 6, 8 ], [ 5, 7 ], [ 6, 8 ], [ 5, 7 ] ] )
gap> VertexNames(gh);
[ [ 1, 1 ], [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 2, 1 ], [ 2, 2 ], [ 2, 3 ],
        [ 2, 4 ], [ 3, 1 ], [ 3, 2 ], [ 3, 3 ], [ 3, 4 ] ]
```

## **B.1.166** TorusGraph

```
▷ TorusGraph(n, m) (function)
```

Returns (the underlying graph of) a triangulation of the torus on n.m vertices. This graphs is constructed using  $\{1,2,\ldots,n\}$  as the vertex set; two of them being adjacent if their difference belongs to  $\{(1,0),(0,1),(1,1)\}$  module  $\{Z_n\times\}$ . Hence, in the category of simple graphs, TorusGraph is a 6-regular graph when n.m

```
TorusGraph(4,4);

Graph( Category := SimpleGraphs, Order := 16, Size := 48, Adjacencies :=
[ [ 2, 4, 5, 6, 13, 16 ], [ 1, 3, 6, 7, 13, 14 ], [ 2, 4, 7, 8, 14, 15 ],
        [ 1, 3, 5, 8, 15, 16 ], [ 1, 4, 6, 8, 9, 10 ], [ 1, 2, 5, 7, 10, 11 ],
        [ 2, 3, 6, 8, 11, 12 ], [ 3, 4, 5, 7, 9, 12 ], [ 5, 8, 10, 12, 13, 14 ],
        [ 5, 6, 9, 11, 14, 15 ], [ 6, 7, 10, 12, 15, 16 ], [ 7, 8, 9, 11, 13, 16 ],
        [ 1, 2, 9, 12, 14, 16 ], [ 2, 3, 9, 10, 13, 15 ], [ 3, 4, 10, 11, 14, 16 ],
        [ 1, 4, 11, 12, 13, 15 ] ])
```

When  $n,m \neq 4$ , TorusGraph ( n, m ) is actually a Whitney triangulation: Every triangle of the graph is a face of the triagulation. The clique behavior of these graphs were extensively studied in [9]. However, this operation constructs the described graph for all  $n,m \neq 1$ .

```
gap> TorusGraph(2,4);
Graph( Category := SimpleGraphs, Order := 8, Size := 20, Adjacencies :=
[ [ 2, 4, 5, 6, 8 ], [ 1, 3, 5, 6, 7 ], [ 2, 4, 6, 7, 8 ], [ 1, 3, 5, 7, 8 ],
        [ 1, 2, 4, 6, 8 ], [ 1, 2, 3, 5, 7 ], [ 2, 3, 4, 6, 8 ], [ 1, 3, 4, 5, 7 ] ] )
gap> TorusGraph(2,3);
Graph( Category := SimpleGraphs, Order := 6, Size := 15, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3, 4, 5, 6 ], [ 1, 2, 4, 5, 6 ], [ 1, 2, 3, 5, 6 ],
        [ 1, 2, 3, 4, 6 ], [ 1, 2, 3, 4, 5 ] ] )
```

Note that in these cases, TorusGraph(n, m) is not 6-regular nor a Whitney triangulation.

## **B.1.167** TreeGraph

```
▷ TreeGraph(arity, depth) (operation)
▷ TreeGraph(ArityList) (operation)
```

Returns a tree, a connected cycle-free graph. In its second form, the vertices at height k (the root vertex has height 1 here) have ArityList[k] children. In its first form, all vertices, but the leaves, have arity children and the height of the leaves is depth+1.

```
gap> TreeGraph(2,3);
Graph( Category := SimpleGraphs, Order := 15, Size := 14, Adjacencies :=
[ [ 2, 3 ], [ 1, 4, 5 ], [ 1, 6, 7 ], [ 2, 8, 9 ], [ 2, 10, 11 ], [ 3, 12, 13 ],
        [ 3, 14, 15 ], [ 4 ], [ 4 ], [ 5 ], [ 6 ], [ 6 ], [ 7 ], [ 7 ] ] )
gap> TreeGraph([3,2,2]);
Graph( Category := SimpleGraphs, Order := 22, Size := 21, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 5, 6 ], [ 1, 7, 8 ], [ 1, 9, 10 ], [ 2, 11, 12 ], [ 2, 13, 14 ],
        [ 3, 15, 16 ], [ 3, 17, 18 ], [ 4, 19, 20 ], [ 4, 21, 22 ], [ 5 ], [ 6 ], [ 6 ],
        [ 7 ], [ 7 ], [ 8 ], [ 8 ], [ 9 ], [ 9 ], [ 10 ], [ 10 ] ] )
```

#### **B.1.168** TrivialGraph

```
    □ TrivialGraph (global variable)
```

The one vertex graph.

```
gap> TrivialGraph;
Graph( Category := SimpleGraphs, Order := 1, Size := 0, Adjacencies :=
[ [ ] ] )
```

#### **B.1.169 UFFind**

```
▷ UFFind(UFS, x) (function)
```

For internal use. Implements the find operation on the union-find structure.

#### **B.1.170 UFUnite**

```
▷ UFUnite(UFS, x, y) (function)
```

For internal use. Implements the unite operation on the union-find structure.

## **B.1.171** UndirectedGraphs

```
\triangleright UndirectedGraphs(G) (function)
```

UndirectedGraphs is a graph category in YAGS. A graph in this category may contain edges and loops, but no arrows. The parent of this category is Graphs.

```
Example

gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=Graphs);

Graph( Category := Graphs, Order := 3, Size := 4, Adjacencies :=

[ [ 1, 2 ], [ 1 ], [ 2 ] ])

gap> GraphByWalks([1,1],[1,2],[2,1],[3,2]:GraphCategory:=UndirectedGraphs);

Graph( Category := UndirectedGraphs, Order := 3, Size := 3, Adjacencies :=

[ [ 1, 2 ], [ 1, 3 ], [ 2 ] ])
```

## **B.1.172** UnitsRingGraph

```
▷ UnitsRingGraph(Rng) (operation)
```

Returns the graph G whose vertices are the elements of Rng such that x is adjacent to y iff x+z=y for some unit z of Rng.

```
Example

gap> UnitsRingGraph(ZmodnZ(8));

Graph( Category := SimpleGraphs, Order := 8, Size := 16, Adjacencies :=

[ [ 2, 4, 6, 8 ], [ 1, 3, 5, 7 ], [ 2, 4, 6, 8 ], [ 1, 3, 5, 7 ],

[ 2, 4, 6, 8 ], [ 1, 3, 5, 7 ], [ 2, 4, 6, 8 ], [ 1, 3, 5, 7 ] ])
```

## **B.1.173** VertexDegree

```
    VertexDegree(G, x) (operation)
```

Returns the degree of vertex x in Graph G.

```
gap> g:=PathGraph(3);
Graph( Category := SimpleGraphs, Order := 3, Size := 2, Adjacencies :=
[ [ 2 ], [ 1, 3 ], [ 2 ] ] )
gap> VertexDegree(g,1);
1
gap> VertexDegree(g,2);
2
```

## **B.1.174** VertexDegrees

▷ VertexDegrees(G)

(operation)

Returns the list of degrees of the vertices in graph G.

```
Example

gap> g:=GemGraph;

Graph( Category := SimpleGraphs, Order := 5, Size := 7, Adjacencies :=

[ [ 2, 3, 4, 5 ], [ 1, 3 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4 ] ] )

gap> VertexDegrees(g);

[ 4, 2, 3, 3, 2 ]
```

#### **B.1.175** VertexNames

```
    VertexNames(G) (attribute)
```

Return the list of names of the vertices of G. The vertices of a graph in YAGS are always  $\{1,2, \dots, G(G)\}$ , but depending on how the graph was constructed, its vertices may have also some names, that help us identify the origin of the vertices. YAGS will always try to store meaninful names for the vertices. For example, in the case of the LineGraph, the vertex names of the new graph are the edges of the old graph.

```
gap> g:=LineGraph(DiamondGraph);
Graph( Category := SimpleGraphs, Order := 5, Size := 8, Adjacencies :=
[ [ 2, 3, 4 ], [ 1, 3, 4, 5 ], [ 1, 2, 5 ], [ 1, 2, 5 ], [ 2, 3, 4 ] ] )
gap> VertexNames(g);
[ [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 2, 3 ], [ 3, 4 ] ]
gap> Edges(DiamondGraph);
[ [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 2, 3 ], [ 3, 4 ] ]
```

#### **B.1.176** Vertices

```
\triangleright Vertices (G) (operation)
```

Returns the list [1..Order(G)].

```
gap> Vertices(Icosahedron);
[ 1 .. 12 ]
```

## **B.1.177** WheelGraph

```
▷ WheelGraph(n)
○ WheelGraph(n, r)
(operation)
```

In its first form WheelGraph returns the wheel graph on n+1 vertices. This is the cone of a cycle: a central vertex adjacent to all the vertices of an n-cycle.

```
Example

WheelGraph(5);

gap> Graph( Category := SimpleGraphs, Order := 6, Size := 10, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3, 6 ], [ 1, 2, 4 ], [ 1, 3, 5 ], [ 1, 4, 6 ],
[ 1, 2, 5 ] ] )
```

In its second form, WheelGraph returns returns the wheel graph, but adding r-1 layers, each layer is a new n-cycle joined to the previous layer by a zigzagging 2n-cycle. This graph is a triangulation of the disk.

```
gap> WheelGraph(5,2);
Graph( Category := SimpleGraphs, Order := 11, Size := 25, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3, 6, 7, 8 ], [ 1, 2, 4, 8, 9 ], [ 1, 3, 5, 9, 10 ],
        [ 1, 4, 6, 10, 11 ], [ 1, 2, 5, 7, 11 ], [ 2, 6, 8, 11 ], [ 2, 3, 7, 9 ],
        [ 3, 4, 8, 10 ], [ 4, 5, 9, 11 ], [ 5, 6, 7, 10 ] ])
gap> WheelGraph(5,3);
Graph( Category := SimpleGraphs, Order := 16, Size := 40, Adjacencies :=
[ [ 2, 3, 4, 5, 6 ], [ 1, 3, 6, 7, 8 ], [ 1, 2, 4, 8, 9 ], [ 1, 3, 5, 9, 10 ],
        [ 1, 4, 6, 10, 11 ], [ 1, 2, 5, 7, 11 ], [ 2, 6, 8, 11, 12, 13 ],
        [ 2, 3, 7, 9, 13, 14 ], [ 3, 4, 8, 10, 14, 15 ], [ 4, 5, 9, 11, 15, 16 ],
        [ 5, 6, 7, 10, 12, 16 ], [ 7, 11, 13, 16 ], [ 7, 8, 12, 14 ],
        [ 8, 9, 13, 15 ], [ 9, 10, 14, 16 ], [ 10, 11, 12, 15 ] ])
```

#### **B.1.178 YAGSExec**

```
▷ YAGSExec(ProgName, InString)
```

(operation)

For internal use. Calls external program *ProgName* located in directory *YAGSDir/bin/* feeding it with *InString* as input and returning the output of the external program as a string. fail is returned if the program could not be located.

```
gap> YAGSExec("time","");
"1415551127\n"
gap> YAGSExec("nauty","l=0$=1dacn=5 g1,2,3. xbzq");
"(4,5)\n(2,3)\n[2,3,4,5,1]\n[\"cb0c\",\"484f264\",\"b0e19f1\"]\n"
```

Currently, this operation is not working on MS Windows.

## **B.1.179 YAGSInfo**

▷ YAGSInfo (global variable)

A global record where much YAGS-related information is stored. This is intended for internal use, and much of this information is undocumented, but some of the data stored here could possibly be useful for advanced users. However, storing user information in this record and/or changing the values of the stored information is discouraged and may produce unpredictable results and an unstable system.

```
gap> YAGSInfo;
rec( AuxInfo := "/dev/null", DataDirectory := "/opt/gap4r7/pkg/yags/data",
  Directory := "/opt/gap4r7/pkg/yags", Internal := rec( ), Version := "0.0.1",
  graph6 := rec( BinListToNum := function( L ) ... end,
    BinListToNumList := function( L ) ... end,
  McKayR := function( L ) ... end, McKayN := function( n ) ... end,
  NumToBinList := function( n ) ... end, PadLeftnSplitList6 := function( L ) ... end,
  PadRightnSplitList6 := function( L ) ... end,
  StringToBinList := function( Str ) ... end ) )
```

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