2.3 Energy lost due to landing

We have not yet included this in our model, but there is an energy lost when the snowboarder lands in the halfpipe.

the snowboarder does not land on the edge of the halfpipe. by watching a video, the snowboarder lands around 20 % down the halfpipe. this means when he lands, some of the kinetic energy is put into the ground and lost. this still needs to be modelled.

2.4 Steering Ed:

2.4.1 Path Construction

To find the optimal path for the rider that maximises airtime, the introduction of steering into the model is necessary. The rider's position is parametrised by the generalised coordinates θ and S, where θ represents the angular position around the halfpipe cross-section and S the longitudinal distance down the pipe. The instantaneous velocity, and therefore the trajectory, is determined by $\dot{\theta}$ and $\dot{S}$.

By prescribing a functional relationship between θ and S, a desired path can be defined. The space of possible trajectories includes linear, parabolic, and more complex S-shaped curves. A convenient and flexible way to represent such routes is with a polynomial of sufficiently high order:

$$\theta_{\text{desired}}(S, C) = C_0 + C_1 S + C_2 S^2 + C_3 S^3 + C_4 S^4 \tag{42}$$

This fourth-order polynomial relates the lateral coordinate θ to the longitudinal coordinate S, and the coefficients C_i define the shape of the rider's route.

Insert example plot of this route

For use within the contact-phase ODEs, we also require expressions for $\dot{\theta}$ and $\ddot{\theta}$ along the desired path. Differentiating with respect to time gives:

$$\dot{\theta}_{\text{desired}} = \frac{d\theta_{\text{desired}}}{dS}\frac{dS}{dt} = \left(C_1 + 2C_2 S + 3C_3 S^2 + 4C_4 S^3\right)\dot{S} \tag{43}$$

and subsequently:

$$\ddot{\theta}_{\text{desired}} = \frac{d^2\theta_{\text{desired}}}{dS^2}\dot{S}^2 + \frac{d\theta_{\text{desired}}}{dS}\ddot{S} \tag{44}$$

2.4.2 Control Term

To model the rider actively steering to follow this desired path, a control term U_θ is introduced in the equation of motion for θ:

$$\ddot{\theta}_{\text{desired}} = \ddot{\theta}_{\text{natural}} + U_{\theta} \tag{45}$$

Here, $\ddot{\theta}_{\text{natural}}$ is the natural angular acceleration determined by gravitational and frictional forces acting on the rider (i.e., the passive dynamics). The control term U_{θ} represents the rider's active turning effort.

Since torque τ and angular acceleration are related by $\tau = I\,U_{\theta}$ (where $I = mR^2$ for a point mass at radius R), this control term effectively corresponds to a torque or turning force applied by the rider:

$$U_{\theta} = \frac{\tau_{\text{turn}}}{I} = \frac{F_{\text{turn}}}{mR} \tag{46}$$

The control input U_{θ} has units of angular acceleration (rad/s^2) and represents how the rider changes their angular motion about the pipe. Physically, this acceleration is generated by applying a turning force F_{turn} tangential to the pipe wall. For a point mass at radius R, the relationship between these quantities is

$$F_{\text{turn}} = mR\,U_{\theta}, \qquad \text{or equivalently,} \qquad U_{\theta} = \frac{F_{\text{turn}}}{mR}. \tag{47}$$

This connects the abstract control term used in the equations of motion to a real, measurable steering force applied by the rider.

2.4.3 Proportional–Derivative (PD) Control

It is unrealistic to assume that the rider can follow the desired path exactly. Instead, a more realistic approach is to model the rider as continuously correcting their motion to reduce the deviation from the desired path. This behaviour can be represented by a proportional–derivative (PD) control law, which combines proportional correction to the angular error and damping through its rate of change:

$$F_{\text{turn}} = mR\big[\,\ddot{\theta}_{\text{desired}} - f_{\theta}(x) - k_p(\theta - \theta_{\text{desired}}) - k_d(\dot{\theta} - \dot{\theta}_{\text{desired}})\,\big] \tag{48}$$

where:

- $f_{\theta}(x)$ is the natural angular acceleration from the passive ODE,
- k_p and k_d are proportional and derivative gains respectively,
- $(\theta - \theta_{\text{desired}})$ is the tracking error,
- $(\dot{\theta} - \dot{\theta}_{\text{desired}})$ is the rate of error.

This control law computes the required turning force F_{turn} that the rider must apply to remain close to the desired route. The gains k_p and k_d determine how aggressively the rider corrects deviations: higher values correspond to tighter path-following and larger steering effort.

2.4.4 Application to Path Optimisation

With the control framework established, the next step is to determine the set of polynomial coefficients $C = [C_0, C_1, C_2, C_3, C_4]$ that define the route $\theta_{\text{desired}}(S, C)$ which maximises the rider's performance. This is achieved through repeated numerical simulation of the rider's motion using the full equations of motion, including the control term that models steering effort.

At each simulation run, the coefficients C_i fully define the desired path and its derivatives:

$$\theta_{\text{desired}}(S; C), \quad \dot{\theta}_{\text{desired}}(S, \dot{S}; C), \quad \ddot{\theta}_{\text{desired}}(S, \dot{S}, \ddot{S}; C).$$

These are used within the contact-phase ODEs:

$$\begin{aligned} \ddot{S} &= f_S(S, \dot{S}, \theta, \dot{\theta}), \\ \ddot{\theta} &= f_\theta(S, \dot{S}, \theta, \dot{\theta}) + \frac{F_{\text{turn}}}{mR}, \end{aligned}$$

where f_S and f_θ are the passive dynamics (gravity, slope, and friction), and F_{turn} is provided by the PD controller:

$$F_{\text{turn}} = mR \left[\ddot{\theta}_{\text{desired}} - f_\theta(x) - k_p(\theta - \theta_{\text{desired}}) - k_d(\dot{\theta} - \dot{\theta}_{\text{desired}}) \right].$$

This controller ensures the rider approximately follows the path defined by C, while the physical model ensures that any deviation or excessive steering effort is captured in the dynamics.

After integrating the ODEs to record performance quantities:

- **Airtime:** total duration during which the rider loses contact with the pipe walls.
- **Turning effort:** energy or squared-force integral $E_{\text{turn}} = \int F_{\text{turn}}^2 \, dt$, representing the physical work needed to follow the route.
- **Route feasibility:** whether the path can be followed without violating physical limits ($|F_{\text{turn}}| < F_{\max}$, θ within pipe boundaries, etc.).

The optimisation objective is then defined as

$$J(C) = \text{Airtime}(C) - \lambda \, E_{\text{turn}}(C),$$

where λ is a weighting factor that controls the trade-off between maximum airtime and minimal steering effort. Alternative formulations can impose $E_{\text{turn}} < E_{\max}$ as a hard constraint.

To locate the optimal coefficients, Bayesian Optimisation, a black-box optimisation method is used. A exhaustive grid search could be used but it becomes very computationally heavy especially with higher order polynomials.

2.5 Bayesian Path Optimisation:

3 Results

This is what happened when did the thing.

4 Discussion

Here are some thoughts on what doing the thing means and about other things we want to do now.

5 Conclusion

We did a thing that is very meaningful because reasons.

References

[1] F. Wolfsperger, F. Meyer, and M. Gilgien. The snow-friction of freestyle skis and snowboards predicted from snow physical quantities. *Frontiers in Mechanical Engineering*, 7, 2021.

[2] F. Wolfsperger, F. Meyer, and M. Gilgien. Towards more valid simulations of slopestyle and big air jumps: Aerodynamics during in-run and flight phase. *Journal of Science and Medicine in Sport*, 24, 2021.

A Here is an appendix

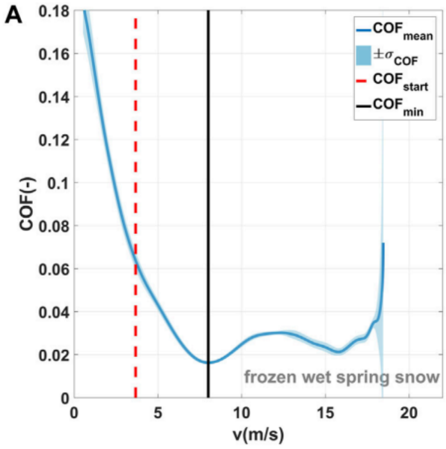

Figure 1: Mean coefficient of friction μ vs speed v for skiers on frozen wet spring snow (source: Fig. 5 of [1]).

Here are some extra details about things.