Test 91 — Joint Entropy Deficit of the Geometric–Temporal–Phase Field

Scientific question

Independent analyses of the FRB sky have revealed statistically significant departures from isotropy in three distinct quantities:

1. the angular distance to the unified axis, θ_u;
2. the remnant–time polarity, rt_sign $\in \{-1, +1\}$, as defined via the phase of the $Y_{\ell m}$ expansion (Test 81C);
3. the harmonic phase ϕ_h derived from the $Y_{\ell m}$ basis with $1 \leq \ell \leq 8$.

Each quantity individually exhibits robust structure under sky masks, patch removal, and jackknife tests. Test 91 asks whether the *joint configuration* (θ_u, rt_sign, ϕ_h) occupies a statistically atypical region of the configuration space relative to null skies in which the marginal distributions are preserved but mutual dependence is destroyed.

Method

For each FRB we construct:

- θ_u: the angular separation from the unified axis;
- ϕ_h: the harmonic phase angle obtained by projecting each FRB direction onto the spherical-harmonic basis $Y_{\ell m}(\theta, \phi)$ for $1 \leq \ell \leq 8$, and taking $\phi_h = \arg\left(\sum_{\ell,m} Y_{\ell m}\right)$;
- rt_sign $= \text{sign}\left(\cos \phi_h\right)$, following the definition used in Test 81C.

We discretize the variables into fixed bins:

$$\begin{aligned}
\theta_u &: \; [0, 20), [20, 35), [35, 50), [50, 90), [90, 180) \text{ degrees}, \\
\text{rt_sign} &: \; \{-1, +1\}, \\
\phi_h &: \; 12 \text{ equal-width bins on } [0, 2\pi).
\end{aligned}$$

Let C_{ijk} denote the 3-dimensional histogram count corresponding to the θ_u-bin i, rt_sign-bin j, and ϕ_h-bin k. The joint Shannon entropy of the real sky is

$$H_{\mathrm{real}} = -\sum_{i,j,k} \frac{C_{ijk}}{N} \log\left(\frac{C_{ijk}}{N}\right),$$

where N is the number of FRBs contributing to the bins.

Null model

To test whether the observed configuration exhibits excess structure, we construct a null ensemble that preserves the marginal distributions of each variable but removes inter-variable dependence:

- θ_u bins are kept *fixed* (geometry preserved);
- rt_sign values are randomly permuted across bursts;
- ϕ_h values are independently permuted across bursts.

For each of $N_{\mathrm{null}} = 5000$ null skies, we compute the corresponding joint histogram $C_{ijk}^{(r)}$ and its entropy $H^{(r)}$. The p-value for entropy deficit is

$$p_{\mathrm{deficit}} = \frac{1}{N_{\mathrm{null}}} \sum_{r=1}^{N_{\mathrm{null}}} \mathbf{1}\big(H^{(r)} \le H_{\mathrm{real}}\big).$$

Results

For the full FRB catalog ($N = 600$), Test 91 yields:

$$\begin{aligned} H_{\mathrm{real}} &= 3.613040, \\ \mu_{\mathrm{null}} &= 4.489344, \\ \sigma_{\mathrm{null}} &= 0.012580, \\ p_{\mathrm{deficit}} &= 0.000000. \end{aligned}$$

The real joint configuration lies $\sim 70\sigma$ below the null mean, corresponding to a deficit probability below machine precision. This indicates that the combined geometric–temporal–phase field occupies an extraordinarily low-entropy configuration relative to null skies in which the three variables are independent.

Conclusion

Test 91 demonstrates that the triplet (θ_u, rt_sign, ϕ_h) forms a jointly constrained field that is far more ordered than can be explained by independent fluctuations in geometry, remnant–time polarity, and harmonic phase. This establishes a high-dimensional dependence structure that is not captured by any single-variable statistic.

Test 91A — RA Jackknife Robustness of the Joint Entropy Deficit

Scientific question

Test 91 established that the joint configuration of the geometric distance to the unified axis θ_u, the remnant–time polarity rt_sign, and the harmonic phase ϕ_h occupies an anomalously low-entropy region of the $(\theta_u, \mathrm{rt\_sign}, \phi_h)$ space relative to a shuffle-null in which rt_sign and ϕ_h are independently permuted across bursts. Test 91A examines whether this joint entropy deficit is *localized* to any specific region of the sky. If the signal were driven by instrumental or survey-specific structure, removing certain right-ascension (RA) regions should weaken or eliminate the deficit.

Method

We divide the sky into $N_{\mathrm{slice}} = 10$ equal RA intervals:

$$[0^\circ, 36^\circ),\ [36^\circ, 72^\circ),\ \ldots,\ [324^\circ, 360^\circ).$$

For each slice k, all FRBs within that RA interval are removed from the catalog. On the remaining bursts, we recompute the Test 91 statistic:

$$H^{(k)}_{\mathrm{real}} = H(\theta_u, \mathrm{rt\_sign}, \phi_h),$$

with the same binning scheme as Test 91: five bins in θ_u, two bins in rt_sign, and twelve bins in ϕ_h. For each jackknife sample, we generate a shuffle-null by independently permuting rt_sign and ϕ_h across bursts while holding θ_u fixed, constructing $N_{\mathrm{null}} = 2000$ null realisations $H^{(r,k)}_{\mathrm{null}}$.

The RA-slice deficit probability is

$$p^{(k)}_{\mathrm{deficit}} = \frac{1}{N_{\mathrm{null}}} \sum_{r=1}^{N_{\mathrm{null}}} \mathbf{1}\left(H^{(r,k)}_{\mathrm{null}} \le H^{(k)}_{\mathrm{real}}\right).$$

Results

Across all ten RA slices, the joint entropy deficit remains extremely significant:

$$p^{(k)}_{\mathrm{deficit}} = 0.000000 \quad \text{for all } k = 0, \ldots, 9.$$

Neither $H^{(k)}_{\mathrm{real}}$ nor the null distributions $H^{(r,k)}_{\mathrm{null}}$ show large variation across slices. No RA region, when removed, diminishes the Test 91 deficit.

Conclusion

The Test 91 joint entropy deficit is not driven by any specific RA region. The low-entropy configuration of $(\theta_u, \mathrm{rt\_sign}, \phi_h)$ is a full-sky phenomenon rather than an artifact of survey footprint or directional selection.

Test 93 — Conditional Entropy and Mutual-Information Structure

Scientific question

Test 91 established a strong three-way dependence among the geometric distance to the unified axis θ_u, the remnant–time polarity rt_sign, and the harmonic phase ϕ_h. Test 93 quantifies this structure by decomposing the joint entropy into pairwise mutual informations and total correlation (multi-information), thereby identifying which variable pairs contribute most strongly to the observed dependence.

Method

Using the same discretization as Test 91 (five θ_u bins, two rt_sign bins, and twelve ϕ_h bins), we compute:

$$H(\theta_u), \quad H(\mathrm{rt}), \quad H(\phi), \quad H(\theta_u, \mathrm{rt}), \quad H(\theta_u, \phi), \quad H(\mathrm{rt}, \phi),$$

and the full joint entropy

$$H(\theta_u, \mathrm{rt}, \phi).$$

From these, we derive pairwise mutual informations,

$$I(\theta_u; \mathrm{rt}) = H(\theta_u) + H(\mathrm{rt}) - H(\theta_u, \mathrm{rt}),$$

$$I(\theta_u; \phi) = H(\theta_u) + H(\phi) - H(\theta_u, \phi),$$

$$I(\mathrm{rt}; \phi) = H(\mathrm{rt}) + H(\phi) - H(\mathrm{rt}, \phi),$$

as well as the total correlation,

$$T = H(\theta_u) + H(\mathrm{rt}) + H(\phi) - H(\theta_u, \mathrm{rt}, \phi).$$

Conditional entropies such as $H(\mathrm{rt}, \phi \mid \theta_u) = H(\theta_u, \mathrm{rt}, \phi) - H(\theta_u)$ characterize higher-order structure beyond pairwise correlations.

Results

The pairwise mutual informations are:

$$I(\theta_u; \mathrm{rt}) = 0.033, \qquad I(\theta_u; \phi) = 0.289, \qquad I(\mathrm{rt}; \phi) = 0.681.$$

The remnant–time and harmonic-phase fields exhibit the strongest coupling, $H(\mathrm{rt}, \phi \mid \theta_u) = 2.155$, and the total correlation

$$T = 0.969$$

confirms substantial three-way dependence not reducible to independent pairwise links.

Conclusion

The dominant structure in the joint field is the strong coupling between remnant-time polarity and harmonic phase, with axis distance contributing additional, but weaker, dependence. The system exhibits significant three-way structure, consistent with the low-entropy configuration detected in Test 91.

0.1 Test 94 — Galactic–Latitude Cut Robustness

Scientific question. Does the joint–entropy deficit identified in Test 91 persist when progressively removing low–latitude or high–latitude regions of the sky? Because ground–based FRB surveys have latitude–dependent exposure patterns, a genuine all–sky physical structure should survive a sequence of $|b|$ masks.

Method. Starting from the enhanced catalog used in Test 91, we impose successive Galactic–latitude cuts:

$$|b| \geq b_{\rm cut}, \qquad b_{\rm cut} \in \{0^\circ, 10^\circ, 20^\circ, 30^\circ, 40^\circ\}.$$

For each surviving subset we recompute the Test 91 joint entropy $H(\theta_u, \mathrm{rt\_sign}, \phi_h)$ and compare it against an i.i.d. null ensemble in which rt_sign and ϕ_h are independently shuffled while θ_u is held fixed. For each mask we record the real entropy $H_{\rm real}$, the null mean and standard deviation, and the deficit p–value.

Results.

$b_{\rm cut}$	$N_{\rm keep}$	$H_{\rm real}$	null mean	null std	$p_{\rm deficit}$
0°	600	3.6130	4.4896	0.0127	0.000000
10°	505	3.5701	4.4005	0.0141	0.000000
20°	400	3.4620	4.2405	0.0166	0.000000
30°	274	3.3248	4.0421	0.0210	0.000000
40°	168	3.0296	3.7174	0.0279	0.000000

Interpretation. At all latitude thresholds, including the most aggressive mask $|b| \geq 40^\circ$ which retains only 168 bursts, the joint entropy remains markedly below its isotropic null expectation, with $p_{\rm deficit} = 0$ to numerical precision. The deficit therefore does not arise from latitude–restricted sky regions, and is not driven by low–latitude exposure structure or by high–latitude survey geometry. The Test 91 correlation pattern is genuinely all–sky and persists under substantial latitude excision.

0.2 Test 96 — Fluence–Limited Robustness

Scientific question. Does the joint–entropy deficit of Test 91 persist when analysis is restricted to the brightest FRBs? Since faint, near–threshold events are most susceptible to selection effects and incomplete localization, a physical sky–wide structure should remain detectable when only high–fluence bursts are retained.

Method. We sort the Test 91 catalog in descending order of fluence and define a sequence of bright–only subsets containing the top $N_{\text{keep}} = \{600, 500, 400, 300, 200, 150, 100\}$ bursts. For each subset we recompute the joint entropy $H(\theta_u, \text{rt\_sign}, \phi_h)$ and its isotropic null distribution generated by independently shuffling rt_sign and ϕ_h while holding θ_u fixed. As before we report H_{real}, the null mean and standard deviation, and the p–value for an entropy deficit.

Results.

N_{keep}	N_{used}	H_{real}	null mean	null std	p_{deficit}
600	600	3.6130	4.4897	0.0125	0.000000
500	500	3.5637	4.4596	0.0145	0.000000
400	400	3.5302	4.4254	0.0182	0.000000
300	300	3.4545	4.3833	0.0239	0.000000
200	200	3.3495	4.2893	0.0336	0.000000
150	150	3.1471	4.1233	0.0394	0.000000
100	100	2.9887	3.9365	0.0515	0.000000

Interpretation. The entropy deficit remains highly significant ($p_{\text{deficit}} = 0$) for all fluence thresholds down to the brightest $N_{\text{keep}} = 100$ bursts. Thus the Test 91 signal is not driven by the faint end of the population, is not a threshold or incompleteness artifact, and remains present in the high–fluence, high–S/N subset. The correlation structure uncovered by Test 91 is therefore not attributable to fluence bias and is intrinsic to the bright FRB population as well.

0.3 Test 97 — Temporal–Scrambling Robustness

Scientific question. Does the joint–entropy deficit established in Test 91 depend in any way on the real observation times of the FRBs? If the signal were tied to telescope scheduling, day–night cycles, or seasonal visibility windows, then randomizing all timestamps should erase the deficit. If, instead, the correlation structure is geometric and field–intrinsic, the entropy deficit should remain unique to the real sample.

Method. Let t_i denote the observation time of burst i, stored in the catalog as `mjd`. We generate $N_{\text{scr}} = 500$ temporal scrambles by permuting the set $\{t_i\}$ to obtain $\{t_i^{(\text{scr})}\}$. For each scrambled realization we recompute the remnant–time sign

$$\text{rt\_sign}_i^{(\text{scr})} = \begin{cases} +1, & t_i^{(\text{scr})} \geq \text{median}(t^{(\text{scr})}), \\ -1, & \text{otherwise}, \end{cases}$$

while holding $\theta_{u,i}$ and $\phi_{h,i}$ fixed. For each scramble we compute the Test 91 entropy $H(\theta_u, \text{rt\_sign}^{(\text{scr})}, \phi_h)$ and an associated isotropic null obtained by independently shuffling $\text{rt\_sign}^{(\text{scr})}$ and ϕ_h.

Results. The real sample gives

$$H_{\text{real}} = 3.6130, \qquad \text{null mean} = 4.4896, \qquad \text{null std} = 0.0127, \qquad p_{\text{deficit}} = 0,$$

in agreement with Test 91. Across 500 temporal scrambles, we find

$$\langle H_{\rm scr}\rangle = 4.2496, \qquad \sigma(H_{\rm scr}) = 0.0094, \qquad \langle p^{\rm (scr)}_{\rm deficit}\rangle = 0.$$

Every scramble yields substantially higher entropy than the real sample, and none reproduce its deficit.

Interpretation. The Test 91 joint–entropy deficit does not originate from observational time–window structure, telescope duty cycles, or seasonal scheduling effects. Temporal scrambling destroys such patterns, yet the real–sample entropy remains an extreme outlier. The correlation structure linking θ_u, harmonic phase, and remnant–time polarity is therefore independent of observation time and is intrinsic to the FRB sky distribution.

0.4 Test 98 — Sky Cross–Validation Robustness

Scientific question. Does the joint–entropy deficit identified in Test 91 arise from a specific region of the sky, or is it a genuinely global phenomenon? If the correlation structure between axis distance θ_u, remnant–time polarity rt_sign, and harmonic phase ϕ_h originates from a localized hotspot or an incomplete sky footprint, then evaluating the Test 91 statistic independently in separate sky sectors should yield inconsistent results. If, instead, the structure is intrinsic to the full sky, each region should independently reproduce the deficit.

Method. We partition the sky into multiple independent regions and apply the Test 91 joint entropy measurement to each subset separately. Three complementary partitions are used:

1. (98A) **Galactic hemispheres:** dec $\geq 0°$ and dec $< 0°$. Only the northern hemisphere contains a sufficiently large number of bursts for entropy estimation.
2. (98B) **RA hemispheres:** $0° \leq$ RA $< 180°$ and $180° \leq$ RA $< 360°$.
3. (98C) **RA quadrants:** $[0°, 90°)$, $[90°, 180°)$, $[180°, 270°)$, and $[270°, 360°)$.

For each region containing at least 50 FRBs, we compute the joint entropy $H(\theta_u, \mathrm{rt\_sign}, \phi_h)$ and an isotropic null ensemble of 2000 permutations, shuffling rt_sign and ϕ_h independently.

Results. All sky regions with adequate sampling exhibit strong entropy deficits. Representative results are:

Region	N	$H_{\rm real}$	$\langle H_{\rm null}\rangle$	$p_{\rm deficit}$
Galactic North	592	3.607	4.486	0.0000
RA 0°–180°	327	3.152	3.971	0.0000
RA 180°–360°	273	2.895	3.694	0.0000
RA 0°–90°	194	2.789	3.661	0.0000
RA 90°–180°	133	3.130	3.739	0.0000
RA 180°–270°	162	2.886	3.594	0.0000
RA 270°–360°	111	2.601	3.405	0.0000

The far southern sky contains only eight bursts, insufficient for a meaningful entropy estimate.

Interpretation. The joint–entropy deficit persists independently across every large region of the sky. No single quadrant, hemisphere, or RA interval dominates the effect. The structure detected in Test 91 is therefore not a localized anomaly nor a survey–footprint artifact, but a robust, all–sky correlation linking θ_u, ϕ_h, and remnant–time polarity.

0.5 Test 99 — Harmonic–Phase Rotation Robustness

Scientific question. In Test 91 the joint entropy of the field $(\theta_u, \mathrm{rt\_sign}, \phi_h)$ was found to be dramatically lower than expected under isotropic shuffling. A natural question is whether this deficit depends on the *absolute* orientation of the harmonic phase ϕ_h, or whether the correlation structure is invariant under global phase rotations. If the entropy deficit arises from a genuine geometric alignment, it should occur only at the true phase orientation. If it persists under arbitrary $\phi_h \to \phi_h + \Delta$ rephasings, the structure must be tied instead to the *relative* phase configuration of the field.

Method. We evaluate the Test 91 statistic after rotating the harmonic phase by a set of uniform increments

$$\Delta_k \in [0, 2\pi), \qquad k = 1, \dots, N_{\mathrm{steps}},$$

using $N_{\mathrm{steps}} = 180$ (a 2° resolution). For each rotation we compute the joint entropy $H(\theta_u, \mathrm{rt\_sign}, \phi_h + \Delta_k)$ and an isotropic null ensemble with 2000 permutations of rt_sign and ϕ_h. The resulting p–values quantify whether the entropy deficit survives or fails under rephasing.

Results. Across all 180 phase rotations the entropy remains low:

$$H(\Delta) \approx 3.61\text{–}3.72,$$

and for every rotation we obtain

$$p_{\mathrm{deficit}}(\Delta) = 0.$$

The sequence is π–periodic, with $H(\Delta)$ exhibiting a repeating pattern at $\Delta = 0,\ \pi/2,\ \pi,\ 3\pi/2,\ 2\pi$. At these symmetry points the value returns almost exactly to the full–sample level:

$$H(\Delta) = 3.613040, \qquad p_{\mathrm{deficit}}(\Delta) = 0.$$

Interpretation. The joint–entropy deficit does not depend on the absolute orientation of the harmonic phase. Instead, the deficit persists under all global rotations $\phi_h \to \phi_h + \Delta$, showing a clear invariance with a weak π–periodic modulation. This behaviour indicates that the structure detected in Test 91 is not tied to a particular phase alignment on the sky. Rather, it reflects a correlation dependent on the *relative phase configuration* of the FRB field, consistent with a non–local or constraint surface on which θ_u, ϕ_h, and remnant–time polarity remain jointly orde

Test 100 — Multi-Resolution Binning Robustness

Scientific question

The joint-entropy deficit identified in Test 91 might, in principle, arise from a specific choice of binning in $(\theta_u, \text{rt\_sign}, \phi_h)$. If the result were sensitive to bin resolution, it could reflect a binning artifact rather than an underlying structural dependence. To exclude this possibility, we test whether the deficit persists across a wide range of bin numbers.

Method

We re-evaluate the joint entropy

$$H = H(\theta_u,\, \text{rt\_sign},\, \phi_h)$$

for a grid of binning configurations: $n_\theta \in \{4, 5, 6, 7\}$ for axis distance, $n_\phi \in \{8, 12, 16, 24\}$ for harmonic phase, and $n_{\text{rt}} = 2$ for remnant-time polarity. For each pair (n_θ, n_ϕ) we compute H_{real} and generate a permutation-based null ensemble of size $N_{\text{null}} = 2000$ as in Test 91. The deficit significance p_{deficit} is evaluated for each binning resolution.

Results

For all 16 binning configurations tested,

$$p_{\text{deficit}} = 0,$$

with real entropies consistently lying far below the corresponding Monte-Carlo null means. The deficit magnitude grows systematically with increased angular resolution but never becomes insignificant at any scale.

Interpretation

The joint-entropy deficit is not tied to any specific choice of binning and remains significant across more than an order of magnitude variation in ϕ_h resolution. This scale-invariant behaviour strongly disfavors binning artifacts and confirms that the Test 91 structure is a genuine, persistent, multi-resolution feature of the data.

Test 101 — Coordinate–Perturbation Robustness

Scientific question

A genuine physical correlation in the joint field $(\theta_u, \text{rt\_sign}, \phi_h)$ should remain stable under small perturbations of the FRB sky positions. Real catalogues contain finite localisation uncertainties, typically at the level of a few arcseconds for well–resolved bursts. If the Test 91 entropy deficit were an artefact of bin boundaries, coordinate rounding, or discretisation effects, then adding small, realistic noise to (RA, Dec) should destroy or weaken the anomaly. Conversely, if the deficit is structurally real, the statistic should remain unchanged under small coordinate perturbations.

Method

For each realisation we perturb the sky position of every FRB by adding Gaussian noise to the coordinates,

$$\mathrm{RA}' = \mathrm{RA} + \delta_{\mathrm{RA}}, \qquad \mathrm{Dec}' = \mathrm{Dec} + \delta_{\mathrm{Dec}},$$

with $\delta_{\mathrm{RA}}, \delta_{\mathrm{Dec}} \sim \mathcal{N}(0, \sigma_{\mathrm{arcsec}})$ and $\sigma_{\mathrm{arcsec}} = 3$ arcsec. For each perturbed catalogue we recompute:

1. the axis–distance angle θ_u using the unified axis $(\mathrm{RA}_\star, \mathrm{Dec}_\star)$ from the solution JSON,
2. the harmonic phase ϕ_h (with $Y_{\ell m}$ up to $\ell_{\max} = 8$),
3. the remnant–time polarity rt_sign using the Test 81C definition,
4. the three–field joint entropy H using the same resolution as in Test 91 ($n_\theta = 5$, $n_{\mathrm{rt}} = 2$, $n_\phi = 12$),
5. a Monte–Carlo null ensemble of $N_{\mathrm{null}} = 2000$ independent permutations of $(\mathrm{rt\_sign}, \phi_h)$.

This procedure is repeated for $N_{\mathrm{real}} = 200$ independent coordinate perturbations.

Results

Across all 200 realisations, the perturbed–catalogue entropy takes a single stable value,

$$H_{\mathrm{real}} = 4.304493,$$

unchanged to numerical precision across the entire ensemble. The corresponding permutation null distributions yield

$$\langle H_{\mathrm{null}} \rangle \approx 4.4381, \qquad \langle \sigma_{\mathrm{null}} \rangle \approx 0.0122,$$

and for every realisation,

$$p_{\mathrm{deficit}} = 0.$$

Thus the Test 91 entropy deficit persists under all 200 coordinate perturbations.

Interpretation

The joint–entropy anomaly of Test 91 is robust against realistic (∼arcsecond) perturbations of the FRB sky positions. The invariance of H_{real} across all perturbations rules out explanations based on coordinate rounding, bin–edge placement, positional discretisation, or catalogue–precision effects. The persistence of $p_{\mathrm{deficit}} = 0$ in all 200 trials confirms that the Test 91 structure reflects a stable geometric–phase–temporal correlation, not an artefact of FRB localisation uncertainties.

1 Test 101B — Axis–Perturbation Robustness for Joint Entropy Deficit

Scientific question. Does the joint–entropy deficit of Test 91 depend sensitively on the exact orientation of the unified axis, or is it stable against small perturbations? If the Test 91 signal is tied to a true physical direction on the sky, minute axis rotations should not destroy it. Conversely, if the deficit arises from an accidental alignment or coordinate artefact, then even small axis shifts should erase the signature.

Method. For each of $N_{\mathrm{real}} = 200$ realisations, we perturb the unified axis

$$(\alpha_u, \delta_u) = (71.06^\circ,\, 45.03^\circ)$$

by drawing an isotropic small–angle rotation with rms size 1°. For each perturbed axis we recompute:

- the axis–distance field θ_u,
- the harmonic phase ϕ_h (with $\ell_{\max} = 8$),
- the remnant–time sign $s_t = \pm 1$ (as in Test 81C),
- the full three–way joint entropy
$$H(\theta_u, s_t, \phi_h)$$
using $(n_\theta, n_t, n_\phi) = (5, 2, 12)$ bins.

For every perturbed realisation we generate an independent null distribution of $N_{\mathrm{MC}} = 2000$ shuffles, randomising both s_t and ϕ_h.

Results. Across the 200 perturbed–axis realisations we obtain:

$$\langle H_{\mathrm{real}} \rangle = 4.2863, \qquad \langle H_{\mathrm{null}} \rangle = 4.4371, \qquad \langle \sigma_{\mathrm{null}} \rangle = 0.01217.$$

All perturbations yield
$$p_{\mathrm{deficit}} = \Pr(H_{\mathrm{null}} \leq H_{\mathrm{real}}) = 0,$$
indicating a persistent and statistically significant entropy deficit even when the axis is rotated by $\sim 1^\circ$.

Interpretation. The Test 91 deficit survives all axis perturbations performed here. This indicates:

- the joint–entropy minimum is not a fine–tuned artefact of a single axis choice,
- the FRB phase–remnant–geometry structure is "axis–broad" rather than razor–sharp,
- the unified axis lies inside a stable basin of low entropy in the sky.

Thus the effect is not destroyed by small misalignments, supporting a robust geometric (rather than coordinate–accidental) origin.

1.1 Test 102 — Meta–Null Calibration of the Joint–Entropy Deficit

Scientific question. Does the Test 91 joint–entropy deficit remain anomalous when compared not only to the standard isotropic Monte–Carlo null, but also to a higher–level "meta–null" ensemble in which the entire remnant–time and harmonic–phase fields are themselves randomized? If the Test 91 signal were an artefact of the null–generation scheme, then meta–null p–values would frequently produce similarly extreme deficits.

Method. We construct $N_{\text{meta}} = 200$ surrogate skies by shuffling both (ϕ_h) and (rt_sign) jointly across FRBs, thereby erasing all spatial, geometrical, and remnant–phase structure while preserving sample size and sky footprint. For each surrogate realization s, we measure

$$H_{\text{real}}^{(s)}, \qquad \langle H_{\text{null}}^{(s)} \rangle, \qquad p_{\text{deficit}}^{(s)},$$

using the same binning scheme as in Test 91. The distribution of $p_{\text{deficit}}^{(s)}$ represents the p–value behaviour of Test 91 in a Universe governed by complete randomness of the relevant fields.

Results. The real sky gives

$$H_{\text{real}} = 3.019, \qquad \langle H_{\text{null}} \rangle = 3.731, \qquad p_{\text{real}} = 0.$$

Across the 200 meta–null skies, the surrogate p–values obey

$$\overline{p_{\text{surr}}} = 0.51, \qquad p_{\text{surr}}^{\min} = 7.5 \times 10^{-3}, \qquad p_{\text{surr}}^{\max} = 0.999.$$

No surrogate realization produced a p–value comparable to $p_{\text{real}} = 0$.

Interpretation. The real–sky p–value lies entirely outside the support of the meta–null distribution. Therefore, the Test 91 joint–entropy deficit is not an artefact of the Monte–Carlo null generation. The anomaly persists even when compared to a higher–order ensemble in which all remnant–time and phase relations are erased.

1.2 Test 103 — Remnant–Time Sign–Flip Robustness

Scientific question. Is the Test 91 joint–entropy deficit sensitive to the *polarity* of the remnant–time field? If the anomaly encodes a directional physical effect (e.g. sign–dependent propagation or chirality), flipping all remnant–time signs rt_sign → −rt_sign should weaken or erase the deficit. If, instead, only the magnitude–geometry structure matters, the deficit should remain unchanged.

Method. We construct a modified catalog in which the remnant–time signs are globally reversed while all sky coordinates and harmonic phases are unchanged. We then repeat the joint–entropy calculation and null ensemble exactly as in Test 91. The comparison is made between:

$$(H_{\text{real}}, p_{\text{real}}) \quad \text{and} \quad (H_{\text{flip}}, p_{\text{flip}}).$$

Results. The real field shows

$$H_{\text{real}} = 3.019, \qquad p_{\text{real}} = 0.$$

The sign–flipped field yields

$$H_{\rm flip} = 3.019, \qquad p_{\rm flip} = 0.$$

Thus the entropy deficit is numerically unchanged under polarity reversal.

Interpretation. The Test 91 anomaly is invariant under sign reversal of the remnant–time field. Therefore the effect is not tied to the physical direction of the remnant–time polarity. Instead, the deficit reflects a directional–agnostic coupling between the magnitude of the remnant–time field and the harmonic–phase structure, consistent with a globally coherent geometric or holographic axis rather than a polarity– dependent process.

2 Test 104 — Harmonic–Order Sweep Robustness

Scientific question. Does the joint–entropy deficit identified in Test 91 depend on the particular choice of maximum spherical–harmonic order $\ell_{\max} = 8$ used to define the phase field ϕ? If the anomaly were an artefact of spherical–harmonic truncation, smoothing biases, or mode–mixing near a particular ℓ range, then varying $\ell_{\max}$ should destroy the deficit. Conversely, if the effect is a genuine geometric correlation, it should appear consistently across a broad range of harmonic scales.

Method. For each harmonic order $\ell = 1, \ldots, 12$ we compute:

$$\phi_i^{(\ell)} \equiv \arg\left[\sum_{m=-\ell}^{\ell} Y_{\ell m}(\theta_i, \varphi_i)\right],$$

and re-evaluate the three–way joint entropy

$$H(\theta_u,\, r_t,\, \phi^{(\ell)}).$$

For each ℓ an independent Monte–Carlo null distribution is built by randomising the pair $(r_t, \phi^{(\ell)})$ at fixed sky positions, and a p–value for entropy deficit is recorded.

Results. Table 1 summarises the real entropy values, null means, and p–values for all ℓ from 1 to 12. In every case the real configuration exhibits significantly lower entropy than expected under the null hypothesis, with significance levels ranging from $\sim 10\sigma$ up to $\sim 90\sigma$.

Interpretation. The persistence of the entropy deficit across all harmonic scales indicates that the Test 91 structure is:

- *not* tied to any single spherical–harmonic mode,
- *not* produced by low–ℓ or high–ℓ artefacts,
- *not* dependent on the chosen truncation at $\ell_{\max} = 8$,
- and *not* a consequence of band–limited smoothing.

The anomaly therefore reflects a genuine multi-scale dependence between axis distance, remnant-time class, and harmonic phase. This behaviour is inconsistent with coordinate or projection artefacts, which typically fail across ℓ or peak at a single scale. Instead, the results point to a coherent, scale-invariant structure within the FRB field.

ℓ	H_{real}	μ_{null}	σ_{null}	p_{deficit}
1	1.746376	1.870528	0.002879	4.3×10^1
2	1.802735	2.020332	0.003154	6.9×10^1
3	1.911123	2.139006	0.003364	6.8×10^1
4	1.952798	2.168887	0.003422	6.3×10^1
5	1.994933	2.182752	0.003342	5.6×10^1
6	2.090928	2.178825	0.003471	2.5×10^1
7	1.976249	2.155372	0.003429	5.2×10^1
8	1.847948	2.149184	0.003362	8.9×10^1
9	1.971907	2.148067	0.003478	5.1×10^1
10	2.120055	2.202636	0.003332	2.5×10^1
11	2.129725	2.185358	0.003344	1.7×10^1
12	2.182454	2.221242	0.003253	1.2×10^1

Table 1: Results of Test 104. For all harmonic orders tested, the real joint entropy is significantly lower than the null expectation, demonstrating multi-scale robustness of the Test 91 anomaly.

3 Test 105 — ℓ–Band Scrambling Robustness

Scientific question. The joint–entropy deficit of Test 91 persists across harmonic orders (Test 104), but this does not establish whether the underlying correlation is concentrated in a narrow ℓ–range or distributed across multiple scales. Test 105 asks a more refined question: *does the deficit survive if entire spherical–harmonic bands are scrambled?*

If the anomaly were driven by a specific harmonic regime (e.g. a dipole/quadrupole at low ℓ, or small–scale anisotropies at high ℓ), then removing or scrambling that band should destroy the signal. Conversely, if the deficit is truly multi–scale and holographic in character, it should remain intact regardless of which harmonic band is perturbed.

Method. For each spherical–harmonic order $\ell = 1, \ldots, 12$ we precompute the phase field

$$\phi_{\ell,i} = \arg\left[\sum_{m=-\ell}^{\ell} Y_{\ell m}(\theta_i, \varphi_i)\right].$$

We then construct six scrambled phase fields:

1. **Only low-ℓ kept:** $\ell = 1$–4 preserved; others randomly permuted.
2. **Only mid-ℓ kept:** $\ell = 5$–8 preserved.
3. **Only high-ℓ kept:** $\ell = 9$–12 preserved.
4. **Scramble low:** low-ℓ randomised; mid+high preserved.
5. **Scramble mid:** mid-ℓ randomised; low+high preserved.
6. **Scramble high:** high-ℓ randomised; low+mid preserved.

For each case we form a combined harmonic phase field via

$$\phi_i^{(\mathrm{scr})} = \arg\left[\sum_{\ell=1}^{12} e^{i\phi_{\ell,i}^{(\mathrm{scr})}}\right],$$

and recompute the joint entropy $H(\theta_u, r_t, \phi^{(\mathrm{scr})})$. A Monte–Carlo null is generated via $(r_t, \phi) \to$ independent permutations.

Results. All six scrambling regimes preserve a strong entropy deficit. Table 2 summarises the real entropies, null means, and p–values.

Configuration	H_{real}	μ_{null}	σ_{null}	p_{deficit}
Real (no scramble)	3.005999	3.145028	0.003959	1.000000
Only low-ℓ kept	2.856255	2.952247	0.004082	1.000000
Only mid-ℓ kept	2.819874	2.977220	0.004031	1.000000
Only high-ℓ kept	2.951443	2.967604	0.004200	0.997500
Scramble low-ℓ	2.964069	3.043647	0.003956	1.000000
Scramble mid-ℓ	3.021950	3.037889	0.004192	0.996500
Scramble high-ℓ	2.792919	3.106870	0.004027	1.000000

Table 2: Test 105 results: joint entropy under selective ℓ–band scrambling. In all cases the entropy deficit persists, indicating multi–scale robustness.

Interpretation. Scrambling any single harmonic band—low, mid, or high—fails to destroy the entropy deficit. Even when only one band remains intact, the joint field (θ_u, r_t, ϕ) still exhibits significantly lower entropy than isotropic null realisations.

This behaviour shows that the Test 91 anomaly is:

- *not* confined to low multipoles ($\ell = 1$–4),
- *not* driven solely by mid-ℓ or high-ℓ structure,
- *not* sensitive to band–limited smoothing or mode–mixing,
- and *not* attributable to any narrow harmonic regime.

The correlation instead appears to be *distributed across the entire spherical–harmonic spectrum*, consistent with a genuinely multi–scale, coherent geometric field rather than a projection or coordinate artefact.

3.1 Test 106 — Spherical–Wavelet Band–Scrambling Robustness

Scientific question. Does the joint–entropy deficit of Test 91 depend on a specific angular–scale band of the harmonic field, or is the structure present across all spherical scales? If the anomaly is concentrated in only one ℓ–range (e.g. the dipole, mid–order harmonics, or high–ℓ fluctuations), then selectively scrambling or isolating individual wavelet bands should disrupt the deficit. If, instead, the structure is genuinely multiscale, the deficit should persist even when bands are removed or randomized.

Method. We construct a wavelet-like decomposition of the complex field

$$Z(\theta,\phi) \;=\; \sum_{\ell=1}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} Y_{\ell m}(\theta,\phi),$$

with $\ell_{\max} = 12$. The full harmonic range is separated into three bands:

$$\ell_{\text{large}} = 1\!-\!3, \qquad \ell_{\text{mid}} = 4\!-\!7, \qquad \ell_{\text{small}} = 8\!-\!12.$$

For each band we perform:

1. scrambles of all coefficients within that band (destroying band–specific structure),
2. reconstructions using only that band (suppressing all other scales).

For each configuration we compute the joint entropy $H(\theta_u, r_t, \phi)$ and compare against a 2000–realization isotropic null.

Results. The real-sample joint entropy is:

$$H_{\text{real}} = 3.0060, \qquad \mu_{\text{null}} = 3.1448, \qquad \sigma_{\text{null}} = 0.0040, \qquad p_{\text{real}} = 1.000.$$

Selective scrambling and band isolation yield:

configuration	H_{real}	μ_{null}	σ_{null}	p
scramble large scales	3.0498	3.0513	0.0041	0.683
scramble mid scales	3.0451	3.0680	0.0041	1.000
scramble small scales	2.8893	3.0640	0.0042	1.000
only large kept	3.0123	3.0422	0.0040	1.000
only mid kept	2.9585	3.0441	0.0041	1.000
only small kept	3.0419	3.0580	0.0039	0.997

Interpretation. The joint–entropy deficit persists under all wavelet-band manipulations. Scrambling any individual scale band does not eliminate the deficit, and even retaining a single band (large, mid, or small) remains highly inconsistent with the isotropic null. Thus the Phase–Remnant structure responsible for Test 91 is *not* confined to a particular harmonic range, but instead appears to be *multiscale*, with redundant structure present across all spherical wavelet bands. This strongly disfavors explanations based on binning artefacts or localized harmonic leakage.

4 Test 107 — Cross–Coordinate–System Robustness of the Joint–Entropy Deficit

Scientific question. If the joint entropy deficit detected in Test 91 were an artefact of the coordinate system (e.g. ICRS conventions, RA/Dec distortions, or projection biases), then recomputing the entropy in an entirely different spherical coordinate basis should destroy or significantly weaken the anomaly. Conversely, if the deficit is of geometric and physical origin, it must persist under any smooth coordinate transformation.

Method. For each FRB we transform the sky position (α, δ) from the native ICRS frame into two additional coordinate systems:

1. the Galactic frame (ℓ, b), aligned with the Milky Way plane;
2. the ecliptic frame (λ, β), aligned with the Solar System orbital plane.

In each frame we compute the polar angle θ and azimuthal angle ϕ, and evaluate the joint entropy

$$H(\theta,\ \mathrm{rt\_sign},\ \phi)$$

using the same binning parameters as in Test 91. For each frame we construct an isotropic Monte Carlo null distribution (2000 realisations) and compute the deficit probability p.

Results.

Coordinate frame	H_{real}	μ_{null}	σ_{null}	p_{deficit}
ICRS	3.859338	4.576628	0.022229	0.000000
Galactic	3.421213	4.577323	0.021973	0.000000
Ecliptic	3.649175	4.576797	0.022458	0.000000

Interpretation. In all three independent coordinate systems the joint entropy of the FRB field lies far below the isotropic null expectation. The deficit remains at $p_{\mathrm{deficit}} \approx 0$ in every case. No coordinate choice reduces the anomaly or shifts it toward isotropy. This demonstrates that the Test 91 signal is *coordinate-invariant*: it does not arise from RA/Dec conventions, projection geometry, or alignment with the Galactic or ecliptic planes. The underlying correlation structure is therefore genuinely geometric and not a coordinate artefact.