Semantic aspects of modality

Viola Schmitt

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1 Introduction

Declarative sentences provide us with information... about what the world is like, i.e. about what's going on in the world around us.

(1) a. *Carlos is eating 30 steaks!*
    b. *Carlos is sick.*
    c. *There is an alligator in the hallway.*

However, sometimes this is not so straightforward. Some declaratives don' tell us straightforwardly about what's going on... but rather, about possibilities or necessities. These contain modal expressions.

(2) a. *Carlos must have eaten 30 steaks!*
    b. *Carlos might eat 30 steaks!*
    c. *Carlos can ate more 30 steaks!*
    d. *Carlos probably ate more 30 steaks!*
    e. *Carlos has to eat these 30 steaks, or else...*

Our goal in this course: Find out more about the meaning of these expressions. What are we actually talking about when using the sentences in (2)? What is the information conveyed by such sentences?

What we will do first: Put this task in perspective about what we might already know about the meanings of NL expressions – and get some background that we need later on.

1.1 A very straightforward notion of meaning: Reference to things around us

One of the basic questions of semantics: What is the meaning of particular NL expressions? And what is the range of meanings for NL expressions, generally?

What we can all agree on: Whatever meanings are, the meanings of complex expressions are derived compositionally.

(3) A
   B   C
$$[\![A]\!] = [\![B]\!] \bullet [\![C]\!]$$

explanatory adequacy . . .

But what are these meanings? And what can NL express? All we know...

- meanings must be intersubjectively accessible
- meanings must be generalisable

(4) a. *Some boy is eating 30 steaks.*
b. *Every boy is eating 30 steaks.*

1.1.1 Extensions

Can we identify meaning with reference to objects around us – with reference to the facts of our world? We usually refer to this aspect of meaning as extensions.

- clear intuitions
- no problem with intersubjectivity
- no problem (??) with generalisation

(5) a. *Carlos smokes.*
b. *Carlos lives next to Jovana.*
c. *Every boy smokes.*
d. *Some boy smokes.*
e. *No boy smokes.*

1.1.2 Examples

Referential DPs

(6) a. $[\![Carlos]\!]$ = Carlos — individual
b. $[\![Jovana]\!]$ = Jovana — individual

Predicates (of individuals)

(7) a. $[\![smokes]\!] = \{x_e : \text{x smokes}\}$ — set of individuals
$[\![smokes]\!] = \lambda x_e.x$ smokes — characteristic function of that set
b. $[\![lives\text{-}next\text{-}to]\!] = \{\langle x_e, y_e\rangle$: y lives next to x $\}$ — set of individuals
$[\![lives\text{-}next\text{-}to]\!] = \lambda x_e.\lambda y_e.y$ lives next to y — curried function on basis of this set

Declaratives

(8) a. $[\![Carlos\ smokes]\!] = 1$ — truth-value
b.
c. $[\![Carlos\ lives\ next\ to\ Jovana]\!] = 1$ — truth-value

Quantifiers

(9) a. $[\![every\ boy]\!] = \lambda P_{\langle e,t\rangle}.\{x_e : x \text{ is a boy }\} \subseteq \{x_e : P(x) = 1\}$
$= \lambda P_{\langle e,t\rangle}.\forall x_e[x \text{ is a boy} \rightarrow P(x) = 1]$
b. $[\![some\ boy]\!] = \lambda P_{\langle e,t\rangle}.\{x_e : x \text{ is a boy }\} \cap \{x_e : P(x) = 1\} \neq \emptyset$
$= \lambda P_{\langle e,t\rangle}.\exists x_e[x \text{ is a boy } \wedge P(x) = 1]$
c. $[\![no\ boy]\!] = \lambda P_{\langle e,t\rangle}.\{x_e : x \text{ is a boy }\} \cap \{x_e : P(x) = 1\} = \emptyset$
$= \lambda P_{\langle e,t\rangle}.\neg\exists x_e[x \text{ is a boy } \wedge P(x) = 1]$ — characteristic function of set of sets of individuals

Quantificational determiners

(10) a. $[\![every]\!] = \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.\{x_e : P(x) = 1\} \subseteq \{x_e : Q(x) = 1\}$
$= \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.\forall x_e[P(x) = 1 \rightarrow Q(x) = 1]$
b. $[\![some]\!] = \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.\{x_e : P(x) = 1\} \cap \{x_e : Q(x) = 1\} \neq \emptyset$
$= \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.\exists x_e[P(x) = 1 \wedge Q(x) = 1]$

c. $[\![no]\!] = \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.\{x_e : P(x) = 1\} \cap \{x_e : P(x) = 1\} = \emptyset$
$= \lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.\neg\exists x_e[P(x) = 1 \wedge Q(x) = 1]$

$\Longrightarrow$ I.e. meanings are: individuals, sets thereof, truth-values

1.1.3 Rules of interpretation (so far)

Our ontology: The set of sets that contain the meanings of expressions.

(11) a. D_e := the non-empty set all all individuals
b. D_t := the set of truth-values, $\{0, 1\}$
c. $D_{\langle a,b\rangle}$:= the set of all functions from $D_{\langle a,b\rangle}$.

Set of expressions: Lexicon (constants and variables), well-formed LFs

(12) a. $\mathcal{F}$ is that function which assigns meanings to constants. $[\![]\!]$ expands this function.
b. An assignment g is a function that assigns values to variables (In general, I assume the domain of any assignment g to be $\mathbb{N}_a$)

Rules of composition:

(13) a. If α is a terminal node of type a, then, for any assignment g, $[\![\alpha]\!]^g = \mathcal{F}(\alpha)$ if α is a constant and $g(\alpha)$ if α is a variable.
b. If α is a non-branching node, β its only daughter, then, for any assignment g, $[\![\alpha]\!]^g = [\![\beta]\!]^g$
c. If α is a branching node with daughters β, γ, then, for any assignment g, if $[\![\gamma]\!]^g \in dom([\![\beta]\!]^g)$, then $[\![\alpha]\!]^g = [\![\beta]\!]^g([\![\gamma]\!]^g$. **functional application (FA)**
d. If α is a branching node with daughters β, γ, where β is an index $i \in \mathbb{N}_a$, then, for any assignment g, $[\![\alpha]\!]^g = \lambda x_a.[\![\gamma]\!]^{x/i}$ **abstraction (ABS)**

1.2 Why this isn't sufficient

Using NL, we don't only talk about the here and now: displacement

1.2.1 Temporal and modal dispacement

We can talk about situations located different from our present time: temporal displacement

(14) a. *Jovana is taking a nap.*
b. *Yesterday afternoon, Jovana was taking a nap.*
c. *Five years ago, Jovana was taking a nap.*
d. *Last Tuesday, Jovana had just taken a nap when she saw Viola trying to steal her bike.*

We can talk about situations located in a different world, i.e. embedded in circumstances:
modal displacement

(15) *Jovana is building an igloo.*

(16) *If it had snowed this morning, Jovana would be building an igloo.* Conditionals

(17) a. *It is possible that Jovana is building an igloo.* modals
b. *It is unlikely that Jovana is building an igloo.*
c. *Jovana might be building an igloo.*
d. *Jovana must be building an igloo.*
e. *Jovana is able to build an igloo.*
f. *Jovana could build an igloo.*

(18) *Carlos is the mayor of Buenos Aires.*

(19) a. *Marika believes that Carlos is the mayor of Buenos Aires.*
b. *Viola hopes that Carlos is the mayor of Buenos Aires.*
c. *Jovana is certain that Carlos is the mayor of Buenos Aires.*

$\Longrightarrow$ A number of expressions trigger displacement in terms of times or worlds. We have no way, so far, to deal with this.

1.2.2 A very concrete example why what we have is insufficient

We assumed that the meaning of a declarative is a truth-value.

(20) a. $[\![$*Viola is 1.65 m tall*$]\!] = 0$
b. $[\![$*Tony Blair is from Mars*$]\!] = 0$

This predicts...

(21) $[\![$*Marika believes that Viola is 1.65 m tall*$]\!] = [\![$*Marika believes that Tony Blair is from Mars*$]\!]$

What's wrong with this?

$\Longrightarrow$ Our old hypothesis not only insufficient to capture meanings of all NL expressions - also makes the wrong predictions!

1.3 Worlds

What we have seen is usually considered as evidence that meanings are intensions, i.e. that expressions are evaluated w.r.t. parameters (such as world parameter and a time parameter). For simplification, I only consider the world parameter.

(22) The set *W* is the set of all possible worlds.

W contains every possible world. A possible world can differ from ours only slightly (Jovana's shirt has a different colour) or more extremely (Viola is an alligator). Obviously, *W* is infinite.

Psychological reality?

The world we live in is a very inclusive thing. Every stick and every stone you have ever seen is part of it. And so are you and I. And so are the planet Earth, the solar system, the entire Milky Way, the remote galaxies we see through telescopes, and (if there are such things) all the bits of empty space between the stars and galaxies. There is nothing so far away from us as not to be part of our world. [...] But things might have been different, in ever so many ways. This book of mine might have been finished on schedule. Or, had I not been such a commonsensical chap, I might be defending not only a plurality of possible worlds, but also a plurality of impossible worlds, whereof you speak truly by contradicting yourself. Or I might not have existed at all [...]. Or there might never have been any people. Or the physical constants might have had somewhat different values, incompatible with the emergence of life. Or there might have been altogether different laws of nature; and instead of electrons and quarks, there might have been alien particles, without charge or mass or spin but with alien physical properties that nothing in this world shares. There are ever so many ways that a world might be: and one of these many ways is the way that this world is. (Lewis: 1986, 1f)

The basic idea behind intensions: The meaning of an expression is a function mapping worlds to the extensions of the expression in that world.

Example:

(23) w1: Jovana is hunting alligators and ...
w2: Jovana is an alligator and Jovana is asleep and ...

w3: Jovana is a human and Jovana is asleep and ...
w4: Jovana is a human and Jovana is having a cup of coffee ...
w5: ...

(24) *Jovana is asleep.*

(25) $[\![$*Jovana is a asleep*$]\!]$ (w1) = 0
$[\![$*Jovana is a asleep*$]\!]$ (w2) = 1
$[\![$*Jovana is a asleep*$]\!]$ (w3) = 1
$[\![$*Jovana is a asleep*$]\!]$ (w4) = 0

I.e. there is an inherent connection between intensions and extensions: An extension is what an expression picks out in a particular world. An intension is a function from worlds to such extensions.

1.3.1 How do we represent intensions?

From now on, we use the following notational conventions.

(26) a. $[\![\alpha]\!] = \lambda w \in W.$ the extension of α in w. intension of α
b. $[\![\alpha]\!]^w = [\![\alpha]\!](w) = \lambda w' \in W.$ the extension of α in $w'(w)$ = the extension of α in w
extension of α in w

Let's briefly consider some intensions. For some expressions, we find that their extension does not vary across worlds: For every world, we get the same extension.

(27) a. $[\![$*Jovana*$]\!] = \lambda w.$ Jovana.
b. $[\![$*no*$]\!] = \lambda w.\lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.\neg\exists x_e[P(x) \wedge Q(x)]$.

For some other expressions, we find that their extensions vary across worlds:

(28) a. $[\![$*smokes*$]\!] = \lambda w.\lambda x_e.x$ smokes in w
b. $[\![$*lives-next-to*$]\!] = \lambda w.\lambda x_e.\lambda y_e.y$ lives next to x in w

Particularly relevant for us: declarative intensions: propositions: Functions from worlds to truth values.

(29) $[\![$*Jovana is asleep*$]\!] = \lambda w.$ Jovana is taking a nap in w

Note: this function actually characterizes a set of possible worlds – all and only those worlds where Jovana is a asleep.

(30) $\{w :$ Jovana is asleep in $w\}$

1.3.2 Propositions

Here are some obvious applications of this new view of sentence meanings.

Word of warning: I will often used sets and their characteristic functions interchangeably. I.e. very often, when an expression denotes a function $f \in D_{\langle a,t\rangle}$, I will simply represent the denotation by the set $S = \{x_a : f(x) = 1\}$. W.r.t. propositions, I will often use (31-a) and (31-b) interchangeably.

(31) a. $\lambda w.$ Jovana is asleep in w
b. $\{w :$ Jovana is asleep in $w\}$

We first look at some configurations of declarative sentences and how they are reflected in their meaning.

Negation

(32) a. *Jovana is asleep.*
b. *Jovana is not asleep*

(33) For any declarative sentence S, $\{w : [\![S]\!](w) = 1\} \cap \{w : [\![not\ S]\!](w) = 1\} = \emptyset$
(simple set talk: $[\![S]\!] \cap [\![not\ S]\!] = \emptyset$)

(34) For any declarative sentence S, if $[\![S]\!]$ is a total function $f : W \rightarrow \{0, 1\}$, then
$\{w : [\![S]\!](w) = 1\} \cup \{w : [\![not\ S]\!](w) = 1\} = W$
(simple set talk: $[\![S]\!] \cup [\![not\ S]\!] = W$)

Conjunction

(35) *Jovana is asleep and Marika is drinking beer.*

(36) For any two declarative sentence S, S', $[\![S\ and\ S']\!] = \{w : [\![S]\!](w) = 1\} \cap \{w : [\![S']\!](w) = 1\}$
(simple set talk: $[\![S\ and\ S']\!] = [\![S]\!] \cap [\![S]\!]$

Disjunction

(37) *Jovana is asleep or Marika is drinking beer.*

(38) For any two declarative sentence S, S', $[\![S\ or\ S']\!] = \{w : [\![S]\!](w) = 1\} \cup \{w : [\![S']\!](w) = 1\}$
(simple set talk: $[\![S\ and\ S']\!] = [\![S]\!] \cup [\![S']\!]$

However, we also have the intuitions about relation between sentence meanings, i.e. between propositions.

Logical consequence A proposition can follow from another proposition....

(39) a. *Jovana owns a grey dog.*
b. *Jovana owns a dog.*

(40) A proposition p follows from a proposition q iff $\{w : q(w) = 1\} \subseteq \{p : p(w) = 1\}$
(simple set talk: $q \subseteq p$)

...or from a set of propositions

(41) a. *Jovana owns exactly one alligator.*
b. *Jovana owns exactly one dog.*
c. *Jovana owns exactly one shark.*
d. *Jovana owns less than four animals.*

(42) *Jovana owns exactly three animals.*

(43) A proposition p follows from a set of propositions A iff
$\{w : \forall q[q \in A \rightarrow q(w) = 1]\} \subseteq \{w : p(w) = 1\}$
(simple set talk $\bigcap A \subseteq p$)

Consistency Propositions can be consistent....

(44) a. *Jovana owns an alligator.*
b. *Jovana was born in Florida.*

(45) A set of propositions A is consistent iff $\{w : \forall q[q \in A \rightarrow q(w) = 1]\} \neq \emptyset$
(simple set talk: $\bigcap A \neq \emptyset$)

or not:

(46) a. *Jovana was born in Florida.*
b. *Jovana was born in Montana.*

1.3.3 Rules of interpretation

We now need so expand our system for interpretating LFs s as to include intensions. [1]

Our ontology:

(47) W is the set of possible worlds
a. D_e := the non-empty set all all individuals
b. D_t := the set of truth-values, $\{0, 1\}$
c. $D_{\langle a,b\rangle}$:= the set of all functions from $D_{\langle a,b\rangle}$
d. $D_{\langle s,a\rangle}$:= the set of all functions from W to D_a

Set of expressions: Lexicon (constants and variables), well-formed LFs

(48) a. $\mathcal{F}$ is that function which assigns meanings to constants. $[\![]\!]$ expands this function.
b. An assignment g is a function that assigns values to variables (In general, I assume the domain of any assignment g to be $\mathbb{N}_a$)

Rules of composition.

(49) a. If α is a terminal node of type a, then, for any assignment g, $[\![\alpha]\!]^g = \mathcal{F}(\alpha)$ if α is a constant and $g(\alpha)$ if α is a variable.
b. If α is a non-branching node, β its only daughter, then, for any assignment g, $[\![\alpha]\!]^g = [\![\beta]\!]^g$

The only thing that is new is that we now have two versions of FA:

Idea: For some expressions α, $\alpha = \{\beta, \gamma\}$, want the intension of α to map worlds w to the extension of β in that world combined with the <u>extension</u> of γ in that world.

(50) If α is a branching node with daughters β, γ the set of its daughters, then, for any world w and assignment a: If $[\![\beta]\!]^{w,g}$ is a function whose domain contains $[\![\gamma]\!]^{w,g}$, then $[\![\alpha]\!]^{w,g} = [\![\beta]\!]^{w,g}([\![\gamma]\!]^{w,g})$
functional application

For some other expressions α, $\alpha = \{\beta, \gamma\}$, we want β and γ we want the intension of α to map worlds w to the extension of β in that world combined with the <u>intension</u> of γ in that world.

(51) If α is a branching node with daughters β, γ the set of its daughters, then, for any world w and assignment a: If $[\![\beta]\!]^{w,g}$ is a function whose domain contains $[\![\gamma]\!]^{g}$, then $[\![\alpha]\!]^{w,g} = [\![\beta]\!]^{w,g}([\![\gamma]\!]^{g})$

[1] I actually assume representation of worlds in the object language, which in fact means I use an extensional system – but you don't need to worry about this