Semantic aspects of modality

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2 Modals and conversational backgrounds

[mostly based on Kratzer (1977, 1981)/revised versions in Kratzer (2012) and Kratzer (1991)]

point of departure: Some expressions in NL convey possibility or necessity: modal expressions

(1) a. *Carlos* **must** *have committed a murder! Look at his bloody hands!*
b. *Good Lord! Carlos* **may** *have committed a murder!*
c. *Carlos* **can** *commit a murder without flinching.*
d. *As a contract killer, Carlos is* **required** *commit 6 murders a week.*
e. ...

intuitive characterisation of what's going on here?

Languages often have modal expressions of different syntactic categories, e.g. English (non-exhaustive)

(2) a. *Carlos* **must/** *have eaten 20 steaks!*
b. *Carlos* **can/should** *eat 30 steaks in an hour.*

(3) a. *Carlos is* **able** *to eat 30 steaks in an hour.*
b. *It is* **impossible** *for anyone to eat 30 steaks in an hour.*
c. *It is* **possible** *that Carlos is hiding a steak from me.* adjectives

(4) *Carlos is* **probably/possibly** *hiding a steak from me.* adverbs

(5) *This book is unread-***able**. affixes

Sometimes, no overt lexical modal element (?):

(6) *Niemand isst 30 Steaks in 10 Minuten*.
Nobody eats 30 steaks in 10 minutes
'Nobody is able to eat 30 steaks in 10 minutes. German, adapted from Kratzer (1981)

Our basic question: What is the meaning of modals??

Kratzer's point (partially based on Lewis (1973)): Modals involve reasoning on the basis of premises.

- modals sentences are evaluated w.r.t. two parameters:
  Parameter-setting 1 determines the set A of worlds we should generally be looking at
  Parameter-setting 2 orders the worlds in A, telling us how 'relevant' they are for our evaluation
- modals involve quantification over possible worlds: Require that a given proposition holds in all or some of the worlds specified by the parameter-settings

2.1 'Flavours' of modality

Several modal expressions seem to have more than one meaning, e.g. *must*

(7) a. *All Maori children* **must** *learn the names of their ancestors.* 'deontic'
b. *The ancestors of the Maori* **must** *have arrived from Tahiti.* 'epistemic'
c. *If you* **must** *sneeze, at least use a handkerchief.* 'dispositional'
d. *When Kahukura-nui died, the people of Kahungo said: Rakaikpaka* **must** *be our chief.* 'bouletic'

Kratzer (1977)

2.1.1 Ambiguity?

Is *must* mulitply ambiguous? Common core of all the examples in (7)- consider paraphrases in (8).

(8) a. **In view of what the duties are**, *the Maori children must learn the names of their ancestors.*
b. **In view of what is known**, *the Maori must have arrived from Tahiti.*
c. *If,* **in view of what your dispositions are**, *you must sneeze, at least use a handkerchief.*
d. **In view of what is good** *for us, Rakaikpaka must be our chief.*

Hypothesis (assuming modal with sentential scope, glossing over 'necessary"):

(9) a. $[\![must_1]\!] \approx \lambda w.\lambda p_{\langle s,t\rangle}.p$ is necessary in w in view of the duties in w
b. $[\![must_2]\!] \approx \lambda w.\lambda p_{\langle s,t\rangle}.p$ is necessary in w in view of what is known in w

Implausible – what would be the role of *view of...* in (8)?

Kratzer (1977, 1981): *must* makes the same semantic contribution in all cases, but is evaluated w.r.t. to different conversational backgrounds ('CBs'), i.e. different premises. CBs can be made explicit, cf. (8).

This does not only hold for necessity modals like *must*, but also possibility modals, e.g. (10):

(10) a. **In view of what we know about his behavior**, *Carlos may have committed a murder.*
b. **In view of the house rules**, *Carlos may go to bed after 10.*
c. ...

Therefore: Modals are always evaluated w.r.t. CBs. CBs may differ, but the modal always makes the same contribution – relative to CB $\Longrightarrow$ **analogy to attitude verbs?**
$\Longrightarrow$ **which role do premises play w.r.t. other expressions?**

CBs can be overtly represented (see above) or not, (11)– in which case it must be contextually provided (we return to this).

(11) a. *All Maori children* **must** *learn the names of their ancestors.*
b. *The ancestors of the Maori* **must** *have arrived from Tahiti.*
c. *If you* **must** *sneeze, at least use a handkerchief.*
d. *When Kahukura-nui died, the people of Kahungo said: Rakaikpaka* **must** *be our chief.*

2.1.2 Lexical entries for *must* and *may* (first attempt)

Point: Modals require a certain relation to hold between a certain proposition and a certain CB.

(12) a. *In view of what I know about her habits, Jovana must be asleep.*
b. *In view of the rules of this house, Jovana must eat a piece of cake in the morning.*

(13) a. *In view of what I know about her habits, Jovana may be asleep.*

b. *In view of the rules of this house, Jovana may eat a piece of cake.*

MUST (necessity modals): (14-a) and (14-b) paraphrase (12-a) and (12-b) uttered in world *w*.

(14) a. It follows from what the speaker knows in *w* that Jovana is asleep.
b. It follows from what the house rules say in *w* that Jovana eats a piece of cake.

Step 1: Conversational background In (12-a): What the speaker knows in world of evaluation (about J's habits) – true beliefs the speaker holds.

(15) It follows from **what the speaker knows in** *w* that Jovana is asleep.

For each world, we get a set of propositions, (16).

(16) [[in view of what I know about her habits]] = $\lambda w.\lambda p.$ the speaker knows p in w

CB in (12-b): what rules the house rules say in the world of evaluation – again a set of propositions.

(17) It follows from **what the house rules say** in *w* that Jovana eats a piece of cake.

(18) [[In view of the rules of this house]] = $\lambda w.\lambda p.$ the house rules say p in w

I.e. a CB is always a set of propositions, (19).

(19) A conversational background, relative to a world *w*, is a set $A \subseteq \wp(W)$

Step 2: Proposition: What is the proposition we evaluate w.r.t. the conversational background?

(20) a. *In view of what I know about her habits, Jovana must be asleep.*
b. *In view of the rules of this house, Jovana must eat a piece of cake.*

(21) a. It follows from what the speaker knows in *w* **that Jovana is asleep**.
b. It follows from the house rules in *w* **that Jovana eats a piece of cake**.

Proposition corresponds to what is expressed by 'rest' of the sentence , without the modal ('S'). So let's assume the modal has sentential scope. (For the moment, we ignore tense.)

(22) a. LF for (12-a): [*must* [$_S$ *Jovana be asleep*]] to be revised
[[S]] = $\lambda w.$ Jovana is asleep in w
b. LF for (12-b): [*must* [$_S$ *Jovana eat a piece of cake*]] to be revised
[[S]] = $\lambda w.$ Jovana eats a piece of cake in w

Step 3: Relation proposition – CB: How do we evaluate [[S]] w.r.t. the CB in case of *must*? It must follow from CB.

(23) a. It **follows** from what the speaker knows in *w* that Jovana is asleep.
b. It **follows** from what the house rules say in *w* that Jovana eats a piece of cake.

Accordingly (I switch to set language to simplify):

(24) a. [[*In view of what I know about her habits, Jovana must be asleep.*]]^w = 1 iff
$\bigcap\{p$: the speaker knows p in $w\} \subseteq \{w''$: Jovana is asleep in $w''\}$
b. [[*In view of the rules of this house, Jovana must eat a piece of cake.*]]^w = 1 iff
$\bigcap\{p$: the house-rules say p in $w\} \subseteq \{w''$: Jovana eats a poc in $w''\}$
$\forall w'[w' \in \bigcap\{p$: house-rule says p in $w\} \rightarrow w' \in \{w''$: Jovana eats poc in $w''\}]$

Note:

(25) a. $\bigcap\{p : \text{the speaker knows } p \text{ in } w\} \subseteq \{w'' : \text{Jovana is asleep in } w''\} =$
$\forall w'[w' \in \bigcap\{p : \text{speaker knows } p \text{ in } w\} \rightarrow w' \in \{w'' : \text{Jovana is asleep in } w''\}]$
b. $\bigcap\{p : \text{the house-rules say } p \text{ in } w\} \subseteq \{w'' : \text{Jovana eats a poc in } w''\} =$
$\forall w'[w' \in \bigcap\{p : \text{house-rule says } p \text{ in } w\} \rightarrow w' \in \{w'' : \text{Jovana eats poc in } w''\}]$

The extension of (12-a) is 1 in w iff the knowledge of the speaker in w is such that every world w' which is compatible with all of this knowledge is a world where Jovana is asleep. (Analogously for (12-b))

Intensions:

(26) a. $[\![\textit{In view of what I know about her habits, Jovana must be asleep.}]\!] =$
$\lambda w.\forall w'[w' \in \bigcap\{p : \text{speaker knows } p \text{ in } w\} \rightarrow w' \in \{w'' : \text{Jovana is asleep in } w''\}]$
b. $[\![\textit{In view of the rules of this house, Jovana must eat a piece of cake.}]\!] =$
$\lambda w.\forall w'[w' \in \bigcap\{p : \text{house-rule says } p \text{ in } w\} \rightarrow w' \in \{w'' : \text{Jovana eats poc in } w''\}]$

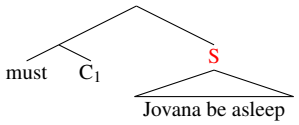
Step 4: preliminary lexical entry for *must* (to be revised!)

(i) How is the CB is represented. It does not have to be overtly realized:

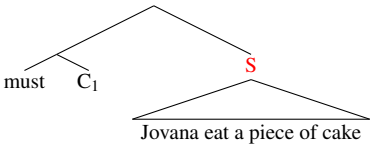
(27) a. *Jovana must be asleep.*
b. *Jovana must eat a piece of cake.*

Accordingly: Sister of modal a variable 'C', ranging over functions from worlds to sets of propositions. Variabe gets its value from the context – assignment g that maps elements of $\mathbb{N}$ to functions from worlds to sets of proposition – i.e. C, is affixed with numerical index.

(28) a. *Jovana must be asleep.*
b. LF



(29) a. *Jovana must eat a piece of cake.*
b. LF



(30): Our lexical entry for *must*. Only one entry – whether the sentence is 'epistemic' or 'deontic' etc. determined exclusively by the value assigned to the variable.

(30) **lexical entry for *must*, prelimary version**
$[\![must]\!] = \lambda w.\lambda Q_{\langle s,\langle\langle s,t\rangle,t\rangle\rangle}.\lambda p_{\langle s,t\rangle}.\forall w'[w' \in \bigcap Q(w) \rightarrow w' \in p]$

Note: *must* denotes a universal quantifier. It requires that all worlds in which the conjunction of all the propositions in the CB are true are worlds in $[\![S]\!]$ is true. I.e. analogous to *every*.

domain restrictions with other quantifiers?

MAY (possibility modals) Analogous: A certain relation must hold btw. proposition and CB

(31) a. *In view of what I know about her habits, Jovana may be asleep.*
b. *In view of the rules of this house, Jovana may eat a piece of cake.*

(32-a) and (32-b) paraphrase (31-a) and (31-b), respectively, if uttered in a world w. Only difference to *must* is relation btw. CB and $[\![S]\!]$: $[\![S]\!]$ has to be compatible with CB.

(32) a. It is **compatible** with what the speaker knows in w that Jovana is asleep.
b. It is **compatible** with what the house rules are in w that Jovana eats a piece of cake.

Accordingly, we get the sentence meanings in (33)...

(33) a. $[\![$*In view of what I know about her habits, Jovana may be asleep.*$]\!] =$
$\lambda w.\exists w'[w' \in \bigcap\{p : \text{speaker knows } p \text{ in } w\} \wedge w' \in \{w'' : \text{Jovana is asleep in } w''\}]$
b. $[\![$*In view of the rules of this house, Jovana must eat a piece of cake.*$]\!] =$
$\lambda w.\exists w'[w' \in \bigcap\{p : \text{house-rule says } p \text{ in } w\} \wedge w' \in \{w'' : \text{Jovana eats poc in } w''\}]$

from which we can derive the following lexical entry:

(34) **lexical entry for *may*, prelimary version**
$[\![must]\!] = \lambda w.\lambda Q_{\langle s,\langle\langle s,t\rangle,t\rangle\rangle}.\lambda p_{\langle s,t\rangle}.\exists w'[w' \in \bigcap Q(w) \wedge w' \in p]$

Note: *may* denotes a universal quantifier. It requires that some world in which the conjunction of all the propositions in the CB are true is a world in which $[\![$S$]\!]$ is true. I.e. it is analogous to *some*. **duals?**

2.1.3 Interim Summary

- different 'flavors' of possibility, necessity
- different 'flavors' due to different premises, i.e. conversational backgrounds
- modals always evaluated w.r.t. premises, i.e. conversational backgrounds
- no ambiguity: semantic contribution of the modal is always the same, differences in sentence meanings due to nature of conversational background
- modals require that a certain relation holds btw. proposition denoted by the part of the structure they c-command (S) and conversational background
- In the case of *must*, the proposition must follow from the conversational background. In the case of *may*, it must be compatible with the conversational background
- we could give a parallel story for other necessity and possibility modals, respectively

2.2 Why this is not sufficient...

2.2.1 Inconsistent conversational backgrounds and The good samaritan

problem 1 We assumed that *in view of...* makes the CB explicit, (35).

(35) *In view of what the law says, murderer Carlos must go to jail.*

Our analysis: (35) is true in w iff in every world w' where the conjunction of all the propositions that are part of the law in w are true, Carlos goes to jail, (36).

(36) 1 in w iff $\bigcap\{p : \text{the law says } p \text{ in } w\} \subseteq \{w : \text{murderer Carlos goes to jail in } w\}$

Assume the law says the following – it is very clear on murder, but inconsistent w.r.t. car theft:

(37) a. Everyone who commits a murder goes to jail. p1
b. Everyone who steals a car pays a fine of exactly 100 Euros. p2
c. Everyone who steals a car pays a fine of exactly 200 Euros. p3

If the world is such, then the sentences in (38-a) and (38-b) are true ... as well as many others, e.g (39):

(38) a. *In view of what the law says, murderer Carlos must go to jail.*
b. *In view of what the law says, murderer Carlos must not go to jail.*

(39) a. *In view of what the law says, murderer Carlos must buy a baby cat.*
b. *In view of what the law says, murderer Carlos must murder more people.*
c. ...

This is clearly counterintuitive!

How does the problem arise? Intersection of all propositions in CB, relative to *w* – CB is inconsistent!

(40) p1 ∩ p2 ∩ p3 = ∅

The condition introduced by *must* is that ⟦S⟧ follows from the CB....

(41) a. ∅ ⊆ {*w* : murderer Carlos goes to jail in *w*}
b. ∅ ⊆ {*w* : murderer Carlos does not go to jail in *w*}
c. ∅ ⊆ {*w* : murderer Carlos buys a baby cat in *w*}
d. ...

problem 2 Kratzer's version of a well-known puzzle from deontic logic, 'The good Samaritan'.

Assume that the law requires two things – that people don't murder one another and that murder is punished by a jail sentence:

(42) a. There is no murder. p1
b. If a murder occurs, the murderer will to jail. p2

For the moment: Conditionals as material implication, '→' (we will look at other treatments of later).

(43)

p	q	p → q
1	1	1
1	0	0
0	1	1
0	0	1

If we translate *if... then* into '→': (44) is true whenever B is true or A is false.

(44) *If A then B*

Accordingly, p2 looks as follows:

(45) {*w* : everyone who committed a murder goes to jail in *w*} (the worlds where B is true)
∪ {*w* : there is no murder in *w*} (worlds where A is false)

I.e. as opposed to problem we looked at before, our CB consistent, (46).

(46) p1 ∩ p2 = {*w* : there is no murder in *w*}

Problem: Given p1, p2 in *w* any of the sentences are true in *w*. This is clearly counterintuitive!

(47) *It is necessary that...*
a. *if a murder occurs, the murderer will go jail.*
b. *if a murder occurs, the murderer will get an award.*
c. *if a murder occurs, the murderer will marry an alligator.*
d. *if a murder occurs, the murderer.....*

Why is this? (48) must hold for sentence to be true: ⟦S⟧ true in all the worlds where no murder occurs. Any S is of form *If a murder occurs, then...*, it is true in all such worlds, because its antecedent is false.

(48) $p1 \cap p2 \subseteq [\![S]\!]$

Interim Summary Sometimes, 'parts' of the CB seem irrelevant for the evaluation of the proposition.

2.2.2 Graded modality

Observation Some expressions in natural language are gradable: Seem to make reference to degrees. Symptom: They can combine with comparative morphemes *-er/more*, similar expressions.

(49) a. *My cat is fatter than your cat.*
The degree of fatness of my cat exceeds the degree of fatness of your cat.
b. *My cat is more beautiful than your cat.*
The degree of beauty of my cat exceeds the degree of beauty of your cat.

Kratzer: Some modal expressions exhibit hallmarks of gradability.

(50) *Given the available evidence, Carlos is more likely to be the murderer (than Jovana).*

Intuitively: More of the available evidence points to Carlos being the murderer than to Jovana.

Can we model this? Let's assume available evidence is the following in *w*

(51) a. Jovana was at the place of the murder at the time of the murder. p1
b. Carlos was at the place of the murder at the time of the murder. p2
c. The murder weapon was found in Carlos' apartment. p3

Problem: According to our analysis, (50) true iff (52) holds – i.e. iff the CB is compatible with ⟦S⟧. Compatible as long as there is a world in $p1 \cap p2 \cap p3$ where Carlos is the murderer. Compatibility is not a graded concept: Either there is such a world or there isn't.

(52) $p1 \cap p2 \cap p3 \cap \{w : \text{Carlos is the murderer in } w\} \neq \emptyset$

Interim Summary Sometimes, some 'parts' of the CB seem more relevant for the evaluation of the sentence than others.

2.3 Modal base and ordering source

Observed: Two different roles of CB.

a. 'symmetric' version: ⟦S⟧ must either follow from CB (necessity modals: $\bigcap$CB $\subseteq$ ⟦S⟧) or be compatible with $\bigcap$CB (possibility modals: $\bigcap$ CB $\cap$⟦S⟧ $\neq \emptyset$)
b. 'asymmetric' version: Some 'parts' of CB are more relevant than others for evaluation of ⟦S⟧

Kratzer: The CB must have more than one function. Interpretation of modals requires two parameters. *In view of...* adjuncts can make either explicit.

a. **Parameter 1: Modal base (MB)** ≈ What the speaker knows in world of evaluation / facts in world of evaluation.
b. **Parameter 2: Ordering source (OS)** ≈ Imposes an ordering on the modal base, i.e. tells us, which worlds we should look at

2.3.1 Ordering source

Observed: Some 'parts' of CB are more relevant than others for evaluation of ⟦S⟧.

Different formulation: Some worlds in the CB are more 'relevant' than others for the evaluation of ⟦S⟧.

Informal illustration:

(53) SITUATION: I am a dictatorial club owner. Everyone who enters and leaves my club has to identify her/himself by fingerprints and the information is immediately sent to me. So far, 3 fingerprints have been recorded at the entrance (and none at the exit): Those of Carlos, those of Marika and those of Jovana.
Being a horrible person, I constantly check the premises. There are 2 rooms: the blue room and the red room. I start my tour in the red room where I see a person who looks exactly like Marika and a person who looks exactly like Jovana and noone else. I say to my aide:
Carlos must be in the blue room.

(53) intuitively true in scenario, but shouldn't be. Elements of the CB (propositions I know):

(54)
a. Exactly 3 people are in the club. p1
b. The fingerprints of these people are those of Carlos, Jovana and Marika. p2
c. My club has 2 rooms, a blue room and a red room. p3
d. There are exactly 2 people in the red room. p4
e. A person who looks like Marika is in the red room. p5
f. A person who looks like Jovana is in the red room. p6

Some of the worlds in $\bigcap$ CB will contain strange facts, e.g. Carlos had cosmetic surgery that makes him look like Jovana, someone – Hedde! – got ahold of Carlos' fingertips and used them to get into the club, etc. Therefore: ⟦S⟧ not true in all $\bigcap CB$

Our intuition: We ignore such far-fetched worlds. We only consider 'normal' worlds, i.e. worlds where noone cut of Carlos' fingertips, where he didn't get surgery etc. We can even make this explicit:

(55) *In view of what I know,* **given nothing absurd has happened***, Carlos must be in the blue room.*

How do we get rid of these 'absurd' or remote worlds? That's the role of the OS.

Formally: OS is a set A of propositions that imposes a partial ordering on a set of worlds (cf. Lewis (1981)), (56).
Informal intuition: The OS specifies an 'ideal' world, where all the propositions in A are true and then orders worlds according to how 'close' they come to this ideal.

(56) For all $w, w' \in W$, for any $A \subseteq \wp(W) : w \leq_A w'$ iff $\{p : p \in A \& w' \in p\} \subseteq \{p : p \in A \& w \in p\}$

(57) says that relative to a set of propositions A, w is a least as A-close ('$\leq_A$') as w' iff the set of propositions that are in A and hold in w' is a subset of the set of propositions that are in A and hold in w.

(57) Let $A = \{p1, p2, p3\}$, $W = \{w1, w2, w3\}$ and $p1 = \{w1\}$, $p2 = \{w1, w2\}$, $p3 = \{w1, w2, w3\}$, then $w1 \leq_A w2 \leq_A w3$

Idea: Worlds in which all the propositions in the modal base are true are ordered by the OS.

The OS with epistemic modals: OS to represents what is 'normal' (corresponds to the normal course of events) or 'stereotypical'. E.g. for (53) above, the OS would contain:

(58) a. Linguists don't have face lifts that make them look like other linguists.
b. Linguistis don't cut of other linguists' fingertips.
c. ...

In (53) the modal base would be the set I have as CB (we return to this). Accordingly: Worlds in $\bigcap$ CB would be ordered by such an OS.

The OS with deontic modals: OS is the 'deontic' or 'bouletic' or 'dispositonal' CB. Informally.

(59) a. *Carlos must go to jail.* OS: the laws of the country
b. *Jovana may get up after 10.* OS: the rules of this house
c. *Jovana can swim very fast.* OS: Jovana's dispositions
d. *Jovana must be our new leader.* OS: What is good for us

Clarification: In some cases, what we identified as the CB before acts the the OS, (60).

(60) a. **In view of the laws of the country**, *Carlos must go to jail.*
b. **In view of of the rules of this house**, *Jovana may get up after 10.*
c. ...

In other cases, what we identified as the CB before doesn't act as the OS, but as the MB, (61):

(61) **In view of what I know**, *Carlos must be in jail.*

Which kinds of CB act as OS, which as modal base? Roughly: Types of CB that can be inconsistent, will be OS (laws, rules, desires...). We will see: This is difficult to define (also.. dispositions?). Kratzer:

(62) a. CB that represents an individuals's knowledge in a world of evaluation or facts about the world $\Longrightarrow$ MB
b. CB that represents laws, rules, desires, dispositions ... $\Longrightarrow$ OS

In general: The OS is a set of propositions that will tell us which worlds in the MB are the relevant ones (...given the rules/laws/normal course of events etc. Since the rules/laws/normal course of events differ from world to world, a contextually provided OS is always world-dependent)

Before we implement this: *Which* worlds are ordered by the OS? I.e. what is the modal base?

2.3.2 The modal base

Earlier: Universal/existential quantification over worlds, CB always yields domain of quantification.

(63) a. *Jovana must be asleep.*
b. *Jovana must eat a piece of cake.*

Now: Some CBs in fact act as OS – ordering on domain of quantification – rather than domain of quantification. Others yield us domain of quantification, i.e. worlds we generally look at: MB.

So what are the worlds that are ordered by the OS? The worlds identified by the modal base. Modal base: identifies the worlds we are generally looking at (OS then imposes an ordering on them.)

Kratzer: Only two types of CB can act as MB:

(64) a. epistemic CB $\Longrightarrow$ epistemic MB
b. circumstantial CB $\Longrightarrow$ circumstantial MB

We know what the epistemic CB is: Represents (a part of) what the speakers knows in the world w of evaluation (about w) – i.e. a set of true beliefs the speaker holds in w (about w).

always the speaker? cf. Yalcin (2007)

Accordingly, epistemic MB: set of propositions.

(65) $\lambda w.\lambda p.$ the speakers knows p in w

So what is a circumstantial CB? Basically represents (a part of) the facts of the world – i.e. a set of propositions that is true in the world. Accordingly, circumstantial MB: set of propositions.

(66) $\lambda w.\lambda p.p(w)$

Difference? Kratzer tries to make point wrt: remote island-scenario.

(67) a. *There might be hydrangeas growing here.* Kratzer (1991)
'Given what I heard and read about this country, I consider it possible that we find some hydrangeas here.'
b. *Hydrangeas can grow here.* Kratzer (1991)
'Given what the soil and weather in this country are like, ?(it is possible that) if you plant hydrangeas here, they will grow.'

(68) a. MB (68-a): { that islander Peter has hydrangeas in his garden, that this country took part in a flower-competition, . . . }
b. MB (68-a): { that the soil contains sulphur, that there is more than 10 inches of rain a month, . . . }

Does this distinction make sense? Is this a distinction in terms of 'types of evidence'?

Kratzers' idea is that the distinction between epistemic and root modality lies in the MB. Epistemic modality: epistemic MB. Root modality: circumstantial MB.

(69) a. *There might be hydrangeas growing here.* epistemic MB
b. *Hydrangeas can grow here.* circumstantial MB

Simplifcation: So far, we viewed the MB, relative to a world w, as a set of propositions. Modals evaluated w.r.t. the set of worlds where all propositions in MB are true. We could just assume that knowledge/facts are represented as a conjunction of propositions – i.e. an MB will be proposition.

(70) a. epistemic MB, relative to world w: $\lambda w'.w' \in \bigcap\{p$: the speakers knows p in $w\}$
b. epistemic MB, relative to world w: $\lambda w'.w' \in \bigcap\{p : p(w) = 1\}$

2.3.3 Putting it all together

We know that every modal is evaluated w.r.t. two parameters: MB and OS. The MB gives us the worlds we are generally looking at. The OS will tell us which of these modals are the 'most relevant'.

Categorizing modals Modals can differ in three respects.

- w.r.t. modal force: Existential vs. universal quantification (over possible worlds)
- w.r.t. MB: Can be lexically specified w.r.t. type of MB (epistemic vs. circumstantial)
- w.r.t. OS: Can be lexically specified w.r.t. type of OS ('stereotypical', 'bouletic',....)

(71)

	modal force	MB	OS
must	universal	no restrictions	no restrictions
may	existential	no restrictions	no restrictions
can	existential	circumstantial	'deontic', 'teleological', ...(?)
be allowed	existential	circumstantial	'deontic'
...	...	...	...

(72) a. *Carlos must have eaten 30 steaks*
MB: epistemic, OS: 'stereotypical'
b. *Carlos must go to bed now*
MB: circumstantial, OS: 'deontic'

(73) a. *Carlos may have eaten 30 steaks*
MB: epistemic, OS: 'stereotypical'
b. *Carlos may go to bed now*
MB: circumstantial, OS: 'deontic'

(74) a. *Carlos can have a piece of cake.*
MB: circumstantial, OS: 'deontic'
b. *Carlos can turn on the TV now.*
MB: circumstantial, OS: 'teleological'
c. *Carlos can swim.*
MB: circumstantial, OS: 'stereotypical'? 'dispositional'?

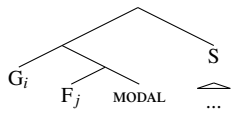
(75) *Carlos is allowed to have a piece of cake.*
MB: circumstantial, OS: 'deontic'

Are there restrictions on the correlation of what acts as the MB/what acts as the OS?

Lexical entries Two examples, *must* (necessity), *may* (possibility)

Syntactic structure MB, OS contextually provided, i.e. represented by variables in the object language. I'll use 'F' as the variable for the MB and 'G' as the variable for the OS. Values assigned by assignments.

(76)



Make-up of MB and OS

a. MB represents conjunction of (part of) what the speaker nows / facts of the world. Function mapping worlds to propositions.

An MB is a function $f : W \to \wp(W)$ (i.e. a function $f \in D_{\langle s,\langle s,t\rangle\rangle}$)

I.e. the relevant assignment maps elements of $\mathbb{N}$ to elements in $D_{\langle s,\langle s,t\rangle\rangle}$

b. OS represents how the world is order w.r.t. some ideal, i.e. what is stereotypical/ what the laws says etc. Function from worlds to sets of propositions. (Why no reduction? Because the OS can be inconsistent.)

An OS is a function $f : W \to \wp(\wp(W))$ (i.e. a function $f \in D_{\langle s,\langle\langle s,t\rangle,t\rangle\rangle}$)

I.e. the relevant assignment maps elements of $\mathbb{N}$ to elements in $D_{\langle s,\langle\langle s,t\rangle,t\rangle\rangle}$

Lexical entries, necessity modals

For *must*, etc.

(77) [[nec-mod]] =
$\lambda w.\lambda f_{\langle s,\langle s,t\rangle\rangle}.\lambda g_{\langle s,\langle\langle s,t\rangle,t\rangle\rangle}.\lambda p_{\langle s,t\rangle}.$
$\forall u[u \in f(w) \rightarrow \exists v[v \in f(w) \wedge v \leq_{g(w)} u \wedge \forall z[z \in f(w) \wedge z \leq_{g(w)} v \rightarrow z \in p]]]$

In words: NEC-MOD denotes a function, which, relative to a world w, a modal base $f(w)$ and an ordering source $g(w)$ maps a proposition p to 1 iff
for every world u in the $f(w)$, there is at least one world v that is also in $f(w)$ and which is at least as $g(w)$-close as u and for which the following holds:
every world z in $f(w)$ that is at least as $g(w)$-close as v is a p-world
(careful: this will include v itself)

Example: (78). And some assumptions below:

(78) *Murderer Carlos must go to jail.*

(79) g(w) = { p1, p2, p3}
- a. p1: Murderers go to jail.
- b. p2: Thieves are fined exactly 10 Euros.
- c. p3: Thieves are fined exactly 20 Euros.

(80) f(w) = { w1, w2, w3, w4, w5 }
- a. w1: propositions true in it: p1, p2
- b. w2: propositions true in it: p1, p3
- c. w3: propositions true in it: p1
- d. w4: propositions true in it: p2
- e. w5: propositions true in it: p3

Let's assume that Carlos goes to jail in w1, w2, w3. Is the sentence true?

Let's check whether each world $w \in$ { w1, w2, w3, w4, w5 } whether the following **Condition**: There is a world $w' \in$ { w1, w2, w3, w4, w5 } that is at least as close as w and in every world $w'' \in$ { w1, w2, w3, w4, w5 } that is at least as close as w'

(81)
- a. w1
  - (i) worlds at least as close as w1: w1
  - (ii) meets condition: √ (Carlos goes to jail in w1)
- b. w2
  - (i) worlds at least as close as w2: w2
  - (ii) meets condition: √ (Carlos goes to jail in w2)
- c. w3
  - (i) worlds at least as close as w3: w3, w1, w2
  - (ii) meets condition: √ (Carlos goes to jail in w1, w2, w3)
- d. w4
  - (i) worlds at least as close as w4: w4, w1
  - (ii) meets condition: √ (Carlos goes to jail in w1)
- e. w5
  - (i) worlds at least as close as w5: w5, w2
  - (ii) meets condition: √ (Carlos goes to jail in w2)

Sentence is true!

The following sentence would not be true in the world above, assuming the MB and the OS remain the same. Can you see why?

(82) *Thief Carlos must pay exactly 10 Euros.*

Lexical entries, necessity modals

For *may*, etc.

(83) [[poss-mod]] =
$\lambda w.\lambda f_{\langle s,\langle s,t\rangle\rangle}.\lambda g_{\langle s,\langle\langle s,t\rangle,t\rangle\rangle}.\lambda p_{\langle s,t\rangle}.$
$\neg\forall u[u \in f(w) \rightarrow \exists v[v \in f(w) \ \wedge \ v \leq_{g(w)} u \wedge \forall z[z \in f(w) \wedge z \leq_{g(w)} v \rightarrow z \in \bar{p}]]] =$
$= \lambda w.\lambda f_{\langle s,\langle s,t\rangle\rangle}.\lambda g_{\langle s,\langle\langle s,t\rangle,t\rangle\rangle}.\lambda p_{\langle s,t\rangle}.$
$\exists u[u \in f(w) \wedge \forall v[v \in f(w) z \leq_{g(w)} u \rightarrow \exists z[z \leq_{g(w)} v \wedge z \in p]]]$

In words: NEC-MOD denotes a function, which, relative to a world w, a modal base $f(w)$ and an ordering source $g(w)$ maps a proposition p to 1 iff
there is a world u in the $f(w)$, s.th. for all v that is also in $f(w)$ and which is at least as $g(w)$-close as u
there is a world z in $f(w)$ that is at least as $g(w)$-close as v is a p-world
(careful: this will include v itself)

Take our example from above. Both sentences in (84) are true w.r.t. specification given.

(84) a. *Murderer Carlos may go to jail*
b. *Murderer Carlos may pay exactly 10 Euros.*

Lexical entries, possibility modals

(85) [[poss-mod]] =
$\lambda w.\lambda f_{\langle s,\langle s,t\rangle\rangle}.\lambda g_{\langle s,\langle\langle s,t\rangle,t\rangle\rangle}.\lambda p_{\langle s,t\rangle}.$

2.4 Summary

- modal expressions are evaluated to premises
- 2 types of premises: Modal base, ordering source
- modals quantify over worlds identified by the premises
- existential quantification (possibility modals), universal quantification (necessity modals)

In the following, we will address some of the problems / open questions of this approach

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